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Microscopic Theory of Pion Production and Sidewards Flow in Heavy Ion Collisions

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Abstract:

Energetic nucleus- nucleus collisions are studied in a microscopic approach based on the Boltzmann equation, which includes a Vlasov-Landau self-consistent mean field and an Uehling- Uhlenbeck collision term which respects the Pauli principle. The theory explains simulateously both the observed collective flow and the pion multiplicity and gives their dependence on the nuclear equation of state.

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One of the most intriguing motivations for studying relativistic nucleus- nucleus collisions is the unique opportunity to explore compressed and excited nuclear matter in the laboratory. A signature of compression is the collective sidewards flow [1] predicted on the basis of nuclear fluid dynamics. The number of pions created in central collisions of heavy nuclei depends in this theory on the compression energy. Hence, measurements of pion yields can be used to extract the nuclear equation of state at high densities [2].

Recently, the predicted collective sidewards flow has been observed in high multiplicity selected collisions of Ca + Ca and Nb + Nb at 400 MeV/N [3]. The pion multiplicities have been measured for near central collisions of Ar on KCl from 400 to 1800 MeV/N [4]. Both datasets, though in agreement with the macroscopic fluid dynamic approach [5], present a challenge to microscopic theories: Intranuclear cascade calculations [6,7] result in forward peaked angular distributions [3,5], in contrast to the data [3], and the calculated pion multiplicities [4,6,7] drastically overestimate the experimental yields [4] . These large discrepancies are surprising in view of the success of the cascade model in describing inclusive data [6,7].

In this letter we present a microscopic theory which explains for the first time simultaneously both the observed collective flow and the pion multiplicity and gives their dependence on the nuclear equation of state. The present theoretical approach is based on the Boltzmann equation which includes a Vlasov-Landau self-consistent mean field and an Uehling-Uhlenbeck collision term which respects the Pauli principle. This extended Boltzmann equation can be written as [8,9]

$$D_t \, f(\vec{r},\vec{v},t) = \left(\frac{\partial f}{\partial t}\right)_{coll} \tag{1}$$

where

$$D_t f = \frac{\partial}{\partial t} f + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} f + \vec{a} \cdot \frac{\partial}{\partial \vec{v}} f = 0 \qquad (2)$$

is Vlasov's equation for the evolution of the single particle distribution function f of a collisionless plasma with a selfconsistent mean potential field and

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = \int \frac{d^3p_2 d^3p_1' d^3p_2'}{(2\pi)^6} \sigma v_{12} \times$$

$$\times \left[f f_2 (1-f_1)(1-f_2) - f_1 f_2 (1-f)(1-f_2) \right] \delta^3(p+p_2-p_1'-p_2'). \qquad (3)$$

is Uehling- Uhlenbeck's quantum mechanical extension of Boltzmann's collision term.

The Boltzmann equation is solved by simultaneous numerical integration of the classical equations of motion of 15 ensembles of $A_P + A_T$ test particles, which move on curved trajectories in the mean potential field. The single particle distribution function is approximated by local ensemble averaging over the test particles' phase space distribution f [8,9]. Smooth density distributions are necessary to compute the acceleration of the test particles from the gradient of the density dependent, ensemble averaged potential U and to calculate the Pauli blocking factors (1-f)(1-f), which are given by the six dimensional phase space occupancy f [8,9]. Two test particles from a given ensemble may undergo s-wave scattering if they approach each other within a distance $d^2 = \sigma/\pi$ [7] and the resulting final states are not Pauli blocked. Protons, neutrons, deltas and pions of different isospin are distinguished and the measured elastic and inelastic scattering cross sections σ are used [9].

Three different Skyrme parametrizations have been chosen to represent the mean field U due to a stiff (compression constant K=375 MeV), a medium (K=200 MeV) and a supersoft nuclear equation of state:

$$\text{stiff} \quad (K=375\ \text{MeV}) \quad U(n) = -124\ n/n_0 + 70.5(n/n_0)^2 \quad \text{MeV} \tag{4a}$$

$$\text{medium} \quad (K=200\ \text{MeV}) \quad U(n) = -356\ n/n_0 + 303(n/n_0)^{7/6} \quad \text{MeV} \tag{4b}$$

The supersoft potential equals (4a) at $n < n_0$, and is constant for $n > n_0$. It allows us to study metastable nuclei with zero compression energy. U is directly related to the nuclear equation of state via

$$U = \frac{\partial(nE)}{\partial n} \tag{5}$$

A potential approach is inherently nonrelativistic and the calculations at the highest energies, E_{cm}=400 MeV/N, are strictly speaking beyond the scope of this method.

We have extensively tested the present method and the newly developed computer program [9]. First, inclusive spectra of protons emitted from high energy heavy ion reactions have been calculated. They compare well with recent data [9,10].

Second, turning to pion production, we have studied conventional proton induced pion production on nuclear targets. Fig. 1 shows the target mass dependence of π^+ and π^- yields experimentally observed [11] together with the present results. The absolute yields, the target mass dependence and the large (factors of 5) difference between the π^+ and π^- yields as well as the pion spectra (not shown) are well reproduced.

The present approach has further been tested by comparison with the standard Cugnon cascade [7]: When Pauli blocking and potential are turned off, the parallel ensembles decouple. Then the test particles move on straight line trajectories until they scatter. Thus the conventional intranuclear cascade model is recovered. In order to simulate Cugnon's calculations as closely as possible his 'Pauli principle' prescription of excluding n-n collisions with c.m. energies below 50 MeV/N is used in our 'cascade mode'. The cascade nuclei can be kept from artificially expanding as a result of their Fermi motion [7,12] by the supersoft potential (there is no repulsive short range potential). The pion yields calculated with the present program in the 'cascade mode' (with the simple Pauli blocker and no field) agree quantitatively with the cascade results [4,7]. Both results differ substantially from those obtained with the present theory with nuclear equation of state and phase space Pauli blocking:

Take the 360 MeV/N data, for instance: The negative pion yield is 1.05 in the 'cascade mode', but drops by a factor of two - to 0.56 - if the compression energy (4a) is included - the suggested large influence of the nuclear matter equation of state [4] is observed. The pion yield drops further (to 0.46) when the Uehling-Uhlenbeck Pauli blocking is applied (see ref. [13]). We would like to refer to other papers on pion production using different approaches [14-17].

The pion multiplicities are shown in Fig. 2 as a function of the bombarding energy. The present theory with equation of state (4a) plus phase space Pauli blocker compares well with the data [4], while the 'cascade mode' overestimates the pion yields by factors > 2 at energies up to 1 GeV/N, in agreement with the results of refs. [4,7]. The drop in the pion yield is found to be due to the transformation of kinetic energy into potential energy during the high density phase of the reaction [2,4] as well as due to Pauli blocking. To check the sensitivity of the pion yields to the equation of state, the calculations have been repeated with the medium potential (4b). We find 2.45±0.09 and 2.13±0.07 π^- at 772 MeV/N with the medium and the stiff equation of state, respectively. At lower energies, statistical errorbars of 15% preclude an accurate assessment of the influence of the potential. At all other energies, where the statistical error bars are 3%, the yields are systematically higher (by about 10%) with the medium equation of state than with the hard equation of state. For the time being we feel unable, though, to extract a nuclear equation of state from the data [4].

We have also investigated the observed sidewards peaking (Fig3b)in multiplicity selected Nb(400 MeV/N)+Nb collisions [3]: The flow tensor F_{ij},

$$F_{ij} = \sum_{r} \frac{p_i p_j}{2m_\nu} \quad , \tag{6}$$

is determined on an event by event basis and the direction of the maximum kinetic energy flow is determined. Cascade calculations [3,5,12] produce forward peaked distributions. The role of the binding potential should be emphasized: The potential field keeps the nuclei from expanding before collisions can occur. If the binding potential is neglected as in Cugnon's original program [7,12], finite flow angles occur [7] due to sidewards expansion of the unbound projectile and target nucleons (see Fig. 3c).

Fig. 3a shows the flow angle distribution for Nb(400MeV/N)+Nb at b=1 fm with the medium and the stiff equation of state. Observe the strong sidewards maximum at large angles in both cases and the drastic influence of the equation of state: The average flow angle is 43 degrees with the stiff equation of state, while it is 28 degrees with the medium one. Thus, it may be possible to extract the nuclear equation of state from a comparison of the data with high statistics calculations, which simulate the experimental trigger conditions and efficiencies.

In conclusion, a microscopic theory based on the Boltzmann equation with a self-consistent mean field and a collision term which respects the Pauli principle explains both pion multiplicity and collective flow observed in central nucleus-nucleus collisions. Both the nuclear equation of state and the Pauli principle have a large influence on the results.

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This cascade version assumes a zero resonance lifetime and neglects pion absorption, which results in largely different pion yields [7].

Figure Captions

Fig. 1 Pion yields from 730 MeV protons as a fuction of the target mass[17].

Fig. 2 Pion multiplicity for central collisions (b<2.4 fm [6]) of Ar + KCl. The data [6] are compared to the present theory with compression energy and phase space Pauli blocker and with the 'cascade mode'.

Fig. 3 Flow angle distributions $dN/d\cos\theta_F$ for $^{93}Nb(400\ MeV/N)+^{93}Nb$.

a) The present theory with the medium and hard equation of state.

b) Data [6] for various multiplicity bins.

c) Cascade results with unbound [7] and stable [12] initial nuclei.

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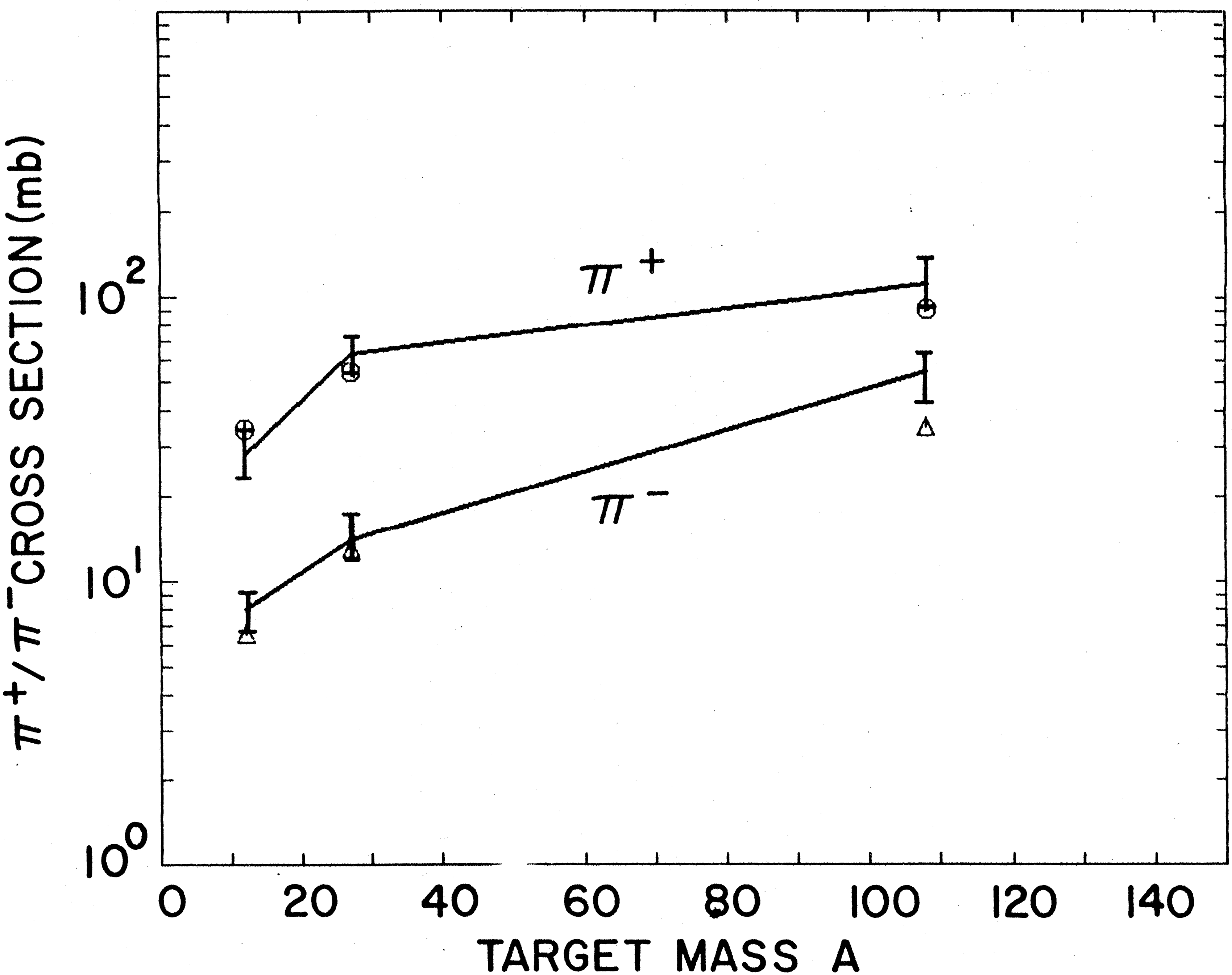

Fig 1

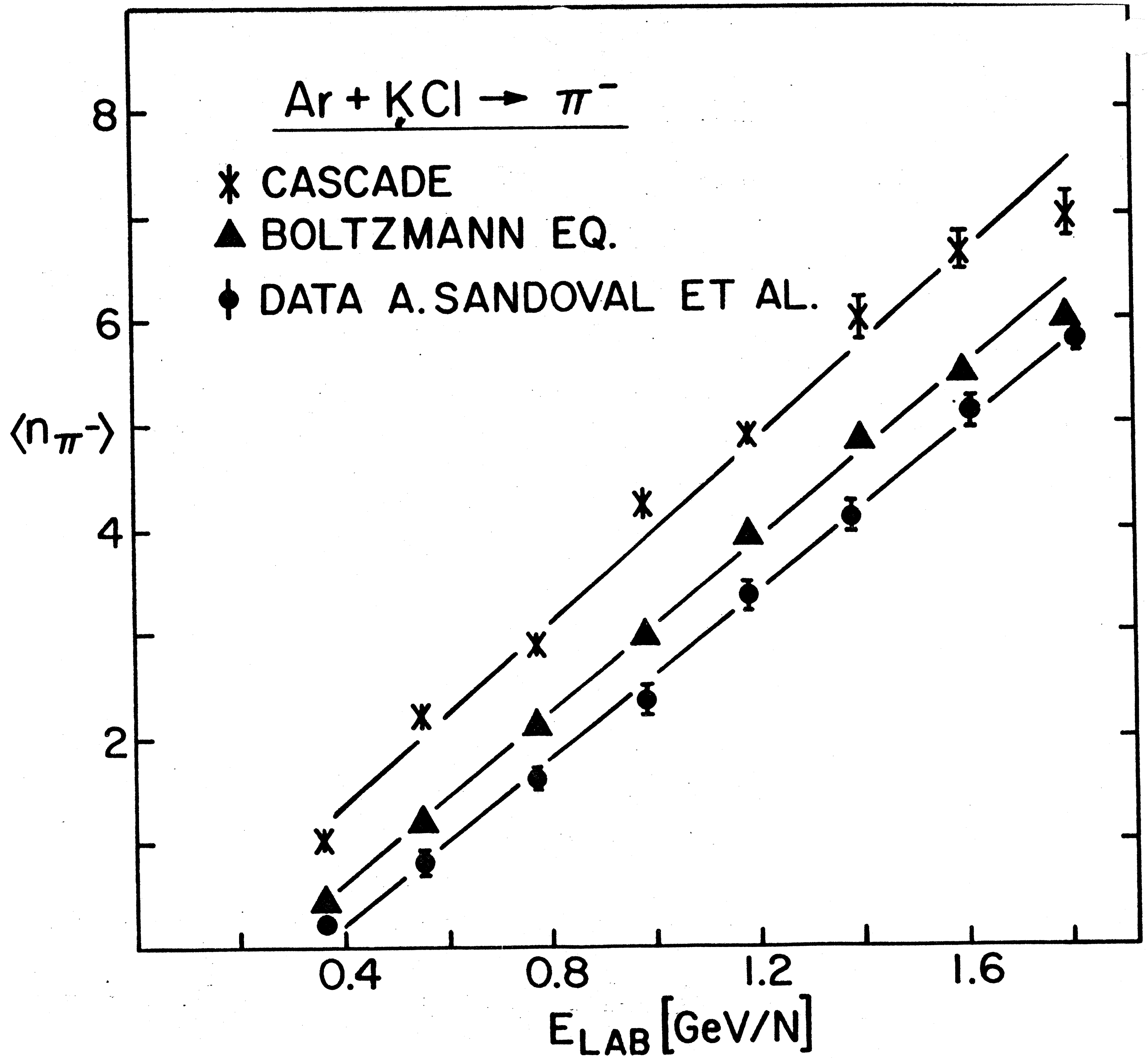
Ar + KCl → π⁻
CASCADE
BOLTZMANN EQ.
DATA A. SANDOVAL ET AL.
8
6
4
2
⟨n π⁻⟩
0.4
0.8
1.2
1.6
E LAB [GeV/N]

Fig 2

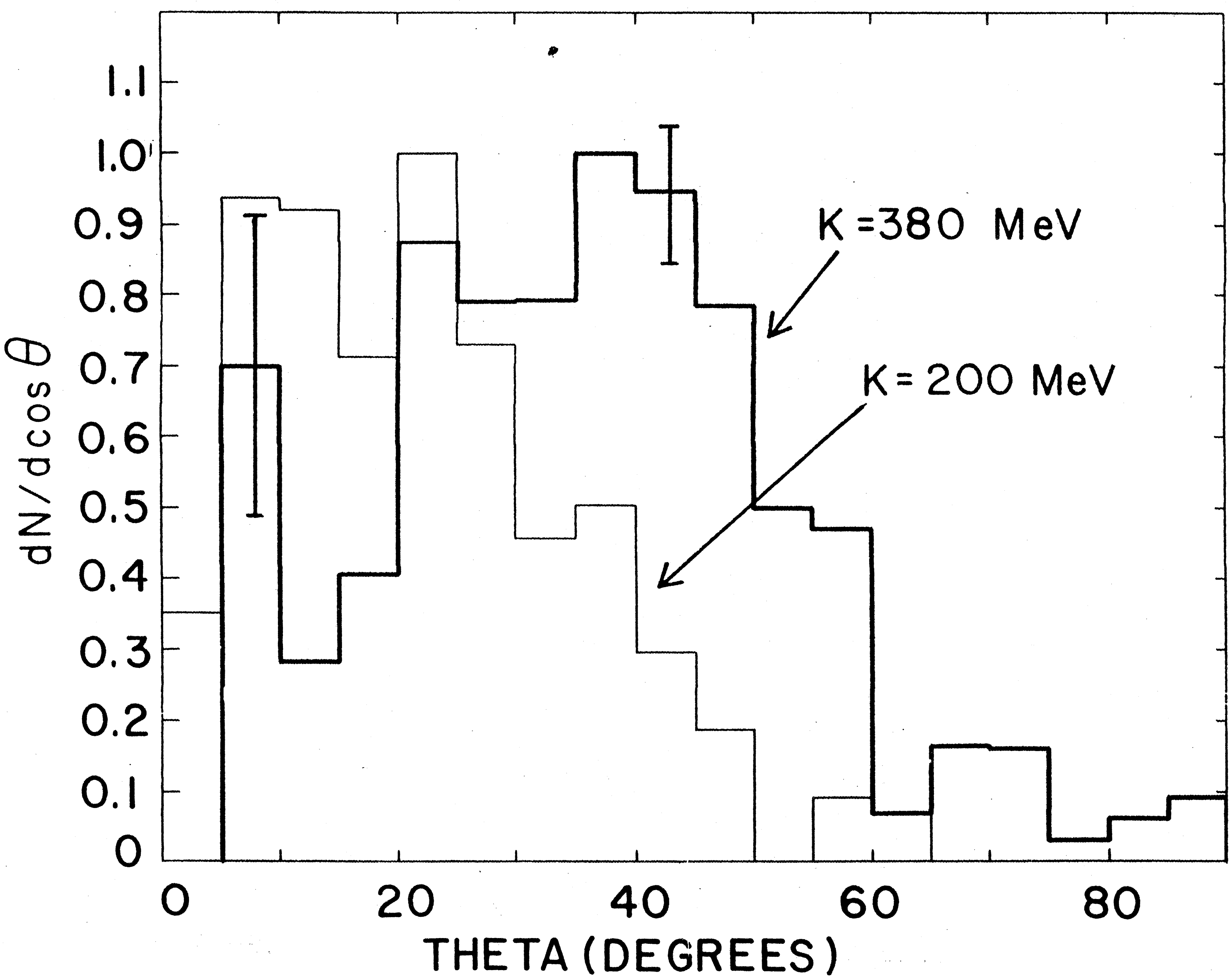
K =380 MeV
K= 200 MeV
dN/dcos θ
THETA (DEGREES)
0
20
40
60
80
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

Fig3a

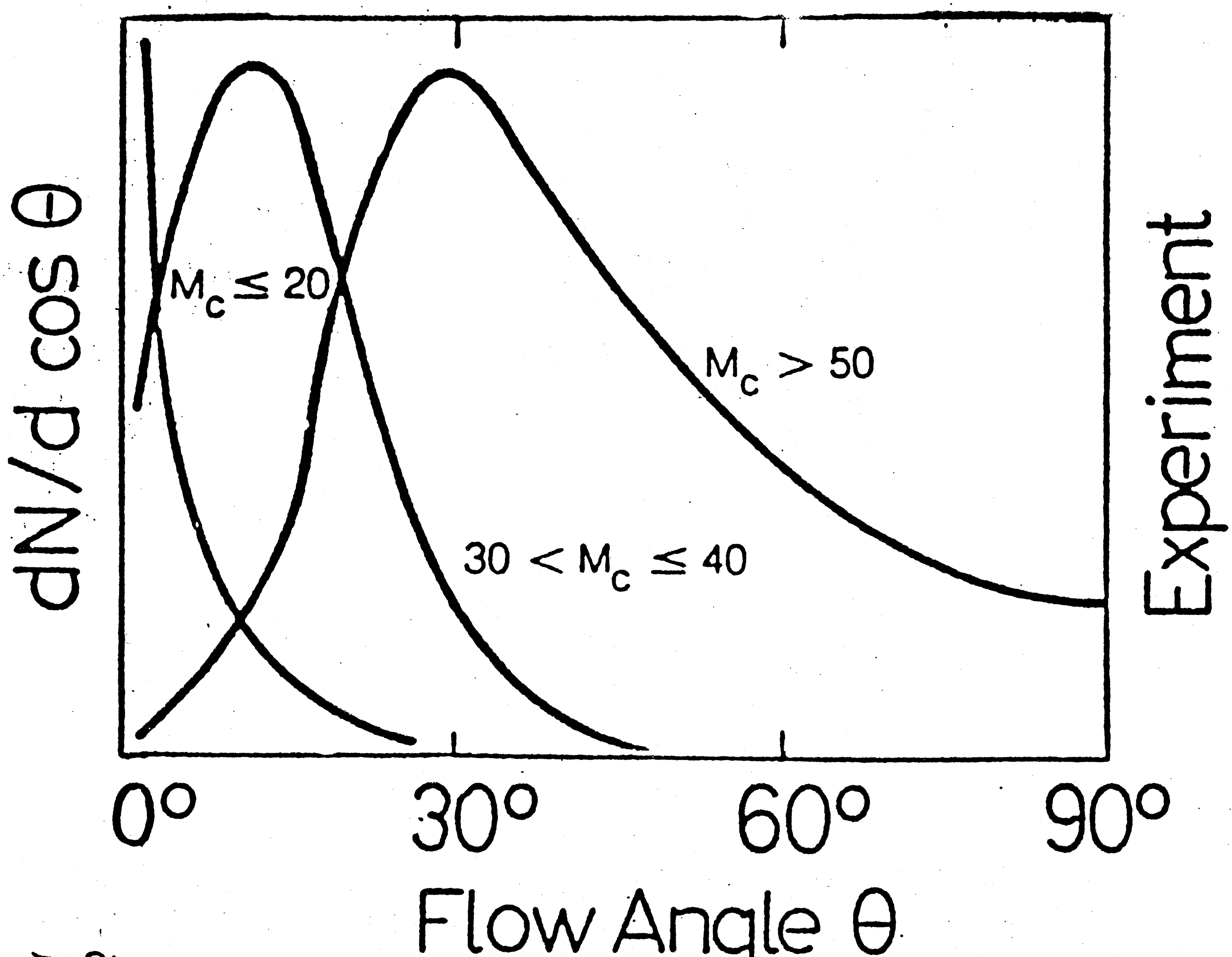

Fig3b

MSU-84-305

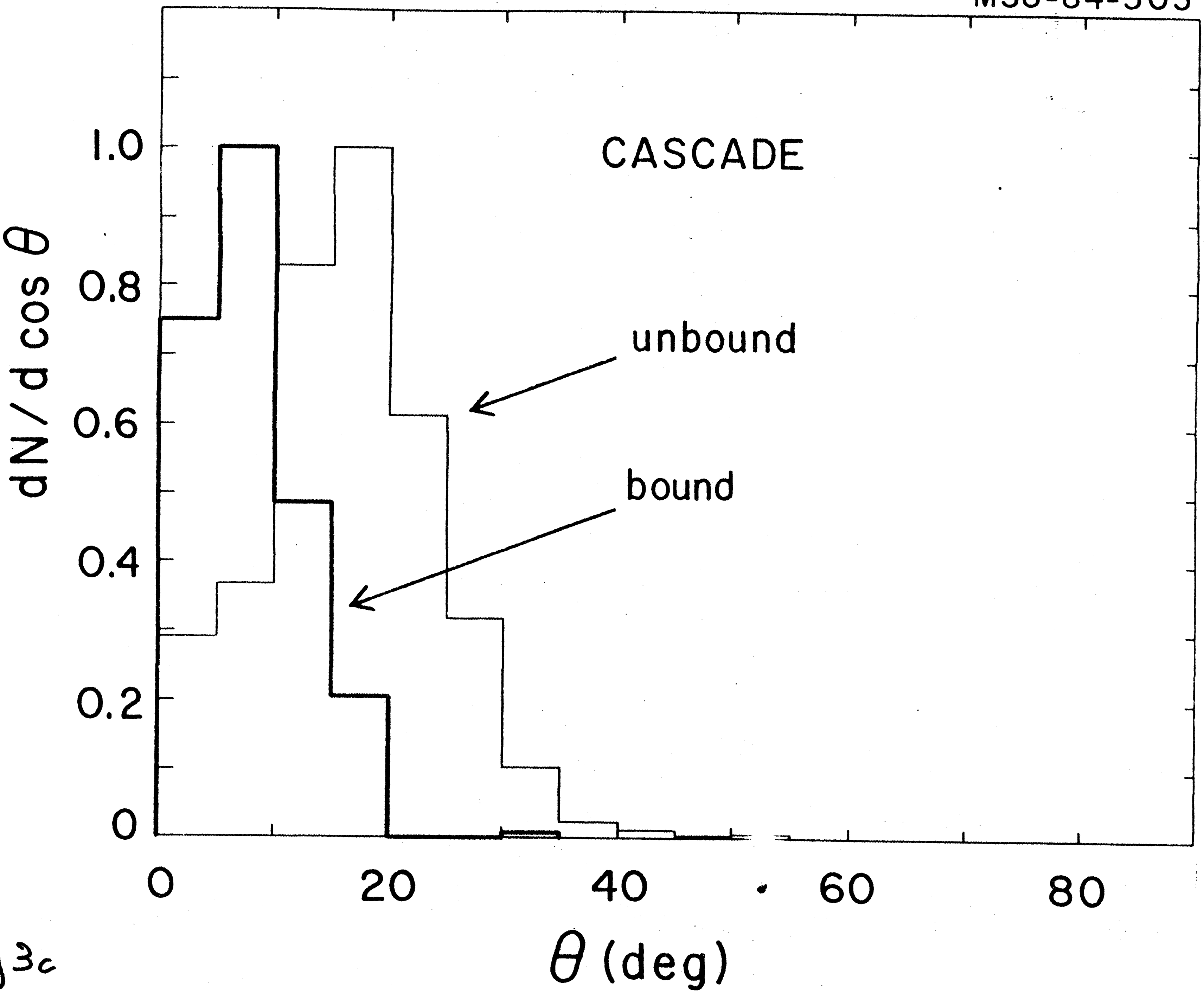

Fig 3c