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THE GIANT DIPOLE AND SPIN-DIPOLE RESONANCE: A COEXISTANCE PROBLEM

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SEPTEMBER 1984

MSUCL-483

MSUCL-483
September 1984

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Abstract: We present numerical and analytical results both indicating that *giant dipole states have no appreciable transition strengths for spin-dipole operators if these states exhaust almost all their sum rule*. We also show that two-body spin-orbit and tensor interaction components have a fairly large influence on spin-dipole states but a very limited one on (S=0) giant dipole states. Implications regarding the relative scale of cross section for S = 0 and S = 1 dipole excitations are also discussed.

1. Introduction

The systematics of giant dipole resonances (GDR) is by now well established[1] and has given us important information regarding the effective interaction of the nuclear many-body system. Recently new information regarding spin-dipole resonances with multipolarities $\lambda^{\pi} = 0^-$, 1^- and 2^- has become available mainly through (p,n)[2)-4)] and pion induced reactions[5]. The problem of the coexistence

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of spin-independent and spin-dependent giant dipole resonances has thus become frequently invoked[6-8]. We will devote the following paper to discussions of the formal and practical implications of such a coexistence in the context of recent experiments investigating the spin excitation resonances in the vicinity of the GDR.

11. The Coexistence Problem

Spin dipole resonances are excited through the transition operators

$$\hat{T}^{SP}_{\lambda\mu} = \sum_i r_i^{\ell=1} [\, Y_{\ell=1}(\hat{r}_i) \times \vec{\sigma}_i]_{\lambda\mu} \cdot \vec{\tau} \;, \; \lambda = 2^-, 1^-, 0^- \tag{1}$$

in intermediate energy hadron scattering at forward angle ($\theta = 5 - 15^{o}$). One multipole of the spin-dipole states has the same angular momentum and parity $J^{\pi} = 1^-$ as the giant dipole resonance,itself induced by the operator

$$\hat{T}^{NS}_{\lambda\mu} = \sum_i r_i^{\lambda=1} \; Y_{\lambda=1}(\hat{r}_i) \,.\, \vec{\tau},\; \lambda = 1^- \tag{2}$$

in electron scattering or photoreaction. In a previous shell model study of $J = 1^-$ states in ^{16}O, we decomposed the nuclear interaction into its central, spin-orbit and tensor components.[7] The results, displayed in Table I, indicated that the central part of the interaction alone yields one state at 21.8 MeV exhausting the total S = 0 strength while another state at 19.8 MeV accounts for 95% of the total transition strength for the spin dependent operator. When the two-body spin-orbit and the tensor interactions are included in the calculation, the spin-dependent transition strenth is largely split out, but the spin-independent transition strength still remains concentrated in one state. This difference can be understood in the light of the following discussion.

The tensor interaction can be given in the form

$$V_T(r) = F(r) \; \{ \, [\vec{\sigma}_1 \times \vec{\sigma}_2]^{(2)} \times [r^2 Y_2(\hat{r})]^{(2)} \}^{(0)} \tag{3}$$

Taking the expansion formula of the solid spherical harmonics $r^2 Y_{2\mu}(\hat{r})$ for two separate coordinates, we can express the tensor interaction in terms of the spin-dipole operators[8)]

$$V_T(r) = F(r) \sum_{\lambda} (-) \frac{\sqrt{4\pi}}{6} [\frac{10}{3}]^{1/2} \qquad \left\{ \begin{array}{c} 2\sqrt{5} \\ -\sqrt{15} \\ 1 \end{array} \right\}$$

$$* \{ r_1 [\vec{\sigma}_1 \times Y_1(\hat{r}_1)]^{(\lambda)} \times r_2 [\vec{\sigma}_2 \times Y_1(\hat{r}_2)]^{(\lambda)} \}^{(0)} \quad \lambda^{\pi} = \left\{ \begin{array}{c} 0^- \\ 1^- \\ 2^- \end{array} \right\} \qquad (4)$$

The spin-orbit two-body operator $\vec{L} \cdot \vec{S}$ is also proportional to the spin-dipole operator as follows

$$\vec{L} \cdot \vec{S} \propto r_1 \{ [\vec{\sigma}_1 \times Y_1(\hat{r}_1)]^{(1)} \times \vec{p}_2 \}^{(0)}$$

$$+ r_2 \{ [\vec{\sigma}_2 \times Y_1(\hat{r}_2)] \times \vec{p}_1 \}^{(0)} \quad . \qquad (5)$$

In Eq. (5), the spin-dipole operator can couple to $\lambda^{\pi} = 1^-$ only. We can now see from Eqs. (4) and (5) that the direct two-body matrix element of the tensor and spin-orbit forces is *finite only for the spin dependent configurations*. Spin-independent excitations receive contributions from exchange matrix elements which are in general several times smaller than the direct ones. Thus, the *spin-independent excitations are practically insensitive to tensor and spin-orbit forces*, while the *spin-dependent ones are greatly affected both in excitation energies and transition strengths*. We should note that since the spin-isospin channel of the central interaction is repulsive, the effects of the tensor and central force components would interfere constructively since F(r) as given in Eq. (4) is repulsive. In fact we can see from Table I that the average excitation energy with the full interaction $\bar{E}_x = 23.8$ is 4.2 MeV higher than with the central interaction alone ($\bar{E}_x = 19.6$ MeV).

One interesting feature of the results displayed in Table I is that the *giant dipole state has always a very small spin transition strength with or without the tensor and the spin-orbit forces*. This is also the case in other nuclei like ^{12}C, ^{40}Ca[7], and even ^{208}Pb[2]. In the following, we will demonstrate that if the spin-independent giant resonance exhaust almost all the sum-rule strength, the same state cannot have a large transition strength for the spin-dependent transition operator (with the same spin and parity). First, let us assume that the giant dipole state (GDS) can be expressed by the following particle-hole (p-h) wave function

$$|GDS> = \sum_{p,h} N^{-1} <(h^{-1}p)1^- \quad ||rY_{\lambda=1}\tau_z \; ||0>$$

$$|(h^{-1}p) \; 1^-> \tag{6}$$

where the normalization N is given by

$$N^2 = \sum_{p,h} \quad |<(h^{-1}p) \; 1^- \quad ||rY_{\lambda=1}\tau_z \; || \; 0> |^2.$$

The phase of the p-h state is taken to be the one adopted by Bohr and Mottelson[9]. The GD -state (Eq. 6) exhausts the non-energy weighted sum rule since

$$|<GDS \; || \sum_i r_i Y_{\lambda=1}(\hat{r}_i)\tau_z(i) \; ||0> |^2$$

$$= \sum_{p,n} | <p \; || \; rY_{\lambda=1} \quad \tau_z \; || \; \bar{h}> |^2 \tag{7}$$

On the other hand, the same giant dipole state connects to the ground state via a spin-dependent matrix element in the following form

$$<GDS \,||\, \sum_i r_i \, [Y_{\ell=1}(\hat{i})\sigma(i)]_{\lambda=1} \, \tau_z(i) \,||0> \; = \sum_{p,h} C(2j_h + 1)$$

$$<j_h \; 1/2 \, \lambda \, 0 | j_p \; 1/2>^2 \; <j_p |r| j_h>^2 \; \{(-)^{\ell_p + 1/2 - j_p} (j_p + 1/2)$$

$$- (-)^{\ell_h + 1/2 - j_h} (j_h + 1/2)\} \qquad (8)$$

since

$$<(h^{-1}p) \; 1^- \;||\; rY_{\lambda=1} \;||\; 0> \; = (-)^{j_p + j_h - 1} <j_p \;||\; rY_{\lambda=1} \;||\; \bar{j}_h >$$

$$= - [(2\lambda + 1)(2j_p + 1)/4\pi]^{1/2} <j_h \; 1/2 \; \lambda \; 0 \; |j_p \; 1/2> <j_p \,|\, r | j_h> \qquad (9)$$

and

$$<(h^{-1} \; p) \; 1^- \;||\; r \; [Y_{\ell=1} \; \sigma \;]^{\lambda=1} \;||0 > \; = - [(2\lambda + 1)(2j_p + 1)/(\lambda(\lambda+1)4\pi)]^{1/2}$$

$$* <j_h \; 1/2 \; \lambda \; 0 \; | \; j_p \; 1/2 > \; <j_p |r| \; j_h> \{(-)^{\ell_p + 1/2 - j_p} (j_p + 1/2)$$

$$- (-)^{\ell_h + 1/2 - j_h} (j_h + 1/2)\}$$

$$(10)$$

The number C in Eq. (8) depends only on the normalization N and the multipole λ. In the ^{16}O case, there are five p-h components forming 1^- states in the $1\hbar\omega$ space, i.e. $[1d_{5/2}, 1p_{3/2}{}^{-1}]$, $[1d_{3/2}, 1p_{3/2}{}^{-1}]$, $[2s_{1/2}, 1p_{3/2}{}^{-1}]$, $[1d_{3/2}, 1p_{1/2}{}^{-1}]$ and $[2s_{1/2}, 1p_{1/2}{}^{-1}]$. If one sums up Eq. (8) with these five p-h configurations, one obtains an exact cancellation of these five contributions if one adopts harmonic oscillator wave functions for the radial integrations. Since ^{16}O is a closed core in both L-S and j-j coupling, all spin couplings $S = S_1 + S_2$ are zero in the ground state. Thus *the giant dipole state which has no intrinsic* spin-component (since the operator $r\, Y_{1\mu}\, \tau_z$ has no spin) cannot have any matrix element for the spin-dependent operator. It is also well known that ^{12}C is not a good j-j coupling core, but remains close to the L-S coupling core[10]. In this sense, the giant dipole state in ^{12}C would also not have large transition strength for the spin-dependent operator, a fact consistent with the small cross sections observed in (p,n)[4] and pion reactions[7], in the vicinity of the GDR.

III. The $|S=0\rangle$ and $|S=1\rangle$ Coexistence: A General Case

Let us finally investigate a more general expression dealing with the orthrogonality of a giant dipole state and a spin-dipole state in a spin-saturated nucleus. Letus start with a generalized energy-weighted sum rule,

$$m_1(\hat{T}_1, \hat{T}_2) = \sum_n \hbar\omega_\lambda^n \langle 0 \mid \hat{T}_1^+ \mid n\rangle \langle n \mid \hat{T}_2 \mid 0\rangle$$

$$= \frac{1}{2} \langle 0 \mid [\hat{T}_1^+ , [H, \hat{T}_2]] \mid 0 \rangle \qquad (11)$$

where $\hat{T}_1$ and $\hat{T}_2$ stand for any one-body operator. The dominant contribution to the m_1 - moment comes from the kinetic energy term as follows,

$$m_1^K (\hat{T}_1 = \hat{T}_2) = \frac{\hbar^2}{2m} \langle 0 \mid \sum_i \nabla_i \hat{T}_1^+ \cdot \nabla_i \hat{T}_2 \mid 0\rangle$$

$$= \frac{\hbar^2}{2m} \frac{\lambda(2\lambda + 1)^2}{4\pi} A \langle r^{2\lambda-2} \rangle, \text{for } \hat{T}_1 = \hat{T}_2 = \sum_{i,\mu} r_i^\lambda Y_{\lambda\mu}(\hat{i}).$$

$$= \frac{\hbar^2}{2m} \frac{\ell(2\ell+1)(2\lambda+1)}{4\pi} \; A < r^{2\lambda-2} >, \text{for } \hat{T}_1 = \hat{T}_2 = \sum_{i,\mu} r_i^{\ell} \, [Y_\ell(i) \times \vec{\sigma}]_{\lambda\mu} \tag{12}$$

For the operators $\hat{T}_1 = \sum_i r_i^{\lambda} Y_{\lambda\mu}(\hat{r}_i)\tau_z(i)$ and $\hat{T}_2 = \sum_i r_i^{\lambda} Y_{\lambda\mu}(\hat{r}_i)\, \sigma_z(i)\tau_z(i)$

the m_1 - moment (Eq. 11) becomes

$$m_1^K (\hat{T}_1, \hat{T}_2) = \frac{\hbar^2}{2m} \cdot \frac{\lambda(2\lambda+1)}{4\pi} \sum_i <i \mid \sigma_z(i) r_i^{2\lambda-2} \mid i> \tag{13}$$

The operator $\hat{T}_2$ in Eq. (13) has no angular momentum projection, but the excited state in Eq. (11) must have the multipolarity $J^{\pi} = 1^-$ due to the $\hat{T}_1$ operator. We can thus see that the l.h.s. of Eq. (13) becomes zero for a spin-saturated nucleus where the ground state has total spin $J^{\pi} = 0^+$. Therefore *a state which exhausts one of the sum rules (12) cannot have a transition strength contributing to the other sum rule.* (This is also the case for higher multipole giant resonances such as $\lambda^{\pi} = 2^+$ and 3^- if these resonances exhaust one of the sum rules (12)).

The momentum-dependent terms of the interaction (for example, the t_1 and t_2 terms of the Skyrme interaction) increase appreciably the m_1-moments (12) both for spin-independent [11)] and spin-dependent[12)] operators. The terms t_1 and t_2 give also additional contributions to the moment (13) which involve the spin-densities $\rho_s(r) = \sum_i <i| \sigma_z(i) \quad \delta(\vec{r} - \vec{r}_i)\{\overset{1}{\tau_z}(i)\} \, |i>$. Since the spin-density might be very small in the spin-saturated system, the r.h.s. of eq. (13) might be negligible in comparison with the moment (12).

IV. Conclusion

In summary, we have shown using sum rule techniques that a giant dipole state |GDS; S = 0> will have no transition strength for spin-dipole operators in a spin-saturated system if that state exhausts all of its sum rule value. Spin-dipole transitions however are much affected by the tensor and spin-orbit interactions. This is clearly exemplified by shell model calculations in ^{16}O showing *large spreading of spin-dipole strength* due to the influence of spin-orbit and tensor components. The strength of the |GDS; S = 0> state remains however fairly immune to changes in spin-orbit and tensor components. This obviously explains why, historically speaking, giant dipole states have been so easily detected whereas spin-dipole excitations have required the advent of sophisticated analyses of low-background (p,n) and pion scattering experiments to manifest themselves.

V. Acknowledgements

We thank J.P. Elliot for enlightening discussions. This work is supported by the National Science Foundation under grant no. PHY-83-12245 and the National Science and Engineering Research Council of Canada.

TABLE I: Energies and Transition Strengths of T = 1, $J = 1^-$ States in ^{16}O[a]

V_c (central) only

E(MeV)	B(E1; S = 0) (%)	B(E1; S = 1) (%)
15.2	0	5
16.1	0	0
17.4	0	0
19.8	0	95
21.8	100	0

$V_c + L_S + V_T$

E(MeV)	B(E1; S = 0) (%)	B(E1; S = 1) (%)
11.70	0	4
17.6	1	17
19.3	4	9
21.4	88	1
27.1	7	68

[a]calculated with a Sussex interaction - details on the separability of V_c, V_T and V_{LS} are given in Ref. 7.

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