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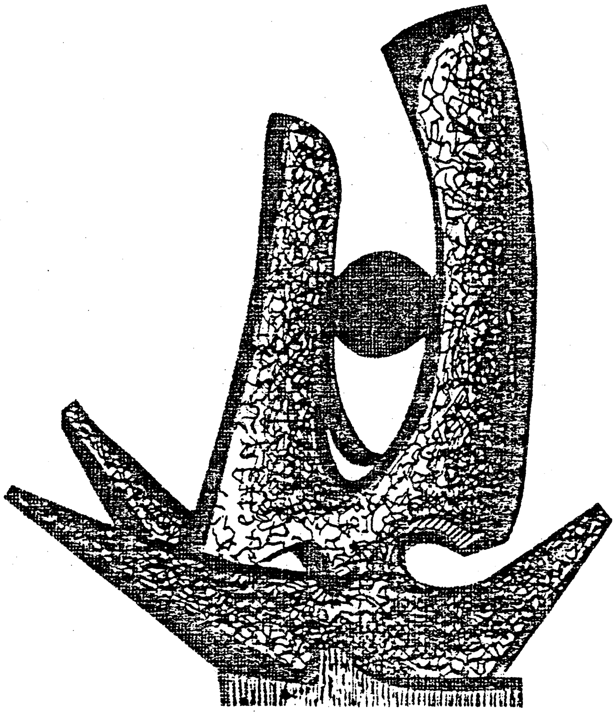

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Microscopic Calculations of Nuclear Matter-Collective Flow in Nb(400 MeV/N) + Nb

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Abstract

Collisions between two nuclei have been modeled by numerically solving classical approximations to the equations of motion of the constituent nucleons. For the reaction Nb(400 MeV/N)+Nb, a correlated sidewards emission of nucleons is observed. This is attributed to the repulsive short range component of the nucleon-nucleon potential. A strong dependence of the flow angle on the impact parameter is observed in accord with recent experimental results.

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The recent experimental observation[1] of sidewards peaks in the emission pattern of fragments emitted in collisions of heavy nuclear systems has stimulated a dispute among theorists about how to interpret these data. It has been shown[2] that the observations are in agreement with the results of macroscopic nuclear fluid dynamical calculations, but several microscopic calculations done to simulate the sidewards emission (via the intranuclear cascade (INC) approach[3,4]) failed - the angular distributions obtained where always forward peaked.[1,2]

We have recently developed a many body equations of motion (EOM) approach to study heavy ion collisions. Our approach is analogous to the early work of Bodmer et al.,[5] and Wilets et al.[6] Hamilton's equations of motion are solved for an ensemble of nucleons with simultaneous mutual two-body interactions between all particles:

$$\dot{\vec{p}}_i = -\frac{\partial H}{\partial r}$$

$$\dot{\vec{r}}_i = \frac{\partial H}{\partial p} .$$

This approach is nonrelativistic and neglects even the basic quantum mechanics, such as the Pauli exclusion principle and Heisenberg uncertainty principle. On the other hand, this approach allows for a study of the simultaneous (classical) interactions between many particles in sharp contrast to the independent particle models such as the intranuclear cascade, which treat nuclear collisions as a simple superposition of successive free space nucleon - nucleon collisions. Also, the EOM approach allows for a systematic study of the repulsive core of the nucleon nucleon interaction (due to its deterministic nature, the EOM calculation produces an excluded volume effect) which is of interest in connection with the possible study of nuclear matter properties at high baryon densities.

In the EOM approach, nuclei are described as an ensemble of protons and neutrons initially distributed randomly throughout a sphere with the nuclear radius $R=1.2\ A^{1/3}$ fm. However, the obtained nuclei are not stable: they tend to collapse and evaporate many nucleons when the classical equations of motion are integrated over not too long a time interval (t=10-30 fm/c). In the present approach, a metastable ground state has been obtained by allowing the nucleons to drift toward the configuration of minimum energy of the chosen nucleon-nucleon potential (Fig. 1). The state of minimum energy is found to be a crystalline structure (Fig. 2).

The nucleon-nucleon potential consists of two terms, an attractive long range Yukawa interaction, and a repulsive short range core.

$$V = (V_R\ e^{-K_R \cdot r} - V_A\ e^{-K_A \cdot r})/r,$$

where

$$V_R = 2970 \text{ MeV-fm} \qquad V_A = 765 \text{ MeV-fm}$$
$$K_R = 2.66 \text{ fm}^{-1} \qquad K_A = 1.75 \text{ fm}^{-1}\ .$$

The parameters in the potential were chosen in a compromise between reproducing in the EOM calculation the n-p differential scattering cross section at large angles $\theta_{cm}=90°$ (which influences the transverse momentum transfer the most) and at the same time giving reasonable nuclear radii and binding energies. This leads to nuclei sufficiently stable for a collision calculation to be meaningful rather than resulting in a disruption of the nuclei before a collision can actually take place. This potential has a

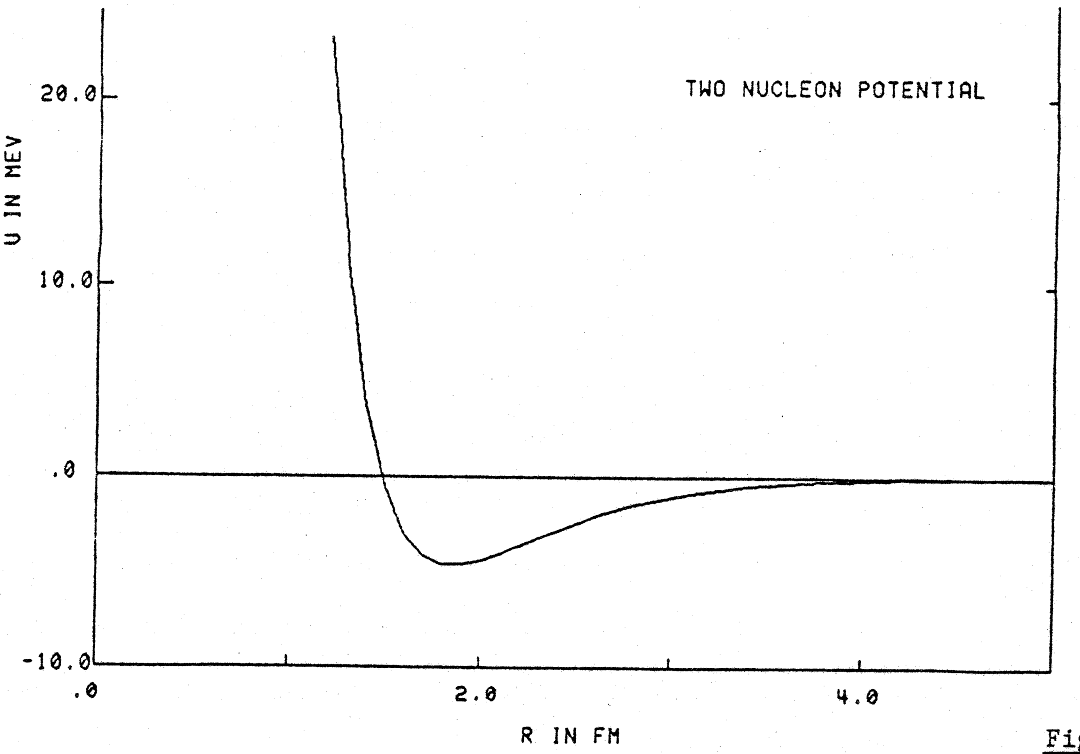

Figure 1

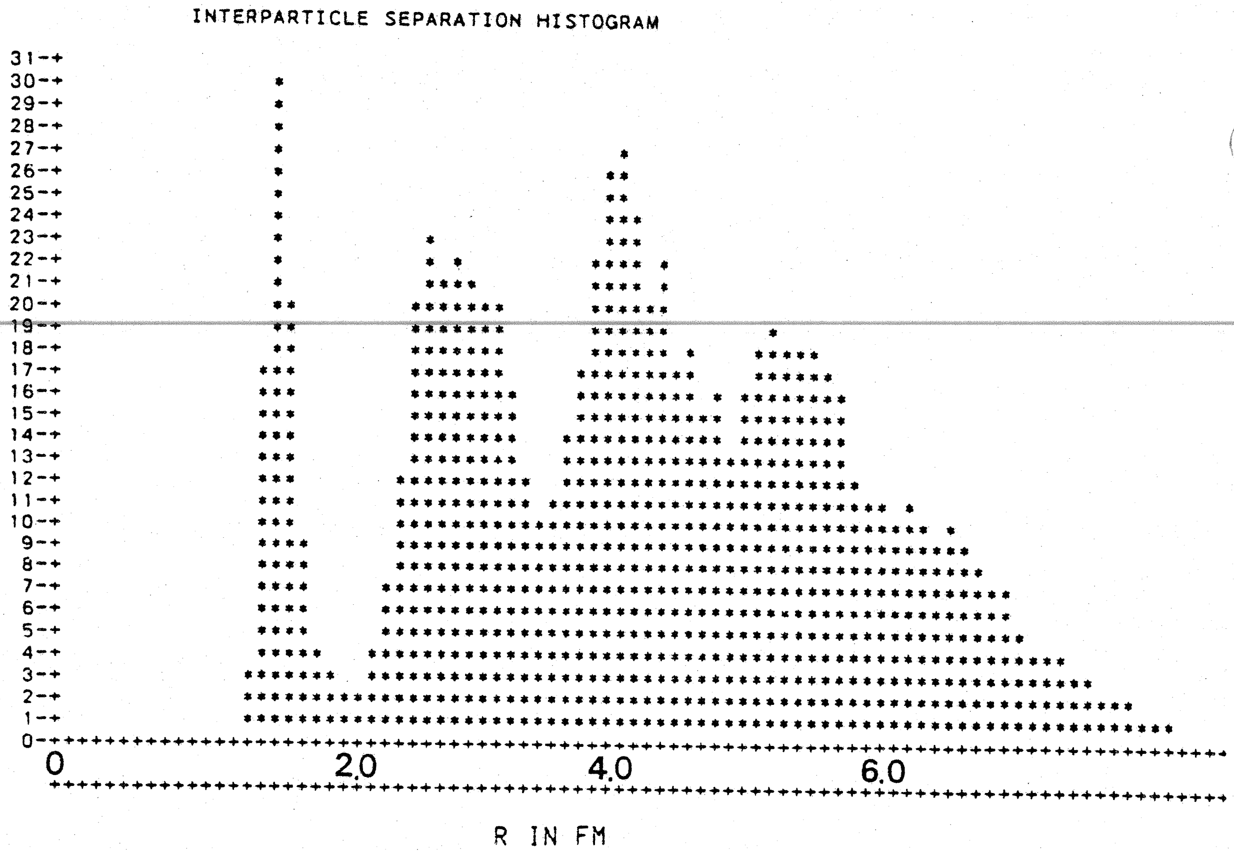

Figure 2

minimum of -4.67 MeV at R = 1.85 fm. The resulting crystalline ground state configuration has an average binding energy of -31 MeV/N. The nucleons then are given random Fermi momenta with an average Fermi energy of 23 MeV/N which results in an average binding energy of 7 MeV/N. These nuclei are stable for t>20 fm/c, i.e. typical collision times.

To numerically simulate a collision process, these nuclei are Galilei-boosted with the respective center of mass momenta at given impact parameters. The equations of motion are integrated using a technique described elsewhere.[7] The second-order differential equations are solved by a fourth order Adams-Moulton predictor-corrector method which is started by a fourth order Runge-Kutta integration procedure. Energy conservation to better than 1% has been demanded. A total of 535 collisions of Nb (400 MeV/N) + Nb have been performed. This provides reliable statistics for the global event-by-event analysis subsequently performed on the numerical 'data'. The computations are stopped after t = 30-50 fm/c, since the results are insensitive to the exact 'break-up time'. The typical late stage of a collision at b = 4 fm impact parameter is shown in Fig 3. The resulting sidewards flow can clearly be seen. We also display the evolution of the center-of-mass trajectory for four collisons at the same impact parameter in Fig. 4. Note that the deflection angle (25°) has approximately the same value as the flow angle (23°).

The individual collisions are analyzed by diagonalizing the kinetic energy flow tensor,[8]

$$F_{ij} = \sum_{v} p_i(v)p_j(v)/2m(v) ,$$

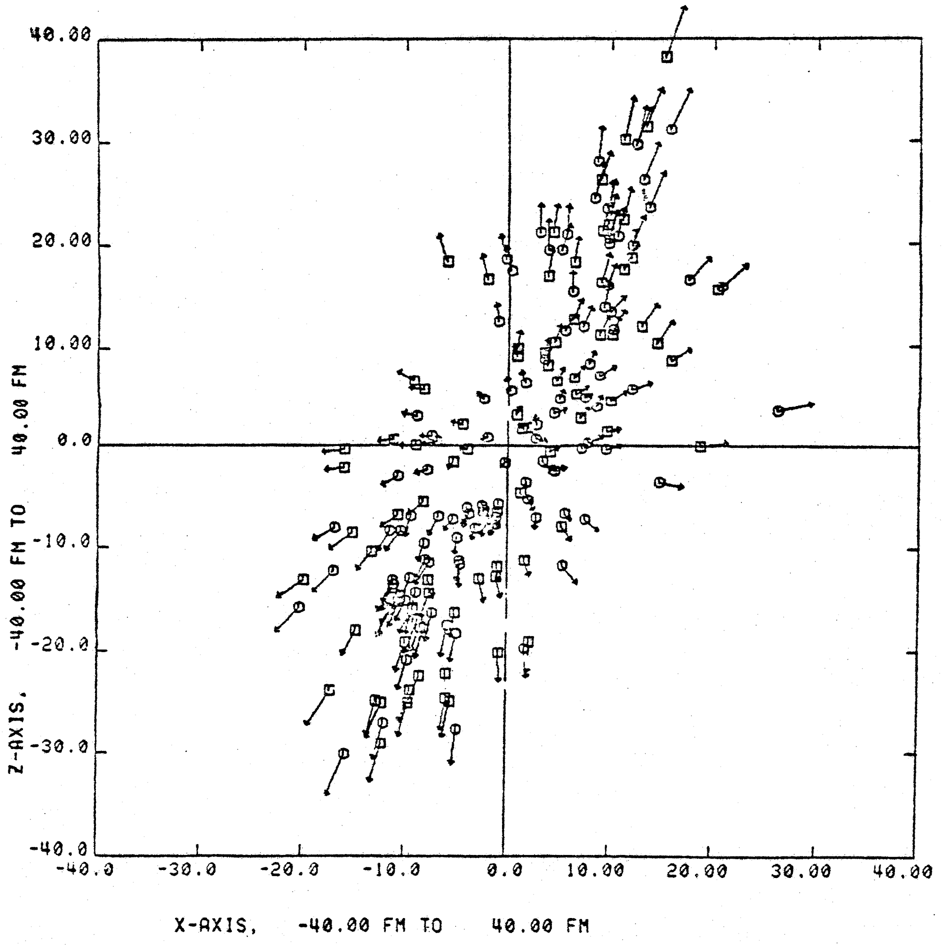

Figure 3

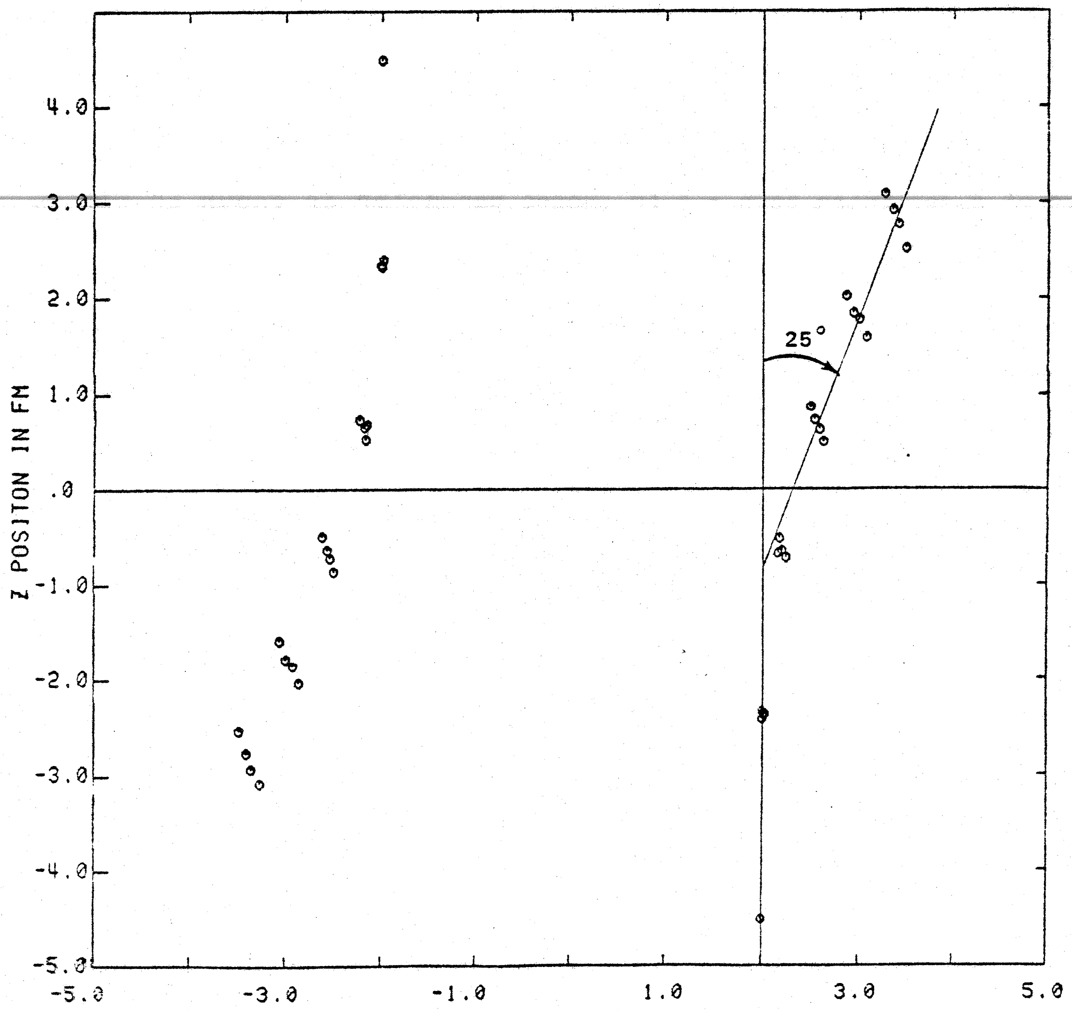

Figure 4

where the sum is over all particles in a given event. By diagonalizing this tensor, the flow angle θ_F and kinetic flow aspect ratio R_{13} is obtained for each event. The distribution of flow angles is presented in Fig. 5 for various impact parameter intervals. The qualitative behavior of the flow pattern in the EOM model is as follows: the flow angle θ_F rises smoothly from 0° at large impact parameters to 90° at $b=0$. However, the contribution of zero impact parameter collisions to the observable cross sections is negligible. Thus a finite range of impact parameters is sampled to compute the angular distributions of the flow angles, $dN/d\cos\theta_F$, which is to be compared to the experimental data of the GSI/LBL collaboration. Fig. 5 also shows the experimental data for the Nb(400 MeV/N)+Nb case discussed above, together with the predictions of the intranuclear cascade[1,2] and fluid dynamical[2] calculations. The data exhibit nonzero average flow angles once high multiplicity, i.e. small impact parameter collisions, are selected. This is in contrast to the intranuclear cascade calculation, which yields zero flow angles even at the highest multiplicities.

The present microscopic EOM model, on the other hand, predicts peaks in the angular distributions of the flow angles. The peak shifts to larger angles with increasing multiplicity which is in qualitative agreement with the experimental data. The difference in the physics of the INC model and the EOM approach leading to the distinct differences in the predictions is twofold. The INC applies a stochastic 4π scattering at the point of closest approach of straight line trajectories which allows for substantial transparency. In contrast, the short range repulsion in the EOM approach, results in an excluded volume effect. The nuclei are not as transparent and easily compressible in the EOM as in the INC. This causes the incident nucleons to be deflected towards sidewards angles. The apparent success of

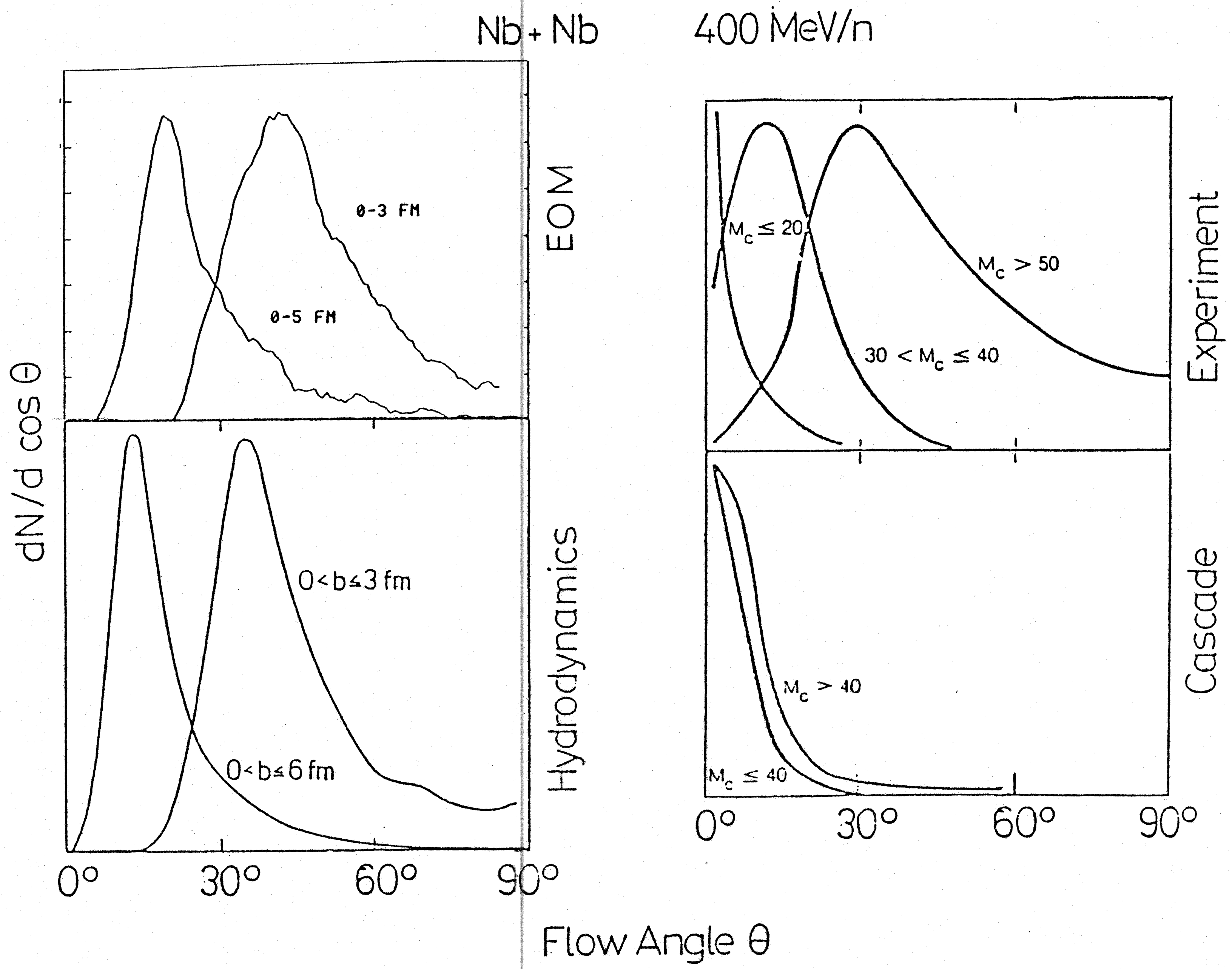
Nb + Nb
400 MeV/n
dN/d cos Θ
EOM
0-3 FM
0-5 FM
Hydrodynamics
0<b≤3 fm
0<b≤6 fm
Experiment
$M_c \leq 20$
$M_c > 50$
$30 < M_c \leq 40$
Cascade
$M_c > 40$
$M_c \leq 40$
0°
30°
60°
90°
Flow Angle θ

Figure 5

the EOM simulations in providing a microscopic basis for the study of the sidewards flow makes us confident that the implementation[9] of Pauli and Uncertainty Principles, as well as relativistic corrections will be worth the effort. This may well result in a tractable model useful for a quantitative understanding of the dynamics of nuclear collisions at medium and high energy. A study of the influence of the nucleon nucleon potential on the dynamics is presently underway.

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