M3 TRANSITIONS IN THE INTERACTING BOSON MODEL

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The properties of the magnetic octupole operator are discussed in the framework of the neutron–proton interacting boson model. It is predicted that in deformed nuclei the $I^\pi = 3^+$ member of a low-lying $K = 1^+$ band can be excited with an appreciable strength.

It has been suggested by various authors [1–6] that if in collective models a distinction is made between neutron and proton degrees of freedom a low-lying collective $I^\pi_\nu = 1^+$ state is present. In particular in the deformed region this state is the bandhead of a $K^\pi = 1^+$ band and has the property that it is connected to the ground state by a relatively large M1 matrix element. The geometric interpretation of such a $K^\pi = 1^+$ mode is that of a small amplitude oscillation in therms of the angle between the two symmetry axes of an axially symmetric deformed neutron and proton distribution [1.3].

Stimulated by a simple sum rule for the expected M1 strength derived in terms of the neutron–proton interacting boson model [3,4] (IBA-2), electron scattering experiments were performed [7] to search for this strength. At present there exist several indications for appreciable M1 strength ($1-2\mu_N^2$) at $E_x = 3$ MeV in several rare-earth nuclei (^{154}Sm, ^{156}Gd, ^{158}Gd, ^{164}Dy, ^{166}Er, ^{174}Yb) [8,9]. Also the electron scattering form factors [4,7] suggest that these states correspond to an orbital rather than a spin-flip excitation in agreement with the collective picture.

These developments suggest further investigations of the higher angular momentum members of these $K^\pi = 1^+$ bands. In particular one may ask whether, in addition to the collective magnetic dipole strength, there also exist collective magnetic octupole transitions. It is the aim of this letter to discuss this question in terms of the IBA-2 model both from a phenomenological and a microscopic point of view.

In the IBA-2 approach the M3 operator is the lowest order boson operator of rank three,

$$T^{M3}_{B\mu} = (35/8\pi)^{1/2}\{\Omega_\pi(d^\dagger_\pi \tilde{d}_\pi)^{(3)}_\mu + \Omega_\nu(d^\dagger_\nu \tilde{d}_\nu)^{(3)}_\mu\}$$
$$= (35/8\pi)^{1/2}\{\Omega_S[(d^\dagger_\pi \tilde{d}_\pi)^{(3)}_\mu + (d^\dagger_\nu \tilde{d}_\nu)^{(3)}_\mu]$$
$$+ \Omega_A(2/N)[N_\nu(d^\dagger_\pi \tilde{d}_\pi)^{(3)}_\mu - N_\pi(d^\dagger_\nu \tilde{d}_\nu)^{(3)}_\mu]\}\,, \quad (1)$$

where the parameters Ω_S and Ω_A are defined in terms of the magnetic octupole moments, $\Omega_\rho(\rho =$

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ν, π), of the neutron and proton d-boson as

$$\Omega_S = (1/N)(N_\pi \Omega_\pi + N_\nu \Omega_\nu), \quad \Omega_A = \tfrac{1}{2}(\Omega_\pi - \Omega_\nu)\,, \tag{2}$$

and N_π and (N_ν) denote the number of proton (neutron) bosons ($N = N_\pi + N_\nu$). In the second line in eq. (1) the M3 operator is decomposed into two terms. The first, the isoscalar term, connects only states which are fully symmetric in the neutron and proton degrees of freedom, i.e. states that have maximal F-spin [10]. The second, the isovector term, connects the fully symmetric states with states that have $F = (F_{max} - 1)$, hereafter referred to as anti-symmetric (a.s.) states.

In the U(5) limit (spherical limit) of the IBA-2 model, the ground state is a pure s-boson state and the matrix elements of the M3 transition operator, eq. (1), between the ground-state and lowest 1^+ anti-symmetric state therefore vanish. In the SU(3) limit (the axially symmetric rotor limit), the picture is more complex. The operator of eq. (1) connects the ground state, $(\lambda, \mu) = (2N, 0)$, $I^\pi = 0^+$ with $I^\pi = 3^+$ states in both the gamma-band [i.e. the symmetric SU(3) representation $(\lambda, \mu) = (2N-4, 2)$] in the $(\lambda, \mu) = (2N-2, 1)$, $K^\pi = 1^+$ band, and in the anti-symmetric $(\lambda, \mu) = (2N-4, 2)$, $K^\pi = 2^+$ band.

We now turn to a discussion of a microscopic estimate of the coupling constants Ω in eq. (1). In the spirit of the microscopic picture underlying IBA one equates the matrix elements between the lowest seniority states in the boson space and the collective fermion space [11] (consisting of S_ρ and D_ρ pairs), i.e.

$$\Omega_\rho = \tfrac{1}{7}(\tfrac{8}{5}\pi)^{1/2} \langle S_\rho^{N-1} D_\rho \| T^{M3} \| S_\rho^{N-1} D_\rho \rangle \quad (\rho = \pi, \nu)\,, \tag{3}$$

where [12]

$$T^{M3}_{F\mu}(q) = iqe\hbar/2mc\sqrt{7}$$

$$- \sum_i \{\sqrt{3}\, j_4(qr_i)[Y^{(4)}(r_i) \times (\tfrac{2}{3} g_\ell l_i - g_s s_i)]^{(3)}_\mu$$

$$+ 2 j_2(qr_i)[Y^{(2)}(r_i) \times (\tfrac{1}{2} g_\ell l_i + g_s s_i)]^{(3)}_\mu\}\,. \tag{4}$$

Note that at the photon point, the limit of $q \to 0$, only the term proportional to $j_2(qr)$ contributes. The microscopic structure of the S and D fermion pair state, can be determined using various methods.

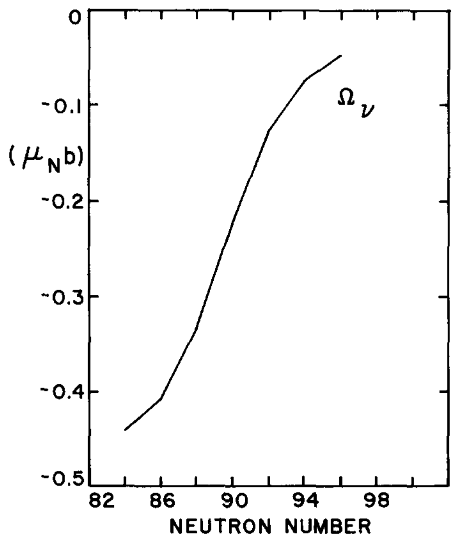

Fig. 1. The calculated octupole moment for the neutron d-boson as a function of neutron number. The spin g_s-factor is quenched ty 70%.

Here we present results using the generalized seniority scheme [13]. The calculated values for the d-boson octupole moments, Ω_ρ are given in fig. 1 for neutrons in the first half of the 82–126 shell and in fig. 2 for protons in the beginning of the 50–82 major shell. In the calculations the neutrons and protons have

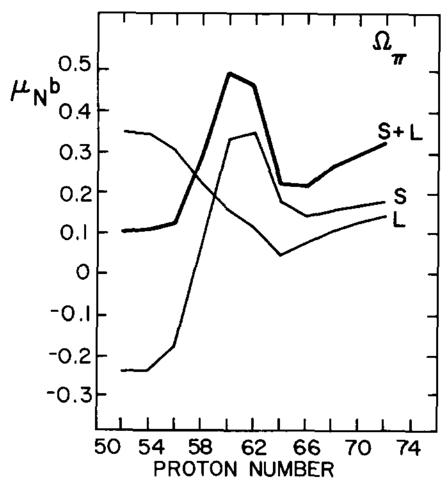

Fig. 2. Same as fig. 1, but for the proton d-boson as a function of proton number. The orbital (L) and spin (S) contribution to the octupole moment are plotted separately.

been considered separately, thus ignoring the effects of the neutron–proton interaction. The values of the M3 matrix element calculated (using a quenching factor of 0.7 for the neutron and proton spin magnetic moments) for the fermion pair states are roughly twice the single particle value (0.4 $\mu_N b$ in this mass region) which indicates a collective effect. The calculated values however show considerable fluctuations with nucleon number. This suggests that the precise value may depend strongly on the detailed description of the intrinsic structure of the fermion pairs.

In fig. 2 the spin and orbital contributions to the proton octupole moment are plotted separately. In a similar analysis of magnetic moments [4,14] the net spin contribution from the collective d-boson was found to almost vanish and only the orbital part of the operator gave a considerable contribution. In the present case the spin contribution is comparable to the orbital part and fluctuates strongly with N. The importance of the spin contribution for spherical nuclei has been confirmed in other shell model calculations [15]. However one might expect that, by including the effect of the n–p interaction, which leads to increased collectivity of the S and D pairs in the microscopic calculation, the net spin contribution will decrease.

For all calculated cases the sign of Ω_ν is consistently negative while that of Ω_π is positive. This means that the M3 operator has a strong isovector component and thus predominantly excites a.s. states. To illustrate this the calculated B(M3) values for ^{154}Sm are given in table 1, using the values given in fig. 1 and fig. 2 for Ω_ν and Ω_π. The numerical calculations were performed using the standard IBA-2 hamiltonian as is given for example in ref. [16], using the parameters of ref. [17]. Only the strength

Table 1
Calculated excitation energies and $B(\mathrm{M3}, 0^+ \to 3^+)$ values for the first four collective $I^\pi = 3^+$ states in ^{154}Sm in units of $\mu_N^2 b^2$.

State	Band	E_x[MeV]	$B(\mathrm{M3}\uparrow)$
3_1^+	2_1^s	1.51	0.23
3_2^+	2_2^s	2.46	0.001
3_3^+	1_1^a	2.99	0.56
3_4^+	2_1^a	3.26	0.31

of the Majorana force has been readjusted to $\xi_1 = \xi_2 = \xi_3 = 0.15$ MeV such that the energy of the 1^+ state is about 3 MeV, where it has been observed [7] in ^{156}Gd. In table 1 the levels are labelled by their K values and the symmetry character [totally symmetric (S) or antisymmetric (A)]. Note, however, that the IBA-2 hamiltonian will in general lead to some mixing of both K and symmetry character and therefore the labels refer only to the dominant components. As expected three $I^\pi = 3^+$ levels are excited in the SU(3) limit. It should be noted that although the energy difference between the $K = 2_2^S$ and the $K = 1_1^A$ levels in ^{154}Sm is small the selection rules are still rather well obeyed. This implies that M3 transitions can be used to identify the position of the a.s. bands in deformed nuclei. The $I_i^\pi = 3_3^+$ state which is a member of the $K^\pi = 1^+$ band is most strongly excited. The single particle value for a M3 transition is 0.13 $\mu_N^2 b^2$ while the typical strength of a transition to the first 2q.p. 3^+ state, as calculated in the generalized seniority model [13], is only of the order of 0.03 $\mu_N^2 b^2$. The M3 transition probability to the collective a.s. states is thus large.

Since we feel that the most promising tool for experimentally observing these states is transverse electron scattering, we have also calculated in PWBA the form factors of the collective M3 transitions using the program DENS [18]. As an example we present in fig. 3 the results for the three strongest $0 \to 3^+$ transitions in ^{154}Sm. Since for ^{154}Sm the individual contributions of the neutrons and the protons are very similar in shape, the resulting form factors for the three states are rather similar. The calculation indicates that since the relative importance of the intrinsic spin (g_s) and orbital (g_ℓ) contribution varies strongly with proton number (see fig. 2) the shape of the proton form factor is Z-dependent. For smaller values of Z the minimum near $q_{\mathrm{eff}} = 1\ \mathrm{fm}^{-1}$ quickly disappears.

Admixture of the a.s. states with states based on a g-boson excitation, as was introduced by Pittel [19] in the study of the spreading of the collective M1 strength, could enhance the collectivity of the M3 transitions. In the case of M3 transitions the introduction of g-bosons in the picture introduces additional terms in the M3 operator.

It has been shown that M3 transitions offer an alternative way of exciting a.s. states. The strength dis-

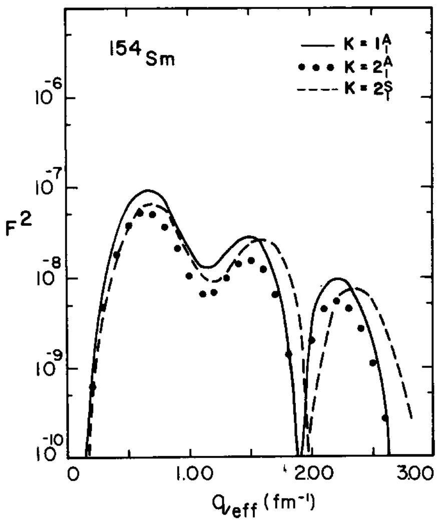

Fig. 3. The calculated form factors for the three strongest excited 3^+ states in the spectrum of ^{154}Sm.

tribution to the various $I^\pi = 3^+$ levels is predicted. Microscopic calculations based on the generalized seniority model indicate that the M3 operator is predominantly isovector in character. Combined with the calculated matrix elements of the M3 operator in the IBA-2 model this implies that the $I^\pi = 3^+$ member of the $K^\pi = 1^+$ band is the strongest state in the M3 spectrum for deformed nuclei. The predicted strength is of the order of 5 s.p. units but depends on details of the calculation.

The experimental determination of the M3 strength in deformed nuclei is thus of two fold interest: (i) it provides important information on the position of anti-symmetric states in the IBA model; (ii) it provides a sensitive test of the microscopic structure of the bosons.

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