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118

EFFECTS OF INTERNAL DEGREES OF FREEDOM ON THE HEAVY—ION FUSION CROSS SECTION

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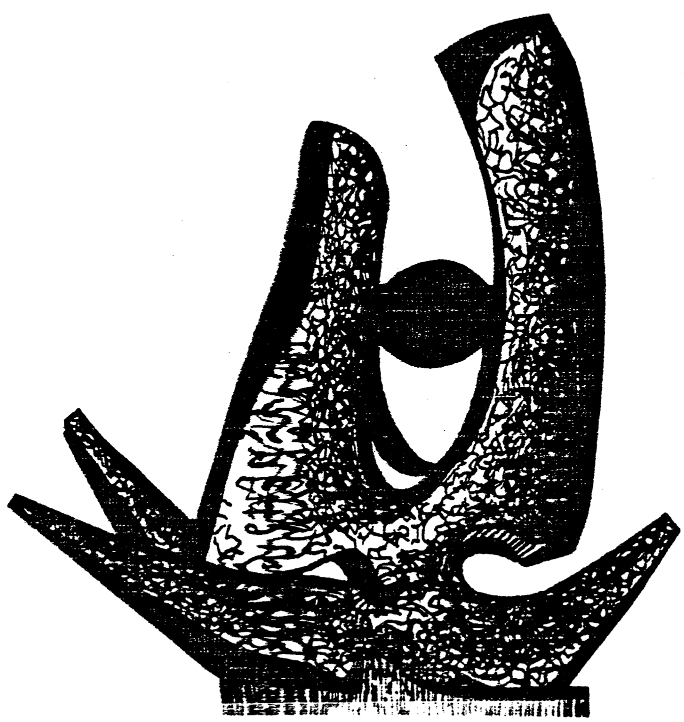

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Effects of Internal Degrees of Freedom on the Heavy-Ion Fusion Cross Section

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Abstract

The large enhancement of the sub-barrier fusion cross section observed in a number of mediate-mass heavy ion collisions has been attributed to the effects of coupling of the relative motion to internal degrees of freedom. In this connection, I discuss several aspects of the tunnelling using the influence functional method. I first discuss the properties of the potential renormalization caused by a linear oscillator coupling, showing that in certain limits it has significant energy-dependence. Then, I discuss a factorization property of the influence functional when several internal modes of excitation are simultaneously involved, showing that there is a saturation effect in the enhancement of the transmission probability. Then, I show that the transmission probability can be factorized as a product of the adiabatic transmission probability and a dissipation factor if the relative motion couples to harmonic oscillators with high frequency. It is suggested that quite different correlations between the dissipation factor and the friction coefficient for a classically accessible process arises depending on the properties of the Hamiltonian of the internal coordinates. Finally, I show that a strong and localized coupling around the potential barrier yields a resonant fusion excitation function.

1. Introduction

It has been observed in a number of mediate-mass heavy ion collisions that the fusion cross section at subbarrier energies is much larger than the prediction of a potential model. This motivated many theoretical studies on the effects of internal degrees of freedom on the heavy ion fusion cross section. In general the coupling between the relative motion and internal degrees of freedom will introduce several distinct effects, e.g. the potential renormalization, the dissipation effect and the renormalization of the mass. The potential renormalization seems to be most important in the problem of subbarrier fusion cross section.[1] After reviewing the influence functional method in sect. 2, I discuss in sect. 3 the properties of the potential renormalization caused by linear coupling to a harmonic oscillator. I discuss the dependence on the frequency of the harmonic oscillator,[2] and on the incident energy.[3]

In actual heavy ion collisions, several internal modes of excitation are simultaneously excited.[4-6] A question then arises concerning the relation between the total enhancement of the transmission probability and the enhancement due to individual excitation. It would be nice if the enhancements for distinct degrees of freedom factored, but we shall find in sect. 4 that this is not so.

The dissipation effect has been an intriguing subject in connection with the hindrance of the phase tunnelling in Josephson-junctions.[7] In this problem, Caldeira and Leggett[8] have pointed out a strong correlation between the hindrance factor of the quantum tunnelling probability and the friction coefficient in a classically accessible process. On the other hand, in a recent paper,[9] Balantekin and I have studied the effects of coupling of the relative motion to a damped harmonic oscillator. The damped harmonic oscillator mimics the excitation of a giant resonance, and naturally leads to the concept of friction in a classically accessible process. We found, however, that the dissipation factor for the tunnelling probability has only small connection to the friction coefficient. This contrasts to the result of Ref. 8. To clarify the situation, I discuss in sect. 5 how a different correlation between the dissipation factor for the tunnelling probability and the friction coefficient arises for the models considered in Refs. 8 and 9. In sec. 6, I discuss the behavior of the excitation function of the transmission probability across a potential barrier when the coupling between the relative motion and the internal degrees of freedom is localized around the potential barrier region. As a special case, I consider a model,[10] where the coupling form factor is a δ-function at the barrier top position. I thus demonstrate that a strong and localized coupling leads to a resonant fusion excitation function.[11] This model shows also that the Q-value effect of the coupling is strongly influenced by the property of the coupling form factor.

2. Influence functional formalism

2.1. Inclusive transmission probability

We consider a one dimensional relative motion, and denote the coordinate as R. It corresponds to the distance between the centers-of-mass of the colliding heavy ions. If we denote the internal degrees of freedom as q, the Schrödinger equation is given by,

$$\left[-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial R^2} + U(R) + \hat{H}_0(q) + \hat{H}_{int}(q,R)\right]\psi = E\psi \qquad (2.1)$$

In this section, we represent the internal degrees of freedom by a harmonic oscillator with frequency ω_0.

The influence functional formalism of Feynman's path integral method provides a powerful theoretical framework to study the effects of coupling. In this method, the inclusive transmission probability is given by,[9]

$$P(E) = \lim_{\substack{R_i\to\infty \\ R_f\to-\infty}} \left(\frac{P_i P_f}{\mu^2}\right) \int_0^\infty dT\, e^{iET/\hbar} \int_0^\infty d\tilde{T}\, e^{-i\tilde{E}\tilde{T}/\hbar}$$

$$\int \mathcal{D}[R(t)]\mathcal{D}[\tilde{R}(\tilde{t})]\, e^{i/\hbar[S_t(R,T)-S_t(\tilde{R},\tilde{T})]}\, \rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T), \tag{2.2}$$

where S_t and ρ_M are the classical action integral of the relative motion and the two-time influence functional, respectively. They are given by,

$$S_t(R,T) = \int_0^T \left(\frac{1}{2}\mu\dot{R}^2 - U(R)\right)dt \tag{2.3}$$

and

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \sum_{n_f} \mathcal{W}^*_{n_f,n_i}[\tilde{R}(\tilde{t});\tilde{T},0]\,\mathcal{W}_{n_f,n_i}[R(t);T,0], \tag{2.4}$$

where $\mathcal{W}_{n_f,n_i}[R(t);T,0]$ is a transition amplitude of the internal state along a given path R(t). It is given by,

$$\mathcal{W}_{n_f,n_i}[R(t);T,0] = \langle n_f|\hat{U}(R(t);T,0)|n_i\rangle, \tag{2.5}$$

where the Green's function for the internal state $\hat{U}$ is determined by the differential equation

$$i\hbar\frac{\partial}{\partial t}\hat{U} = [\hat{H}_0 + \hat{H}_{int}(q,R(t))]\hat{U} \tag{2.6}$$

subject to the boundary condition $\hat{U}(t=0)=1$.

2.2 Examples of the influence functional

When the relative motion linearly couples to a harmonic oscillator as,

$$\hat{H}_{int} = \alpha_0 f(R)(a^\dagger + a), \tag{2.7}$$

α_0 being the amplitude of the zero point motion of the harmonic oscillator, the corresponding two time influence functional can be easily obtained as,[9)]

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = e^{-i\frac{1}{2}\omega_0(T-\tilde{T})} \exp\{-(\tfrac{\alpha_0}{\hbar})^2[\int_0^T dt \int_0^t ds\, f(R(t))f(R(s))e^{-i\omega_0(t-s)}$$

$$+ \int_0^{\tilde{T}} dt \int_0^t ds\, f(\tilde{R}(t))f(\tilde{R}(s))e^{i\omega_0(t-s)}$$

$$- e^{i\omega_0(\tilde{T}-T)} \int_0^T dt f(R(t))e^{i\omega_0 t} \int_0^{\tilde{T}} ds f(\tilde{R}(s))e^{-i\omega_0 s}]\}. \tag{2.8}$$

We have assumed that the harmonic oscillator is initially in the ground state, i.e. $n_i=0$.

If the harmonic oscillator has zero frequency, Eq. (2.8) and the Hubbard-Stratonovich transformation leads to,[9]

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx\, e^{-\frac{1}{2}x^2}\, e^{-\frac{i}{\hbar}x\alpha_0\left[\int_0^T dt f(R(t)) - \int_0^{\tilde{T}} d\tilde{t} f(\tilde{R}(\tilde{t}))\right]} . \quad (2.9)$$

The influence functional for the coupling to a high lying harmonic oscillator will be well approximated by the adiabatic limit,

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) \approx \mathcal{W}^*_{n_i n_i}[\tilde{R}(\tilde{t});\tilde{T},0]\mathcal{W}_{n_i n_i}[R(t);T,0] \quad (2.10)$$

This is equivalent to ignoring the last term inside the exponent in eq (2.8). The transition matrix is given by,

$$\mathcal{W}_{00} = \exp\left[-\frac{i}{\hbar}\int_0^T W(R(t);t)dt\right], \quad (2.11)$$

where

$$W(R(t);t) = \frac{1}{2}\hbar\omega_0 - i\frac{\alpha_0^2}{\hbar} f(R(t))\int_0^t f(R(t_1))e^{-i\omega_0(t-t_1)}dt_1. \quad (2.12)$$

Repeating the partial integration for the time integral in Eq. (2.12), the influence potential $W(R(t);t)$ can be expressed as,

$$W(R(t);t) = \frac{1}{2}\hbar\omega_0 - \frac{\alpha_0^2}{\hbar\omega_0} f(R(t))\sum_{n=0}^{\infty}\left(\frac{i}{\omega_0}\right)^n \frac{d^n f(R(t))}{dt^n}. \quad (2.13)$$

We have discarded the terms proportional to $f(R(0))$ and $\left(\frac{d^m f}{dt^m}\right)_{t=0}$ to be consistent with the initial condition $n_i=0$. To the leading order, we thus obtain,

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) \approx e^{i/\hbar\left[\int_0^T dt \frac{(\alpha_0 f(R(t)))^2}{\hbar\omega_0} - \int_0^{\tilde{T}} d\tilde{t}\frac{(\alpha_0 f(\tilde{R}(\tilde{t})))^2}{\hbar\omega_0}\right]} . \quad (2.14)$$

3. Potential renormalization due to linear oscillator coupling

The original coupled channel problem reduces to a single channel problem in two limiting cases, namely when $\omega_0=0$ (sudden limit), and when ω_0 is large (adiabatic limit). The latter condition is satisfied when ω_0 is much larger than the inverse of the transmission time or the curvature of the potential barrier. In both cases the dominant effect of coupling is to renormalize the potential barrier. Eq. (2.14) shows that the effective potential in the adiabatic limit is given by,

$$U_{ad}^{eff}(R) = U(R) - \alpha_0^2[f(R)]^2/\hbar\omega_0 . \quad (3.1)$$

which is apparently energy independent. In the sudden limit, Eq. (2.9) leads to the following zero-point motion formula[12] for the inclusive transmission probability,

$$P(E_{cm}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx\ e^{-\frac{1}{2}x^2}\ P_0(E,U(r) + H_{int}(\alpha_0 x,R)), \tag{3.2}$$

where $P_0(E,U(R))$ is the transmission probability of a one dimensional problem across the potential barrier U(R). For the linear oscillator coupling, Eq (3.2) becomes,

$$P(E_{cm}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx\ e^{-\frac{1}{2}x^2}\ P_0(E_{cm},U(R) + x\ \alpha_0 f(R)). \tag{3.3}$$

In order to see more clearly the properties of the potential renormalization implicit in Eq. (3.3), let us consider the case, when the incident energy is well below the potential barrier. In this case, the WKB approximation is fairly accurate and gives,

$$P_0(E,V(R)) \approx \exp[\frac{2}{\hbar}(ET_0 + S_0)]\ . \tag{3.4}$$

where $-iT_0$ is the transmission time for the tunnelling process, and $-iS_0$ is the classical action for the potential V(R), i.e.

$$S_0 = -\int_0^{T_0} \{\frac{1}{2}\mu\left(\frac{dR}{d\tau}\right)^2 + V(R(\tau))\}d\tau\ . \tag{3.5}$$

Note that a classically inaccessible process can be described by considering the time evolution along the negative imaginary time axis.[8,13-16] Eqs. (3.3) through (3.5) lead to,

$$P(E_{cm}) \approx e^{-2\int_0^{T_0} [\frac{\mu}{2}\left(\frac{dR}{d\tau}\right)^2 + U(R(\tau)) - E_{cm}]d\tau/\hbar}\ e^{2[\alpha_0 \int_0^{T_0} f(R(\tau))d\tau/\hbar]^2}\ . \tag{3.6}$$

Eq. (3.6) agrees with the result in Refs. 2 and 15, where P(E) has been obtained by studying the imaginary time propagator. It indicates that the renormalized effective potential $\tilde{U}^{eff}_{\omega=0}(R)$ is given by,

$$\tilde{U}^{eff}_{\omega=0}(R(\tau)) = U(R(\tau)) - \frac{2\alpha_0^2}{\hbar} f(R(\tau)) \int_0^{\tau} d\tau_1\ f(R(\tau_1)). \tag{3.7}$$

On the other hand, the effective potential, which appears in the classical equation of motion determining the dominant tunnelling path, is given by,[2,15]

$$U^{eff}_{\omega=0}(R) = U(R) - \frac{2\alpha_0^2}{\hbar} f(R) \int_0^{T_0} d\tau_1 f(R(\tau_1)). \tag{3.8}$$

The classical equation of motion and Eq. (3.8) can be obtained by minimizing the exponent of Eq. (3.6) with respect to $R(\tau)$ for fixed boundary conditions.

Figure 1 compares the effective potential $U^{eff}_{\omega=0}(R)$ given by Eq. (3.8) at two different energies, (the thin solid and the dot-dashed lines), the effective potential $U^{eff}_{ad}(R)$ (the dashed line) and the bare potential (the thick solid line).

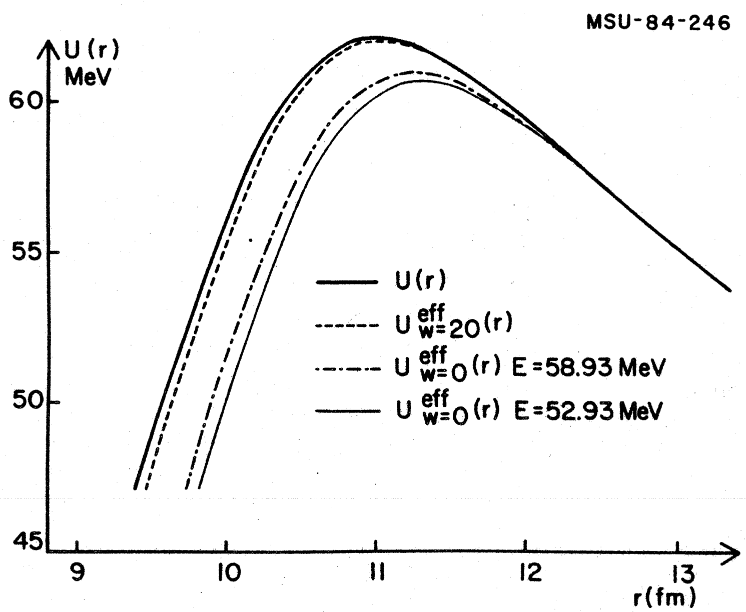

Figure 1. Effective potential barrier.

To calculate $U^{eff}_{ad}(R)$, we have chosen the energy quanta of the high lying harmonic oscillator as $\hbar\omega_0$=20 MeV. The amplitude of the zero point motion α_0 was chosen to be 0.2 fm for both high lying and zero-frequency harmonic oscillators. We have assumed the same bare potential and the coupling form factor as those in Ref. 2, which are realistic to describe heavy ion collisions. We see that the linear oscillator coupling lowers the height of the potential barrier and makes the potential barrier thinner. This is what is needed[1] to explain the enhancement of the subbarrier fusion cross section. The potential renormalization due to the excitation of a degenerate harmonic oscillator is more pronounced than that due to the excitation of a high lying harmonic oscillator. Another feature shown in Fig. 1 is that the potential renormalization due to the excitation of a degenerate harmonic oscillator becomes more significant as the incident energy decreases. This offers an explanation of the *dynamical effect* discovered phenomenologically in Ref. 17. Also, this is consistent with the conclusion in Ref. 18 which extracted a local potential from the fusion data: namely, the assumption of an energy-independent local effective potential is inadequate in describing subbarrier fusion cross section.

4. Multiple modes of excitation-saturation of enhancement

4.1. Factorization property of the influence functional

The influence functional method is especially convenient in order to study the effects of multiple modes of excitation. The *multiplication property* of the influence functional is clear. Namely, if the relative motion couples with several independent internal degrees of freedom, then the total influence functional is given as the product of the influence functional for each coupling. However, this does not imply that the total enhancement of the transmission probability is given by the product of the enhancement due to individual excitation.

As an example, let us consider the case, when the relative motion linearly couples to a high lying and a zero-frequency harmonic oscillator simultaneously. In this case, the total influence functional is given by the product of the right-hand side of Eqs. (2.9) and (2.14)

$$\rho_M(\tilde{R}(\tilde{t}),\tilde{T};R(t),T) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx\, e^{-\frac{1}{2}x^2}\, e^{-i\frac{1}{2}\omega_0^{(H)}(T-\tilde{T})}$$

$$\exp[-\frac{i}{\hbar}\{\int_0^T dt[x\alpha_0^{(L)} f_L(R(t)) - \frac{(\alpha_0^{(H)} f_H(R(t)))^2}{\hbar\omega_0^{(H)}}]$$

$$- \int_0^{\tilde{T}} d\tilde{t}[x\alpha_0^{(L)} f_L(\tilde{R}(\tilde{t})) - \frac{(\alpha_0^{(H)} f_H(\tilde{R}(\tilde{t})))^2}{\hbar\omega_0^{(H)}}]\}] \;, \quad (4.1)$$

with trivial notations. Accordingly, the transmission probability is given by,

$$P(E_{cm}) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx\, e^{-\frac{1}{2}x^2}\, P_0(E_{cm}, U(R) + x\alpha_0^{(L)} f_L(R) - \frac{(\alpha_0^{(H)} f_H(R))^2}{\hbar\omega_0^{(H)}}). \quad (4.2)$$

We will now illustrate the transmission probability obtained with Eq. (4.2) with a numerical example.

4.2 Saturation of enhancement

We define the enhancement factor ζ by

$$\zeta = P(E_{cm})/P_0(E_{cm}, U(R)). \quad (4.3)$$

In Fig. 2, we compare the enhancement factor for the simultaneous linear coupling ζ, that for individual coupling ζ_0 and ζ_{20} and their product $\zeta_0 \times \zeta_{20}$. The R is the ratio of ζ to $\zeta_0 \times \zeta_{20}$. Eqs. (4.2), (3.3) and (2.14) have been used to calculate ζ, ζ_0 and ζ_{20}, respectively. The frequency of the high lying harmonic oscillator was assumed to be $\hbar\omega_0$=20 MeV. The value of the amplitude of the zero point motion was chosen to be 0.2 fm and 0.4 fm for the zero frequency and for the high lying harmonic oscillators, respectively. We have used the same bare potential and the coupling form factor as for Fig. 1.

Figure 2 clearly shows the *saturation property* of the enhancement. Namely, the enhancement factor for the simultaneous excitations is smaller than the product of the enhancement factor for individual excitation. This can be understood as follows. The coupling reduces the potential barrier and the transmission time. Let us denote these due to the excitation of i-th harmonic oscillator as δU_i and $-\delta T_i$, respectively. The multiplication property of the influence functional then indicates,

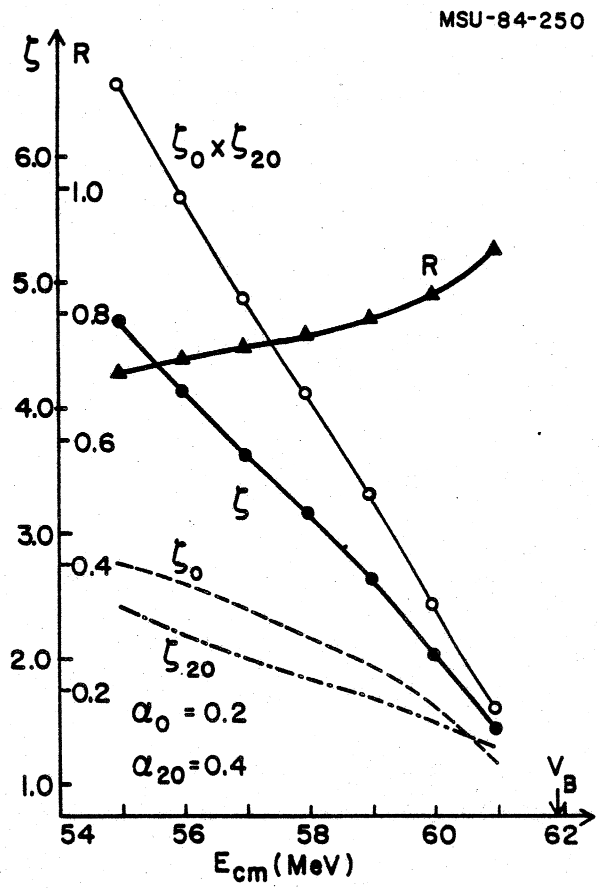

Figure 2. Comparison of the enhancement factor for multiple modes of excitation with that for individual excitation.

$$P(E_{cm}) = \exp\{-2 \int_0^{T_0^{(0)} - \sum_i \delta T_i} [\frac{\mu}{2} \dot{R}^2 + U(R) + \sum_i \delta U_i - E_{cm}] d\tau/\hbar\} . \qquad (4.4)$$

where $-iT_0^{(0)}$ is the transmission time through the bare potential barrier. Up to the second order, we thus obtain

$$\zeta = (\prod_j \zeta_j) \times R, \qquad (4.5)$$

where ζ_j is the enhancement factor in the case when the relative motion couples to only j-th harmonic oscillator, and R is given by,

$$R = \exp\{2 \sum_j \delta T_j \sum_i \delta U_i/\hbar\} \qquad (4.6)$$

Note that the $(\delta T)^2$-term disappears because of the equation of motion. For linear oscillator coupling, $\delta T_j > 0$ and $\delta U_i < 0$. Therefore,

$$R < 1, \text{ i.e. } \zeta < \prod_j \zeta_j. \qquad (4.7)$$

It is essential to take into account the change of the transmission time due to coupling in order to obtain the saturation property. The constant coupling form factor and the succeeding use of the quadratic barrier approximation to calculate the transmission probability do not satisfy this condition. This explains why the saturation is absent in the model considered in Ref. 5.

5. Dissipation factor and the correlation with the friction coefficient

5.1. Coupling to a damped harmonic oscillator

In order to study the effects of coupling to a damped harmonic oscillator, we have considered[9] a model of heavy ion collisions, where the relative motion linearly couples to a collective harmonic oscillator, which further couples to many other non-collective harmonic oscillators. The internal and the interaction Hamiltonians are taken to be

$$\hat{H}_0 = \hbar\omega_0(a_0^+a_0 + \frac{1}{2}) + \sum_{i=1}^{m} \hbar\omega_i(b_i^+b_i + \frac{1}{2}) + \hbar\kappa \sum_{i=1}^{m} (a_0^+b_i + a_0b_i^+) \tag{5.1}$$

and

$$\hat{H}_{int} = \alpha_0 f(R)(a_0^+ + a_0), \tag{5.2}$$

where (a_0^+, a_0) and (b_i^+, b_i) are the (creation, annihilation) operators of the collective and the i-th non-collective harmonic oscillators, respectively. The internal Hamiltonian $\hat{H}_0$ can be easily decoupled into normal modes. If we denote the creation and the annihilation operators of the j-th normal mode as $\tilde{a}_j^+$ and $\tilde{a}_j$, and the corresponding eigenfrequency as $\tilde{\omega}_j$, then

$$\hat{H}_0 = \sum_{j=1}^{m+1} \hbar\tilde{\omega}_j(\tilde{a}_j^+\tilde{a}_j + \frac{1}{2}) \tag{5.3}$$

and

$$\hat{H}_{int} = \sum_{j=1}^{m+1} \alpha_0 \chi_j f(R)(\tilde{a}_j^+ + \tilde{a}_j^+), \tag{5.4}$$

where χ_j is the amplitude of the collective oscillator in the j-th normal mode. The problem thus reduces to calculating the transmission probability in the case, when the relative motion linearly couples to (m+1) independent harmonic oscillators according to the strength distribution of the collective oscillator. We assume that the frequency of the collective oscillator ω_0 is so high that the *adiabatic approximation* for the influence functional, Eq. (2.10), is valid. The generalization of Eq. (2.12) to the present problem is straightforward,

$$W(R(t);t) = \frac{1}{2}\sum_j \hbar\tilde{\omega}_j - i\frac{\alpha_0^2}{\hbar}\sum_j \chi_j^2 \, f(R(t)) \int_0^t f(R(t_1))e^{-i\tilde{\omega}_j(t-t_1)} dt_1 \, . \tag{5.5}$$

As a special model, we consider the case when the non-collective harmonic oscillators are distributed with an equal spacing Δ from $-\infty$ to ∞, and assume that the strong coupling condition, $\frac{\pi\kappa}{\Delta} >> 1$, is satisfied. In this case, the strength distribution of the collective oscillator is given by the Lorentzian distribution,[19)]

$$J(\tilde{\omega}_j) = \frac{\chi_j^2}{\Delta} \approx \frac{1}{2\pi} \frac{\Gamma}{(\tilde{\omega}_j-\omega_0)^2+(\frac{\Gamma}{2})^2} , \tag{5.6}$$

where Γ corresponds to the width of the damped harmonic oscillator. Replacing the sum over the normal modes in Eq. (5.5) by the integral over the normal frequency $\tilde{\omega}$ from $-\infty$ to ∞ by using Eq. (5.6), we obtain

$$W(R(t);t) = \frac{1}{2}\sum_j \hbar\tilde{\omega}_j - i\frac{\alpha_0^2}{\hbar} f(R(t)) \int_0^t dt_1\, f(R(t_1)) e^{-\frac{\Gamma}{2}(t-t_1)} e^{-i\omega_0(t-t_1)} . \tag{5.7}$$

We now perform partial integration for the time integral in Eq. (5.7), and keep the terms up to the first order derivative. Inserting the result into Eq. (2.2), we obtain

$$P(E) = \lim_{\substack{R_i\to\infty \\ R_f\to-\infty}} \left(\frac{P_iP_f}{\mu^2}\right) \left| \int_0^\infty dT\, e^{\frac{i}{\hbar}(E-\sum_j \frac{1}{2}\hbar\tilde{\omega}_j)T} \int \mathcal{D}[R(t)]\, e^{\frac{i}{\hbar} S_{eff}^{(\Gamma)}(R,T)} \right|^2 \tag{5.8}$$

where

$$S_{eff}^{(\Gamma)}(R,T) = S_t(R,T) + \delta S_t^{(\Gamma)}(R,T), \tag{5.9}$$

$$\delta S_t^{(\Gamma)}(R,T) = \frac{\alpha_0^2}{\hbar} \int_0^T dt \left\{ [f(R(t))]^2 \frac{\omega_0+i\frac{\Gamma}{2}}{\omega_0^2+(\frac{\Gamma}{2})^2} + \frac{1}{2} i \frac{df^2}{dt} \frac{(\omega_0+i\frac{\Gamma}{2})^2}{[\omega_0^2+(\frac{\Gamma}{2})^2]^2} \right\} . \tag{5.10}$$

This indicates[9,16)] that the tunnelling probability is given by the following *factorization formula*,

$$P(E) \approx P_{ad}(E)\cdot P_D \tag{5.11}$$

where

$$P_{ad}(E) = P_0(E - \frac{1}{2}\sum_j \hbar\tilde{\omega}_j,\ U_{ad}(R)) \tag{5.12}$$

with

$$U_{ad}(R) = U(R) - \frac{\alpha_0^2}{\hbar}[f(R)]^2 \frac{\omega_0+i\frac{\Gamma}{2}}{\omega_0^2+(\frac{\Gamma}{2})^2} \tag{5.13}$$

and

$$P_D = \exp\left[-\left(\frac{\alpha_0}{\hbar}\right)^2 \Delta f^2 \frac{\omega_0^2-(\frac{\Gamma}{2})^2}{[\omega_0^2+(\frac{\Gamma}{2})^2]^2}\right] \tag{5.14}$$

where Δf^2 is the square of the coupling strength at the end of the tunnelling process. To the leading order of Γ, the imaginary part in $U_{ad}(R)$, given by Eq. (5.13), does not affect the transmission probability. $P_{ad}(E)$ is, therefore, nearly equal to the transmission probability across the adiabatic potential barrier given by the real part of $U_{ad}(R)$.

5.2 Caldeira-Leggett model

References 8 and 20 have discussed the problem that a macroscopic degree of freedom, which undergoes a quantum tunnelling, couples to many independent harmonic oscillators. Let us assume that the dominant effect is caused by those oscillators whose frequencies are so high that the adiabatic approximation to the influence functional can be applied. The ground state transition amplitude W_{00} is given by,

$$W_{00} = \exp[-i \frac{1}{2} \sum_j \omega_j T]\exp[-\frac{1}{\hbar} \int_0^T dt \int_0^t dt_1 f(R(t))f(R(t_1)) \sum_j \frac{1}{2m_j\omega_j} e^{-i\omega_j(t-t_1)}] \quad (5.15)$$

where we have used the mass m_j instead of the amplitude of the zero point motion α_j. We have assumed a common coupling form factor for all harmonic oscillators.

Following Ref. 8, we rewrite Eq. (5.15) as,

$$W_{00} = e^{-i \frac{1}{2} \sum_j \omega_j T} W_{00}^{(P)} W_{00}^{(D)} , \quad (5.16)$$

where

$$W_{00}^{(P)} = \exp[-\frac{1}{\hbar} \int_0^T dt \int_0^t dt_1 \{[f(R(t))]^2 + [f(R(t_1))]^2\} \sum_j \frac{1}{4m_j\omega_j} e^{-i\omega_j(t-t_1)}] \quad (5.17a)$$

$$= \exp[-\frac{i}{\hbar} \int_0^T dt[f(t)]^2 \sum_j \frac{1}{2m_j\omega_j^2} \{1 - \frac{1}{2}[e^{-i\omega_j t} + e^{-i\omega_j(T-t)}]\}] \quad (5.17b)$$

and

$$W_{00}^{(D)} = \exp[\frac{1}{\hbar} \int_0^T dt \int_0^t dt_1 [f(R(t)) - f(R(t_1))]^2 \sum_j \frac{1}{4m_j\omega_j} e^{-i\omega_j(t-t_1)}] \quad (5.18)$$

We now replace the sum in Eq. (5.18) by an integral by introducing the strength distribution $J(\omega)$,

$$\sum_j \frac{1}{m_j\omega_j} e^{-i\omega_j(t-t_1)} \to \lambda = \frac{2}{\pi c^2} \int_0^\infty d\omega \, J(\omega)e^{-i\omega(t-t_1)}, \quad (5.19)$$

where we have introduced a quantity c for later use. The strength distribution function assumed in Refs. 8 and 20 could be expressed as,

$$J(\omega) = \eta\omega e^{-\omega/\omega_c}. \quad (5.20)$$

The λ is then given by,

$$\lambda = \frac{2}{\pi c^2} \eta \frac{1}{[\frac{1}{\omega_c} + i(t-t_1)]^2} \tag{5.21}$$

If ω_c is much larger than the inverse of the tunnelling time along the imaginary time axis, one is allowed to approximate λ by

$$\lambda \approx - \frac{2\eta}{\pi c^2} \frac{1}{(t-t_1)^2} \tag{5.22}$$

We now expand $f(R(t_1))$ in Eq. (5.18) around $t_1=t$ up to the first order, and assume that the strength of the coupling form factor linearly increases with time, i.e.

$$f(t) = at \; . \tag{5.23}$$

The coefficient $W_{oo}^{(D)}$ then becomes,

$$W_{oo}^{(D)} = e^{-\frac{\eta}{4\pi c^2 \hbar} \Delta f^2} \; . \tag{5.24}$$

Eqs. (2.2), (2.10), (5.16), (5.17) and (5.24) thus indicate that the transmission probability is given by the factorization formula Eq. (5.11) in this model as well, where

$$U_{ad}(R) = U(R) - \sum_j \frac{1}{2m_j\omega_j^2} [f(R)]^2 = U(R) - \sum_j \frac{[\alpha_j f(R)]^2}{\hbar\omega_j} \tag{5.25}$$

and

$$P_D = \exp[- \frac{\eta}{2\pi c^2} \frac{1}{\hbar} \Delta f^2] \; . \tag{5.26}$$

We have disregarded the exponential terms inside the curly bracket in Eq. (5.17b), which should be negligible under the circumstances when Eq. (5.22) is valid. The potential renormalization in Eq. (5.25) exactly corresponds to the renormalization of the conservative force in the equation of motion for a classically allowed process[8] (see Eq. (A.6)).

References 8 and 20 have considered the tunnelling decay from a metastable state at R=0, and assumed a linear coupling form factor, i.e. $f(R)=cR$. In this case,

$$P_D = \exp[- \frac{1}{2\pi\hbar} \eta\cdot\Delta R^2] \; , \tag{5.27}$$

where ΔR is the distance under the barrier. Eq. (5.27) is essentially the formula obtained in Refs. 8 and 20. Contrary to Eq. (5.14), the dissipation factor strongly depends on the friction coefficient in the classically accessible process η (see Eq. (A.9)).

The formulae in this section have been derived based on the adiabatic approximation to the influence functional. The ignored term, i.e. the last term inside the exponent in Eq. (2.8), is of the order of $\frac{1}{\omega_0 T}$, T being the transmission time, compared to the leading term. On the other hand, the ratio of the exponent in the dissipation factor to the potential renormalization is also of the order of $\frac{1}{\omega_0 T}$. Therefore, the last term inside the exponent might introduce a dissipation factor, which is of the same order as those obtained under the present adiabatic approximation. In fact, the coefficient in front of η in Eq. (5.27) is different from that in the corresponding equation in Ref. 20. This discrepancy might be related to the accuracy of the adiabatic approximation discussed above.

6. Barrier-Top-Resonance and Q-value effect

As we have discussed in the previous sections, an important effect of the coupling of relative motion to internal degrees of freedom is to renormalize the potential barrier. If the coupling is strong and is localized around the potential barrier, then the effective potential barrier could become a double humped barrier. This could then yield a resonant fusion excitation function. In order to illustrate this phenomena, we have considered a model,[11)] where the internal degrees of freedom have only two levels and the coupling form factor is the δ-function at the barrier top position.

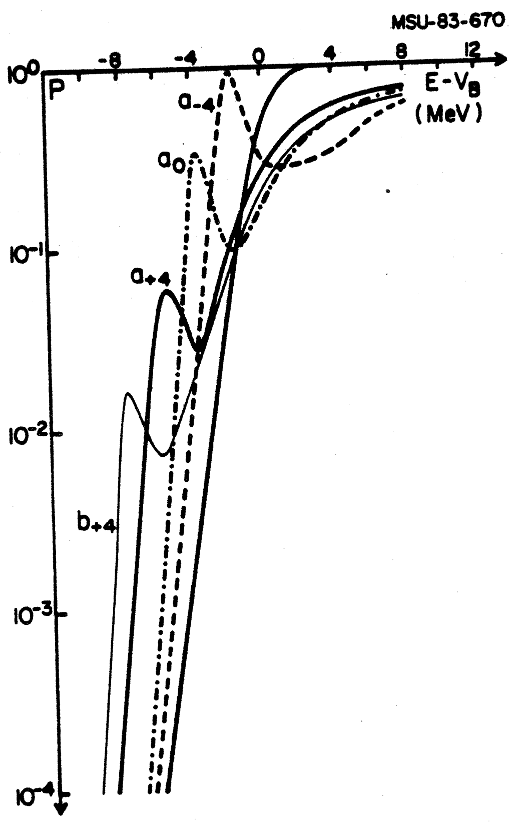

Figure 3. Fusion excitation function for a δ-function coupling at the barrier top position (two level model).

Figure 3 shows an example of the corresponding transmission probability, which is plotted as a function of the incident energy measured relative to the barrier height V_B (see Ref. 11 for detail). The solid monotonic line is the transmission probability when there is no coupling. The lines denoted by a and b were calculated for a weak and a stronger coupling Hamiltonian. The suffix refers to the reaction Q-value, i.e. the energy difference between the two internal states. The figure shows a clear resonance peak in each excitation function, which has been calculated for a given strength of the coupling Hamiltonian. Also, the figure shows that the transmission probability is enhanced by the coupling not only for positive Q values, but also for negative Q-values. This shows the important influence of property of the coupling form factor on the Q-value effect.

In this conference, several speakers[21-23] have reported resonant fusion excitation function in light heavy ion collisions. It will be an interesting subject to study whether they are examples of the resonances, which are produced around the barrier region due to the coupling to internal degrees of freedom.

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Appendix A - Induced force in a classically accessible process

The force induced by the linear coupling to N independent harmonic oscillators is given by,[9)]

$$F(t) = \frac{2}{\hbar} \sum_{j=1}^{N} \alpha_j^2 \left(\frac{df}{dR}\right)_t \int_0^t dt_1\, f(R(t_1)) \sin \omega_j (t-t_1), \quad (A.1)$$

where α_j is to be specified later.

A.1 Coupling to a damped harmonic oscillator

The α_j in Eq. (A.1) should be replaced by $\alpha_0 \chi_j$ for the model, which has been discussed in Sect. 5.1. We then replace the sum over j with the integral over the frequency ω by introducing the strength distribution given by Eq. (5.6). If the width Γ is sufficiently large, then one can approximate $F(t)$ by

$$F(t) = F^{ind}(t) - \gamma(t)\dot{R}(t), \quad (A.2)$$

where

$$F^{ind}(t) = \alpha_0^2 \frac{1}{\hbar} \frac{\omega_0}{\omega_0^2 + (\frac{\Gamma}{2})^2} \frac{d}{dR(t)} [f^2(R(t))] \quad (A.3)$$

and

$$\gamma(t) = \alpha_0^2 \frac{2}{\hbar} \frac{\Gamma\omega_0}{[\omega_0^2 + (\frac{\Gamma}{2})^2]^2} \left[\frac{df(R(t))}{dR(t)}\right]^2. \quad (A.4)$$

The coefficient in front of the squared derivative of the form factor in $\gamma(t)$ is nothing but the slope of the Lorentzian strength distribution $J(\omega)$ at $\omega=0$.

A.2 Caldeira-Leggett model

For the model considered in Sect. 5.2, we replace α_j^2 in Eq. (A.1) by $\frac{\hbar}{2m_j\omega_j}$, and first perform the partial integration to obtain,[24)]

$$F(t) = F^{ind}(t) + F^{(D)}(t), \quad (A.5)$$

where $$F^{ind}(t) = \sum_j \frac{1}{m_j\omega_j^2} f(R(t)) \left(\frac{df}{dR}\right)_t \tag{A.6}$$

and $$F^{(D)}(t) = -\left(\frac{df}{dR}\right)_t \int_0^t dt_1\, \dot{R}(t_1)\left(\frac{df}{dR}\right)_{t_1} \sum_j \frac{1}{m_j\omega_j^2} \cos\omega_j(t-t_1) \tag{A.7}$$

We have discarded the term proportional to $f(R(0))$. The $F^{ind}(t)$ is nothing but the induced conservative force corresponding to the potential renormalization given by Eq. (5.25). We now introduce the same trick as Eq. (5.19) to introduce the distribution function in order to rewrite $F^{(D)}(t)$. This leads to,

$$\sum_j \frac{1}{m_j\omega_j^2} \cos\omega_j(t-t_1) \rightarrow \frac{2}{c^2}\,\eta\,\frac{1}{\pi}\,\frac{\frac{1}{\omega_c}}{(\frac{1}{\omega_c})^2+(t-t_1)^2} \tag{A.8}$$

Therefore, in the limit of large ω_c,

$$F^{(D)}(t) \approx -\frac{\eta}{c^2}\left[\left(\frac{df}{dR}\right)_t\right]^2 \dot{R}(t). \tag{A.9}$$

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