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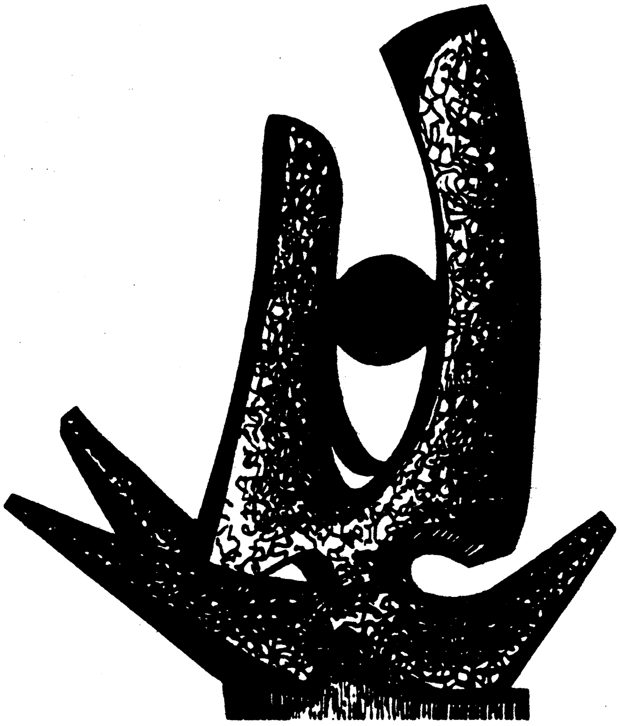

May 1985

MSUCL–503

MSUCL-503
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Abstract

We study the recent claim that the intranuclear cascade model exhibits collective sidewards flow. We show that instability of the target and projectile nuclei produces spurious sidewards flow. Inclusion of the nuclear binding results in small flow angles, which are in contrast to recent experimental data. The lack of substantial flow effects in the cascade model is due to the absence of the essential compressional energy that can cause collective sidewards motion - the intrinsic pressure built-up in the cascade model is too small.

Cugnon and L'Hote have recently claimed[1] that the intranuclear cascade model of Cugnon et al.[2] - from here on referred to as the Cugnon cascade for brevity - can exhibit collective sidewards flow, in agreement with the 4π exclusive data on Nb(400 MeV/nucleon) + Nb obtained by the GSI/LBL Plastic Ball group.[3] Collective sidewards flow had originally been predicted by macroscopic nuclear fluid dynamics[4] and has since then received much theoretical attention because of it's possible connection to the nuclear matter equation of state. Calculations done using the intranuclear cascade code of Yariv and Fraenkel[5] did not exhibit any signs of sidewards flow.[3] Even more surprising, calculations done with the Cugnon intranuclear cascade code by other authors[4,6] also produced little flow. Microscopic models such as the classical equations of motion,[7] the Vlasov-Uehling-Uhlenbeck theory,[6,8] and the time dependent Dirac approach,[9] which explicitly incorporate a compressional energy, are able to explain the data. Here we underline the fact that the intranuclear cascade model does exhibit too little dynamical flow and explain why Cugnon and L'Hote[1] overestimated the flow.

Historically, the intranuclear cascade idea is due to Serber[10]. His idea was that nuclear reactions at high energies might be understood in terms of a quite simple picture different from the description needed at low energies: Because the collision time between an incident high energy nucleon and a nucleon in the nucleus is short compared to the time between collisions of the nucleons in the nucleus, he inferred that the high energy reaction could be modelled as a cascade process. Collisions occur between the incident particle and those particles which are directly struck in the nucleus. This model was first investigated in two dimensions in 1948 by Goldberger[11] who performed his calculations by hand for the case of high energy neutrons interacting with heavy nuclei. The first fully three dimensional calculations were done by Metropolis et. al.[12] in 1958 for incident protons and neutrons using the MANIAC computer; they also added a second stage to the cascade calculation during which the excited residual nucleus evaporates particles, as had also been suggested by Serber.[10]

Many others have contributed to the development of the intranuclear cascade model. The two most commonly used versions of the cascade code in the theory of high energy heavy ion reactions are due to Yariv and Fraenkel[5] and Cugnon et. al.[2] What is the intranuclear cascade model as it is used in these codes? It is a microscopic simulation of a nuclear reaction at high bombarding energies. Nuclear collisions are treated as a superposition of independent two-body nucleon-nucleon collisions. Nucleons move on straight line trajectories (since there is no field) until they collide with a probability given by the free nucleon-nucleon scattering cross section. The creation of deltas, pions, and other particles and the interaction of all these particles occurs according to experimental cross sections. The intranuclear cascade models[2,5] incorporate relativistic kinematics. Target and projectile nucleons are initialized in configuration and momentum space with random Fermi momenta and then Lorentz boosted to an appropriate frame, where the collision simulation proceeds. Momentum and energy are conserved in the particle-particle interactions and the evolution of the system is followed up to a time where the majority of interactions have ceased. Thus the intranuclear cascade model may loosely be viewed as a solution to the Boltzmann equation without mean-field or sophisticated Pauli blocking factors. Collisions are only Pauli blocked according to a simple criterion, say if the total center of mass energy is less than the Fermi energy in ground state nuclear matter[2] or if the outgoing particle would scatter into the momentum space regions originally occupied by projectile or target.[5]

Both the Yariv-Fraenkel and the Cugnon cascade satisfy the above criteria. They differ in two respects:

(i) the particles in the Yariv-Fraenkel simulation sit in a potential well of constant depth V_o ;

(ii) in the original Yariv- Fraenkel approach the incoming particles (projectile nucleons) are cascading independently through a medium (target). In the updated, more recent version, this scheme has been improved by including the so-called cascade-cascade interactions: for a given cascade particle (a particle which has undergone at least one collision), the other cascade particles are acting as a medium superimposed to the original target medium.

If the two approaches are so similar, then how can Cugnon and L'Hote and others[13] claim that the cascade exhibits collective sidewards flow, while the Yariv-Fraenkel code as well as a different version of Cugnon's cascade does not?

To determine the origin of these discrepancies, we have done some intranuclear cascade simulations and analysed the individual events using the coalescence invariant kinetic energy flow tensor:[14]

$$K_{ij} = \sum_{\nu} p_i(\nu) p_j(\nu)/2m(\nu) \qquad (1)$$

The sum is over the charged particles observed in each individual event and the indices (i,j) represent the Cartesian components (x,y,z). The tensor is diagonalized and the flow angle θ_F obtained via a principal axis transformation. Then flow angle distributions, which incorporate the proper

Jacobian,[15] are made. There are two issues where particular care must be taken in interpreting these flow angle distributions and comparing them to the experimental data. The first is the stability or binding of the nuclei. Secondly, the theoretical predictions have to be subjected to the acceptance windows and efficiency cuts imposed by the Plastic Ball.[3]

Concerning the issue of nuclear instability, Cugnon et al. themselves had noted in their original paper[2] that the nuclei do expand as a result of the Fermi momenta of the nucleons since a mean field is absent in the Cugnon cascade. This is illustrated quite dramatically in Fig. 1a for Nb (E_{lab} = 0 MeV/nucleon) + Nb, i.e. the system (at rest) for which Cugnon and L'Hote[1] report supposed collective flow at 400 MeV/nucleon. Notice that the two nuclei, which are at rest and do not collide, become completely obliterated over the course of a typical collision time t = 40 fm/c. There is a rapid expansion of the original nuclei, which are just supposed to sit there. The fact that there is a problem for massive nuclei has been pointed out several years ago,[14] but not much attention was paid to a solution, because predominantly light systems at higher energies were studied then, where the expansion does not play such a dramatic role because of the shorter collision time.

It is possible to bind the nucleons within the context of the Cugnon code by letting each nucleon move only with the beam velocity until it interacts with another nucleon, at which point it 'remembers' it's Fermi motion.[14] Of course, this bound Cugnon cascade does not exhibit the nuclear instability or deleterious expansion that the (unbound) original code does. We show in Fig. 2 flow angle distributions for Nb (400 MeV/nucleon) + Nb at impact parameters b = 1, 3 and 5 fm calculated with these two different (bound and unbound) versions of Cugnon et. al.'s code.[2] Notice that the original - unbound - version of the cascade exhibits some sidewards flow.

This sidewards flow is strongly reduced when the binding of the spectator nucleons is taken into account. Note that the peak position of the flow angular distribution does not depend strongly on the impact parameter, neither for the bound nor the unbound cascade, in sharp contrast to what is predicted by models which incorporate the nuclear compressional energy explicitly.[4,6-8] In fact, the flow angle distributions obtained with the bound cascade are always peaked at angles of about ten degrees or less. This obvious forward peaking had been observed by other authors[3,4,6,14-16] using the revised bound Cugnon code or the Yariv-Fraenkel cascade. Note, however, that the bound cascade is theoretically not very satisfactory either, since in real nuclei, nucleons can travel in all directions. A self-consistent treatment of the nuclear binding and compression potential is required for a more realistic description of the reaction dynamics, for instance in the manner of the Vlasov-Uehling-Uhlenbeck approach.[6,8]

Can we understand from the above the origin of the sidewards flow in the unbound cascade? The nucleons in each unstable nucleus, which move towards the other nucleus due to their Fermi motion, have a large probability of colliding. Hence they tend to form a more or less equilibrated hot participant system (fireball). On the other hand, those nucleons which expand freely away from the beam axis do generate finite flow angles because the projectile expands into the upper hemisphere and the target into the lower hemisphere. This creates an artificial non- dynamical flow effect in the unbound cascade.

To study this point further, we have performed the flow analysis for only those nucleons which have not undergone any collisions (the spectators). The results are shown in Fig. 3 for Nb (400 MeV/nucleon) + Nb using the bound and unbound cascades at b = 3 fm. Notice that the spectator nucleons flow in the unbound cascade, whereas the revised code exhibits forward peaking.

How about the other models that have exhibited collective flow? Is it possible that some flow there is also due to instability and the Fermi momenta? Nuclear fluid dynamics, the classical equations of motion approach, the Vlasov-Uehling-Uhlenbeck theory, and the time dependent Dirac-Hartree theory all have either forces or a mean field which serves to bind the nucleons. Thus the predictions of all these theories are not invalidated due to nuclear instability. This is shown for the Vlasov-Uehling-Uhlenbeck theory in Fig. 1b. We have let ensembles of two Nb nuclei at E_{lab}= 0 (zero) MeV/nucleon evolve for 80 fm/c, just as was done for the Cugnon unbound cascade in Fig. 1a. Note that only a few particles are evaporated during this time.

A comparison of Fig. 2 and 3 shows that there is some flow effect which is not solely due to the treatment of the spectators in the unbound cascade - the participants do also yield some sidewards flow! However, this does not lead to the large angle sidewards maxima observed experimentally.[3] What is causing this residual flow of the participants in the cascade calculation? It is the finite amount of pressure built-up in the interaction zone, which pushes on layers of matter through subsequent collisions. The additional strong repulsive interactions present in the other theories is, however, missing in the cascade approach. That there is a finite pressure build-up in the cascade can be seen when the diagonal elements of the stress tensor in central collisions are evaluated near the center of mass[17].

It is interesting to note that the flow of the participants is further enhanced in the unbound cascade as a consequence of the spurious instability of the initial nuclei: some nucleons move unduly into the interaction zone. This is actually reflected in an increased number of binary collisions. The interaction zone thus is less transparent and an additional artificial pressure build-up occurs. It is amusing to note that for b = 0 fm the situation is somewhat different: in this case more nucleons are moving away from the interaction region, thus decreasing the chance for collisions and making the participant region more transparent. Therefore, the maximum flow is obtained in the unbound cascade for impact parameters around 3 fm, while for exactly central collisions, b = 0 fm, the flow angular distribution is peaked at zero degrees, even for the unstable cascade.

Can the intranuclear cascade model predict - at least in principle, say by going to very massive systems - the same large flow angles as the data or the hydrodynamic model? In the limit of a short mean free path, the cascade model should approach the hydrodynamic limit and the results of cascade and hydrodynamics should be similar. But even for U + U collisions, the standard Cugnon cascade produces near isotropy and does not approach the fluid flow limit as was shown by Gyulassy, Frankel and Stöcker[14]. Calculations with $A_P = A_T = 500$, 1000, and even 2000 confirm this finding[14].

Can fluid behaviour be forced into the cascade by using larger effective NN cross sections to induce fluid behaviour? This has also been tried previously.[14] But, although the flow angle increases somewhat, the flow is still far weaker than the hydrodynamic result.[14] To understand this puzzling finding we must remember that the hydrodynamic calculations have - in addition to the assumption about a short mean free path - also incorporated a repulsive short range nuclear interaction via a compressional potential in the nuclear equation of state. Hence, $\lambda/R << 1$ is not sufficient to simulate the observed fluid behaviour.

It has been shown,[14] though, that the flow is sensitive to the scattering style - recall that the standard scattering style corresponds to a stochastic classical force with a random sign - 'inward' scattering occurs with the same probability as 'outward' scattering. Therefore, the momentum transfer and the relative coordinate between two nucleons are assumed to be completely uncorrelated at the scattering time. Classically, on the other hand, any potential leads to definite correlations between r and p. These correlations have been implemented into the standard Cugnon cascade.[14] It has been shown that only the non- random scattering (repulsive in-plane scattering) with increased scattering cross section leads to collective flow similar to nonviscous hydrodynamics, as was later observed in the classical equations of motion approach which incorporates an excluded volume approximation.[7] As the effective cross section is increased, the flow becomes more pronounced.[14] For five times normal scattering cross section, the flow is even more pronounced than in nonviscous fluid dynamics. Thus, the flow obtained with non- viscous one- fluid hydrodynamics represents only one possible class of flow patterns.

Gyulassy, Frankel and Stoecker have conjectured[14] that the variations of the effective scattering cross section and scattering style correspond to substantial variations in the transport properties and the equation of state in terms of viscous fluid dynamics. Therefore, it seems to be of utmost importance to include the short range nuclear repulsion into any microscopic theory of medium and high energy nuclear phenomena, e.g., via the repulsive nuclear compression potential fields which are neglected in intranuclear cascade model.

We must remember to examine the effect of the acceptance windows and efficiency cuts of the Plastic Ball in order to compare the theoretical and experimental flow distributions in the same multiplicity bins. We have used the original SIMDAT data simulation routine of the LBL-GSI group, which was developed by the Plastic Ball group to simulate the response of the spectrometer system to Monte Carlo events. In the SIMDAT routine the cascade nucleons are randomly assigned isospin, target nucleons which have not collided are omitted, and the residual charged particles are subjected to the acceptance and efficiency of the Plastic Ball. Then the events at finite impact parameters are binned according to the multiplicity of charges that the Plastic Ball would see. Notice that there is a strong dependence of the impact parameter distribution on the selected multiplicity bin as is illustrated in Fig. 4.

We now come to the most important result of this paper: the flow angle distributions for the bound and unbound versions of Cugnon et. al.'s code, subjected to the Plastic Ball filter and binned according to multiplicity, are shown in Fig. 5. Also shown are the experimental data[3] for the two highest multiplicity bins. Note that neither the bound nor unbound cascade match the Nb (400 MeV/nucleon) + Nb data. By comparing Fig. 2, which shows the flow distribution without filter, with Fig. 5, which includes the effects of the filter, we demonstrate that the Plastic Ball filter tends to cause the flow angle distribution to broaden, but does not increase the peak angle substantially.

Several remarks concerning the filter are in order: apparently there is an effect of the filter on the resulting angular distributions; also the multiplicity of charged fragments is not unambiguously obtained from the cascade model (the multiplicity of charges is easily accessible), unless one takes account of both the target spectators (as we have done explicitly in Fig. 5) and of cluster formation (which we have not done, although one can use a six dimensional coalescence of the final state[18]). Therefore it is not immediately obvious which multiplicity binning should be chosen to compare the data with the theory. However, we can rectify this problem by rebinning the data according to the multiplicity of charges (rather than according to the multiplicity of charged fragments, as was done in ref. 3): then there is no significant change of the flow distributions in the same multiplicity bin! It has been claimed in ref. 1 that the position of the maximum in the flow angle distribution can be forced to larger values in the unstable cascade calculation by going to very high multiplicities, $M > 66$. We have applied this same high multiplicity cut to the bound cascade results. Then we obtain a maximum in the flow distribution at about 20 degrees, still substantially smaller than the observed peak position in the intermediate multiplicity bin $40 < M < 50$ of the data. If the multiplicity cut M is lowered, the peak moves to even smaller angles, as shown above. It would be desirable to have information about the detailed experimental cross sections of the emitted fragments in the different multiplicity bins and for the cluster formation and spectra to be self-consistently calculated in the theory.

The invariant cross sections $d^2\sigma/dYd(p_x/m)$ in the scattering plane, which are determined from the finite multiplicity cascade events on an event by event basis, are shown in Fig. 6 for the multiplicity bin $40 < M < 50$ for the bound and unbound cascade, respectively. Y is the rapidity and p_x is the transverse momentum component in the reaction plane, respectively. Observe the strong depletion of the cross section near the target rapidity: This is due to the absorption of low energy particles in the wall of the scattering chamber in the Plastic Ball, which is here simulated by the SIMDAT - routine. Note that the contour plot for the bound cascade is nearly symmetric about the beam axis, while the unbound cascade exhibits some asymmetry.

It is remarkable how similar the results from the bound cascade obtained here are to those which the experimentalists obtained[3] using the Yariv-Fraenkel code. This underlines the fact that the presence of a constant depth potential well also does not lead to dynamical flow, in contrast to the recent claim of Kitazoe et. al.[13] It is possible that Kitazoe et al. do obtain sidewards flow because of an instability of their nuclei which is caused by the drastic readjustment of the nuclear radius. This is due to their requirement that the nuclear density is not allowed to be depleted, as is the case in the Yariv- Fraenkel cascade, but is bound to stay constant. The radius of their residual nucleus shrinks rapidly as collisions between nucleons occur, thus shaking off a substantial amount of uncollided nucleons from the residual nuclei. A quantitative analysis of this effect is not possible at this time, because the computer code[13] has not been accessible to the authors.

In summary, we have shown that intranuclear cascade models lack substantial dynamical flow. Flow effects had been overestimated previously due to a spurious sidewards expansion of the spectators; thus care need be taken to include binding in the cascade model if heavy nuclei are to be studied. Some flow is inherent, however, to the cascade model as a result of the finite pressure build-up. Besides treating nonequilibrium and quantum effects, one needs to include nuclear binding and introduce details of nuclear forces and the nuclear equation of state. To include a compressional pressure into a microscopic theory we need approaches like the Vlasov-Uehling-Uhlenbeck equation, the equations of motion approach and the time-dependent Dirac- Hartree theory. In fact, the Vlasov-Uehling-Uhlenbeck theory has been successfully applied to study the influence of the nuclear compression potential and Pauli Principle on the observables, namely flow effects and pion yields[6,8]. From a comparison to the experimental data, a rather stiff nuclear equation of state has been deduced[6,8], i.e. nuclear matter appears to be rather incompressible and the effects of the compression potential are large.

We express our sincere gratitude to the members of the GSI/LBL Plastic Ball group, in particular Dr. Hans-Georg Ritter, for help in analyzing the effect of the acceptance and efficiencies of the Plastic Ball detector system on the theoretical data. We also acknowledge the help of Randy Rencsok in converting the programs for use on the FPS-164 Attached Processor at the MSU-NSCL's computing center and in demonstrating the stability of the VUU approach.

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Figure Captions

Fig. 1 Evolution of the distribution function in configuration space for Nb + Nb at b = 5 fm at rest, i.e. at E_{lab}= 0 (zero) MeV/nucleon, as a function of time:

a) left - instability of nuclei in the unbound cascade model as used by Cugnon and L'Hote;

b) right - the same calculation done with the Vlasov-Uehling-Uhlenbeck approach.[6,8,18]

Fig. 2 Flow angle distributions for Nb (400 MeV/nucleon) + Nb at impact parameters b = 1, 3, and 5 fm obtained from:

a) the bound version of Cugnon's intranuclear cascade;

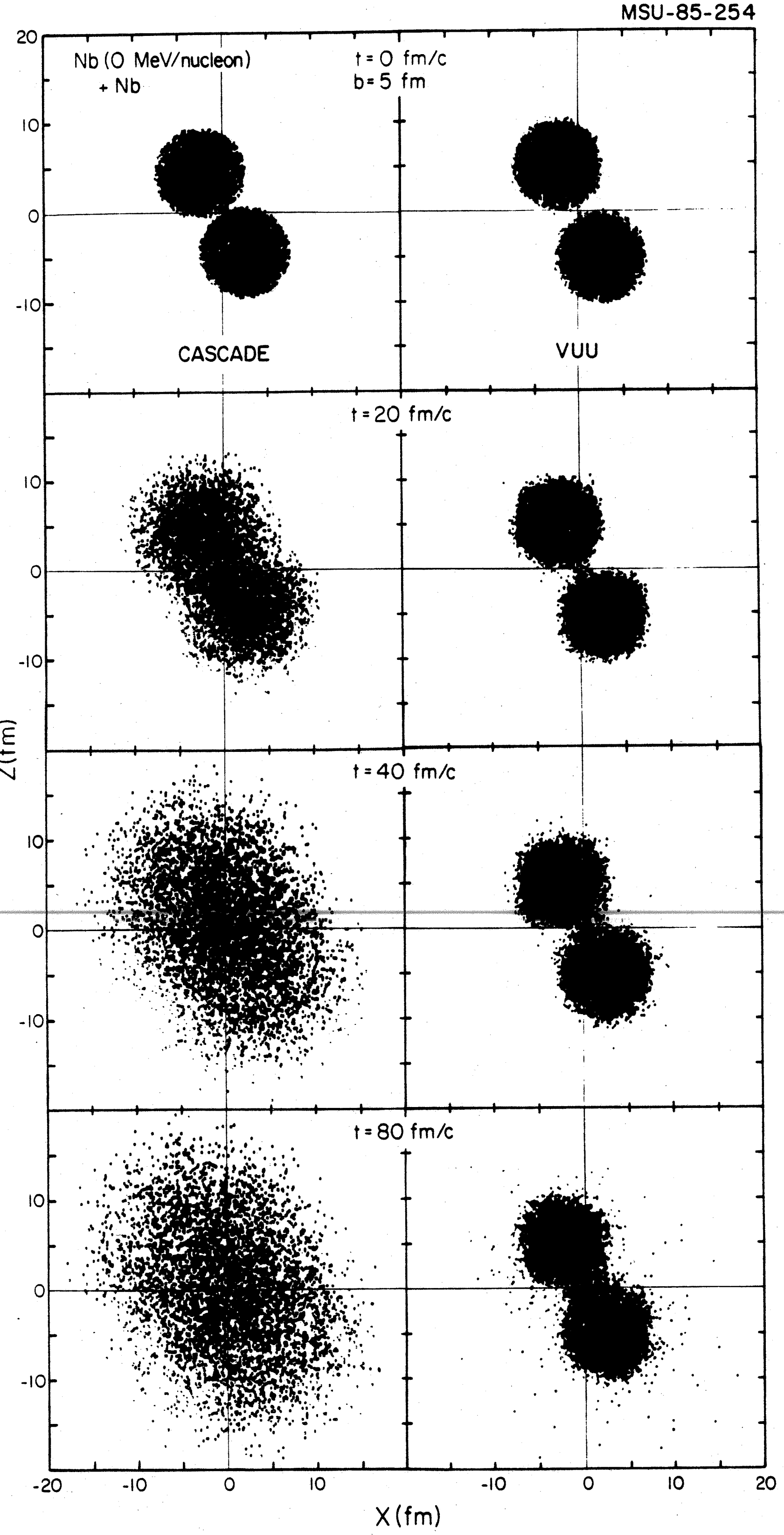
b) the unbound version of Cugnon's code.

Fig. 3 Flow angle distributions of only those particles which have undergone zero collisions, i.e. of the spectator nucleons, for Nb (400 MeV/nucleon) + Nb at b = 3 fm as resulting from the bound and the unbound cascade code, respectively.

Fig. 4 Dependence of the impact parameter distribution on the multiplicity bin chosen, if the bound cascade results are subjected to the Plastic Ball filter SIMDAT.

Fig. 5 Flow angle distribution for Nb (400 MeV/nucleon) + Nb binned according to the multiplicity of charges seen by the Plastic Ball for bound and unbound cascade, respectively. Also shown are the experimental data for the two highest multiplicity bins.

Fig. 6 Invariant cross section $d^2\sigma/dYd(p_x/m)$ for Nb (400 MeV/nucleon) + Nb in the reaction plane for $40 < M < 50$ for the bound and the unbound cascade, respectively.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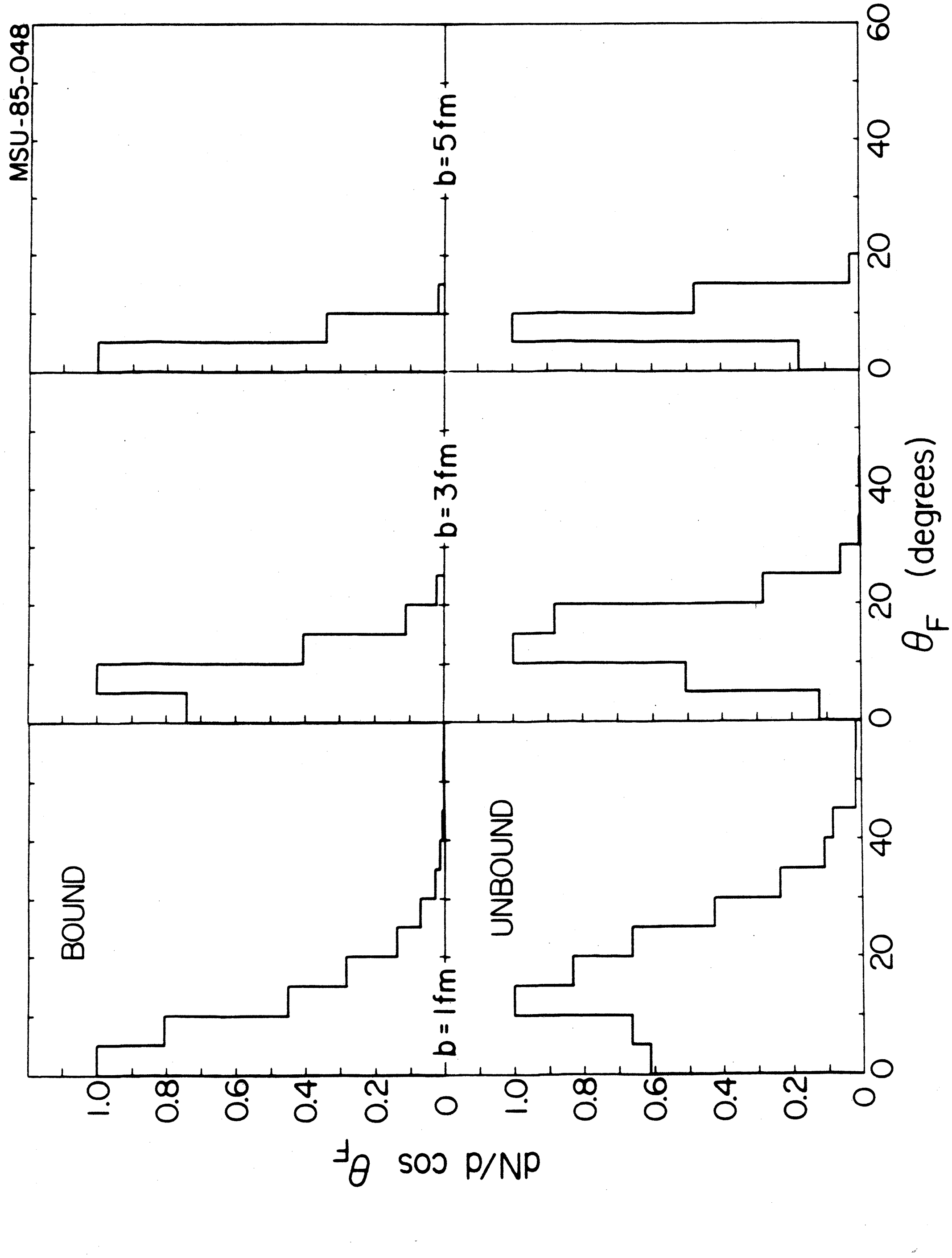
Nb (0 MeV/nucleon)
+ Nb
t = 0 fm/c
b = 5 fm
CASCADE
VUU
t = 20 fm/c
t = 40 fm/c
t = 80 fm/c
Z (fm)
X (fm)
20
10
0
-10
-20
10
20

FIGURE 1

FIGURE 2


MSU-85-048
BOUND
UNBOUND
b = 1 fm
b = 3 fm
b = 5 fm
dN/d cos θ_F
1.0
0.8
0.6
0.4
0.2
0
1.0
0.8
0.6
0.4
0.2
0
0
20
40
0
20
40
0
20
40
60
θ_F (degrees)

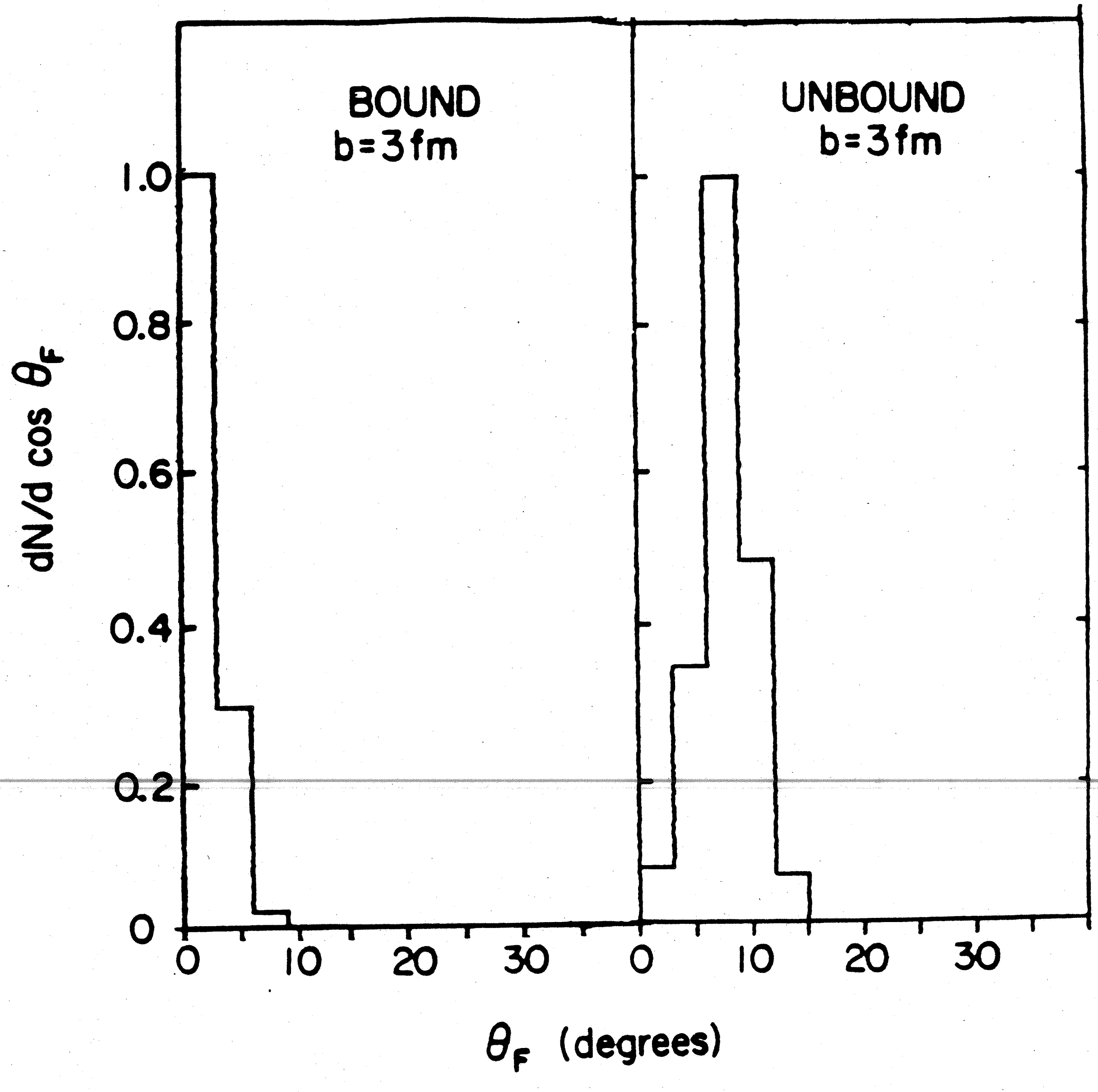
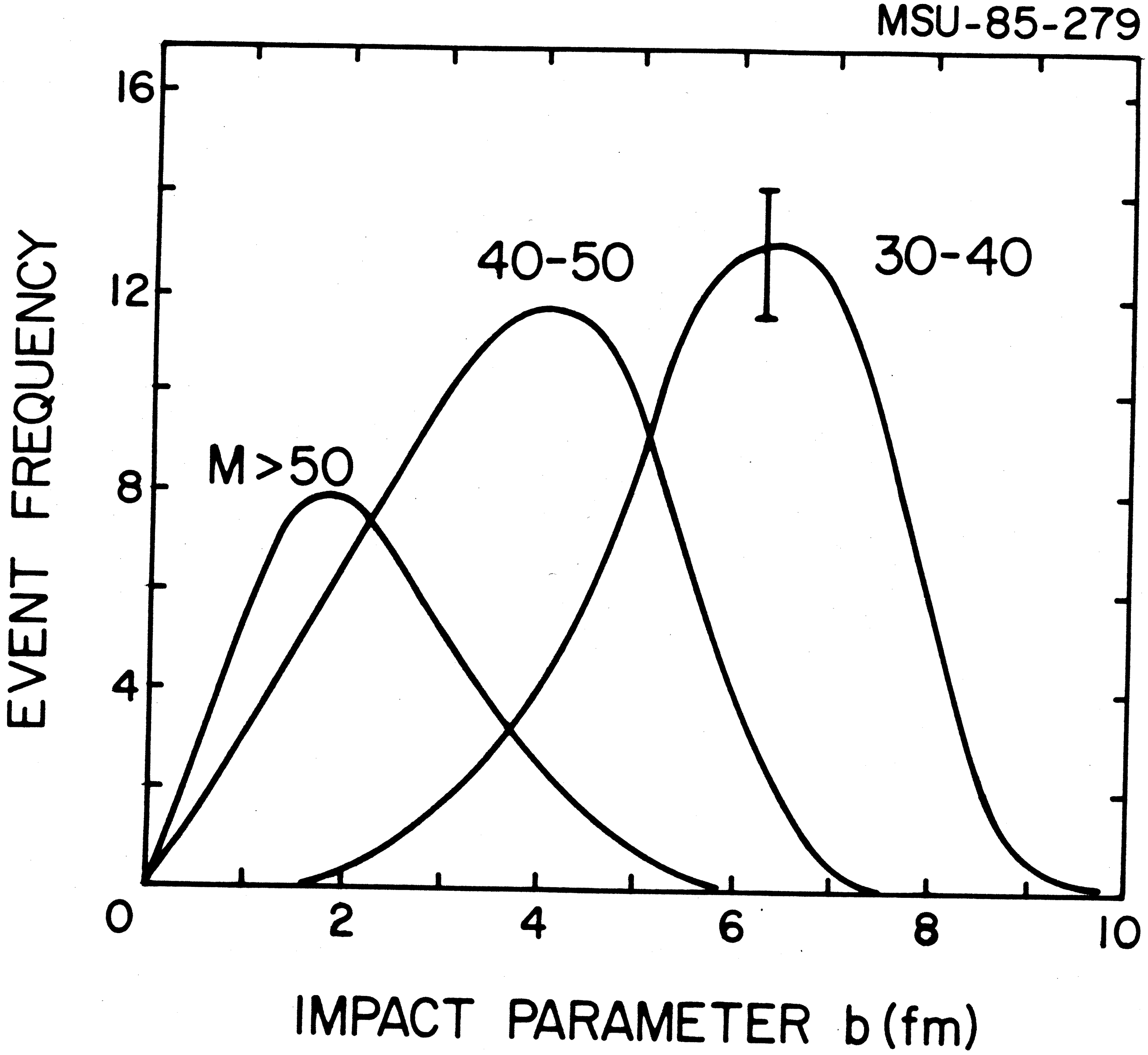

BOUND
b=3fm
UNBOUND
b=3fm
dN/d cos θ_F
1.0
0.8
0.6
0.4
0.2
0
0
10
20
30
0
10
20
30
θ_F (degrees)

FIGURE 3


MSU-85-279
EVENT FREQUENCY
16
12
8
4
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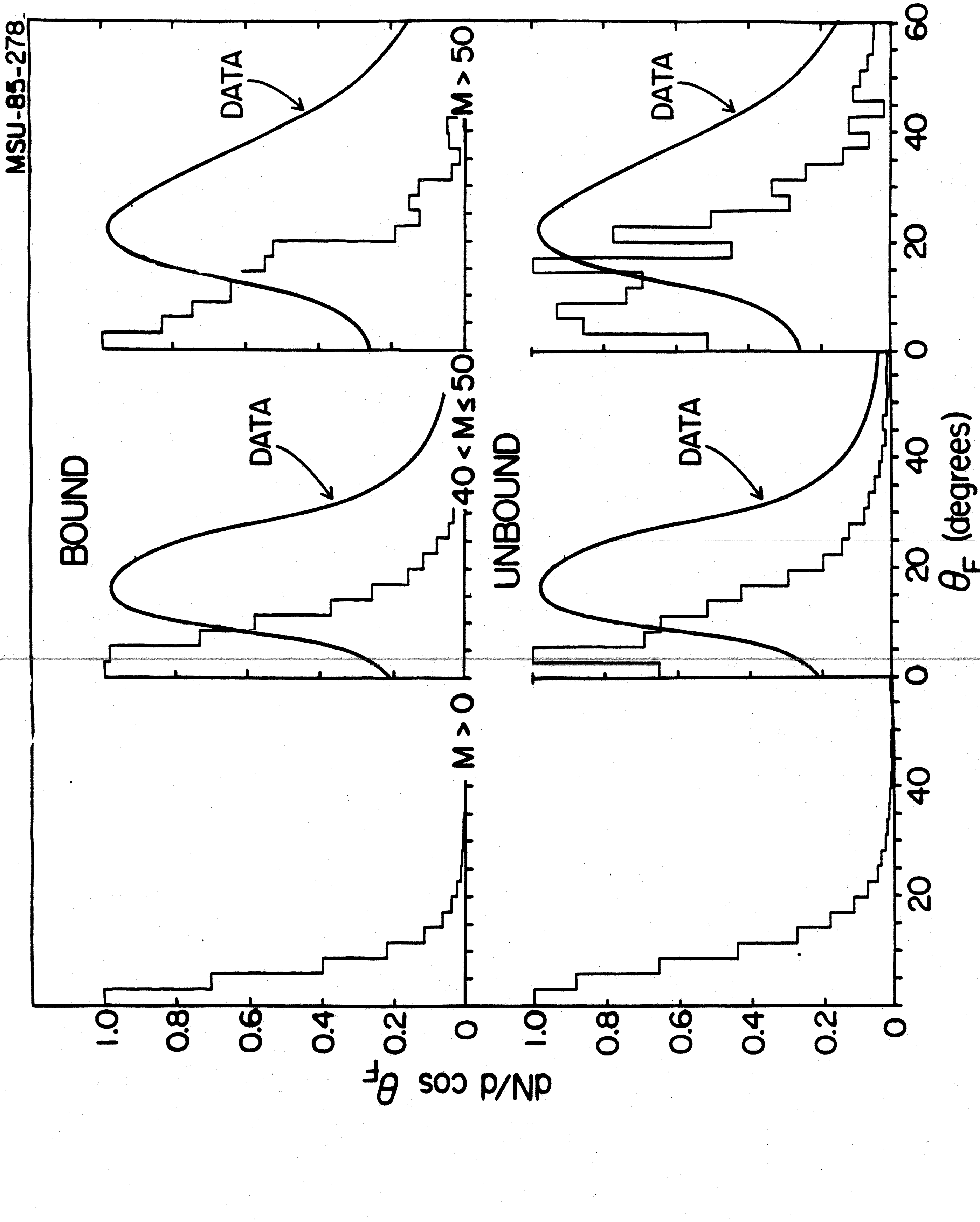
2
4
6
8
10
IMPACT PARAMETER b (fm)
M > 50
40-50
30-40

FIGURE 4

FIGURE 5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BOUND
UNBOUND
DATA
DATA
DATA
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M > 0
40 < M ≤ 50
M > 50
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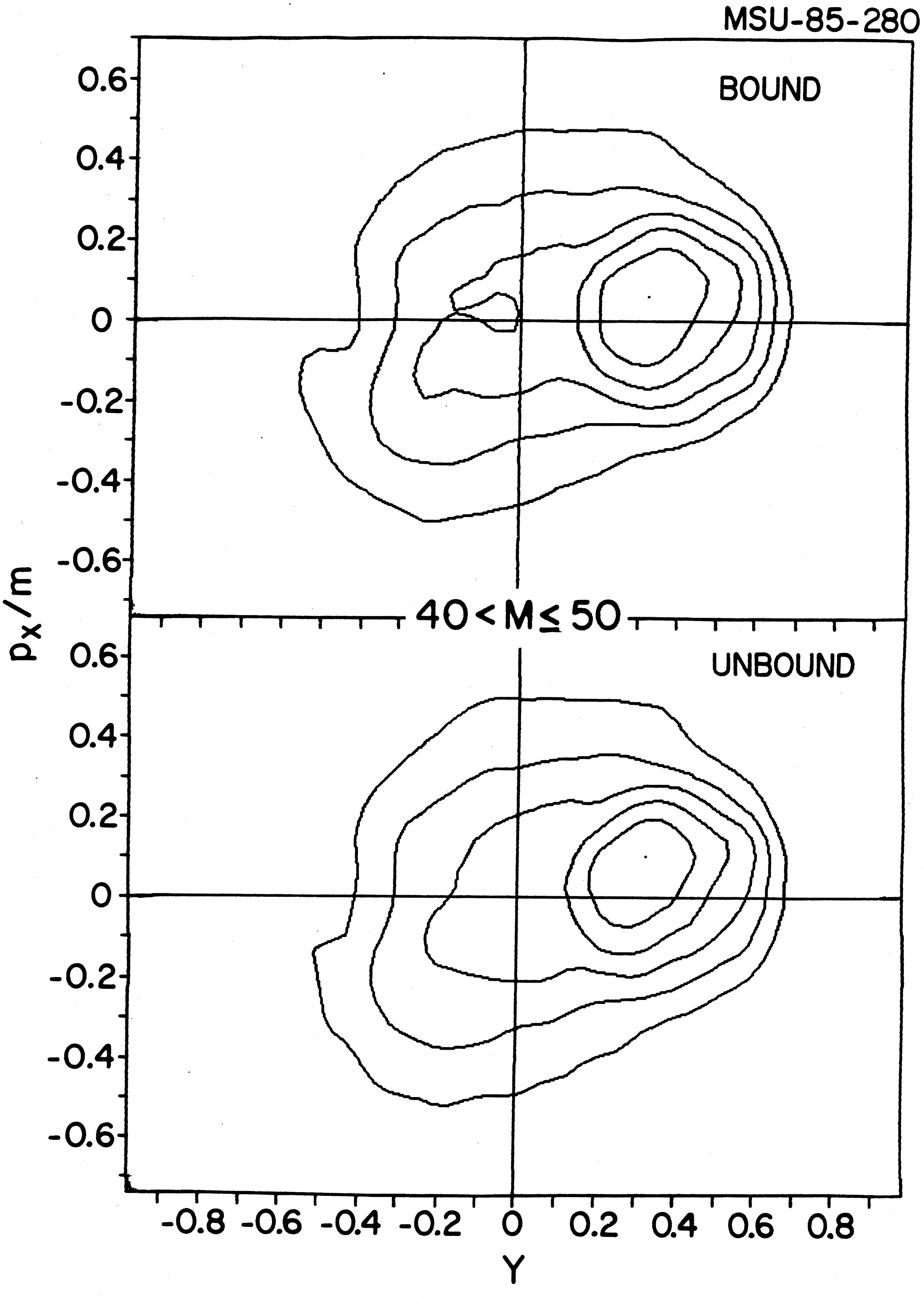
0.4
0.2
0
0
20
40
60
θ_F (degrees)
dN/d cos θ_F

FIGURE 6


MSU-85-280
BOUND
UNBOUND
40<M≤50
p_x/m
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0.4
0.2
0
-0.2
-0.4
-0.6
Y
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8