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The Time Dependent Dirac Equation with Relativistic Mean Field Dynamics Applied to Heavy Ion Scattering

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ABSTRACT

We treat in three dimensions the relativistic propagation of nucleons coupled to scalar and vector meson fields, in a mean field approximation. Results for collisions of $^{16}O+^{16}O$ at E/A(lab)=600 MeV and b=0,1,2 fm impact parameter are presented and show significant differences with conventional TDHF calculations. Positive angle sidewards collective flow is observed. Using the novel transverse momentum analysis proposed by Danielwicz and Odyniec we observe qualitative agreement between the new Streamer Chamber data and the present theory.

The time-dependent Hartree-Fock (TDHF) theory, using Skyrme forces, is widely used in microscopic descriptions of the mean field dynamics of low energy heavy ion reactions.[1)] An approach which allows extensions to energies comparable to the nucleon rest mass raises questions of covariance and retardation in the mean field propagation. We present in this letter a microscopic model of relativistic nuclear mean field dynamics. The model consists of nucleons obeying the time-dependent Dirac equation, of a classical spin zero attractive meson field (sigma) obeying the Klein Gordon equation, of a spin one repulsive meson field (omega) obeying the Proca equation and a meson-baryon interaction between them. The resulting coupled field equations are solved simultaneously in a mean field approximation. This model has been studied for the static case.[2,3)] The theory is treated in the Hartree approximation which yields an effective Lagrangian. The masses and coupling constants for the mesons are phenomenological and are adjusted to fit static nuclear matter properties.[2)]

The model Lagrangian density is

$$\mathcal{L} = \int d^3r \;\; \hbar c\bar{\psi}(i\gamma^{\nu}\partial_{\nu} - \frac{m_B c}{\hbar})\,\psi \; - \frac{1}{2}\,(\hbar c)^2[\partial_{\nu}\Phi\partial^{\nu}\Phi - (\frac{m_S c}{\hbar})^2\,\Phi^2]$$

(1)

$$- \frac{1}{2}\,(\hbar c)^2[\frac{1}{2}\,F_{\mu\nu}F^{\mu\nu} - (\frac{m_V c}{\hbar})V_{\mu}V^{\mu}] \; - \hbar c\; g_s\;\Phi\bar{\psi}\;\psi - \hbar c\; g_{\nu}V^{\nu}\bar{\psi}\;\gamma_{\nu}\psi$$

where $F_{\mu\nu}=\partial_{\nu}V_{\mu} - \partial_{\mu}V_{\nu}$, $\Phi=\sigma$-field, $V=\omega$-field, and ψ=baryon field. The notation follows the conventions of Ref. 4. The model parameters are: baryon mass m_Bc^2=938 MeV, scalar meson mass m_Sc^2=500 MeV, vector meson mass m_Vc^2=780 MeV, scalar coupling constant g_S=18.030 MeV fm, vector coupling constant g_V=33.141 MeV fm. The addition of nonlinear terms to the

Lagrangian (1) would allow a more realistic fit to other nuclear properties, such as the compressibility and effective nucleon mass.[3)]

The following equations of motion emerge from the Lagrangian density (Eq. 1):

$$i\hbar \frac{\partial}{\partial t} \psi = \hbar c\left[-i\gamma_0\vec{\gamma}\cdot\vec{\nabla} + \frac{m_B c}{\hbar}\gamma_0 + g_S\Phi\gamma_0 + g_V V^\nu \gamma_0\gamma_\nu\right]\psi \tag{2a}$$

$$i\hbar \frac{\partial}{\partial t} \Pi = \left[-\hbar^2\Delta + (m_S c^2)^2\right]\Phi + \hbar c\, g_S\langle\bar{\psi}\psi\rangle \tag{2b}$$

$$i\hbar \frac{\partial}{\partial t} \Phi = \Pi$$

$$i\hbar \frac{\partial}{\partial t} P_\mu = \left[-\hbar^2\Delta + (m_V c^2)^2\right]V_\mu + \hbar c\, g_V\langle\bar{\psi}\gamma_\mu\psi\rangle$$

$$i\hbar \frac{\partial}{\partial t} V_\mu = P_\mu \tag{2c}$$

The gauge and continuity equations

$$\partial_\nu V^\nu = 0, \qquad \partial_\nu\langle\bar{\psi}\gamma^\nu\psi\rangle = 0 \tag{3}$$

were used to obtain (2c). They are evaluated during the time evolution as a check of the numerical accuracy. A further test of the accuracy is provided by evaluating the energy E vs. time,

$$E = \int d^3r\{\bar{\phi}_\alpha[-i\hbar c\vec{\gamma}\cdot\vec{\nabla} + \gamma_0 m_B c^2 - g_V V^\nu\gamma_\nu - g_S\Phi]\phi_\alpha$$
$$+ \frac{1}{2}(\hbar c)^2[\dot{\Phi}^2 + (\vec{\nabla}\Phi)^2 + (\frac{m_S c}{\hbar})^2\Phi^2] - \frac{1}{2}(\hbar c)^2[\dot{V}_\nu V^\nu + \nabla V_\nu\cdot\nabla V^\nu + (\frac{m_V c}{\hbar})^2 V_\nu V^\nu]\} \tag{4}$$

The classical equations (2b) and (2c) propagate the mesonic mean fields. Thus the baryonic field equation (2a) can be treated exactly by single nucleon propagation, and Slater determinants will remain so at all times. We restrict, in the present work, the propagation to the A occupied bound states above the baryon vacuum. These approximations result in a theory free of mass and charge renormalization effects.

We represent the meson fields and the baryon wave functions on a three dimensional (32×24×24) mesh in both coordinate and momentum space, with a mesh spacing of 0.5 fm in coordinate space. The momentum space mesh is reached with the help of a fast fourier transform. The time evolution is done with a variable order predictor-corrector scheme.[5] This allows larger time steps (0.05 fm/c) than in the first 1-D attempt of Müller.[6] We employ isospin degenerate nucleon wave functions and treat the spin degrees of freedom in the usual four component spinor formalism. Thus we propagate eight independent four component wave functions for each ^{16}O nucleus.

We use the relativistic static Hartree wave functions for each of the ^{16}O initial states. The nuclei are located at a separation distance of 10 fm from each other on the mesh, and are Lorentz boosted to the appropriate initial energy in the center of velocity frame. The time evolution for the baryon wave functions is done with a fifth order predictor-corrector in momentum space and the meson equations are solved simultaneously using Green functions techniques. The momentum space energy truncation was taken as 2 GeV. This turns out to limit the time step to be 0.05 fm/c.

We have calculated collisions at $E_{kin}/u(lab)=600$ MeV/u, for b=0,1,2 fm impact parameter, and at 300 and 1200 MeV/u, for b=0. The time evolution is followed until the expanding matter reaches the edges of the mesh. We show in Fig. 1 the time evolution of the baryon density

$$\rho_B = \sum_\alpha \phi_\alpha^\dagger(\underset{\sim}{r})\phi_\alpha(\underset{\sim}{r}) \tag{5}$$

for E_{kin}(lab)/u=600 MeV/u, b=2 fm. The early time behavior is similar to that of nonrelativistic Skyrme TDHF.[1,5] A compression zone and sidewards flow then develops from t=6 to 10 fm/c and the system proceeds to spallate. Fig. 1 also shows that the spallation of the spectator fragment is delayed somewhat. The reaction can be contrasted with Skyrme TDHF where projectile-like fragments and target-like fragments emerge after the reaction. The sidewards flow observed in of this work are more reminiscent of fluid dynamical calculations.[7)]

Figure 2 shows the baryon density in momentum space for the same conditions as Fig. 1. The momentum space rotation (scattering) takes place mainly between 6.2 and 9.1 fm/c. The expansion phase follows, as seen in Fig. 1. The filling at zero c.m. momentum is more complete at 300 MeV/u and smaller at 1200 MeV/u. This partial filling of the zero momentum states can be contrasted with hydrodynamic calculations[7)] and Boltzmann equation results[8)] where complete filling is observed. Experiments[9)] with heavier systems at 400 MeV/u and at 800 MeV/u suggest that more particles do indeed fill the zero momentum region than predicted by our calculation. The partial filling seen here could be a result of the light system considered here which shortens the interaction time needed to decrease the longitudinal momenta via scattering. The inclusion of two-body scattering terms in addition to the mean fields should also increase the filling.

We show in Fig. 3 a quantity which is less sensitive to the filling and which illustrates the sidewards flow. The reaction plane mean transverse momentum[10)] shows peaks near the initial momenta of projectile and target, which compare favorably with the recent experimental results[10)] obtained from measurements of the GSI-LBL streamer chamber group. Calculations for $^{40}Ca+^{40}Ca$ would be desirable for a quantitative comparison, but they are presently inhibited because of the numerical expenditure.

It is interesting to see how the collective flow comes about in the present mean field theory. We find that the scalar and vector mean fields develop strong out-of-phase oscillations in space and time, soon after the maximum compression point. The corresponding large change in field energy is of course compensated elsewhere and the total energy (Eq. 4) remains conserved. This oscillation is largely responsible for the small time step needed and serves to deflect the nucleons, causing the sideways flow. This dynamical instability of the meson fields is presently being investigated in its nuclear matter limit.

We have computed relativistic collisions of $^{16}O+^{16}O$ in a relativistic mean field model with scalar and vector mesons coupled to a baryon field. Particle-hole excitations out of the baryon vacuum were neglected. The results show spallation and collective sidewards flow, in agreement with experiment. The meson fields developed strong oscillations which seem to contribute to the breakup. These results raise a number of interesting theoretical questions. We expect that the inclusion of nonlinear terms for the σ field, and other mesons such as π's and ρ's would influence the flow properties. Explicit two-body collisions should also increase the filling of zero momentum states. Finally the inclusion of $p-\bar{p}$ pairs and internal nucleon excitations should become dominant effects at still higher energies.

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References

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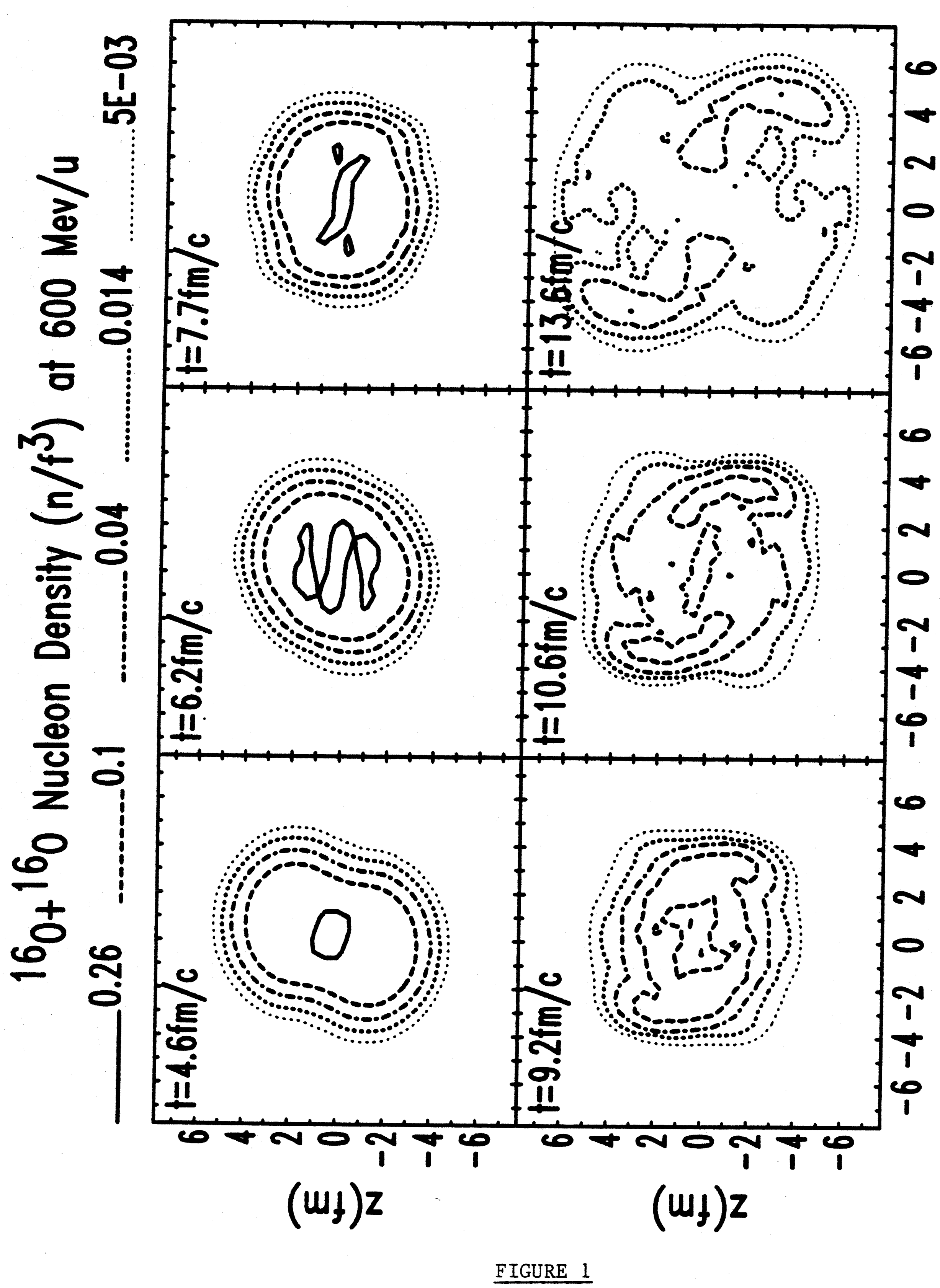
Figure Captions

Fig. 1. The coordinate space baryon equidensity contours in the collision plane at various times during the 600 MeV/u collision, at impact parameter b=2. The five density cuts are as indicated in the figure, in n/fm^3. The absolute maximum density attained is 0.29 n/fm^3. The high compression phase takes place between ~47 and 7.7 fm/c. The expansion phase follows and at the final time shown (13.6 fm/c) the remaining portion of the spectator fragments are expanding while bouncing off. Fig. 2 for the momentum space density shows the direction of the fragments.

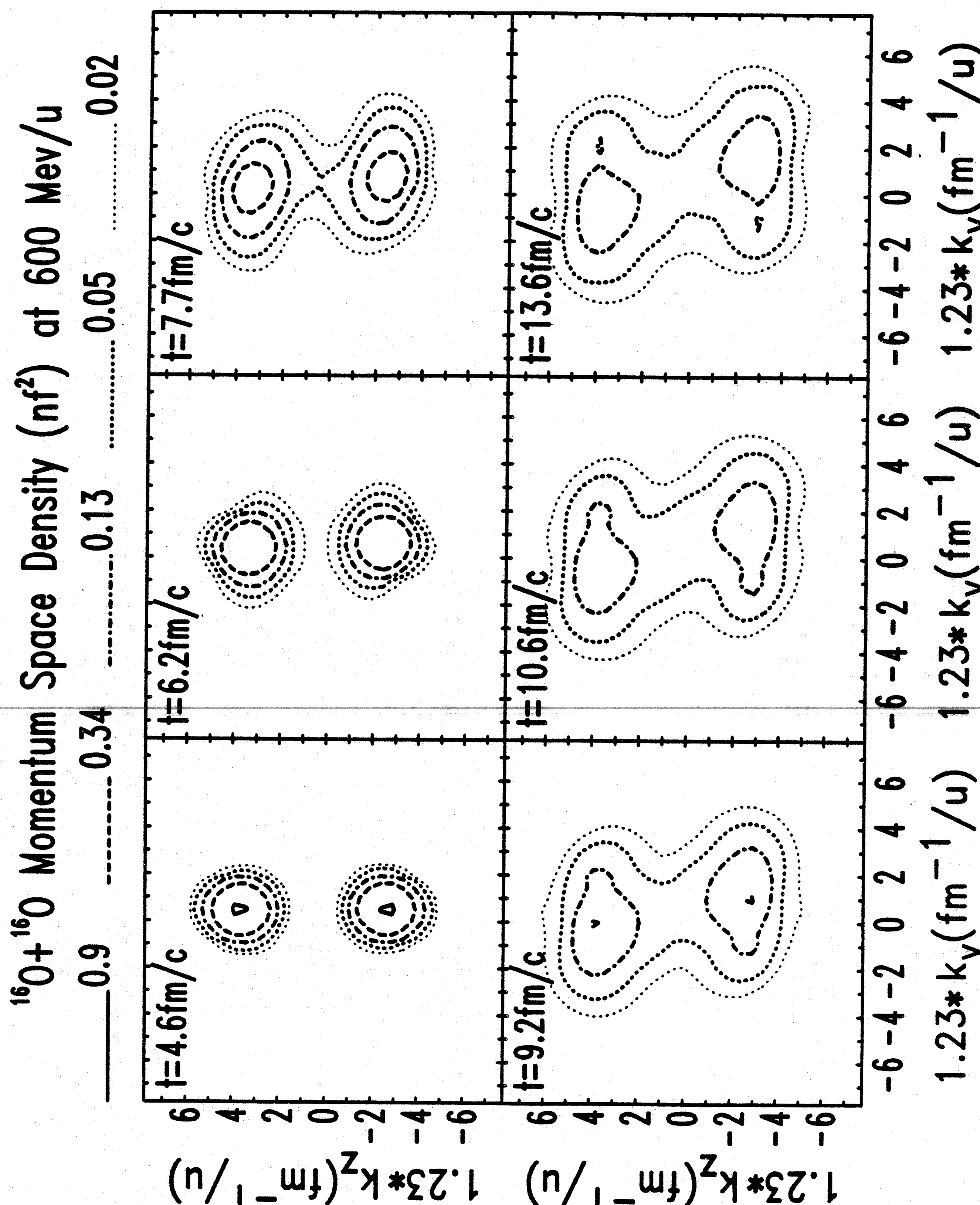
Fig. 2. The momentum space baryon equidensity integrated over k_x, the momentum perpendicular to the reaction plane, in $nucleons/fm^{-2}$, at b impact parameter b=2 and energy 600 MeV/u. The bulk of the momentum change takes place between 6.2 and 9.1 fm/c. Some recondensation of the 0.05 nf^2 contour can be observed between 10.6 and 13.6 fm/c. A comparison with Fig. 1 facilitates the determination of the bounce period.

Fig. 3. The transverse momentum projected into the reaction plane and averaged over all perpendicular momenta, as a function of the longitudinal momentum P_{par}, for $^{16}O+^{16}O$, at 600 MeV/u, for b=1 and 2. The b=2 result corresponds to a mean flow angle of about 16°, while the b=1 flows at about 9 degrees.



16O+16O Nucleon Density (n/f3) at 600 Mev/u
0.26
0.1
0.04
0.014
5E-03
t=4.6fm/c
t=6.2fm/c
t=7.7fm/c
t=9.2fm/c
t=10.6fm/c
t=13.6fm/c
z(fm)
y(fm)

FIGURE 1



16O+16O Momentum Space Density (nf2) at 600 Mev/u
0.9
0.34
0.13
0.05
0.02
t=4.6fm/c
t=6.2fm/c
t=7.7fm/c
t=9.2fm/c
t=10.6fm/c
t=13.6fm/c
1.23*ky(fm-1/u)
1.23*ky(fm-1/u)
1.23*ky(fm-1/u)
1.23*kz(fm-1/u)
1.23*kz(fm-1/u)
-6 -4 -2 0 2 4 6

FIGURE 2

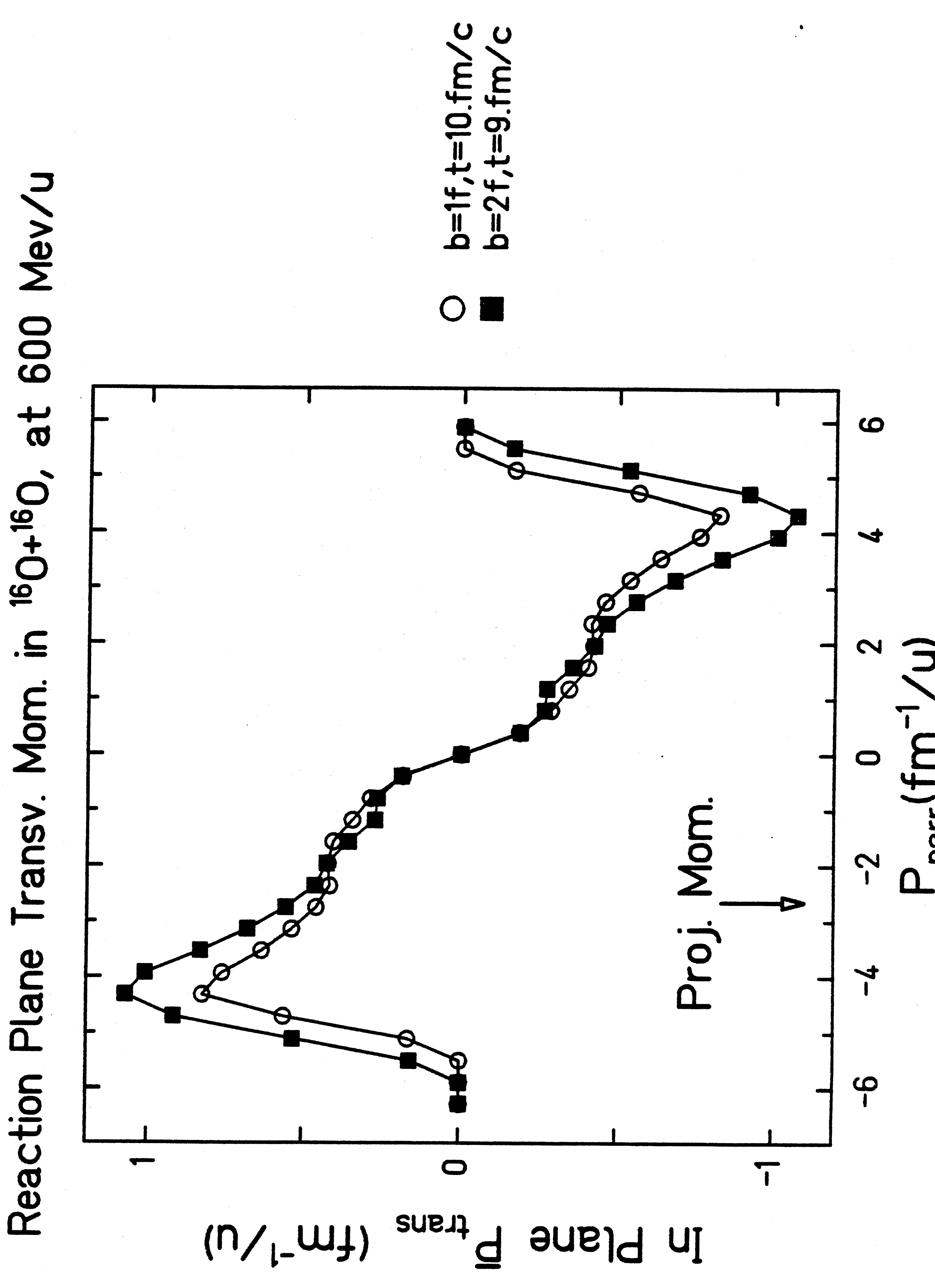

Reaction Plane Transv. Mom. in 16O+16O, at 600 Mev/u
b=1f,t=10.fm/c
b=2f,t=9.fm/c
Proj. Mom.
In Plane P_trans (fm^-1/u)
P_parr (fm^-1/u)
1
0
-1
-6
-4
-2
0
2
4
6

FIGURE 3