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NEUTRON STARS AND THE ELECTRON MASS AT FINITE CHEMICAL POTENTIAL

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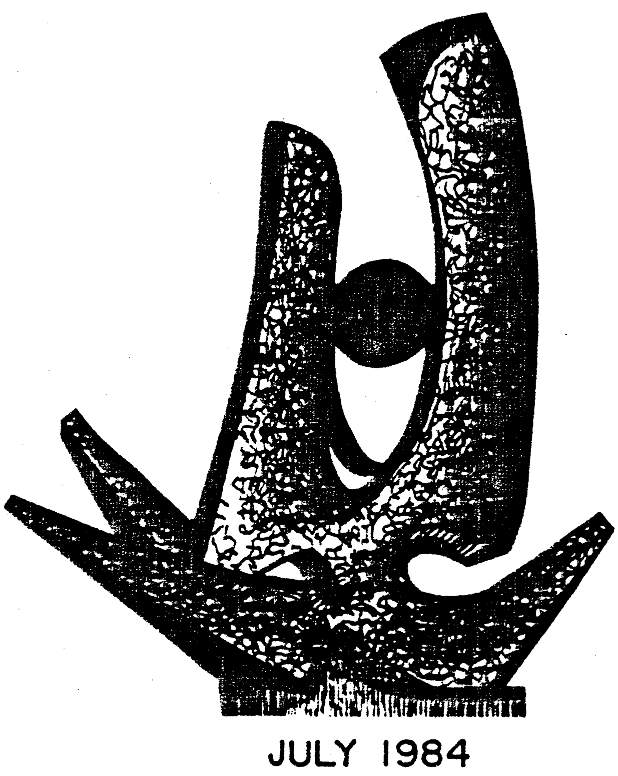

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Neutron Stars and the Electron Mass at Finite Chemical Potential

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Abstract

The shift in the pole of a massless fermion's propagator arising from its electromagnetic self-interaction is calculated at the one loop level for large temperature and/or chemical potential. As an application, the shift of the electron's mass in a neutron star is considered. It is shown that, at neutron star densities, the shift is large compared to the free particle electron mass, but small compared to the electron chemical potential.

I. Introduction

Although the formalism for performing calculations in QED at finite temperature and chemical potential has existed for some time[1,2], only recently has its application to cosmology been investigated in detail[3,4]. In particular, the dependence of the helium abundance produced in the Big Bang model of the Early Universe on quantities such as the electron mass shift at finite temperature (and zero chemical potential) has been studied, and shown[4] to be small. We calculate here the dispersion relation for a massless fermion at large temperature (T) and/or chemical potential (μ) and then apply the results to the electron mass shift, including the T,μ range for a neutron star[5]. We find that specific properties of neutron stars may be changed in a small way, similar to that found for the helium abundance in the early universe.

The real time formalism[6,7] of finite temperature relativistic field theory will be chosen for the calculations presented here. While this formalism has usually been considered unreliable beyond the one loop level [see Appendix B of Ref. 6], it has recently been extended[8] to allow calculations to all orders of perturbation theory. Although there are differences between the formalism expressed in Ref. 6 and 7 and that of Ref. 8, they can be shown to be identical for the calculation of interest here, namely the real part of the self energy to one loop. In the finite temperature, zero chemical potential case, two calculations for the mass shift of a spin 1/2 particle have been performed: Weldon[9] has calculated the shift for a massless fermion at high temperature, while Peresutti and Skagerstam[3] have performed a similar calculation for massive fermions.The two approaches make different assumptions which leads to disagreement at high T. A numerical comparison of their results can be found in Ref. 10.

The following section contains a one loop calculation of the dispersion relation for a massless fermion in the presence of a background characterized by a finite μ and T, and contains a discussion of the effective mass that this dispersion relation implies. [The notation of Ref. 9 is used to facilitate comparison]. In Section III, a simplified neutron star model will be used to obtain the relevant chemical potentials. At these chemical potentials, it will be shown that the electron mass shift will be large compared to its free particle rest mass, but small compared to the chemical potential. The conclusions are summarized in Section IV.

II. Effective Mass at Large T,μ

For a massless fermion at finite temperature and chemical potential, the self energy calculated in Fig. 1 will be of the form (the gamma matrix notation of Ref. 11 is used):

$$\Sigma\,(K) = -a\not{K} - b\not{u} \tag{1}$$

where u is the 4-velocity of the heat bath normalized via $u^{\alpha}u_{\alpha} = 1$. The quantities a and b are Lorentz invariant functions which can depend on two Lorentz scalars

$$\omega = K^{\alpha}\,u_{\alpha} \tag{2}$$

and

$$k = [(K^{\alpha}u_{\alpha})^2 - K^2]^{1/2} \tag{3}$$

such that

$$K^2 = \omega^2 - k^2 \tag{4}$$

The full fermion propagator will then exhibit a pole when

$$(1 + a)^2\,K^2 + 2(1 + a)bK\cdot u + b^2 = 0 \tag{5}$$

Using the real time formalism, the self energy is

$$\Sigma(K) = -ie^2 \int \frac{d^4P}{(2\pi)^4}\;\gamma^{\mu}\,D_{\mu\nu}(-P)S(K+P)\gamma^{\nu} \tag{6}$$

where

$$D_{\mu\nu}(P) = -g_{\mu\nu}\left[\frac{1}{P^2+i\eta} - i\Gamma_b(P)\right]$$

$$S(P) = \not{P}\left[\frac{1}{P^2+i\eta} + i\Gamma_f(P)\right]$$

and

$$\Gamma_b(P) = 2\pi\delta(P^2)n_b(P)$$

$$\Gamma_f(P) = 2\pi\delta(P^2)\left[\theta(P_0)n_f^{+}(P,\mu)+\theta(-P_0)n_f^{-}(P,\mu)\right]$$

The n's are the distribution functions

$$n_b(P) = \{\exp(\beta|P\cdot u|)-1\}^{-1}$$

$$n_f^{\pm}(P) = \{\exp[\beta(|P\cdot u|\pm\mu)]+1\}^{-1}$$

with β the inverse temperature. Ignoring the T=μ=0 piece in Eq. (6) leaves

$$\Sigma'(\kappa) = -ie^2 \int \frac{d^4P}{(2\pi)^4} \gamma_\nu (\not{P}+\not{K})\gamma^\nu \{\frac{i\Gamma_f(P+K)}{P^2 + i\eta} - \frac{i\Gamma_b(-P)}{(P+K)^2+i\eta} - \Gamma_b(-P)\Gamma_f(P+K)\} \quad (7)$$

For the remainder of this work, we will concern ourselves with only the first two terms of Eq. (7), i.e. real part of Σ'(K). The importance and interpretation of the remaining imaginary term have recently been investigated[12,13]) but are not important here. The real part of Σ' is thus

$$\mathrm{Re}\,\Sigma' = \frac{2e^2}{(2\pi)^3}\int d^4P\,\{\frac{\not{P}+\not{K}}{(P+K)^2}\,\delta(P^2)n_b(P)$$

$$-\frac{\not{P}+\not{K}}{P^2}\,\delta([P+K]^2)[\theta(P_0+K_0)n_f^+(P+K) + \theta(-P_0-K_0)n_f^-(P+K)] \quad (8)$$

Two equations are needed to solve for the functions a and b in Eq. (1), which can be obtained from

$$1/4\ \mathrm{Tr}(\not{K}\ \mathrm{Re}\ \Sigma') = -a(\omega^2-k^2)-b\omega, \quad (9)$$

$$1/4\ \mathrm{Tr}(\not{u}\ \mathrm{Re}\ \Sigma') = -a\omega-b. \quad (10)$$

It is more convenient to evaluate Eqs. (9) and (10) rather than (8) directly. The angular integration is performed first, followed by the integral over P_0 of the delta function. After a change of variables in the second term of Eq. (8), one obtains:

$$1/4\ \mathrm{Tr}(\not{K}\ \mathrm{Re}\ \Sigma') = \frac{e^2}{8\pi^2}\int_0^\infty dp\{[2p + \frac{K^2}{2k}(\ln(\frac{\omega_+}{\omega_-}) + \frac{1}{2}[L_1(p)-L_2(p)])]n_f^+(P)$$

$$+\left[2p - \frac{K^2}{2k}\left(\ln\left(\frac{\omega_+}{\omega_-}\right) - \frac{1}{2}[L_1(p)+L_2(p)]\right)\right]n_f^-(P)$$

$$+\left[4p - \frac{K^2}{2k} L_1(p)\right]n_b(P)\Big\} \qquad (11)$$

$$1/4\ \mathrm{Tr}(\not{K}\ \mathrm{Re}\,\Sigma') = \frac{e^2}{8\pi^2 k}\int_0^\infty dp\Big\{p\left[\ln\left(\frac{\omega_+}{\omega_-}\right) - \frac{1}{2}\left(L_1(p)+L_2(p)\right)\right]n_f^+(P)$$

$$+p\left[\ln\left(\frac{\omega_+}{\omega_-}\right) + \frac{1}{2}\left(L_1(p)-L_2(p)\right)\right]n_f^-(P)$$

$$+\left[2p\ln\left(\frac{\omega_+}{\omega_-}\right) - pL_2(p) - \omega L_1(p)\right]n_b(P)\Big\} \qquad (12)$$

where $p \equiv |\vec{P}|$, $\omega_\pm = \frac{1}{2}(\omega \pm k)$ and

$$L_1(p) = \ln\left(\frac{p+\omega_+}{p+\omega_-}\right) - \ln\left(\frac{p-\omega_+}{p-\omega_-}\right)$$

$$L_2(p) = \ln\left(\frac{p-\omega_+}{p-\omega_-}\right) + \ln\left(\frac{p+\omega_+}{p+\omega_-}\right) \qquad (13)$$

The integrals over the logarithmic terms L_1, L_2 and $\frac{K^2}{2k}\ln\left(\frac{\omega_+}{\omega_-}\right)$ will be dropped since we are interested in the large T, μ limit, and these terms involve a smaller power of p than do the other terms. Explicit numerical calculations[10]) indicate that these terms never contribute more than 4% to the dispersion relation and are negligible when considering the mass shift. With this simplification, Eqs. (11) and (12) become

$$1/4\ \mathrm{Tr}(K\ \mathrm{Re}\ \Sigma') = M^2 \qquad (14)$$

$$1/4\ \mathrm{Tr}(\not{K}\ \mathrm{Re}\ \Sigma') = \frac{1}{2k}\ln\left(\frac{\omega_+}{\omega_-}\right) M^2 \qquad (15)$$

where

$$M^2 \equiv \frac{\alpha}{\pi}\int_0^\infty dp\ p\left[\left(n_f^+(P) + n_f^-(P) + 2n_b(P)\right)\right] \qquad (16)$$

and $\alpha \equiv e^2/4\pi$. Evaluating the integral gives

$$M^2 = \frac{\alpha}{2\pi}\,[\mu^2 + \pi^2 T^2] \tag{17}$$

The solutions for the quantities a and b are

$$a = M^2 \;\frac{1}{k^2}\,[1 - \frac{\omega}{2k}\;\ln\,(\frac{\omega_+}{\omega_-})] \tag{18}$$

$$b = M^2 \;\frac{1}{k}\,[-\,\frac{\omega}{k} + \frac{1}{2}\,(\frac{\omega^2}{k^2} - 1)\;\ln\,(\frac{\omega_+}{\omega_-})] \tag{19}$$

These quantities can be substituted back into Eq. (5) to allow ω to be solved in terms of k and M. It is easily shown that as $k \to 0$, $\omega \to M$, so that M appears to be an effective mass at finite temperature and chemical potential. However, to demonstrate that M really does enter as a mass, the relationship between ω and k implied by Eq. (5) must be compared with the on-shell free particle dispersion relation $\omega^2 = k^2 + M^2$. To perform this comparison, the reduced variables $w = \omega/M$ and $\kappa = k/M$ will be introduced. Then Eq. (5) can be rewritten as

$$w - \kappa = \frac{1}{\kappa}\,[1 + (1 - \frac{w}{\kappa})\;\frac{1}{2}\;\ln\,(\frac{w_+}{w_-})] \tag{20}$$

The quantity $(w^2 - \kappa^2)^{1/2}$ is plotted as a function of κ in Fig. 2a. The on-shell dispersion relation reads $(w^2 - \kappa^2)^{1/2} = 1$. In the limit of large κ, $w - \kappa$ goes to zero as $1/\kappa$, so that $w^2 - \kappa^2 \to 2$. Then the momentum dependent effective mass, as defined by $(\omega^2 - k^2)^{1/2}$, will increase from M to $\sqrt{2}$ M as $k \to \infty$, as one can also see from Fig. 2a. Although the effective mass shows deviations of up to 40% as k increases, the ratio of w as obtained from Eq. (20) to $w_{o.s.}\;\; (k^2 + 1)^{1/2}$ will be much closer to unity at $\kappa \to \infty$, since $w/\kappa \to 1$ for either relationship. This is illustrated in Fig. 2b. Hence, the greatest effect on the behavior of the free massless electron's energy caused by introducing it into the medium at finite T,μ will be at low values of k. At large enough values of k, one will always have $\omega \to k$.

The value of M itself has two simple limits. At $\mu = 0$, $T \neq 0$, the quantity M becomes $\sqrt{\pi\alpha/2}$ T, while at the other extreme, namely $T = 0$, $\mu \neq 0$, M becomes $\sqrt{\alpha/2\pi}\;\mu$. The intermediate case of T,μ both non-zero is illustrated in Fig. 3, where contours of constant M are plotted as a function of T and μ.

Before leaving this calculation, mention should be made of the approach to mass shifts used by Peresutti and Skagerstam[3]). This approach was used to calculate the mass shift of a massive fermion (rather than the massless one considered here) but does not include the heat bath term in the propagator. We first check to see whether the heat bath can be neglected in the large T,μ case considered above. As can be seen from Eq. (1) neglect of the heat bath in our calculation would imply that b is small compared to $a\omega$. In the small k limit, Eqs. (18) and (19) give $a\omega/b \to 1/2$, showing that the heat bath does indeed play an important role at large T,μ.

However, at T small compared to the electron mass m_e, one finds numerically[10]) that the general solution approaches that of Ref. 3. For comparison with the mass shift calculated above, we extend the method of Ref. 3 to $T=0$, $\mu\neq 0$. The calculation consists of evaluating Re Σ' from Eq. (8) in the $k=0$ frame. Because $T=0$, the integral over Γ_b vanishes and that over Γ_f simplifies since $n_f^+(P)=0$ and $n_f^-(P)=1$ for $0\leq P_0+K_0\leq\mu$ and zero otherwise. After some algebra, it can be shown that

$$\text{Re}\ \Sigma' = \alpha\frac{1}{\pi}m_e \int_0^{x_0} dx\left[(x^2-1)(x^2+1)^{-1/2}+1\right] \tag{21}$$

where

$$x_0^2 = (\mu/m_e)^2 - 1 \tag{22}$$

For values of μ/m_e large compared to one, Eq. (21) has the approximate solution of

$$\delta m_e/m_e \approx \frac{\alpha}{2\pi}\left(\frac{\mu}{m_e}\right)^2 \tag{23}$$

Comparing the two results for the mass shift, one can see that the mass predicted by Eq. (23) is less than that predicted by Eq. (17) for $\mu \leq \sqrt{2\pi/\alpha}\, m_e$. Of course, for the large values of μ which are relevant in neutron stars (see following section) the heat bath term is expected to be important and so Eq. (17) will be used for the calculation of the mass shift.

III. Neutron Stars

Because the electromagnetic mass shift at finite chemical potential goes like $\sqrt{\alpha/2\pi}\ \mu$, its physical effects are going to be small. To observe such effects, one must either find a physical observable which can be measured very accurately (for example the magnetic moment) such that a small value of μ could yield a detectable result, or find an environment with a very large value for μ. One such possibility would be a neutron star[5]. To determine whether the chemical potentials are large enough to produce an observable effect, we adopt a simplified model of a neutron star and calculate the appropriate electron chemical potential and mass shift. Construction of a detailed model to calculate the changes in neutron star properties arising from the electron mass shift is beyond the scope of this paper.

For the model, a uniform gas of electrons, protons and neutrons will be chosen for the neutron star matter (clearly such a model will not be valid at high densities where coulomb and strong interaction effects will become important). Chemical potentials μ_e, μ_p, and μ_n, which include the mass, are assigned respectively to the electrons, protons and neutrons present with number densities n_e, n_p and n_n. For a Fermi gas, these quantities can be related via $\mu^2 = m^2 + (3\pi^2 n)^{2/3}$. If $\mu_e + \mu_p$ exceeds μ_n, then electrons will be captured by protons until the number densities change such that $\mu_e+\mu_p=\mu_n$. Hence, for each value of n_n, there will be a value of μ_e which satisfies the β-stability condition.

Assuming local electrical neutrality so that $n_e = n_p$, then the chemical potential equality yields

$$\mu_e + (\mu_e^2 + m_p^2 - m_e^2)^{1/2} = ((3\pi^2 n_n)^{2/3} + m_n^2)^{1/2} \qquad (24)$$

The solution of this equation for μ_e as a function of n_n is shown in Fig. 4. At small n_n, $\mu_e - m_n - m_p$, while for large n_n (but not so large that the neutrons are relativistic)

$$\mu_e \approx (3\pi^2 n_n)^{2/3}/2m_n. \qquad (25)$$

Both of these limits are obvious from Fig. 4. This figure includes a region in which μ_e exceeds m_π, although obviously $e^- \rightarrow \pi^- + \nu_e$ would be allowed in this region (see Ref. 14).

From Fig. 4, it can be seen that over much of the neutron density range of interest in the formation of a neutron star, μ_e is large compared to m_e and the large μ expression for M should be reasonably accurate. It is found, then that the mass shift becomes substantial compared to m_e, as is also shown on Fig. 4, but small compared to μ_e ($M/\mu_e \sim 1/30$). The larger electron mass will lead to an earlier onset of electron capture in the formation of the neutron star, and hence an increase in the rate of neutrino emission. Since the neutrino mass is changed only by weak interaction diagrams such as are shown in Fig. 5, its mass shift would be very small indeed and would not compensate for the increased electron mass. Similarly, the abundance of electrons in the neutron star core would be lowered.

IV. Summary

A calculation for the dispersion relation of a massless fermion due to its electromagnetic self interaction has been performed for large T and/or μ. The dispersion relation shows that, in the presence of a medium at finite T and/or μ, the particle behaves as if it has a mass which varies by no more than about 40% as a function of momentum. For comparison, the mass shift for a massive fermion at T=0 and small μ was also calculated. This gave a smaller mass shift at small μ than what would be expected if the large T,μ expression were applied to this region.

In an attempt to find a system which has a large enough chemical potential such that these finite T,μ effects might be observed, the interior of a neutron star was considered. In a simplified model, it was shown that the electron mass shift in a neutron star may be several times its rest mass, and hence the rate of cooling of the star will be changed. However, the mass shift is still small (3.5%) compared to the chemical potential, so one does not expect there to be large changes to any measurable quantities. Other evidence for this conclusion can be found in a calculation of the energy density of a gas of electrons, positrons and photons at finite chemical potential. There, it was shown[15]) that the ratio of the corrections to the energy density to the ideal (free particle) value is of order $\alpha/2\pi$.

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Figure Captions

Fig. 1. One loop contribution to the electron self mass illustrating the momentum labels used in the text.

Fig. 2. a) Effective mass (in units of M) from $w^2-\kappa^2$ shown as a function of κ.

b) Ratio of w calculated from Eq. (20) to $w_{o.s.} = (\kappa^2+1)^{1/2}$, again as function of κ.

Fig. 3. Contours of constant M (in MeV) shown as a function of T,μ.

Fig. 4. Expected value of the electron chemical potential and electron mass shift shown as a function of neutron number density. (Normal nuclear matter density (n_o) is indicated for comparison).

Fig. 5. One loop contribution to the neutrino self energy.

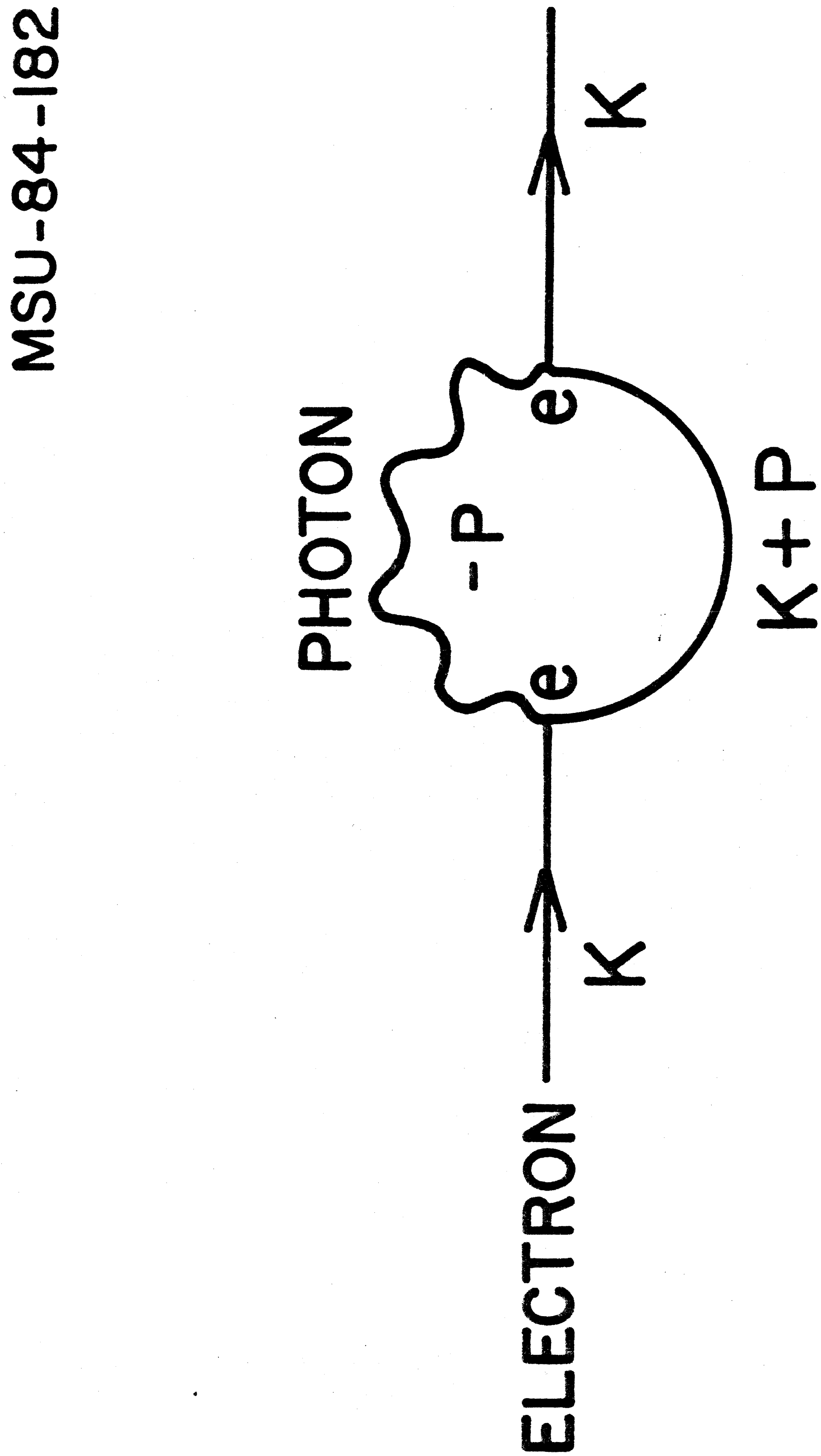
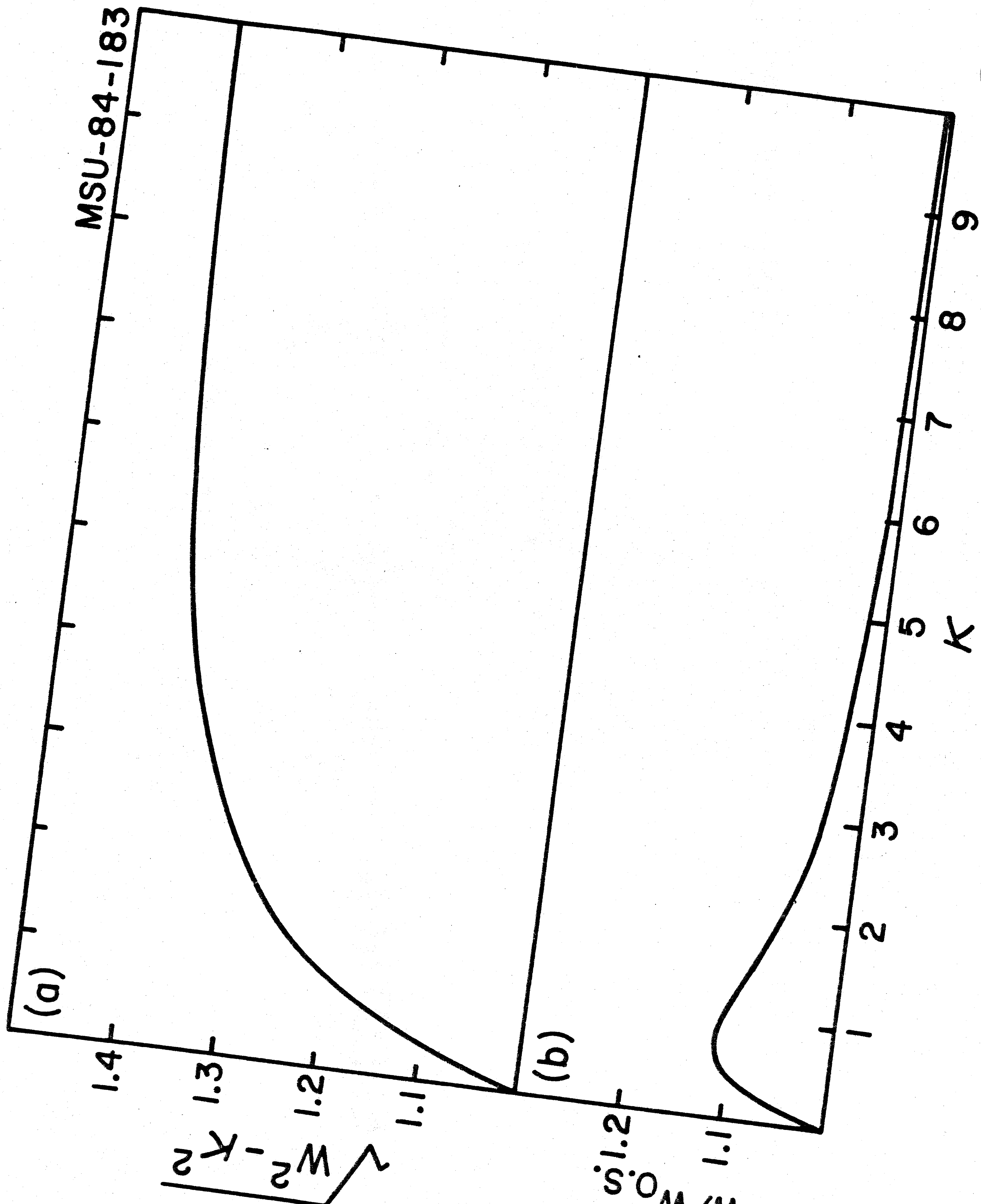


figure 1



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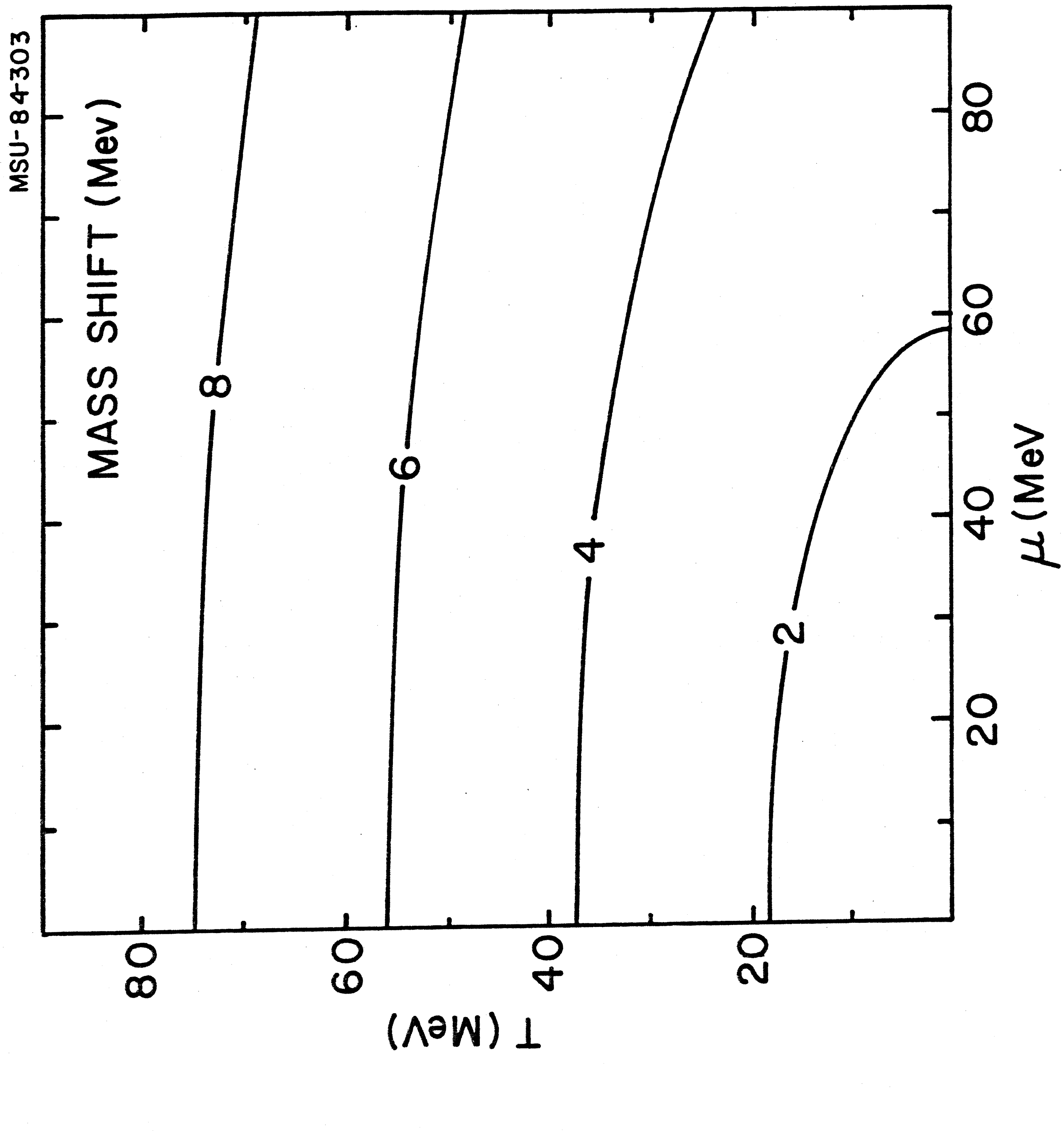

figure 3

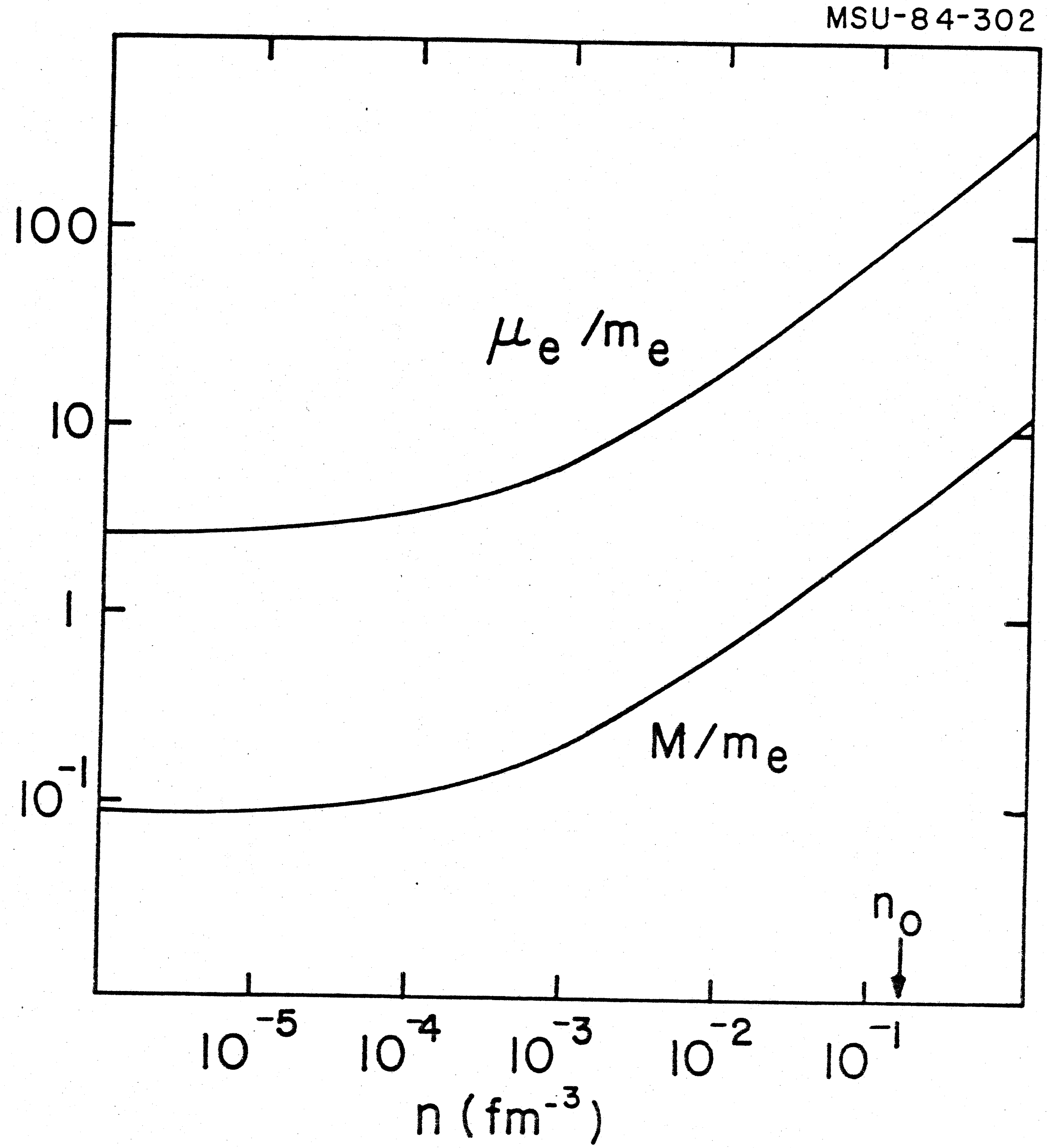

figure 4

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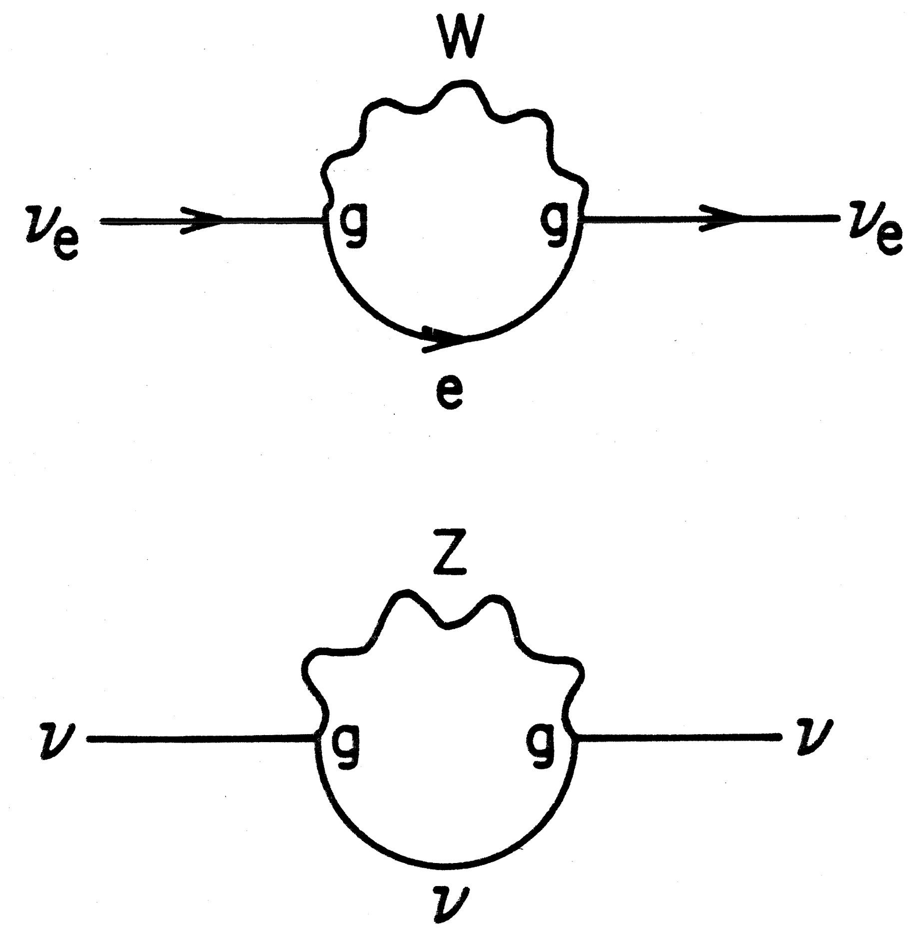

figure 5