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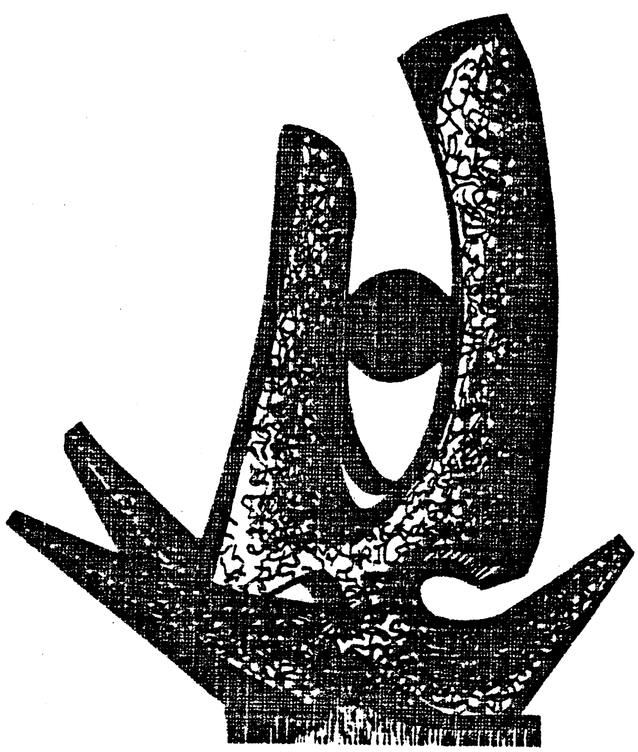
H. SAGAWA and B.A. BROWN



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Coulomb Form Factors of Collective E4 Transitions in S-D Shell Nuclei

H. Sagawa and B. A. Brown

National Superconducting Cycrotron Laboratory
Michigan State University
East Lansing, MI 48824-1321, USA

Abstract

Coulomb form factors of E4 transitions in the s-d shell nuclei are discussed taking into account core-polarization effects due to hexadecupole giant resonances. The calculation has been performed within a framework of microscopic theory and gives remarkably good agreement with experimental form factors both in the absolute strength and the q-dependence.

Recent high-resolution electron scattering data make it possible to obtain model independent transition densities and current densities in many nuclei over a broad region of the mass table.[1] These data provide precise and interesting information which can be used to test the validities of various nuclear models.

A microscopic model has recently been proposed by the present authors[2] in order to study the core-polarization effect on the transition and current densities of single-particle configurations. This model is essentially made from two parts. At the first stage, we calculate the single-particle wave functions and giant resonances by using the self-consistent Hartree-Fock (H-F) + random phase approximation (RPA) theory. Then, we evaluate the core-polarization effect due to these giant resonances by the particle-vibration coupling model. Our model gives quite satisfactory results for describing the E2 core-polarization charges in the s-d shell nuclei in comparison with the empirical ones. Moreover, the coulomb E2 form factor of the $(1d_{3/2}^{-1} \rightarrow 2s_{1/2}^{-1})_{\pi}$ transition in ^{39}K is well reproduced by our calculation in both the q-dependence and the absolute cross section.[2]

Higher multipole transitions might also give interesting and important information about the nuclear wave functions since they are more sensitive to the radial profiles of the wave functions than the lower multipole transitions, especially at the surface region.[3] So far, the E4 transition strengths and the coulomb form factors have been discussed by using s-d shell-model wave functions with phenomenological effective charges[4] and also by using the deformed H-F wave functions.[5,6] We will discuss in this letter the core-polarization effect on the coulomb E4 form factor in the s-d shell nuclei based on a hybrid microscopic theory which combines shell-model wave functions and highly-excited giant resonances by using the perturbation theory.

The reduced one-body matrix element for shell-model wave functions can be expressed as a linear combination of the single-particle matrix elements;

$$\langle J_f || \hat{T}_\lambda || J_i \rangle = \sum_{\alpha,\beta} C_{J_f,J_i}(\alpha,\beta) \langle \alpha || \hat{T}_\lambda || \beta \rangle \tag{1}$$

where J_f and J_i stand for the shell-model states and $C_{J_f,J_i}(\alpha,\beta)$ are the structure factors (one-body transition densities). The particle-vibration coupling model[2,7] gives the modified single-particle matrix element,

$$\langle \tilde{\alpha} || \hat{T}_\lambda || \tilde{\beta} \rangle = \langle \alpha || \hat{T}_\lambda || \beta \rangle + \sum_{\omega_\lambda} \left[2\omega_\lambda / (\varepsilon^2_{\alpha\beta} - \omega^2_\lambda) \right]$$
$$* \langle (\beta \times \omega_\lambda)\alpha | V_{ph} | \beta \rangle \langle \omega_\lambda || \hat{T}_\lambda || 0 \rangle / (2\lambda+1)^{1/2} \tag{2}$$

where ω_λ and $\varepsilon_{\alpha\beta}$ are the excitation energy of giant resonance and single particle energy difference, respectively. The particle-vibration coupling Hamiltonian V_{ph} is derived from the Skyrme-type interaction by replacing the velocity dependent terms by a Fermi-momentum dependent δ-interaction.[2] The modified transition matrix element for the shell-model wave function is now given by inserting $\langle \tilde{\alpha} || \hat{T} || \tilde{\beta} \rangle$ in Eq. (1);

$$\langle \tilde{J}_f || \hat{T}_\lambda || \tilde{J}_i \rangle = \sum_{\alpha,\beta} C_{J_f,J_i}(\alpha,\beta) \langle \tilde{\alpha} || \hat{T}_\lambda || \tilde{\beta} \rangle \tag{3}$$

This effect can be regarded as a polarization of the core protons by the valence protons and neutrons through the proton-proton and proton-neutron two-body interaction. The proton and neutron core-polarization charges are defined by,

$$\delta e_p = 1 - [\langle\tilde{\alpha}||\hat{T}_\lambda||\tilde{\beta}\rangle_\pi / \langle\alpha||\hat{T}_\lambda||\beta\rangle_\pi] \qquad (4)$$

$$\delta e_n = \langle\tilde{\alpha}||\hat{T}_\lambda||\tilde{\beta}\rangle_\nu / \langle\alpha||\hat{T}_\lambda||\beta\rangle_\pi$$

We performed the self-consistent H-F + RPA calculations assuming a $(s^4p^{12}(d_{5/2})^{12})$-core using the Skyrme-interaction SGII.[8] This interaction gives the H-F rms charge radius $\sqrt{\langle r^2\rangle_c}$ (H-F) = 3.107 fm for ^{28}Si which is quite close to the experimental value $\sqrt{\langle r^2\rangle_c}$ (exp.)[9] = 3.125 fm. The single-particle B(E4)-values calculated with harmonic oscillator and the H-F wave functions are listed in Table (1) . The oscillator length of the harmonic-oscillator wave functions is taken to be b= 1.819 fm which provides the rms charge radius $\sqrt{\langle r^2\rangle_c}$(HO) = 3.107 fm. While the rms radii of the harmonic-oscillator and H-F ground state wave functions are the same, there is a significant difference (up to 20%) in the B(E4)-values (see Table (1)). (The H-F wave functions give $\langle 1d_{3/2}|r^4|1d_{3/2}\rangle$ (H-F)= 244 fm^4 and $\langle 1d_{5/2}|r^4|1d_{5/2}\rangle$ (H-F) =170 fm^4, while the harmonic oscillator wave function gives $\langle 1d|r^4|1d\rangle$ (HO)= 172 fm^4.)

The RPA responses for the isoscalar(IS) and isovector(IV) hexadecupole operators are shown in Fig. 1 for ^{28}Si-core. We can see few strong resonances in the IS response at around Ex= (20-25) MeV where 23% of the total strength is existing, while the IV response spreads out in a broad energy region Ex=(20-60) MeV without any strong peaks. The IS strength distributions between Ex=(20-56) MeV are divided into seven energy regions for the calculation of the core-polarization effects. The transition strength in this energy region exhausts 83% of the energy-weighted sum rule value. The IV response is divided into six energy regions between Ex = (22-62) MeV where we found 78% of the IV energy-weighted sum rule value with the enhancement factor κ = 0.09. Remaining transition strengths are in the high energy tail above Ex=60 MeV. In each energy region, the radial shape of the transition density has about the same shape. The calculated IS and IV core-polarization charges are given in Table (1). The averaged δe^{IS} and δe^{IV} are 0.45 and 0.15, respectively. Thus, the proton and neutron core polarization charges are given by $\delta e_n=\delta e^{IS}+\delta e^{IV}=0.60$ and $\delta e_p=\delta e^{IS}-\delta e^{IV}=0.30$. The particle-vibration coupling model with the separable interactions[3,7] gives $\delta e^{IS}=0.50$ and $\delta e^{IV}=0.32$ (for a ^{40}Ca-core). The larger IV core-polarization charge stems from the strong IV separable interaction. On the other hand, the E4 polarization charges are about 30% smaller in the first-order perturbation theory calculations using phenomenological Gaussian-type interactions[3] or using the bare G-matrices.[10] This might be due to the fact that our particle-vibration coupling model takes into account the higher-order RPA perturbation terms in the calculations.[11]

The shell-model calculations have been performed in the full s-d shell model space with the empirical two-body matrix elements of Wildenthal.[4] The excitation energies and B(E4)-values of collective states are listed in Table (2). The shell-model wave functions give a quite satisfactory agreement in the excitation energies in comparison with experimental data. Nevertheless, the calculated B(E4)-values are typically several times smaller than the empirical ones. We have calculated the B(E4)-values including core-polarization charges. The strong hexadecupole transitions in

^{24}Mg, ^{27}Al and ^{28}Si are dominated by the isoscalar part and the B(E4)-values are enhanced by the core-polarizaton effects by a factor of 3.5. The agreement between experiment and the shell model calculations with core polarization is remarkably good. In ^{26}Mg, the $0^+ \rightarrow 4_1^+$ transition has some isovector component and the enhancement factor is relatively small. The fourth 4^+ state has a large proton ($1d_{5/2} \rightarrow 1d_{3/2}$) amplitude and hence the enhancement is also smaller for B(E4)-value.

Castel, Zamick and their collaborators[5,6] claim that hexadecupole deformation is necessary to describe the E4 transitions in s-d and p-f shell nuclei. They performed deformed H-F calculations using two sets of Skyrme interactions (SI and SII). For the first 4^+ state in ^{28}Si, their calculation gives a B(E4)-value $1.69*10^4 e^2fm^8$(SII interaction), while our model gives $B(E4) = 2.74*10^4 e^2fm^8$. These similar values suggest that the two methods have a large overlap. Thus, it would be interesting to study the coulomb form factor using the deformed H-F wave function so that we can compare detailed radial dependences of the transition densities in two models. In those cases where the 0^+ and 4^+ states can be well described by a deformed intrinsic state, both our method and the deformed H-F method seem to be nearly equivalent. However, our method can also be applied to non-deformed states as long as they lie within the sd shell configuration space.

We show the transition density and the coulomb form factor for the ($0^+ \rightarrow 4_1^+$) transition in ^{28}Si in Fig. 2. The form factor is increased by a factor of 2.5 at the maximum around $q = 1.4\ fm^{-1}$, however, there is not much increase in the high q-region above $2.0\ fm^{-1}$. This change is attributed to the enhancement of the transition density at the surface region. In Fig. 3, we show the coulomb form factors for other strong E4 transitions in the vicinity of ^{28}Si. The enhancement factor due to the core-polarization around the peak is almost the same (2.5 times) in every case except for the $(0_1^+ \rightarrow 4_1^+)$ transition in ^{26}Mg. The agreement of the calculation with the experimental data is in general remakably good. (The experimental data for the $(5/2^+ \rightarrow 11/2^+)$ transition in ^{27}Al was obtained at 90°. There is some transverse M3 component, but it is negligible compared to the longitudinal part.) The $(0_1^+ \rightarrow 4_1^+)$ transition in ^{26}Mg has a relatively large isovector component and the enhancement factor around the peak is smaller than for the other cases.

In summary, the B(E4)-values and the coulomb form factors of strong hexadecupole transitions in the middle of s-d shell can be described quantitatively by our microscopic model which takes into account highly excited giant resonances by perturbation theory. Our model gives a large IS polarization charge δe^{IS} =0.45 which is important for reproducing experimental B(E4)-values.

Acknowledgements

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Table (1) IS and IV core polarization charges for E4 transitions in s-d shell configurations. The single particle E4 transition strength is defined by $B(E4)_{s.p.} = |\langle \alpha || \hat{T}_{\lambda=4} || \beta \rangle / (2j_\beta+1)$ with $e(\pi) = e(\nu) = 1$.

	α	β	$B(E4)_{s.p.}$ (e^2fm^8) HO(*10³)	H-F(*10³)	δe IS	IV
ν	$1d_{5/2}$	$1d_{5/2}$	(2.03)	(1.70)	0.456	0.151
	$1d_{3/2}$	$1d_{5/2}$	(4.06)	(4.23)	0.405	0.133
π	$1d_{5/2}$	$1d_{5/2}$	2.03	1.94	0.465	0.153
	$1d_{3/2}$	$1d_{5/2}$	4.06	5.03	0.412	0.133

Table (2) Excitation energies and B(E4)-values for E4 transitions in s-d shell nuclei. The calculated B(E4)-values are obtained by the s-d shell model wave functions with and without core-polarization effects. The experimental B(E4)-values are taken from ref. 9 (^{24}Mg), ref. 17 (^{26}Mg), ref. 16 (^{27}Al) and ref. 13 (^{28}Si).

		Energy		B(E4) (e^2 fm^8)		
nucleus	#	theory	exp.	SM(*10³)	SM+CP(*10³)	exp.(*10³)
^{24}Mg	1	4.38	4.12	0.032	0.159	(2.0 ± 0.3)
	2	5.93	6.01	12.0	40.4	(43 ± 6)
^{26}Mg	1	4.53	4.32	3.14	9.21	(4.1 ± 1.6)
	2	4.93	4.90	4.24	19.3	(15.6 ± 3.4)
	3	5.47	5.47	0.54	0.02	(0.89 ± 0.34)
	4	6.01	5.72	4.48	9.69	(6.4 ± 1.8)
^{27}Al		4.58	4.51	2.59	8.47	
^{28}Si	1	4.66	4.62	8.30	27.4	(27 ± 5)

Figure Captions

Fig. 1 - The RPA strength distributions for the IS and IV hexadecupole operators $\hat{T}^{IS}_{\lambda=4} = [1/2]\ \Sigma_i r_i^4 Y_{4m}(\hat{r}_i)$ and $\hat{T}^{IV}_{\lambda=4} = [1/2]\ \Sigma_i r_i^4 Y_{4m}(\hat{r}_i)\ \tau_{zi}$. The solid curve shows the IS response, while the dashed one corresponds to the IV response.

Fig. 2 - Transition densities and coulomb form factors for the $(0^+ \rightarrow 4_1^+)$ transition in ^{28}Si. The center of mass correction is taken into account in the harmonic oscillator model.[12] and the nucleon finite size correction is incorporated in the dipole approximation.[13] The solid and dashed curves correspond to the results with and without core-polarization effects, respectively. The data are taken from ref. 14.

Fig. 3 - Coulomb E4 form factors for the $(0^+ \rightarrow 4_2^+)$ transition in ^{24}Mg, $(5/2^+ \rightarrow 11/2^+)$ transition in ^{27}Al and $(0^+ \rightarrow 4_1^+)$ and $(0^+ \rightarrow 4_2^+)$ transitions in ^{26}Mg. The solid and dashed curves correspond to the results with and without core-polarization effects, respectively. The data are taken from: ref 15 (triangles), ref. 9 (circles) and ref. 9 (squares) for ^{24}Mg, ref. 16 for ^{27}Al and ref. 17 for ^{26}Mg.

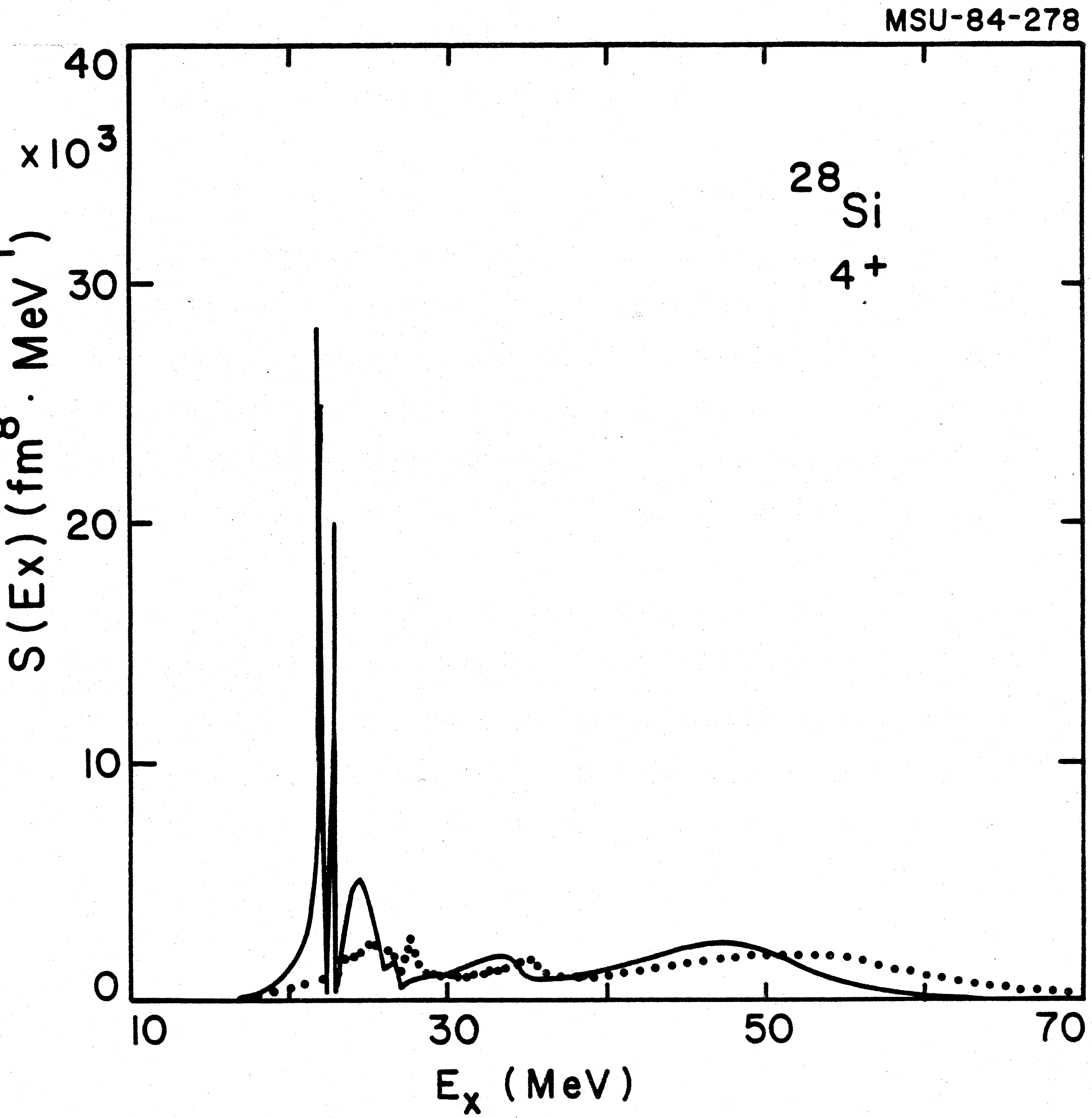

FIGURE 1

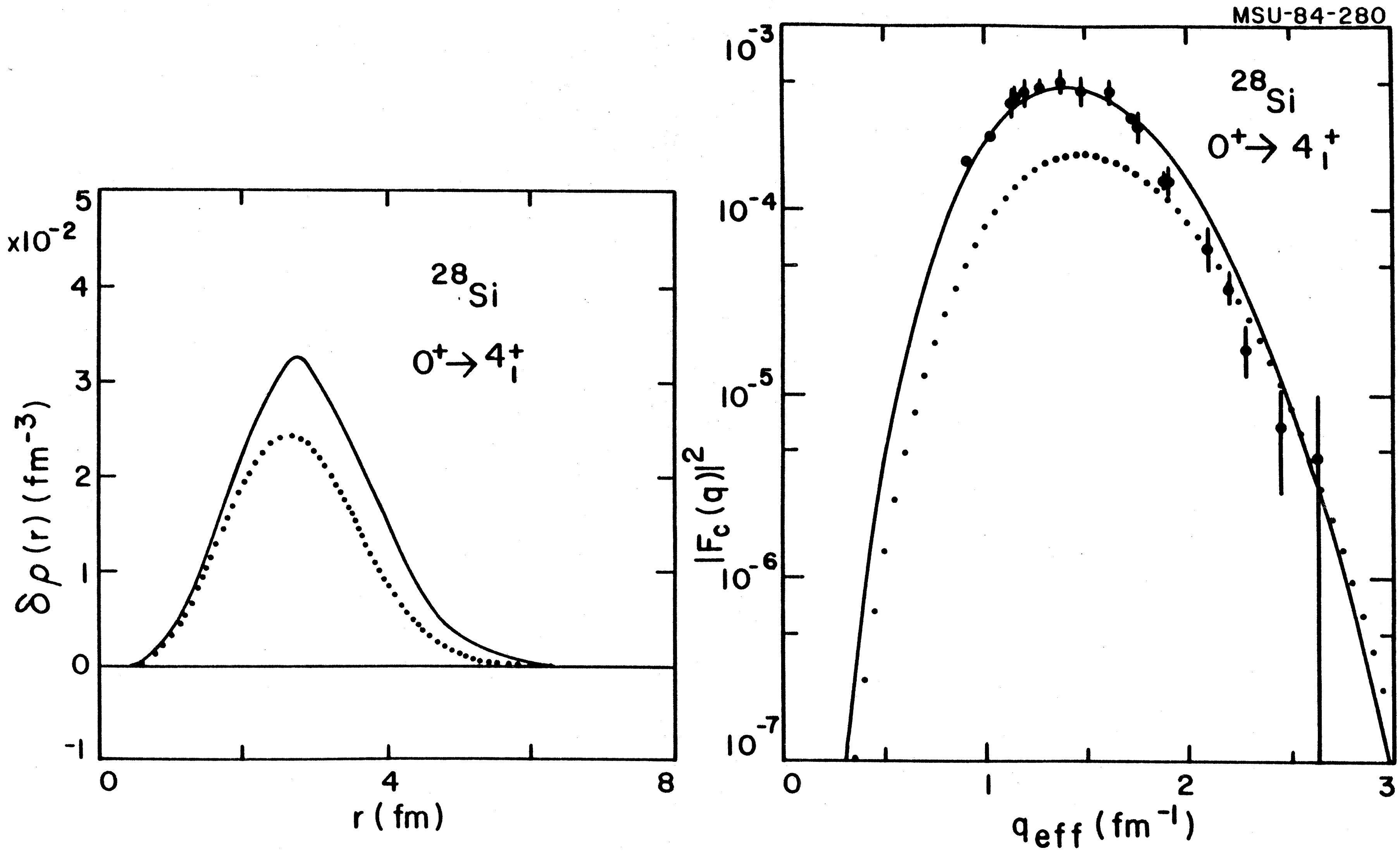
MSU-84-280
^{28}Si
$0^+ \rightarrow 4_1^+$
^{28}Si
$0^+ \rightarrow 4_1^+$
x10^{-2}
$\delta\rho(r)$ (fm^{-3})
r (fm)
$|F_c(q)|^2$
q_{eff} (fm^{-1})

FIGURE 2

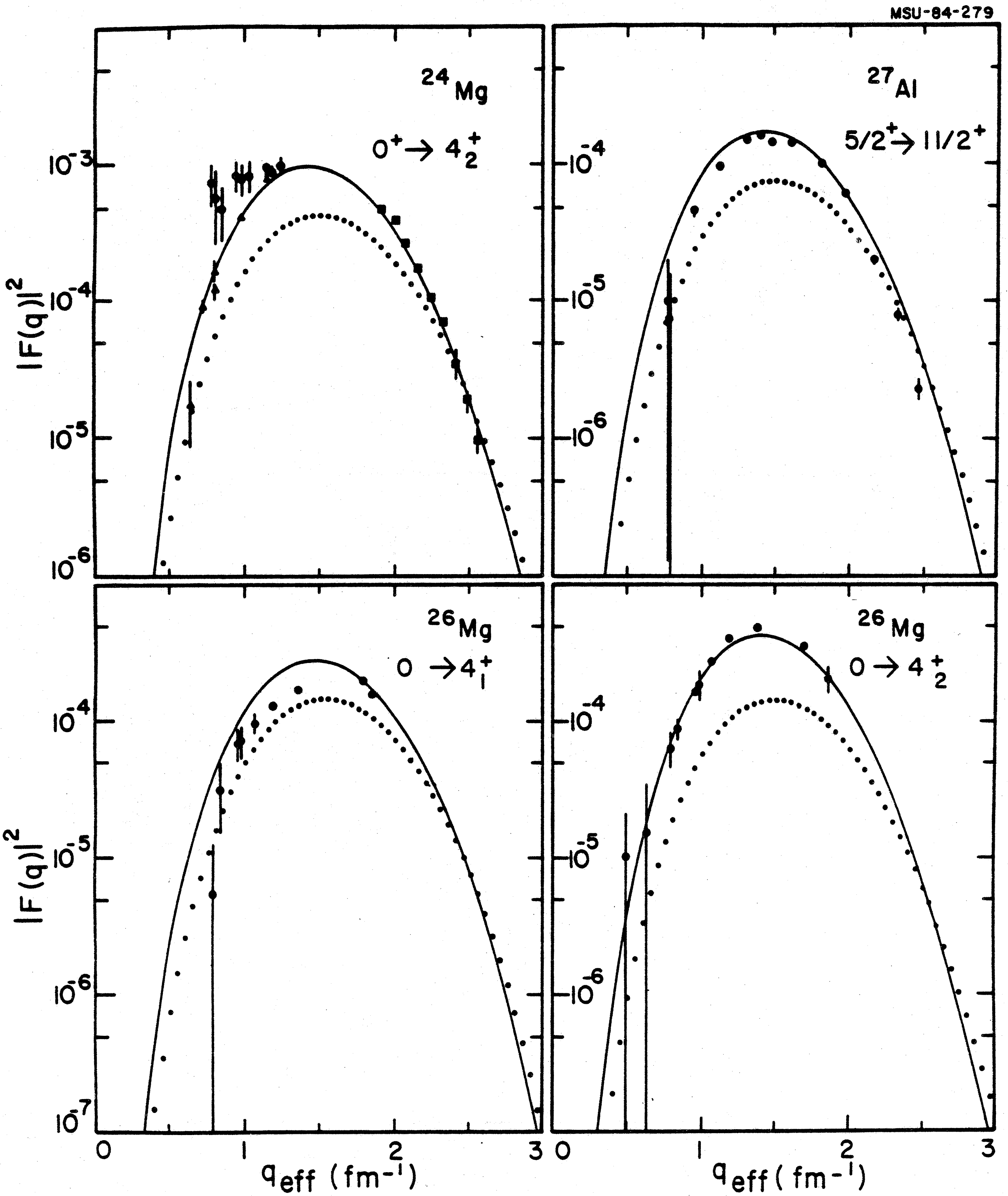
MSU-84-279
^{24}Mg
$0^+ \to 4_2^+$
^{27}Al
$5/2^+ \to 11/2^+$
^{26}Mg
$0 \to 4_1^+$
^{26}Mg
$0 \to 4_2^+$
$|F(q)|^2$
$|F(q)|^2$
q_{eff} (fm^{-1})
q_{eff} (fm^{-1})
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
1
2
3

FIGURE 3