

The Role of Mean Field Dynamics and Two Body Collisions
in Intermediate Energy Heavy Ion Collisions

H. Kruse,^{*} B.V. Jacak, G.D. Westfall

National Superconducting Cyclotron Laboratory
Michigan State University, East Lansing, Michigan 48824

and

H. Stöcker

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

and

Department of Physics and Astronomy and
National Superconducting Cyclotron Laboratory
Michigan State University, East Lansing, Michigan 48824

We present a microscopic description of nuclear collisions based on the Boltzmann equation, which includes the nuclear mean field, two-body collisions and Pauli blocking. The theory is supplemented with a phase space coalescence model of fragment formation. Calculated proton spectra compare well with recent data for Ar(42,92 and 137 MeV/N) + Ca. The mean field dynamics without two-body collisions exhibits forward peaked distributions, in contrast to the data.

* Present address: TELCO Research, Nashville, Tennessee 37203

The recent interest in medium energy ($E = 20-200$ MeV/N) heavy ion collisions is motivated by the opportunity to study the transition from the Pauli principle dominated low energy region to high energies, where two body collisions are important.¹ Time-dependent Hartee-Fock and fluid-dynamical calculations have been applied in this energy region with drastically different results:² The mean field calculations, i.e. TDHF, exhibit transparency, while fluid dynamics predicts compound nucleus formation and rapid disintegration of the highly excited system. There is an obvious need to include the finite mean free path of nucleons, the single-particle viscosity, i.e. the interaction of nucleons with the nuclear mean field, and the two-particle viscosity due to nucleon-nucleon collisions into a microscopic theory appropriate for this energy region.³ In this letter we present a microscopic approach based on the Boltzmann equation which incorporates both the nuclear mean field and nucleon nucleon collisions with an appropriate Pauli blocker. Recent data on inclusive light and heavy particle production from 40-140 MeV/N reactions⁴ provide a testing ground for the theory.

The intranuclear cascade model^{5,6} is a microscopic simulation of the reaction dynamics used at high bombarding energies. Nuclear collisions are treated as a superposition of independent n-n collisions. Nucleons move on straight line trajectories until they collide with a probability given by free nucleon-nucleon scattering cross sections. Momentum and energy are conserved in the collisions and the evolution of the system is followed until the interactions cease. The intranuclear cascade may loosely be viewed⁵ as a solution of the Boltzmann equation without the mean field term and Pauli blocking factors. Recently we have demonstrated that these terms are important even at high bombarding energies, $E > 300$ MeV/N.⁷ At

intermediate energies these effects become increasingly important: A potential field keeps the nuclei from expanding before collisions can occur, and also provides the one-body dissipation effects which dominate the dynamics of heavy ion reactions at lower energies. Furthermore, respecting the Pauli principle is essential at these energies, where the incident nuclei are overlapping in momentum space.

In the present letter intermediate bombarding energies are studied via the Boltzmann equation, including the mean field and the Pauli blocking terms⁷⁻⁹. The single particle distribution function $f(p,r,t)$ is obtained by ensemble averaging over the phase space distribution of test particles⁷⁻⁹. The time evolution of f is given by⁷⁻⁹

$$\frac{\partial}{\partial t} f + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} f + \vec{a} \cdot \frac{\partial}{\partial \vec{v}} f = \int \frac{d^3 p_2 d^3 p_1' d^3 p_2'}{(2\pi)^6} \sigma v_{12} \times \\ \times [ff_2(1-f_1)(1-f_2) - f_1 f_2 (1-f)(1-f_2)] \delta^3(p+p_2-p_1'-p_2'). \quad (1)$$

The motion of the test particles under the influence of the mean field is governed by the Vlasov equation, i.e. the left-hand side of eq. (1) set equal to zero, which is the classical analogue to the TDHF equations⁹.

The test particles are initially assigned random positions in a sphere of nuclear radius. Trajectories in configuration and momentum space are computed by assuming that each particle moves on a curved trajectory⁷ under the influence of an acceleration term generated by the gradient of the mean field. For the density dependent potential field, $U(\rho)$, a local Skyrme interaction is used: $U(\rho) = -124 \rho/\rho_0 + 70.5 (\rho/\rho_0)^2$ MeV, with a compressibility coefficient of $K=380$ MeV.

Fifteen collision simulations are followed in parallel and the ensemble averaged phase space density in a sphere around each particle is computed.⁷

The ensemble averaging ensures a reasonably smooth (about 10% fluctuation at normal density) single particle distribution function, which is used to determine the mean field and the Pauli blocking probability.^{7,8}

A constant time-step integration routine is used to insure synchronization of the ensembles.^{7,8} Within each synchronization time-step (increments of 0.5 fm/c are used) a time-matrix⁶ is constructed for the curved trajectories, i.e. all particles are transported in smaller time intervals to the (lab)-time of the next collision before that collision is allowed to take place. The acceleration of the test particles due to the field gradient is calculated prior to each transport step, and is assumed to be constant within a synchronization time-step. The local gradient of the field is computed via a finite difference method between two hemispheres ($r=2$ fm, with an equivalent mesh size of 0.75 fm) centered around the test particle. This method⁷ is analogous to Lagrange's method in fluid dynamics, in contrast to the space-fixed Eulerian mesh, which has been applied in other attempts to simulate the Boltzmann equation via Monte Carlo methods^{8,9}.

Protons, neutrons, deltas and pions of different isospin are included separately with their experimentally determined scattering cross sections.⁷ Two particles may undergo s-wave scattering if they approach each other with a minimum distance of less than $(\sigma/\pi)^{1/2}$ and if the final states are not Pauli blocked. The Pauli blocking factor is computed via the ensemble averaged density in the 6-dimensional sphere around the phase space coordinates of the scattered particles.⁷ If the nucleons within the sphere are very non-uniformly distributed, we recalculate the test volume by removing a pole cap with the volume of unoccupied space. This is necessary when the particle is near the surface of a nucleus and the test sphere

extends well into the vacuum. The Pauli blocking factor for each nucleon is given by $(1-f)$, and the scattering cross section is then reduced by the Uehling-Uhlenbeck factor $(1-f_1)(1-f_2)$. The Pauli blocker has been tested on groundstate nuclei. It has an efficiency of about 96%.

A generalized 6-dimension coalescence model⁷ is used to find the nucleons bound in clusters, and prevent them from contributing to the proton cross sections. In this scheme, a nucleon is part of a cluster if it is (a) within a configuration space distance, r_0 , from any other member of the cluster, and (b) within a momentum space distance, p_0 , from the center-of-momentum of the cluster. The decay of excited clusters is not yet included at this stage. Hence, evaporation protons are absent in the calculated spectra. This is important at medium energies, where a large fraction of the emitted protons are found to be bound in fragments.^{4,10}

The present program has been tested at higher energies⁷ by comparison to experimental data and to cascade model results and good agreement is found. To test the usefulness of this method to medium energy collisions, we have calculated inclusive proton spectra for Ar (40 - 140 MeV/N) + Ca, for which inclusive data have recently been obtained. The generalized coalescence prescription has been applied to the primordial nucleon distribution after nucleon-nucleon collisions ceased. Nucleons separated by less than 2.2 fm and less than 200 MeV/c in momentum space were collected into a cluster. These two parameters have been adjusted to yield correct total cross sections for observed nucleons, and correspond to reasonable fragment sizes. Since we do not allow for the decay of particle-unstable species, the 6-d coalescence serves only to prevent bound protons from being counted as free protons in the detectors. The calculated neutron and proton distributions are practically identical, and have been combined to decrease

the statistical uncertainty in the calculated double differential proton spectra.

Figure 1a shows the comparison between calculated and measured proton spectra for 137 MeV/N Ar + Ca at six lab angles from 30 to 130 degrees. The calculated cross sections and the slopes of the spectra agree reasonably well with the data. Production of high energy nucleons at 50 and 70 degrees is underpredicted by a factor of 2. Fig. 1b shows the same data compared to the proton spectra calculated with the cascade model of Cugnon et al.⁶, which serves as a reference model to demonstrate the importance of the mean field and phase space Pauli blocking. The cascade calculation includes a simple approximation to the Pauli blocking by excluding collisions with less than 24 MeV c.m. kinetic energy. The resulting nucleon momentum distributions were analyzed via the same procedure as the Boltzmann equation results, including the coalescence step. Variation of the coalescence parameters changes the magnitude of the cross sections, but has a negligible effect on the shape of the spectra. The same coalescence parameters were used in Fig. 1 a) and b). It is clear that the simple cascade simulation, though appropriate for high energies, cannot reproduce the medium energy data.

Figure 1c,d compares the present theory with data for Ar + Ca at 42 and 92 MeV/nucleon. The same coalescence parameters as above are used.

The measured proton cross sections are known to within 20% for the 137 and 92 MeV/nucleon data, but are uncertain by a factor of three for the 42 MeV/nucleon data due to beam monitoring difficulties.⁴ At 92 MeV/nucleon, the calculations agree with the data, in particular note the good agreement at 50 and 70 degrees. The calculation at 42 MeV/nucleon agrees well with the data except for the 30^o spectra, which are underpredicted at the lower

proton energies. This is probably due to our neglect of evaporation protons, which dominate the projectile and target rapidity regions.

The Pauli blocking is found to be very important at these bombarding energies: At 137 MeV/N 80% of the attempted collisions are blocked due to lack of available final state configurations. Many of these attempted collisions are between nucleons of the same nucleus. The spectra of low energy ($E < 80$ MeV) nucleons are also influenced by the improved Pauli blocker, which prevents collisions that would yield one very low and one high energy nucleon. The cascade calculation, on the other hand, suppresses the cross sections of low energy nucleons, in contrast to the measured spectra.

We have also studied this system in the mean field approximation by excluding two body collisions in the present theory, thus mimicking TDHF by solving the Vlasov equation⁹: The lack of two body collisions results in strongly forward peaked angular distributions, in qualitative agreement with TDHF calculations² in this energy regime.

The Boltzmann equation, including the nuclear mean field and Pauli blocking corrections to the collision terms, provides a new approach to intermediate energy heavy ion collisions. We solve the equation in a Monte Carlo framework. Inclusive proton spectra from 42, 92 and 137 MeV/nucleon Ar + Ca collisions agree with the calculated cross sections. This approach is also successfully applied at higher bombarding energies⁷. It can be useful to study the effect of the collision term on the mean field dynamics at even lower energies, where it may also be used to mimic TDHF. We are presently investigating the effect of the nuclear equation of state on intermediate energy collisions as well.

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Figure Captions

Fig.1 Inclusive proton spectra from Ar(42,92 and 137 MeV/N)+ Ca. The data⁴ are indicated by points, and the theory by histograms. The largest statistical errors in the calculation are shown on the histograms. The breaks in the data from 30 to 40 MeV are the result of dead layers in the detectors.

- a) Comparison of the present work with the 137 MeV/N data.
- b) The same data compared to results obtained with the cascade model.⁶
- c) The present theory compared to the 92 MeV/N data.
- d) The present theory compared to the 42 MeV/N data.

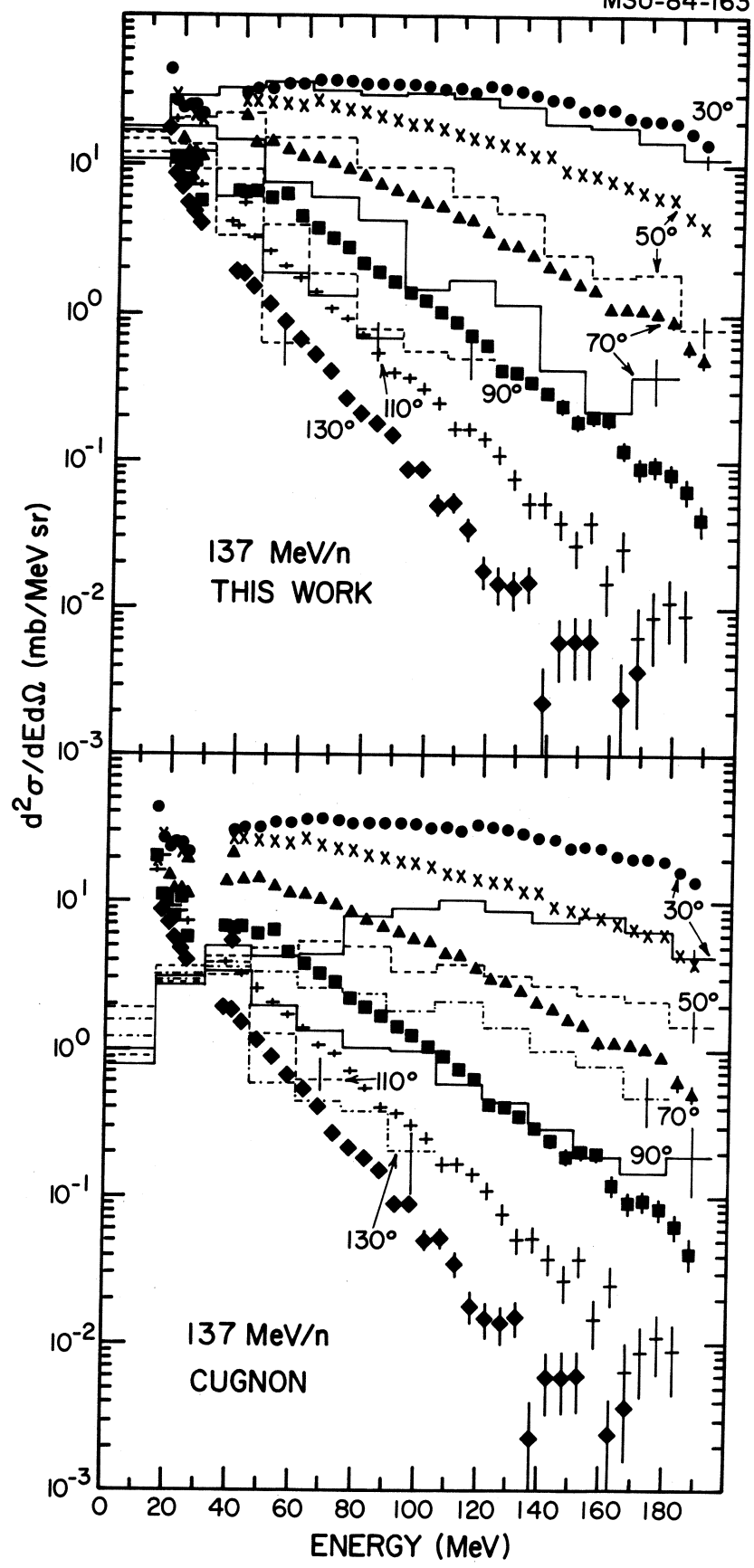


FIGURE 1a,b

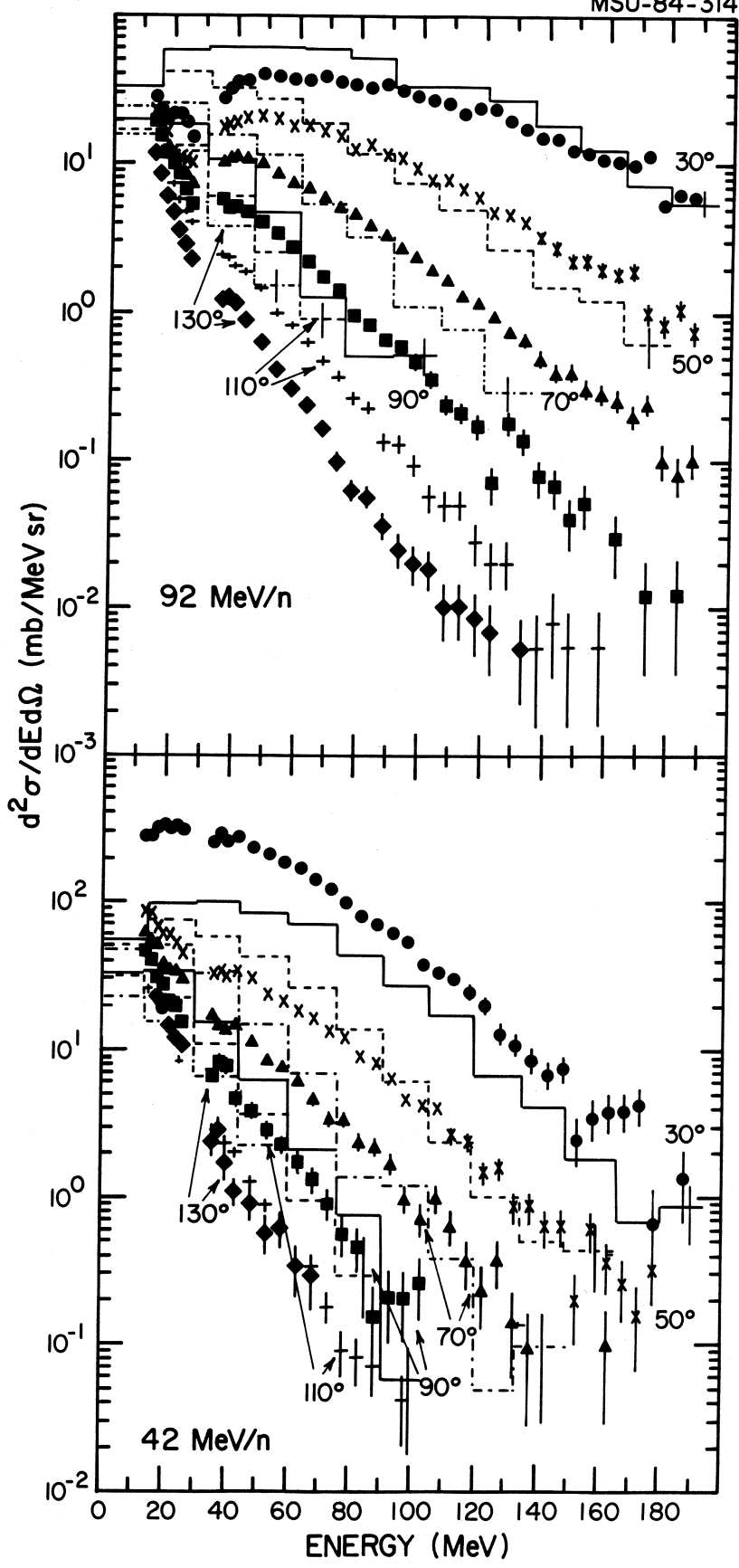


FIGURE 1c,d