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Abstract

Using a semi-classical theory, we show that the coupling to a low lying vibrational excitation yields a potential renormalization for the inclusive heavy ion fusion process, whose energy dependence is consistent with a phenomenological discovery. Also, the problem of double counting is clarified by comparing the potential renormalization with that for elastic scattering.

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The enhancement of the sub-barrier fusion cross-sections for intermediate-mass systems over the predictions of one-dimensional potential models[1] has been attributed to the internal vibrational excitations or transfer reactions. In this connection, some recent works have pointed out the importance of potential renormalization[2-8], which is also indicated by phenomenological analyses of data[9-12]. In the present work we study the energy dependence of the potential renormalization in a semi-classical framework. We also compare the potential renormalization for inclusive fusion process with that for elastic scattering. This clarifies the problem of double counting[13], since part of the potential renormalization due to internal excitations would already be included in the one dimensional "bare" potential which is taken to be the real part of a phenomenological optical potential for elastic scattering. Finally, we comment on the relevance of the counter-term prescription, which has been introduced to cancel out the potential renormalization in the SQUID problem[14].

We calculate the inclusive transmission probability for the system whose total Hamiltonian is given by

$$H = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial R^2} + U(R) + \hat{H}_0(q) + \hat{H}_{int}(q,R), \qquad (1)$$

where R is the distance between colliding nuclei, and $H_0(q)$ is the Hamiltonian representing the internal degree of freedom. We particularly consider the case where the relative motion linearly couples to an internal harmonic oscillator:

$$\hat{H}_0 + \hat{H}_{int} = (a^+a + \frac{1}{2})\hbar\omega_0 + \alpha_0 f(R)(a + a^+), \qquad (2)$$

where α_0 is the amplitude of the zero-point motion. When the oscillator represents a high-lying vibrational excitation of a nucleus, aside from a multiplicative, almost energy-independent factor which is nearly equal to one, we obtain the transmission probability as[8]

$$P(E) \approx P_{ad}(E) = P_0\left(E - \frac{1}{2}\hbar\omega_0,\ U(R) - \frac{\alpha_0^2}{\hbar\omega_0} f^2(R)\right), \quad (3)$$

where $P_0(E, U(R))$ is the transmission probability through a one-dimensional potential barrier $U(R)$. Hence, the effect of a high-lying vibrational excitation can be incorporated into a one-dimensional potential model if one replaces the bare potential $U(R)$ by the adiabatic potential $U_{ad}^{eff}(R)$ given by

$$U_{ad}^{eff}(R) = U(R) - \frac{\alpha_0^2}{\hbar\omega_0} f^2(R). \quad (4)$$

Clearly, the corresponding potential renormalization is independent of the incident energy.

Previous studies[6,15] have shown, however, that low-lying vibrational excitations play a more important role in the enhancement of the sub-barrier fusion cross-section. Therefore, as a limiting case we study the effect of a degenerate harmonic oscillator, i.e. of a harmonic oscillator whose oscillator frequency is zero. The corresponding inclusive transmission probability is given by the so-called zero point motion formula[2,5,8]:

$$P(E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx\, e^{-x^2/2}\, P_0\left(E,\ U(R) + x\alpha_0 f(R)\right). \quad (5)$$

In order to see the potential renormalization more clearly, we derive a semi-classical approximation to Eq. (5) when the incident energy is well below the barrier maximum. In this case, the saddle point approximations[16] to the path integral representation of $P_0(E, U(R))$ gives

$$P_0(E, U(R)) = \exp [+ \frac{2}{\hbar} (ET_0 + S_0)], \qquad (6)$$

where $-iT_0$ is the transmission time for the tunneling process and $-iS_0$ is the classical action for the potential $U(R)$. Inserting Eq. (6) into Eq. (5) we get

$$P(E_{cm}) \approx e^{-2 \int_0^{T_0} [\frac{\mu}{2} \dot{R}^2 + U(R) - E_{cm}] d\tau/\hbar} \; e^{2 [\alpha_0 \int_0^{T_0} f(R(\tau))d\tau/\hbar]^2} . \qquad (7)$$

Eq. (7) agrees with the result in Refs. 4 and 6, where $P(E)$ has been obtained by studying the imaginary time propagator. The quality of the semi-classical formula Eq. (7) has also been discussed in Refs. 17 and 18. Eq. (7) indicates that the renormalized effective potential, $\tilde{U}^{eff}_{\omega=0}(R)$ is given as

$$\tilde{U}^{eff}_{\omega=0} (R(\tau)) = U(R(\tau)) - \frac{2\alpha_0^2}{\hbar} f(R(\tau)) \int_0^{\tau} f(R(\tau_1))d\tau_1 . \qquad (8)$$

On the other hand, the potential that appears in the classical equation of motion to determine the dominant tunneling path[4,6] is

$$U^{eff}_{\omega=0}(R) = U(R) - \frac{2\alpha_0^2}{\hbar} f(R) \int_0^{T_0} d\tau_1 \; f(R(\tau_1)), \qquad (9)$$

Fig. 1 compares the bare potential, the effective potential for an adiabatic coupling ($\hbar\omega_0$=20 MeV), and the effective potential for the coupling to a degenerate harmonic oscillator, given by Eq. (9), for two different energies. The parameters to specify U(R), f(R) and α_0 are given in Ref. 6. The figure shows that the potential renormalization due to the excitation of a degenerate harmonic oscillator, which has been introduced to mimic low-lying vibrational excitations of nuclei, becomes more significant as the incident energy decreases. This offers an explanation of the *dynamical effect* discovered phenomenologically in Ref. 10. Also, this is consistent with the conclusion in Ref. 11 based on an inversion process. Namely, the assumption of an *energy-independent* local effective potential is inadequate in describing sub-barrier fusion cross section.

We now discuss the problem of double counting. The influence potential method is useful in microscopic formulation of the optical potential[19]. We assume that the ground state matrix element of the Green's function takes the following standard form:

$$\mathcal{W}_{00}[R(t);\, t,0] = \langle 0|\hat{U}\,(R(t);\, t,0)|0\rangle = e^{-\frac{i}{\hbar}\int_0^t W_e(t_1)dt_1}, \qquad (10)$$

where $\hat{U}$ satisfies the equation

$$i\hbar\,\frac{\partial\hat{U}}{\partial t} = [\hat{H}_0 + \hat{H}_{int}\,(\hat{q},\, R(t))]\hat{U}, \qquad (11)$$

subject to the initial condition $\hat{U}(t=0) = 1$. The optical potential is then given by

$$U_{opt}\,(R(t)) = U(R(t)) + W_e\,(t). \qquad (12)$$

When the internal Hamiltonian is given by Eq. (2), the influence potential $W_e(t)$ is given by

$$W_e(\tau) = \frac{1}{2}\hbar\omega_0 - \frac{\alpha_0^2}{\hbar} f(R(\tau))\, e^{-\omega_0\tau}\int_0^\tau d\tau_1\, e^{\omega_0\tau_1} f(R(\tau_1)). \qquad (13)$$

In obtaining Eq. (13), we have considered a classically forbidden process, and studied the evolution of the internal system along the negative imaginary time axis by setting $t = -i\tau$. Remark that, in this instanton approximation, the internal vibrational excitation does not yield any imaginary part to the optical potential during the classically forbidden process. The effect of internal vibrational excitations on the inclusive fusion process can also be represented in terms of the influence potential[4,6]. If we denote it as $W_f(\tau)$, then

$$W_f(\tau) = \frac{1}{2}\hbar\omega_0 - \frac{\alpha_0^2}{\hbar}\,[1 + e^{-2\omega_0(T_0-\tau)}]f(R(\tau_1))\, e^{-\omega_0\tau}\int_0^\tau d\tau_1\, e^{\omega_0\tau_1} f(R(\tau_1)). \qquad (14)$$

The comparison between Eqs. (13) and (14) suggests that one should replace the full influence potential W_f by the superfluous influence potential $(W_f - W_e)$ in the influence functional formalism of the inclusive fusion process if the real part of a phenomenological optical potential is used as the "bare" potential barrier. Note that the part of the superfluous influence potential which is associated with a high lying vibrational excitation is negligible. It is, of course, another question whether the data of elastic scattering can give any information of the optical potential at the barrier region at all.

Finally, we wish to comment on the *counter-term* prescription in Ref. 14. The potential renormalization in Eq. (4) is nothing but minus times the counter-term Hamiltonian in Ref. 14. However, the potential renormalization due to the excitation of a degenerate harmonic oscillator is very different from that in Eq. (4). Therefore, the counter-term prescription in Ref. 14 would be adequate at most concerning the effect of high lying vibrational excitations.

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References

[1] L.C. Vaz, J.M. Alexander and G.R. Satchler, Phys. Lett. C69 (1981) 393; and references therein.

[2] H. Esbensen, Nucl. Phys. A352 (1981) 147.

[3] W. Reisdorf et al., Phys. Rev. Lett. 49 (1982) 1811.

[4] S.Y. Lee and N. Takigawa, Phys. Rev. C28 (1983) 1123.

[5] P.M. Jacobs and U. Smilansky, Phys. Lett. 127B (1983) 313.

[6] N. Takigawa and G.F. Bertsch, Phys. Rev. C in press.

[7] N. Takigawa, Proceedings of the Seventh Oaxtepec Symposiuim on Nuclear Physics, Mexico, January 1984.

[8] A.B. Balantekin and N. Takigawa, MIT Report CTP-1161, to be published.

[9] R. Pengo et al., Nucl. Phys. A411 (1983) 255.

[10] L.C. Vaz, J.M. Alexander, M. Prakash and S.Y. Lee, Proc. Int. Conf. on Nuclear Physics with Heavy Ions, Stony Brook, N.Y., April 1983, ed. P. Braun-Munzinger; Vol. VI Nucl. Sci. Research Conf. Ser. (Harwood Academic Publishers, Amsterdam, 1984).

[11] A.B. Balantekin, S.E. Koonin and J.W. Negele, Phys. Rev. C28 (1983) 1565.

[12] H.R. Jaqaman, Phys. Rev. C29 (1984) 335.

[13] M.J. Rhoades-Brown and M. Prakash, SUNY, Stony Brook preprint (1984).

[14] A.O. Caldeira and A.J. Leggett, Ann. Phys. (NY) 149 (1983) 374; and references therein.

[15] H. Esbensen, J.Q. Wu and G.F. Bertsch, Nucl. Pnys. A411 (1983) 275.

[16] D.M. Brink and U. Smilansky, Nucl. Phys. A 405 (1983) 301.

[17] S. Landowne, Phys. Rev. C in press.

[18] V.P. Aleshin and A. Wintner, Niels Bohr Institute, unpublished.

[19] D.M. Brink, Lecture Notes in Physics, 89 (1978) 340.

Figure Caption

Fig. 1. The bare potential (thick solid line), the adiabatic potential (dashed line) and two of the effective potential for the coupling to a degenerate harmonic oscillator are compared.

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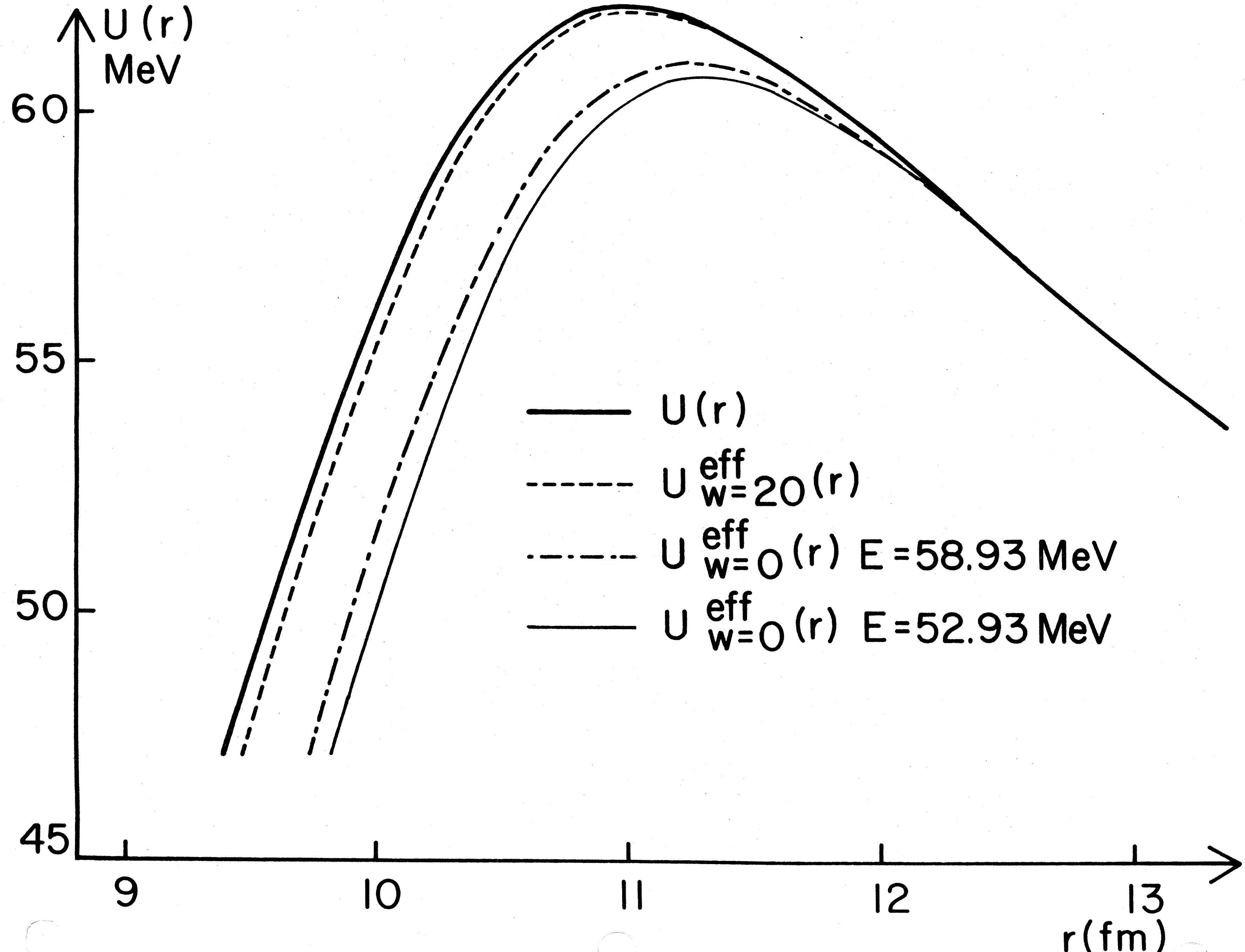

U (r) MeV
60
55
50
45
9
10
11
12
13
r (fm)
U(r)
U eff w=20 (r)
U eff w=0 (r) E=58.93 MeV
U eff w=0 (r) E=52.93 MeV