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LETTER TO EDITOR

Pion production, light-fragment formation and the entropy puzzle in relativistic nuclear collisions

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Abstract. The production of pions and light fragments in fast nuclear collisions is calculated in a relativistic fluid-dynamical model. The linear dependence on bombarding energy and, in particular, the absolute values of the calculated pion, proton and deuteron yields are found to be in agreement with recent experimental data. The previous discrepancy between the calculated entropy values and the measured deuteron-to-proton ratios is resolved.

The dependence on bombarding energy of pion and light-fragment production has been measured recently for central collisions of nuclei of equal mass (Sandoval *et al* 1980a†, Nagamiya *et al* 1981; see also Fung *et al* 1978, Wolf *et al* 1979, Sandoval *et al* 1980b). Such experiments are of great interest for several reasons. Fluid-dynamical calculations show some sensitivity of the pion excitation function to the nuclear equation of state (Stöcker *et al* 1978, 1979a, 1980a, Danielewicz 1979, Kapusta and Strottman 1981); in particular, it has been suggested that the increase in entropy due to a phase transition (pion condensates, density isomers) in dense nuclear matter can result in a threshold increase of the pion excitation function (Stöcker *et al* 1978, 1979a, 1980a). Also, the deuteron-to-proton ratio, d/p, carries information on the entropy produced in the collision (Kapusta and Strottman 1981, Siemens and Kapusta 1979, Mishustin *et al* 1980, Stöcker *et al* 1981a). Such thermal and fluid-dynamical descriptions of high-energy nuclear collisions require the incident energy and longitudinal momentum to be randomised rapidly so that local thermal equilibrium is established. Microscopic calculations indicate some equilibration even for the rather light systems Ne + NaF and Ar + KCl with, however, considerable non-equilibrium contributions (Bodmer and Panos 1977, 1981, Callaway *et al* 1979, Gudima *et al* 1979, Yariv and Fraenkel 1979, Bodmer *et al* 1980, Cugnon 1980, Cugnon *et al* 1981). In the nuclear fluid-dynamical model deviations from local thermal equilibrium can be treated phenomenologically by incorporating the nuclear viscosity (Stöcker *et al* 1978, 1979a, 1980a, Csernai and Barz 1980, Csernai *et al* 1980, Wong *et al* 1977, Tang and Wong 1980, Buchwald *et al* 1981). Fully three-dimensional fluid-dynamical calculations (Amsden *et al* 1977, Nix 1979, Sierk and Nix 1980, Nix and Strottman 1981, Stöcker *et al* 1979b, c, 1980a, b, c, 1983), which did not take into account the nuclear viscosity, as well as one- and two-dimensional viscous calculations (Stöcker *et al* 1978, 1979a, 1980a, Csernai and Barz 1980, Csernai *et al* 1980, Wong *et al* 1977, Tang

† The pion production cross sections of this reference are the basis for the recent attempt to extract the equation of state of nuclear matter from discrepancies in the cascade calculations (Stock *et al* 1982).

and Wong 1980, Buchwald *et al* 1981) have shown that in the bombarding energy range of interest ($E_{lab} = 0.1$–2.1 GeV/nucleon) the nuclear matter is strongly compressed and highly excited during the early stage of the collision. In this compression stage, the densities ρ and temperatures T calculated in the fluid-dynamical models with and without viscosity agree remarkably well with the solution of the relativistic Rankine–Hugoniot equation (Stöcker *et al* 1978, 1979a, 1980a). This agreement is natural since this equation simply expresses the conservation of baryon number, momentum and energy fluxes across the shock fronts which develop for supersonic hydrodynamical flow. Even a substantial spreading of the width of the shock front due to large viscosity values does not influence the state of the compressed matter behind the shock front (Stöcker *et al* 1978, 1979a, 1980a, Csernai and Barz 1980, Csernai *et al* 1980, Wong *et al* 1977, Tang and Wong 1980, Buchwald *et al* 1981). In the present work, the relativistic Rankine–Hugoniot equation, $W^2 - W_0^2 + p(W/\rho - W_0/\rho_0) = 0$, is used to compute ρ and T and the entropy $S(\rho, T)$. Here $W_0 = m_N c^2 - B_0 \approx 931$ MeV and $\rho_0 = 0.17$ fm^{-3}. The internal energy per nucleon, $W(\rho, T) = W_0 + E_C(\rho) + E_T(\rho, T)$, is composed of a compression energy $E_C(\rho)$, for which we use $E_C(\rho) = K/18\rho\rho_0(\rho - \rho_0)^2$ or $\tilde{E}_C = E_C\rho/\rho_0$, respectively (hereafter referred to as the linearly (ρ) and quadratically (ρ^2) increasing $E_C(\rho)$), with the compression constant $K = 200$ MeV, and the thermal energy $E_T(\rho, T)$ of a free Fermi gas. The pressure $p(\rho, T)$ is connected to $W(\rho, T)$ via (Stöcker *et al* 1978, 1979a, 1980a) $p = p_C(\rho) + \frac{2}{3}\rho E_T$. The internal energy in the shock zone, $W(\rho, T)$, is fixed by the laboratory kinetic energy per nucleon, E_{lab}, as $W(\rho, T) = \gamma_{CM}(E_{lab})W_0$ where $\gamma_{CM} = (1 + E_{lab}/2W_0)^{1/2}$. The entropy per nucleon, S, in the compressed system can be well approximated by the low-temperature Fermi-gas expansion (Stöcker *et al* 1981a) S_F for $E_{lab} < 400$ MeV/nucleon, and by the classical Boltzmann-gas expression (Siemens and Kapusta 1979, Mishustin *et al* 1980) S_B at higher energies. At lower energies S_B is much too small and even becomes negative for $E_{lab} < 120$ MeV/nucleon. The exact Fermi-gas expression $S(x)$ (Kapusta and Strottman 1981), which depends only on the quantity $x = E_T\rho^{-2/3}$, is used in the present calculation†.

From the compressed state the system expands because of the internal pressure. As the shock compression is the only mechanism for entropy production in a perfect (non-viscous) fluid, the entropy is conserved during the decompression stage and the expansion can be calculated analytically: $E_T^{\text{expansion}}(\rho, S = \text{constant}) = E_T^{\text{initial}}(\rho/\rho^{\text{initial}})^{2/3}$. However, Csernai and Barz (1980) have demonstrated that for an imperfect fluid the viscosity has important effects on the expansion stage for the light system (Ar + KCl) investigated here: because of the friction, part of the energy is taken out of the fluid motion and is restored in internal excitations. In particular, they have shown that viscous effects increase the mean entropy per nucleon of the fluid during the expansion by a factor $\eta = S_\eta/S_s \approx 1.2$ (Csernai and Barz 1980, Csernai *et al* 1980). Here S_s is the entropy in the compressed stage and S_η is the entropy after the viscous expansion. This result is utilised in the present work to estimate the difference in the particle production rates for the viscous ($\eta = 1.2$) and non-viscous ($\eta = 1.0$) cases, respectively.

Figure 1 shows the trajectories $T(\rho)$ for the compression and subsequent expansion for various bombarding energies and two different equations of state. The density and temperature increase as the incident energy is raised. During the subsequent decompression phase the temperature decreases as a function of the density. The viscous expansion results in higher temperatures than the isentropic expansion as illustrated qualitatively by the shaded areas. The linearly increasing $E_C(\rho)$ results in higher densities

† Note that the formation of pions and delta resonances is neglected in the entropy calculation—their inclusion would increase the entropy values achievable by less than 20% for $E_{lab} < 1$ GeV/nucleon.

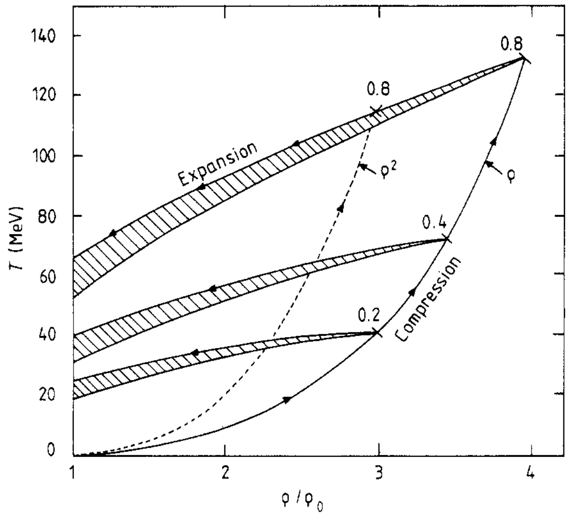

Figure 1. The trajectories $T(\rho)$ of the hydrodynamical compression and expansion of the matter are shown for $E_{lab}=0.2$, 0.4 and 0.8 GeV/nucleon. The shaded areas indicate qualitatively the mean temperature increase due to viscosity. $T(\rho)$ using two different equations of state, ρ (full curve) and ρ^2 (broken curve), are shown for a Fermi gas consisting of nucleons only.

and temperatures than the stiffer quadratically increasing $\bar{E}_C(\rho)$ at any given bombarding energy. However, note that the temperatures at any given density during the expansion depend only slightly on the particular choice of $E_C(\rho)$. This is because the total entropy produced is rather insensitive to the equation of state (Stöcker *et al* 1981a). For example, $S=3.3$ and 3.35 at $E_{lab}=0.8$ GeV/nucleon for the linearly (ρ) and quadratically increasing (ρ^2) equations of state, respectively. Only at lower energies, $E_{lab} \lesssim 0.2$ GeV/nucleon, is there a larger influence of $E_C(\rho)$ on the amount of entropy produced (Stöcker *et al* 1981a).

In the expansion, the fluid eventually reaches the break-up density, $\rho_{BU} \lesssim 0.7\rho_0$ (Montvay and Zimanyi 1979, Mekjian 1977, 1978a, b, Subramanian *et al* 1981, Randrup and Koonin 1981, Fai and Randrup 1982, Gosset *et al* 1978), at which the thermal equilibrium can no longer be maintained. Calculations based on transport theory indicate that chemical equilibration can be reached towards this freeze-out time (Montvay and Zimanyi 1979, Mekjian 1977, 1978a, b, Subramanian *et al* 1981, Randrup and Koonin 1981, Fai and Randrup 1982). In the present work the average number of pions and light nuclei produced is calculated by equating the baryon number and the internal energy of the nuclear fluid at this freeze-out moment with that of a relativistic quantum gas of non-interacting particles in chemical equilibrium (Gosset *et al* 1978). The chemical equilibrium calculation is done using the firestreak code (Gosset *et al* 1978†), which also includes the formation of metastable nuclei A^*. They decay subsequently into lighter stable nuclei by emission of a proton or neutron: $A^* \to (A-1) +$ nucleon (Gosset *et al* 1978). We will show below that the particle-unstable nuclei can have a profound effect on the observed deuteron-to-proton ratio.

Figure 2 shows the mean number of negative pions per charged participant particle, $\langle n_{\pi^-}\rangle/\langle Q\rangle$, measured in high-multiplicity selected ('central') collisions of ^{40}Ar + KCl over

† The author thanks Gary Westfall for providing the firestreak program used to calculate the chemical equilibrium composition in the present work.

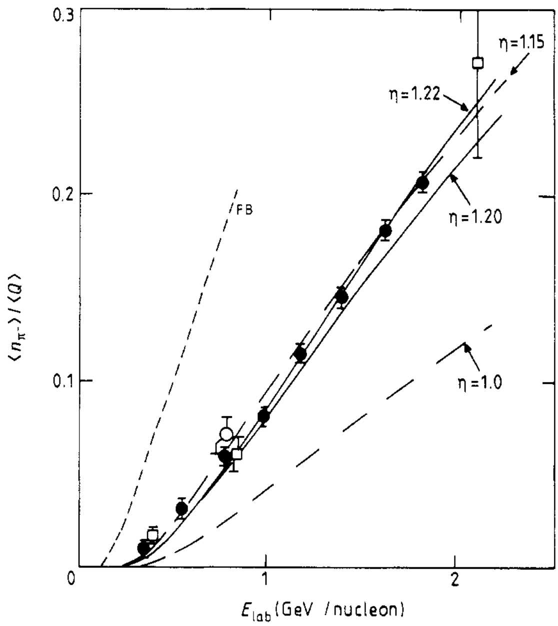

Figure 2. The bombarding energy dependence of the average number of pions per charged nuclear fragment emitted, $\langle n_{\pi^-}\rangle/\langle Q\rangle$ (●, Sandoval *et al* (1980a), Ar + KCl, high multiplicity; ○, Nagamiya *et al* (1981), Ar + KCl, inclusive; □, Nagamiya *et al* (1981), Ne + NaF, inclusive), is compared with the fireball model (FB, short broken curve) and with the present calculation. The full (long broken) curves are for $\rho_{BU}=0.7\rho_0$ ($1.0\rho_0$), respectively.

a wide range of bombarding energies (Sandoval *et al* 1980a). Here Q is defined, in accord with Sandoval *et al* (1980a), as $\langle Q\rangle=\Sigma-2\langle n_{\pi^-}\rangle$ where Σ is the mean total number of charged participants. Also shown are the measured large-angle inclusive π^-/Z ratios (Nagamiya *et al* 1981) where Z is the total number of positive charges in reactions of $^{40}Ar + KCl$ and $^{20}Ne + NaF$. It is interesting that linear cascade calculations (Randrup 1981, Knoll and Randrup 1979) fail to reproduce the absolute values and the observed linear dependence of $\langle n_{\pi^-}\rangle/\langle Q\rangle$ on the bombarding energy. The calculations in the nuclear fireball model shown in figure 2 (short broken curve) and in the related phase-space model (Bohrmann and Knoll 1981) yield absolute values of $\langle n_{\pi^-}\rangle/\langle Q\rangle$ which are too large by more than a factor of three for the whole excitation function range investigated.

In contrast, the absolute values of $\langle n_{\pi^-}\rangle/\langle Q\rangle$ as well as the linear energy dependence obtained in the present work reproduce the data (see figure 2) once the effects of the viscosity, that is, the deviations from the perfect local equilibrium, are incorporated. Also shown are the results of the non-viscous fluid calculation, which underestimates the pion production rates: too large a fraction of the internal microscopic kinetic energy is converted into collective macroscopic motion. For the isentropic expansion $T\sim\rho^{2/3}$; hence, the pion production rates depend on the break-up density as shown in figure 2†.

Another quantity of interest is the π^-/π^+ ratio (Nagamiya *et al* 1981, Frankel *et al* 1981). We find $\pi^-/\pi^+=1.38$ for Ar + Ca at 1 GeV/nucleon, in agreement with the experimental data (Nagamiya *et al* 1981, Frankel *et al* 1981) which give $\pi^-/\pi^+\approx 1.4$.

† The uncertainties in the viscosity effects and the break-up density make an experimental determination of the nuclear equation of state via the pion yield (Stöcker *et al* 1978, Stock *et al* 1982) very difficult.

As a further test of the present model, we now compute the yield of light nuclei. It was suggested by Siemens and Kapusta (1979) that the deuteron-to-proton ratio, $R_{dp} \equiv \langle d \rangle / \langle p \rangle$, can be used to determine the entropy per baryon, S, produced in the collisions via

$$`S' \equiv 3.95 - \ln R_{dp}. \qquad (1)$$

This formula is derived for an ideal classical gas of nucleons assuming $R_{dp} \ll 1$. Figure 3 shows the 'entropy' values 'S', extracted from the data (Nagamiya *et al* 1981) via equation (1) in comparison with the entropy calculated in the present model. The calculated entropy S_η tends towards the experimental data at high energies ($E_{lab} \gtrsim 2$ GeV/nucleon). However, the observed 'entropy' values, 'S'≈ 5–6, are almost constant as a function of the bombarding energy, while S_η and S_s naturally drop to zero as $E_{lab} \to 0$. This seems to indicate an unusual mechanism for entropy production (Siemens and Kapusta 1979, Mishustin *et al* 1980).

On the other hand, our calculated $\langle d \rangle / \langle p \rangle$ ratios are also nearly independent of bombarding energy and agree with the measured values (Nagamiya *et al* 1981), $R_{dp} \approx 0.25$. To resolve this apparent paradox, in figure 3 we show the 'entropy', 'S', obtained via equation (1) from the calculated number of deuterons and protons produced, that is, from the *calculated* R_{dp} values. Note the good agreement between the calculations and the data (Nagamiya *et al* 1981). Also note that the calculated 'S'≈ 5 even for $E_{lab} < 100$ MeV/nucleon! In fact, experiments in the intermediate-energy region, 15 MeV/nucleon $< E_{lab} <$ 100 MeV/nucleon, consistently report (Löhner *et al* 1979, Wu *et*

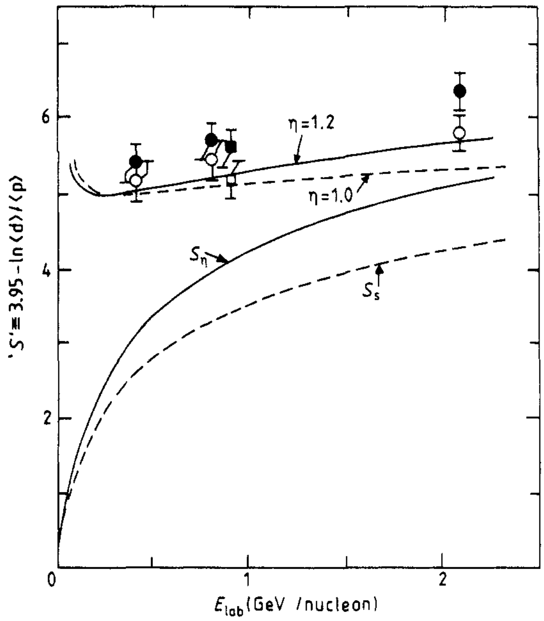

Figure 3. The bombarding energy dependence of the entropy is shown as calculated for the viscous (S_η) and inviscid (S_s) fluids. Also shown are the 'entropy' values, 'S', obtained from the measured (Nagamiya *et al* (1981): ○, Ne + NaF, inclusive; ●, Ne + NaF, 90° CM; □, Ar + KCl, inclusive; ■, Ar + KCl, 90° CM) and calculated deuteron-to-proton ratios R_{dp} via equation (1). The full (broken) curves represent the viscous (inviscid) calculation with $\rho_{BU} = 0.7\rho_0$. Resonance formation has not been taken into account in the determination of S_s.

al 1979, Ball *et al* 1978, Westfall *et al* 1982, 1984) deuteron-to-proton ratios $R_{dp} \approx 0.3$, just as predicted by the present calculation. We find that the disagreement between these values of 'S' (equation (1)) and the entropy calculated by fluid dynamics is due to the contamination of $R_{dp}^{observed}$ by those protons which emerge from the decay of particle-unstable excited nuclei. Hence, we conclude that the connection between the entropy and $R_{dp}^{observed}$ is not given by equation (1). The decay processes become increasingly important at intermediate and low energies where heavier fragments are formed more abundantly. In fact, the resonance decay products dominate the chemical equilibrium contribution

$$\langle p \rangle_{observed} = \langle p \rangle_{equilibrium} + \langle p \rangle_{decay} \quad (2)$$

with $\langle p \rangle_{decay} \gtrsim \langle p \rangle_{equilibrium}$ for $E_{lab} < 400$ MeV/nucleon.

It is important to point out that the calculated R_{dp} ratios are actually found to be nearly independent of the entropy produced. They are also insensitive to the break-up density ρ_{BU}: the calculated 'S' values agree with the data whether we use $\rho_{BU} = 0.1\rho_0$ or $\rho_{BU} = \rho_0$.

In this Letter we have presented calculations on the production of pions and light fragments in high-energy nuclear collisions. The model is based on relativistic fluid dynamics, incorporating the effects of nuclear viscosity, that is, allowing for some deviations from local thermal equilibrium. The calculated linear dependence on bombarding energy and, in particular, the absolute values of the $\langle n_{\pi^-} \rangle / \langle Q \rangle$, π^- / π^+ and $\langle d \rangle / \langle p \rangle$ ratios reproduce the experimental data. The shapes of the pion, proton and deuteron spectra can also be reproduced in the fluid-dynamical model (Kapusta and Strottman 1981, Csernai 1980, Csernai *et al* 1980, Siemens and Rasmussen 1979, Stöcker *et al* 1981b). These results demonstrate that the data are consistent with hydrodynamical compression effects in fast nuclear collisions but viscous effects, formation of metastable clusters and the approach towards chemical and thermal equilibrium play an essential role. Very precise calculations will be necessary to determine the equation of state accurately in the future.

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