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Abstract:

A thermodynamically consistent treatment of the nuclear interaction is employed to study the dependence of pion production on the nuclear equation of state in heavy ion collisions. Massive baryon resonances, heavy mesons and the Bose condensation of pions are incorporated into a macrocanonical relativistic quantum- statistical treatment of the highly excited system. The measured pion multiplicities, which vary over eight orders of magnitude in the bombarding energy range from 30 MeV/nucleon to 4 GeV/nucleon, are reproduced within a simple one dimensional fluid dynamical model if it is assumed that nuclear matter is rather incompressible. The pion yields are in this model directly related to the compression energy, which amounts to one half of the total center of mass energy at all BEVALAC energies. The maximum compression derived is uncertain by about 10 % and 30 % at E_{lab}= 0.4 and 1.8 GeV/nucleon, respectively. The temperatures of the system in the moment of the chemical freeze out of the pion/delta degree of freedom are determined from the measured pion yields and range from 10 MeV to 100 MeV. An extrapolation to CERN/BNL energies, i.e. E_{lab}> 10 GeV/nucleon, yields T = 150 - 200 MeV. A strong energy dependence of cross sections and slopes of hard γ s are predicted by this model. The calculated photon yields are in surprising agreement with the data on γ - production at intermediate energies.

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One of the most important motivations for high energy heavy ion collisions is the possibility to determine the equation of state of nuclear matter at high densities and temperatures from controlled experiments in the laboratory. Fluid dynamical calculations[1,2] predict that the number of pions per baryon created in central collisions is directly related to the compression energy and it's density dependence. Therefore the pion yields may be used to extract the nuclear equation of state from experimental data. Recently the nuclear compression energy has for the first time been determined via a comparison of the pion multiplicities measured by the GSI/LBL Streamer Chamber collaboration for near central collisions of Ar (0.4-1.8 GeV/nucleon) + KCl[3] with cascade[4] and hydrodynamic[5] calculations. The basic ideas in this extraction procedure are[4,5]

1) The pion yield is determined by the thermal energy of the system at high density.

2) The compression energy is given by the difference between the center of mass energy and the thermal energy calculated from the pion yields.

3) The density achieved in a given collision is determined from a suitable dynamical model, say the cascade model[4] or the hydrodynamic model[5].

4) The density dependence of the compression energy is obtained by relating 2 and 3.

The line of argument rests on the assumption of an early chemical freeze-out of the pion degree of freedom, i.e. the pion plus Δ and N^* abundance is assumed to stay constant after the point of highest compression is reached[1,4]. This has been substantiated by calculations using the cascade model[4,6] and the Vlasov- Uehling- Uhlenbeck approach[7,8]. The pion yield would drop as a function of the density if the pions and deltas were in chemical equilibrium with the nucleons during an isentropic expansion of the system, where internal energy is converted into collective flow energy until the late, low density and low temperature phase of the reaction.[9,10]

The nuclear equation of state extracted from the data is surprisingly stiff[4,5]. Independent evidence for a stiff equation of state has emerged from a comparison of selfconsistent microscopic calculations[7,8] to recent 4π detector data on collective flow[11] and transverse momentum transfer[12] (and also on the pion yields[3]).

A simple classical relation between the pressure and the thermal energy has been used in ref.5. Relativistic Bose- Einstein and Fermi- Dirac statistics for pions, nucleons, and deltas, respectively, have been incorporated recently into the relativistic Rankine - Hugoniot approximation to predict the pion yields.[13]

In the present paper we develop a thermodynamically consistent treatment of the nuclear potential, which overcomes corresponding shortcomings in the literature, to study the dependence of pion production on the nuclear equation of state. We also analyze the importance of massive baryon resonances, heavy mesons and the Bose condensation of pions, which decrease the temperature in the system substantially. We study the connection between the pion multiplicities and thermodynamical quantities in the energy region of the BEVALAC and extract the temperatures in the moment of the chemical pion/delta freeze out from the data. We extend our calculations over the bombarding energy range between 30 MeV and 30 GeV per nucleon and compare the results with various inclusive and exclusive pion and hard photon measurements.

The main point of this paper is to extract the temperatures from the data, using only a relativistic quantum- statistical macrocanonical approach to describe the state of the system in the moment of the pion freeze-out. However, to check the consistency of the conclusions drawn in the literature, and to study the influence of resonances heavier than the $\Delta(1232)$ as well as Bose condensation on the extraction of the equation of state, we have to make the same simplifying assumption about the reaction dynamics and the pion production as were employed before[1,5,13]:

i) The pions emerge from the high density stage of the reaction with a multiplicity given by the chemical equilibrium value of the pion plus Δ and N^* abundance at the given thermal energy.

ii) All the available c.m. energy resides exclusively in thermal and compressional energy at one - namely the maximum achievable - density in the moment of the pion, Δ and N^* chemical freeze-out.

iii) The density and temperature in this moment of highest compression and excitation is given by a simplified one dimensional hydrodynamical calculation using the Rankine- Hugoniot equations for two colliding equal nuclei.[1,5]

Although these assumptions are qualitatively supported by cascade calculations[4,6], they are certainly not quantitatively fulfilled in the complicated reaction dynamics of an actual experiment. Nevertheless, they serve as a convenient method to guide our study of the principle physical effects of the compression on the bombarding energy dependence of the pion multiplicity.

Let us first be reminded of the relativistic Rankine-Hugoniot equation which is derived from the conservation of baryon number, energy and momentum on the two sides of a shock front :

$$W^2(\rho,T)-W_0^2 + P \{ \frac{W(\rho,T)}{\rho} - \frac{W_0}{\rho_0} \} = 0 \qquad (1)$$

where W_0= 939 MeV - B, B ≈ 16 MeV and $\rho_0 \approx 0.15/fm^3$. The Rankine equation relates the total energy per baryon, $W(\rho,T)$, in the high density phase - equated in the one dimensional model with the incident c.m. energy per nucleon - to the baryon density ρ. The pressure P is given by the isentropic derivative of W. Then all the properties of the high density region are given through the solution of eq.(1) for a prescribed constitutive equation, say $W(\rho,T)$. The total pion multiplicity is given as the sum of the pions which emerge from the decay of the baryonic and mesonic resonances taken into consideration plus the thermal gas of pions created in the reaction volume.

We have extended the calculations done in refs. 5 and 13 by including in our calculations the 23 lowest-lying well established nonstrange resonances of the nucleon, as well as the pion and the η-meson (m_η=549 MeV). We have also added the blackbody photon radiation to study the gamma yields in the thermal model. We will show below that the inclusion of the heavy resonances plays an important role for the achievable temperature and the absolute pion yields even at rather moderate BEVALAC energies. The finite width of the Δ (1232)-resonance is incorporated via a Lorentzian distribution in energy (see table 1). Masses and statistical weights of the nucleonic resonances are taken from ref. 14 and listed in table 1. We would like to point out that the number of pions per baryon emitted from most of the high-lying nucleonic resonances is substantially larger than one, which causes an increase of the pion multiplicities at higher energies (6 % at E_{lab}= 2 GeV/nucleon). The pion production rates are calculated from the branching ratios of ref. 14. The system Ar + KCl includes 76 baryons - this number determines the reaction volume and therefore the contribution of the Bose condensate states to the total pion yield - also this effect has been neglected in the previous treatments. We will see that the Bose condensate pions play a dominant role at intermediate and low bombarding energies. This Bose ground states and the high lying resonances increase the pion-multiplicity by 15% at 1 GeV, even more at other energies and must therefore be taken into account if the pion yields are to be used to extract the compressional energy. This had been neglected up to date.

To solve the Rankine- Hugoniot equation (1), the energy per baryon as a function of temperature and density $W(\rho,T)$ and the pressure $P(\rho,T)$ have to be related with a particular baryon density at a given total energy E_{lab}. A thermal energy can always be defined by subtracting the compression energy at zero temperature from the total energy per baryon at finite temperature:

$$W(\rho,T) = E_T(\rho,T) + E_C(\rho) + W_0 \qquad (2)$$

where

$$E_C(\rho) = W(\rho,T=0) - W(\rho_0,T=0) = W(\rho,T=0) - W_0 \qquad (3)$$

is defined to be the compressional energy. Two commonly used functional forms for $E_C(\rho)$ are taken from the extended liquid drop model of Scheid and Greiner[15]

$$E_C(\rho) = \frac{K_l(\rho-\rho_o)^2}{18\ \rho\rho_o} \qquad (4a)$$

$$E_C(\rho) = \frac{K_q(\rho-\rho_o)^2}{18\ \rho_o^2} \qquad (4b)$$

the first being referred to as the linear- and the second as the quadratic EOS, respectively, in accord with their different asymptotic increase with density.

The total energy per baryon can, on the other hand, also be written as an expression involving kinetic and potential terms and, using relativistic Fermi- and Bose-distributions, we get

$$W = U + \sum_{i=1}^{\sigma_b} \left(\frac{\overset{o}{\rho_i}\ m_i c^2}{\rho} + \frac{4\pi g_i}{(2\pi\hbar c)^3} \frac{1}{\rho} \int_{m_i c^2}^{\infty} \frac{\varepsilon^2\sqrt{(\varepsilon^2-m_i^2c^4)}}{\exp[\varepsilon/T]-1} d\varepsilon \right)$$

$$+ \sum_{i=\sigma_b+1}^{\sigma} \frac{4\pi g_i}{(2\pi\hbar c)^3} \frac{1}{\rho} \int_{m_i c^2}^{\infty} \frac{\varepsilon^2\sqrt{(\varepsilon^2-m_i^2c^4)}}{\exp[(\varepsilon+U-\mu)/T]+1} d\varepsilon \qquad (5)$$

where the first sum runs over the Bose-degrees of freedom, the pion, the η-meson and the photons, the second sum runs over the 24 states of the nucleon. We assume that all nucleonic resonances feel the same interaction energy per particle U, which depends only on the total baryon density ρ. The potential energy must be included into the Fermi- Dirac distribution function in a selfconsistent treatment. This has not been done in refs. 5 and 13, resulting in an error in the chemical potential, which does cancel out, however, when particle ratios are calculated. See ref. 16 for further details.

The baryons are assumed to be in chemical and thermal equilibrium and have the same chemical potential μ. Both the chemical potential and the interaction potential for the Bosons are taken to be equal to zero. ρ_i^o is the contribution of the Bose ground-state to the density of the boson-phases. The connection between the baryon density and the chemical potential reads

$$\rho = \sum_{i=\sigma_b+1}^{\sigma} \frac{4\pi g_i}{(2\pi\hbar c)^3} \int_{m_i c^2}^{\infty} \frac{\varepsilon \sqrt{(\varepsilon^2 - m_i^2 c^4)}}{\exp[(\varepsilon+U-\mu)/T]+1} d\varepsilon \tag{6}$$

The number of mesons can be calculated via

$$N_i = \frac{g_i}{\exp(m_i c^2/T) - 1} + \frac{4\pi g_i V}{(2\pi\hbar c)^3} \int_{m_i c^2}^{\infty} \frac{\varepsilon \sqrt{(\varepsilon^2 - m_i^2 c^4)}}{\exp[\varepsilon/T]-1} d\varepsilon \tag{7}$$

We also need the connection between U and the compression energy E_C. For $T \to 0$, eq. (6) becomes

$$(\mu-U)^2 = m^2 c^4 + (\rho C/g)^{2/3} \tag{8}$$

with $C = 6\pi^2(\hbar c)^3$. In the same limit, we get for eq. (5)

$$W(T=0) = 0.75\ X + U$$

$$+ \frac{m^2 c^4}{8} \left\{ \frac{3X}{X1^2} - \frac{3m^2 c^4}{X1^3} \ln[(X + X1)/mc^2] \right\} \tag{9}$$

with $g=4$, $mc^2=939$ MeV, $X1=(\rho C/g)^{1/3}$, $X=\sqrt{(m^2c^4 + X1^2)}$, assuming that for T=0 only the nucleonic ground state is populated, which should be true for small densities. This point can be questioned, if the nucleon-Δ-interaction is much stronger than the nucleon-nucleon-interaction[17]. Expanding (8) and (9) for small densities, we obtain the well known approximation for the energy

$$\mu-U \approx mc^2 + 0.5 \frac{X1^2}{mc^2} + \ldots \tag{10}$$

$$W \approx U + mc^2 + 0.3\,(\rho/\rho_0)^{2/3}\hbar^2/m\,(6\pi^2\rho_0/g)^{2/3} + \ldots \tag{11}$$

The difference between the exact expression (9) and the approximation (11), which was used in ref. 13, is only about 1 MeV for $\rho/\rho_0=3$. By comparing (2) end (9) we finally get the relation between U and E_C:

$$U(\rho) = E_C + W_0 - 0.75\,X - \frac{m^2c^4}{8}\left\{\frac{3X}{X1^2} - \frac{3m^2c^4}{X1^3}\ln[(X + X1)/mc^2]\right\} \tag{12}$$

The pressure is given by[16]

$$P = -T\sum_{i=1}^{\sigma_b}\frac{4\pi g_i}{(2\pi\hbar c)^3}\int_{m_ic^2}^{\infty}\varepsilon\sqrt{(\varepsilon^2-m_i^2c^4)}\,\ln(1-\exp[-\varepsilon/T])\,d\varepsilon$$
$$+\,T\sum_{i=\sigma_b+1}^{\sigma}\frac{4\pi g_i}{(2\pi\hbar c)^3}\int_{m_ic^2}^{\infty}\varepsilon\sqrt{(\varepsilon^2-m_i^2c^4)}\,\ln(1+\exp[(\mu-U-\varepsilon)/T])\,d\varepsilon$$
$$+\,\rho^2\frac{\partial U}{\partial\rho} \tag{13}$$

and the entropy per baryon is

$$S/N_B = \frac{P/\rho + W - \mu}{T} - \frac{\rho}{T}\frac{\partial U}{\partial \rho} - 1/N_B\sum_{i=1}^{\sigma_b} g_i\ln(1-\exp[-m_ic^2/T])$$
$$= -\frac{\partial\Omega}{\partial T/\mu, V} \tag{14}$$

Let us now turn to the resulting pion yields as a function of the bombarding energy. Figure 1 shows the total pion multiplicities per baryon, the pion abundance after the decay of the baryonic resonances, versus the bombarding energy, calculated with the linear and the quadratic equations of state (4a) and (4b), respectively. Different values of the compression constant K are compared with the data of ref. 3. The results for the different functional forms are almost the same, if we compare the yields for K_l=1000 and 1400 MeV with the yields for K_q=600 and 800 MeV. If one wonders about the large values of K one should keep in mind that these compression constants are fitted for densities far away from normal nuclear matter density, where the ground state compression constant obtained from giant monopole resonances is K ≈ 210 ± 50 MeV.

We find that the linear equation of state, (4a), with K_l= 1400 MeV and the quadratic one, (4b), with K_q=800 MeV (4b) fit the observed pion yields over the whole BEVALAC bombarding energy range. Therefore, it is difficult to distinguish between the two functional forms for $E_C(\rho)$ by means of the pion multiplicities of ref. 3.

This should not disturb us, though, in the search for $E_C(\rho)$, as long as both the compression energy and the density obtained at the bombarding energies E_{lab} of the data points are in good agreement for both functional forms with K_l=1400 MeV and K_q=800 MeV, respectively. To examine this point, we have calculated for these two best fitting equations of state the compressional energy $E_C(E_{cm})$ and the density $\rho(E_{lab})$.

We find that the compressional energy is nearly identical for the two distinct best fitting equations of state, namely 50 % of the total available center of mass energy for all BEVALAC energies. The calculated densities are in good agreement at medium energies, but they differ by about 30 % at E_{lab}= 2 GeV/nucleon (see Figs. 1c and 2a). The compression energy extracted via this method is an upper bound: A delayed chemical freeze-out of the pion plus Δ and N^* degrees of freedom will yield a nonzero flow energy in the moment of freeze-out, which would lower the obtained compression energy.[9,10] However, as pointed out above, cascade calculations imply that this is a rather small effect.

Let us now take a look from a different point of view, leaving aside the dynamical treatment of the shock compression in the reaction. Now, we do not try to derive the nuclear EOS from the energy dependence of the pion yields but inspect the temperature and density dependence of the pion plus Δ and N^* abundance directly using the quantum- statistical approach only. This yields an extraction of the temperature of the system in the moment of chemical freeze-out as shown below. In Fig. 2b the total pion yields per baryon, including those from the decay of baryonic resonances, are displayed as a function of the temperature for two fixed densities, ρ/ρ_0=1 and 4, respectively. The yields rise extremely rapidly with temperature, in particular at moderate temperatures up to T ≈ 100 MeV. Note that the large difference in the densities has a negligible influence on the pion yields. Therefore, we can relate the pion yield per baryon directly to the temperature of the system in the moment of the chemical pion, Δ and N^* freeze- out. This one-to-one correlation gives the temperature as a function of the experimentally observed pion multiplicities in the moment of the chemical freeze-out of the pions and the baryonic resonances, which is close to the moment of highest density - therefore the derived temperature is close to the highest one in the shock-zone itself. Once the temperature is fixed by means of the experimental data we can compute the thermal energy and, via the available energy E_{cm}, also the nonthermal (compression plus flow) energy, which in the simple one dimensional Rankine- Hugoniot model is set equal to E_C.

We have extended our calculations to other energies. Only the inclusive pion cross sections are measured to date at low energies, where the absolute yields become exceedingly small. However, it was shown[3,5] that the number of pions observed in a collision at fixed energy depends linearly on the number of participant protons, at least for E_{lab} > 400 MeV/nucleon. Therefore the ratio of pions to participant protons in high multiplicity events is the same as the ratio of the inclusive pion cross section to the inclusive cross section of participant protons, where one has integrated over all impact parameters. This is supported by the inclusive data[18] : the ratio of the inclusive cross sections for Ar + KCl at 800 MeV/nucleon coincides with the results of ref. 3. The inclusive data[18] also show that $\sigma(\pi)/\sigma(Z)$ is nearly independent of the examined symmetrical system. Almost the same ratios are obtained for ^{12}C + ^{12}C, ^{20}Ne + NaF and ^{40}Ar + KCl[18,19]. The result can be used to establish a simple procedure for translating inclusive pion cross-sections for these three systems into pion multiplicities per nucleon for central events: take the ratio of the pion cross-section and two times ($A \approx 2Z$) the cross-section for the participant protons of the examined system, with the assumption that the participant proton cross section is roughly energy independent[18], which is of course the basis of the participant spectator model. Remember that the participant protons at intermediate energies are to a large extend emitted as protons bound in intermediate mass fragments, and can not be observed as free protons.

The inclusive pion yields of refs. 20,21 can then be added to the exclusive data of ref. 3,5,22. The results are shown in Fig. 3a.

The validity of the first assumption in this procedure, the linear dependence of the pion multiplicity on the number of participant protons, is doubtful at very low energies: To ensure energy conservation, many nucleons must necessarily share their energy to produce a pion. This must lead to an increase of the pion yield with the number of participants which is steeper than linear. Since the inclusive data only reflect the multiplicity integrated cross section, the assumption of a linear dependence underestimates the pion yields per nucleon for head on collisions. This effect increases with decreasing bombarding energy, which means the pion "data" shown at the lowest energies in Fig. 3a are underestimated.

Also shown in Fig. 3a are the results of the simple shock calculations discussed above. The low energy data points are mostly measured in the ^{12}C + ^{12}C system, where Bose condensation plays an important role[23]. Therefore, these calculations were done for the ^{12}C - ^{12}C system. We show only the results for the quadratic equation of state. It is important to point out that the linear equation of state with the same compression constant gives almost exactly the same results! Therefore we observe that the nuclear equation of state is at the lower energies rather well determined by the compression constant alone. This is, of course, due to the fact that we deal here with densities not to far from ρ_0 : the absolute value, the first derivative and - via the compression constant - the second derivative of the EOS at $\rho=\rho_0$ are fixed. It seems that we can get a reasonable overall description of these data with a compression constant of 250 MeV, which is close to the values extracted from the breathing mode of the giant resonance, K = 210 MeV. One must keep in mind, though, that the low energy pion multiplicities extracted from the inclusive yields are probably underestimated. It seems to be necessary to introduce an additional mechanism for pion production, for example the pion Bremsstrahlung model[24], which has recently been shown to explain the angular distribution of the subthreshold pions rather well.

The hard photon cross sections for the ^{12}C - ^{12}C system, calculated from the photon yield per baryon with the same procedure as described above for the pions, are shown in Fig. 3b. Since the total photon energies are lower - due to the missing mass - than those of the pions, the transformation of the calculated numbers to inclusive cross sections is rather well justified even at the lower bombarding energies. Also shown are recent data which have been extracted assuming an isotropic differential cross-section[25]. The yields shown in Fig. 3b include, due to experimental uncertainties at the lowest energies, only photons with energies larger than 30 MeV. We would expect that the experiment gives much less photons than the theory, as a result of the small electromagnetic coupling constant, which would imply a largely increased equilibration time necessary for statistical equilibrium to be established. In this respect it is surprising that the photon yields are overestimated only by about a factor 1.5 to 3.5 in this simple model calculation. Even more surprising is that the lower energies are reproduced better than the higher energies. One may speculate that this is due to the shorter collision time at higher energies.

In Fig. 4a we show the different contributions to the pion yields and their energy dependence for the ^{12}C + ^{12}C system. For energies below 30 MeV/nucleon the Bose-condensed state alone acounts for roughly 90% of the pion yield. In the BEVALAC energy region, on the other hand, the dominant contribution is due to the Δ-resonance. It is of course a question whether the ground state pions can at all be detected experimentally, since they occupy a zero momentum quantum state, i.e. they are at rest with respect to the system. However, the nucleons which represent the main constituents of the system and therefore dominate the center of mass motion of the system, exhibit a finite flow velocity - therefore even the zero momentum pions ought to have a finite transverse velocity, which corresponds to a very small transverse kinetic energy ($m_\pi/m_N \approx 1/7$). This could show up in the pion spectra at low bombarding energies by a steep increase of the cross section at low pion momenta. Low energy pions are hard to detect and therefore their yield has been neglected in the pion multiplicities shown in Fig. 3a. The high lying nuclear resonances, which were just as the Bose ground state and the photons neglected in refs. 5 and 13, become quite important at bombarding energies around 1 GeV/nucleon and are the main pion source at bombarding energies above 10 GeV/nucleon.

The main information which can be extracted from pion multiplicities, namely the thermal energy content and the temperature of the system at the moment of the pion freeze-out, is shown in Fig. 4b. Here the temperatures are not calculated via the shock model with a trial EOS, but are directly taken from Fig. 2b. Even if the experimental errors and the uncertainty in the freeze-out density are included the temperatures are rather well determined, especially at the low energies, where the variation of the pion yield with the temperature is very rapid (see Fig. 2b).

We would like to point out that the extrapolation of the temperatures from present experiments with $E_{lab} < 4$ GeV/nucleon (Fig.4b) to higher energies yields temperature values of 150 to 200 MeV, i.e. the Hagedorn-temperature[10], for energies of $E_{lab} \approx 10 - 20$ GeV/nucleon. This is in the reach of the relativistic heavy ion accelerator projects at CERN and Brockhaven.

The thus extracted temperatures coincide at the high energies with the slope parameters of ref. 18,21. We would like to point out that this agreement is purely coincidental. The pion slope "temperature" is not related with the fireball temperature[26], except that it rises with incident energy. Its shape is the result of a superposition of free pions and pions from the decay of baryonic resonances. One could compare our temperature, extracted from the total pion yield, with the "proton temperature"[26], which is much higher than the slope parameter of the pions, "T_{proton}" > 120 MeV at 1.8 GeV/nucleon. However, one should keep in mind that not only most of the pions, but also many of the protons stem from the decay of baryonic resonances, disturbing the picture of a pure "Boltzmann slope" of the proton spectra.

In conclusion we have pointed out that pion multiplicities can provide information on the nuclear equation of state. The compression energy and the density can be estimated from the pion yields via models like the Rankine-Hugoniot equation. The temperature of the system can be extracted from the data as a function of the bombarding energy. Calculations with the nuclear fluid dynamic model reproduce the pion yields rather well over 8 orders of magnitude. The photon yields, which have been measured recently at intermediate energies, are overestimated, but in the right ball park. This possibly indicates that the electromagnetic interaction may not be quite sufficient to reach statistical equilibrium.

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Table 1

Mass[MeV]	stat.weight	pion-production rate		
939.	4	0		
1110.4	0.1616	1	)	
1171.2	2.6484	1	)	
1232.	8.655	1	)	Δ-Resonance
1292.8	3.154	1	)	
1353.6	1.381	1	)	
1440.	4	1.3		
1520.	8	1.45		
1535.	4	1.05		
1620.	8	1.7		
1650.	4	1.55		
1675.	12	1.65		
1680.	12	1.4		
1700	8	1.85		
1700	16	1.85		
1710	4	1.9		
1720	8	1.7		
1900	8	2.		
1905	24	1.8		
1910	8	1.95		
1920	16	1.8		
1930	24	1.9		
1950	32	1.6		
2190	16	1.8		
2220	20	1.8		
2250	20	1.9		
2420	48	1.9		
2600	24	1.95		

Figure captions

1.a Pion multiplicities per nucleon versus bombarding energy, calculated with the linear equation of state (4a) and the compression constants K_l=1000 MeV, 1400 MeV and 1800 MeV for the solid, dashed and dotted line, respectively. Circles indicate the data points of ref.3. Figure b displays the results for the EOS (4b) with K_q= 600 MeV, 800 MeV and 1000 MeV for the solid, dashed and dotted line. Figure 1c shows the density achieved in the shock-zone versus the bombarding energy for the two best fitting EOS with K_l = 1400 MeV (solid line) and K_q = 800 MeV (dashed line). The circles and triangles indicate the energies of the measurements of ref.3.

2.a The best fitting equations of state. Compression energy per baryon versus baryon density is drawn, the meaning of lines, circles and triangles is the same as in Fig. 1c. In Figure b we have drawn pion multiplicities versus the temperature for baryon densities one time (solid line) and four times (dashed line) normal nuclear matter density. The curves describe the properties of a hot and dense piece of nuclear matter, no shock-equation needs to be solved.

3.a Pion multiplicities, calculated from neutral (triangles) and charged (squares) pion-cross sections as described in the text are compared to the results of a shock calculation with the quadratic EOS and K_q = 250 MeV (solid line) or 500 MeV (dashed line), together with exclusive data (circles) from refs. 3 and 22. The charged and the neutral pion cross sections are taken from ref. 20 and 21, respectively. In Figure b we have drawn the energy integrated photon cross-section from ref. 25 (circles) and the results of shock-calculations with K_l = K_q = 250 MeV for the solid and the dashed line, respectively. The data and the calculations include only photons with energies larger than 30 MeV.

4.a Contributions to the pion yields per baryon versus the bombarding energy for a $^{12}C - ^{12}C$ system, calculated with the quadratic EOS (4b) and K_q = 800 MeV. The solid line shows the ratio of the pions from the Bose ground state to the total amount of pions, the dashed line refers to the free thermal pions, the dotted line to the contributions from the Δ-resonance and the dashed-dotted one to higher resonances with mass larger than 1.3 GeV. Figure b shows the freeze-out temperatures of the pions, calculated from the pion multiplicity data per nucleon, versus the bombarding energy. For $E_{lab} <$ 400 MeV we have assumed a freeze-out density between 1 and 3 ρ_0, for higher energies between 2 and 5 ρ_0.

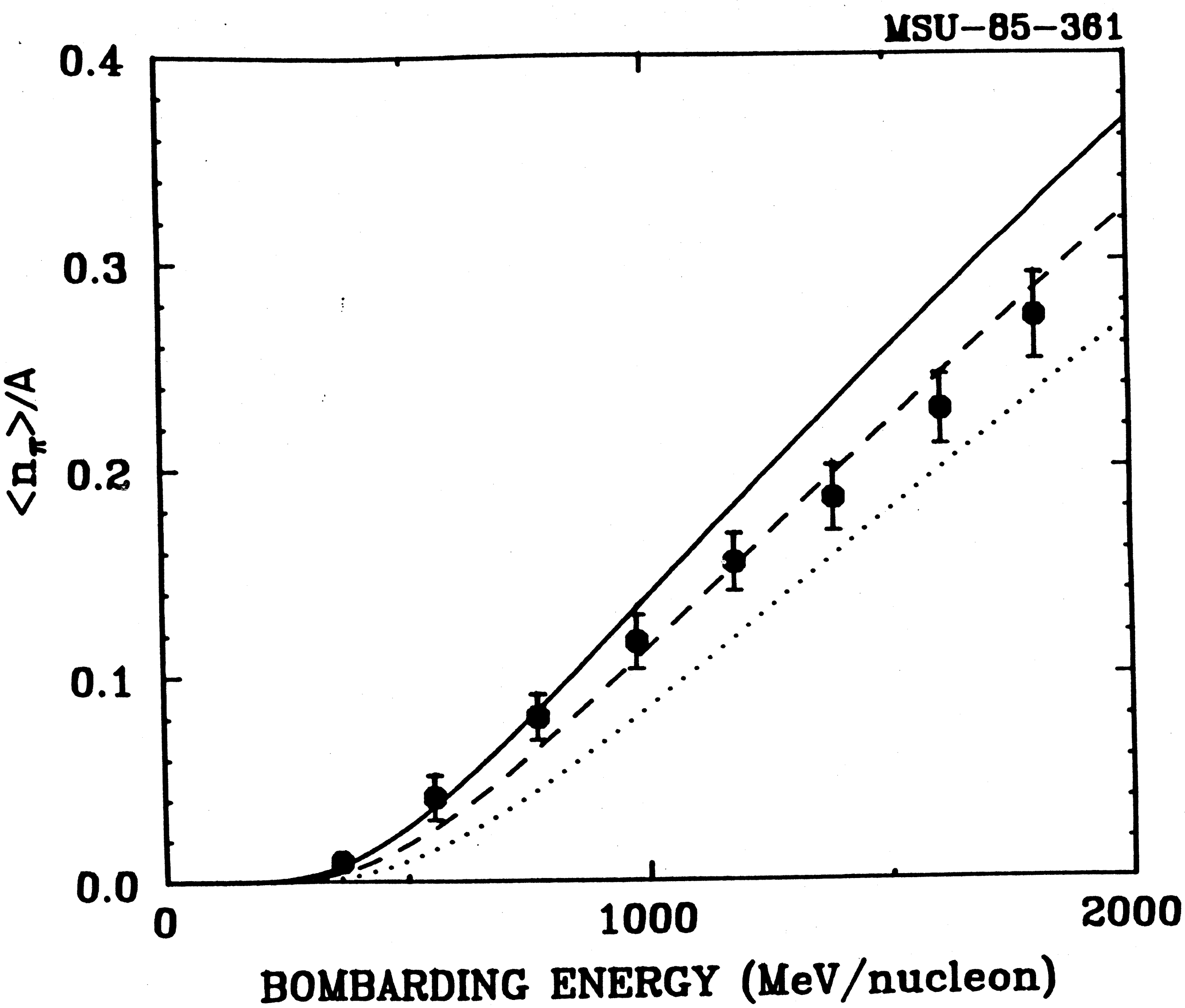
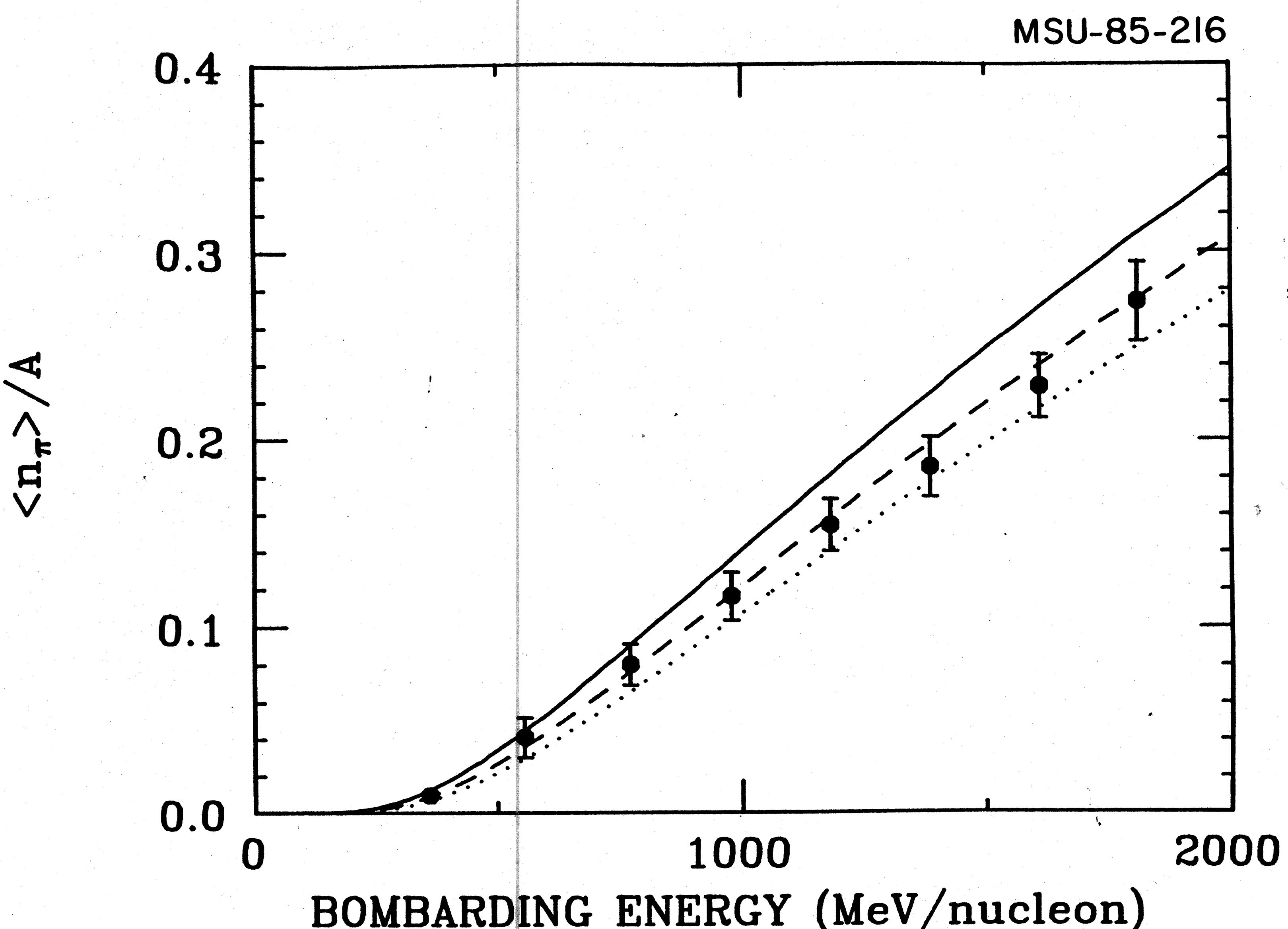

MSU-85-361
0.4
0.3
0.2
0.1
0.0
<n_π>/A
0
1000
2000
BOMBARDING ENERGY (MeV/nucleon)

FIGURE 1A



MSU-85-216
0.4
0.3
0.2
0.1
0.0
<n_π>/A
0
1000
2000
BOMBARDING ENERGY (MeV/nucleon)

FIGURE 1B

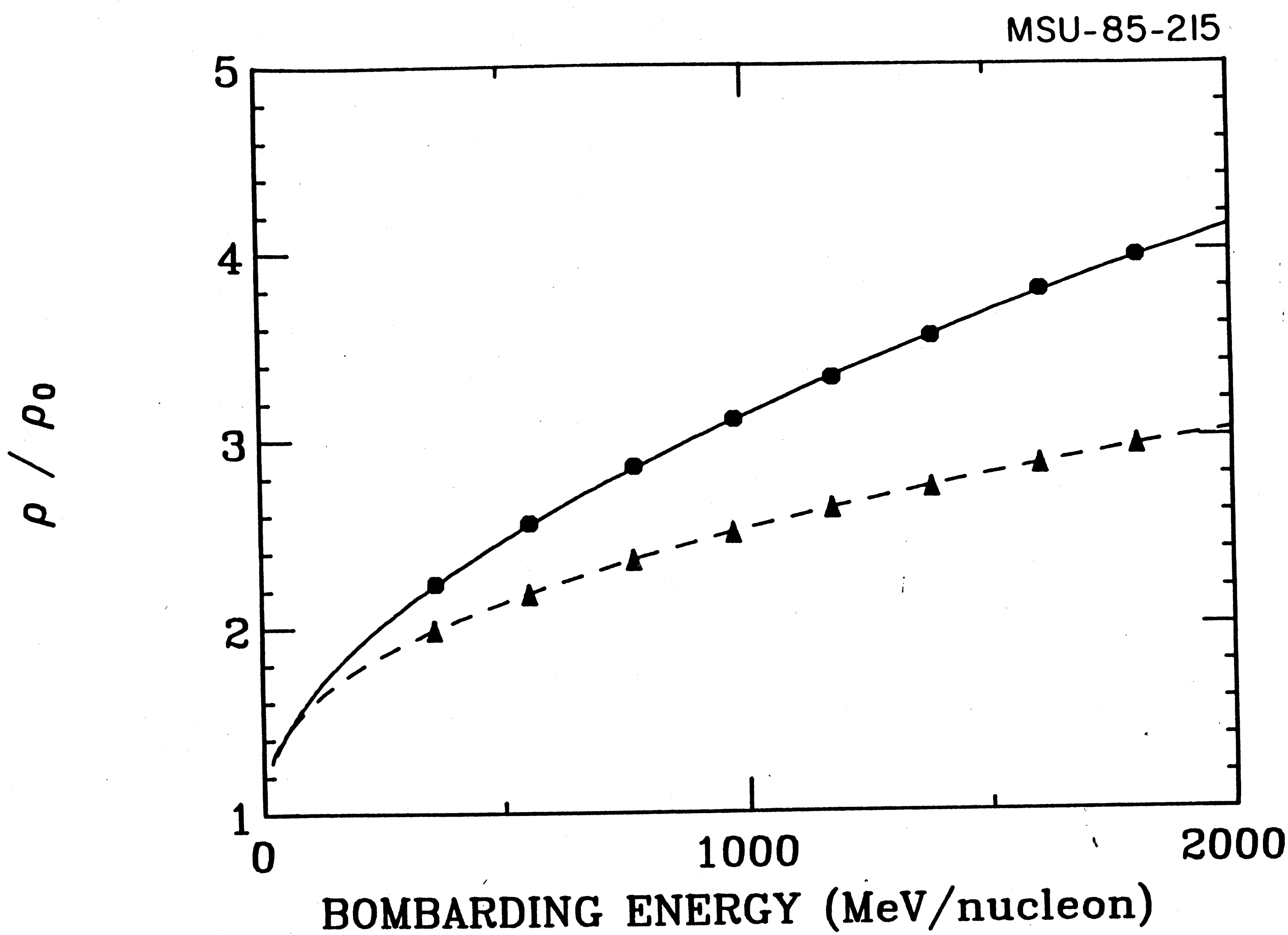

MSU-85-215
5
4
3
2
1
0
1000
2000
ρ / ρ0
BOMBARDING ENERGY (MeV/nucleon)

FIGURE 1C

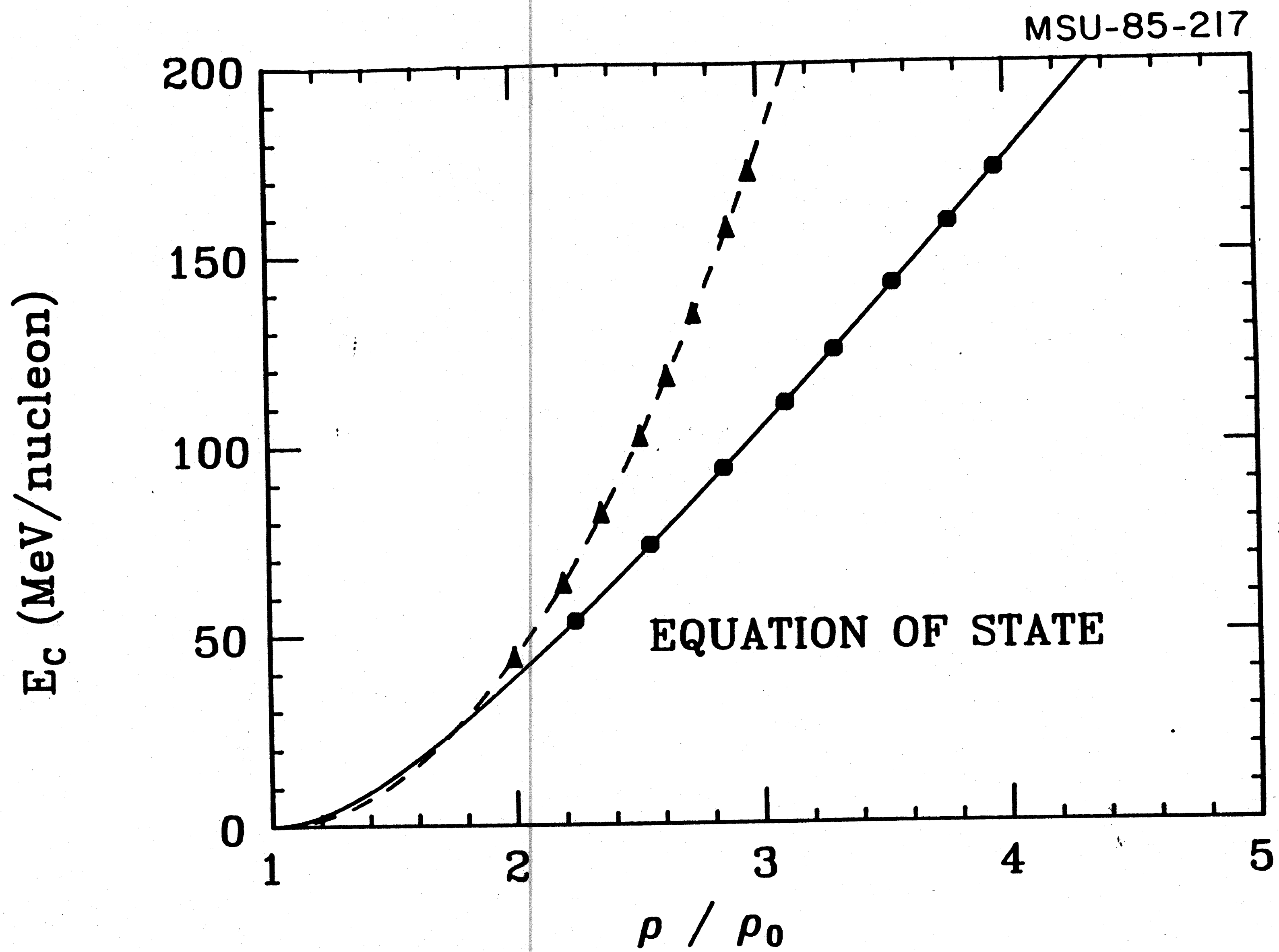

MSU-85-217
200
150
100
50
0
1
2
3
4
5
E_c (MeV/nucleon)
ρ / ρ_0
EQUATION OF STATE

FIGURE 2A

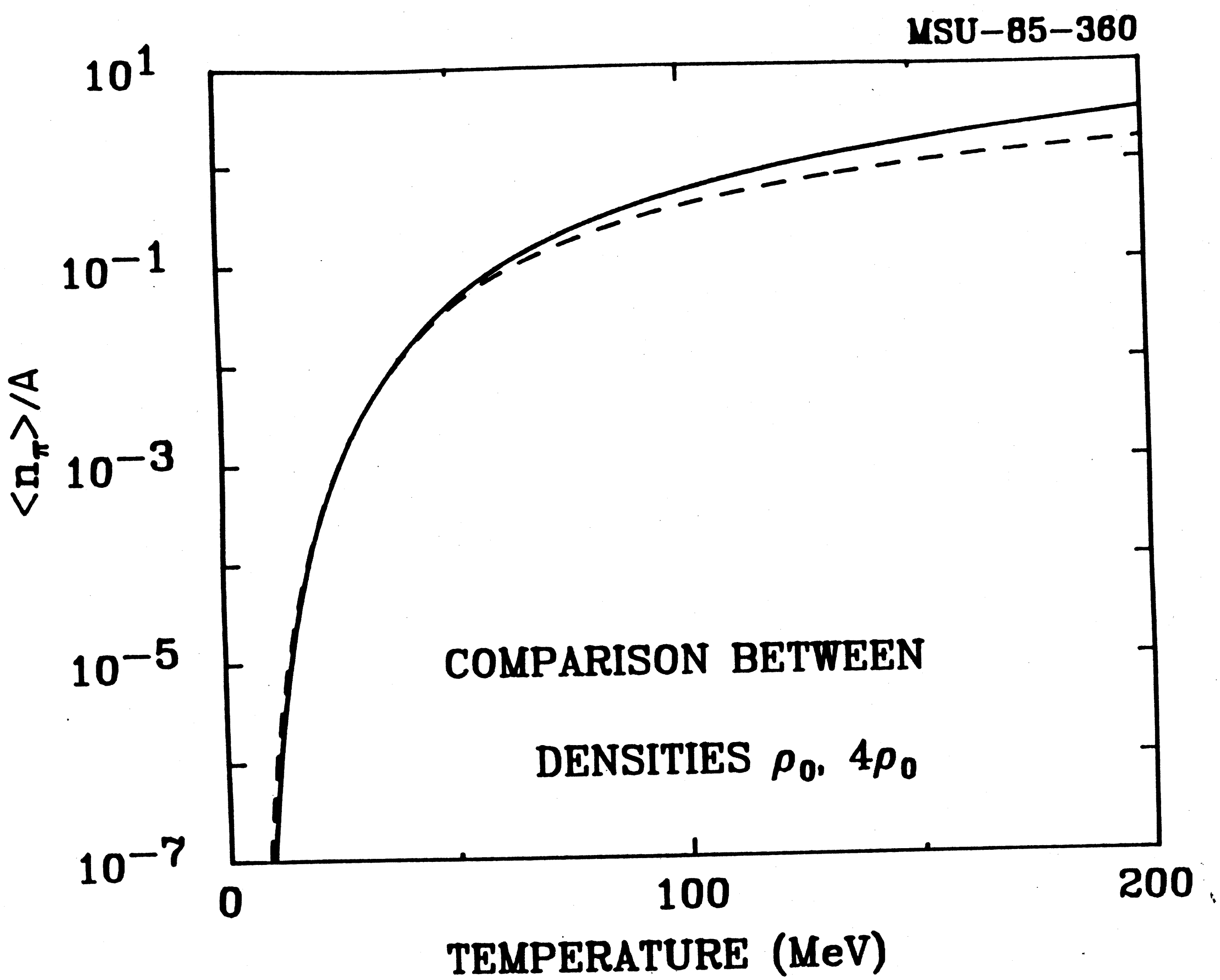
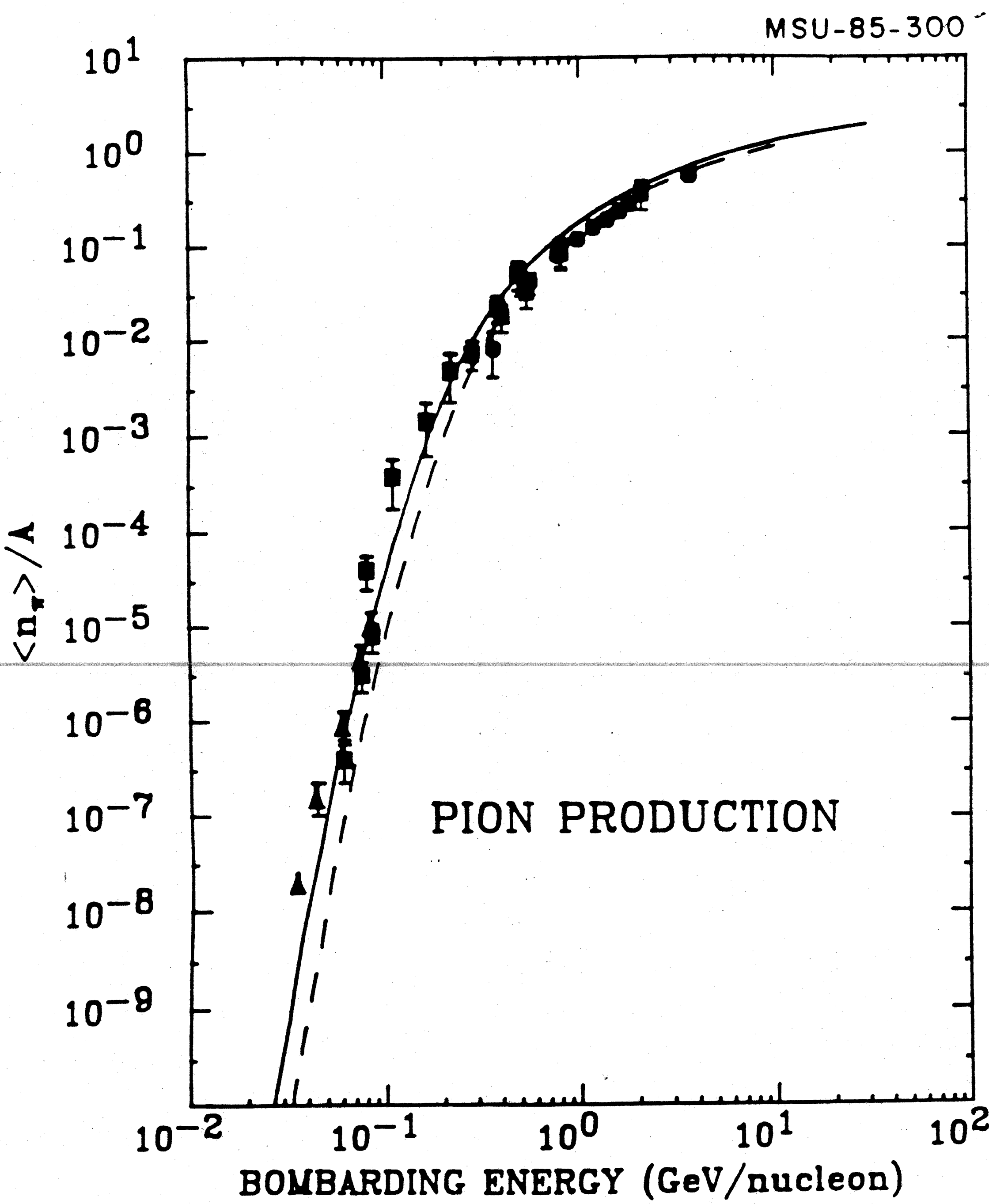

MSU-85-360
10^1
10^{-1}
10^{-3}
10^{-5}
10^{-7}
$\langle n_\pi \rangle / A$
COMPARISON BETWEEN
DENSITIES ρ_0, $4\rho_0$
0
100
200
TEMPERATURE (MeV)

FIGURE 2B



MSU-85-300
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
<n_π>/A
PION PRODUCTION
10^-2
10^-1
10^0
10^1
10^2
BOMBARDING ENERGY (GeV/nucleon)

FIGURE 3A

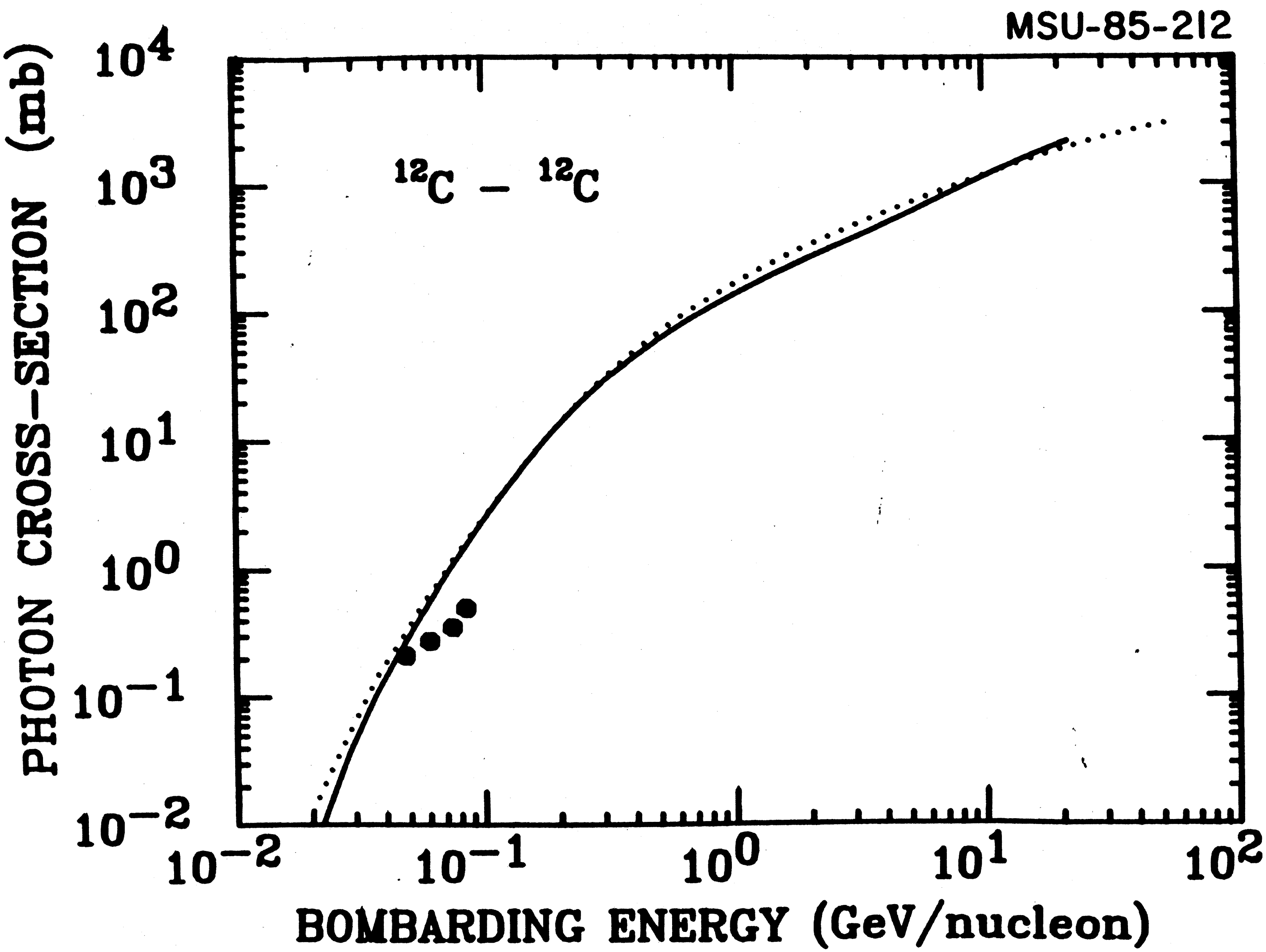

MSU-85-212
$^{12}C - {}^{12}C$
PHOTON CROSS-SECTION (mb)
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
BOMBARDING ENERGY (GeV/nucleon)

FIGURE 3B

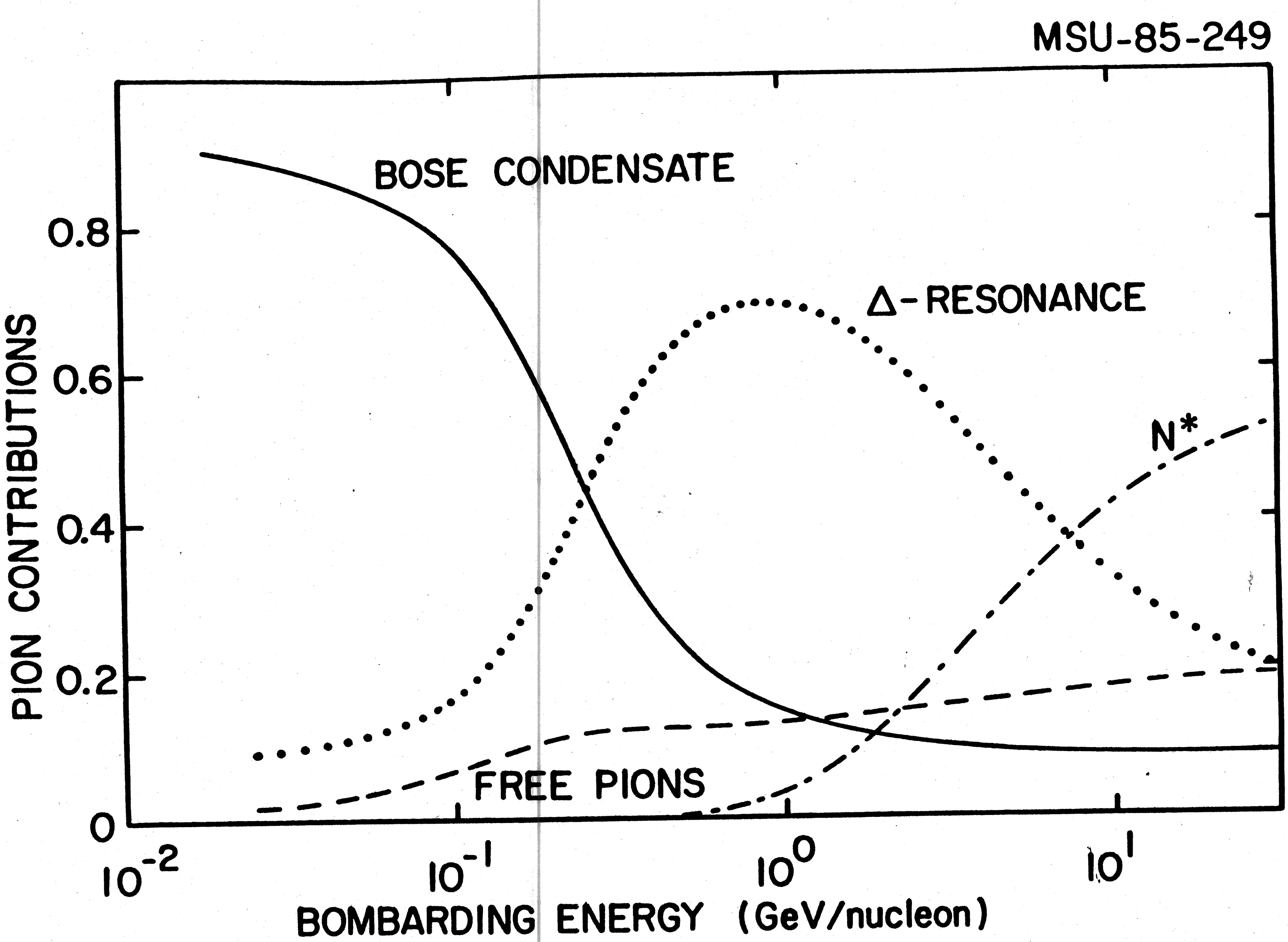

MSU-85-249
BOSE CONDENSATE
Δ-RESONANCE
N*
FREE PIONS
PION CONTRIBUTIONS
0
0.2
0.4
0.6
0.8
10^-2
10^-1
10^0
10^1
BOMBARDING ENERGY (GeV/nucleon)

FIGURE 4A

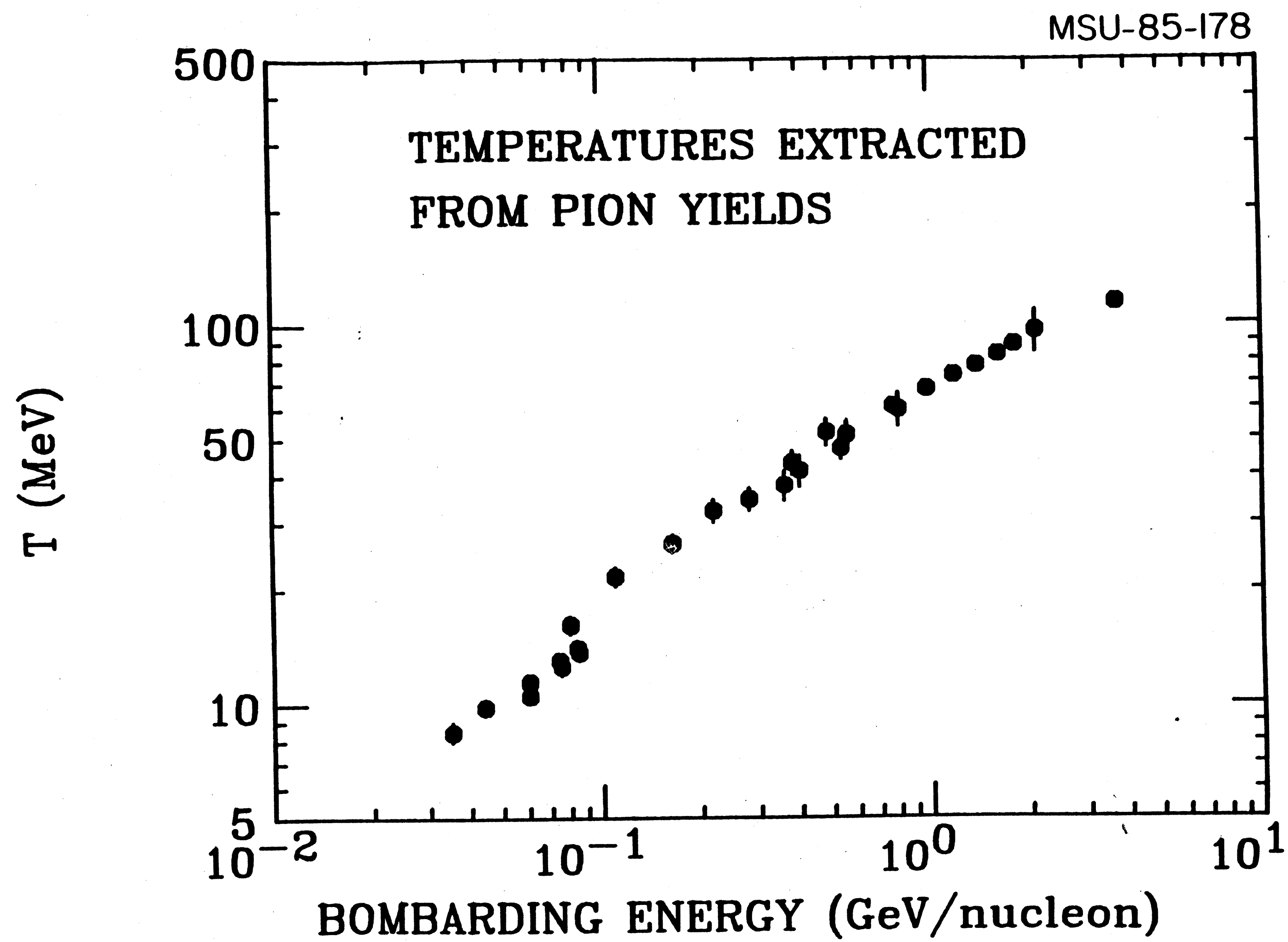

MSU-85-178
TEMPERATURES EXTRACTED
FROM PION YIELDS
T (MeV)
500
100
50
10
5
10^-2
10^-1
10^0
10^1
BOMBARDING ENERGY (GeV/nucleon)

FIGURE 4B