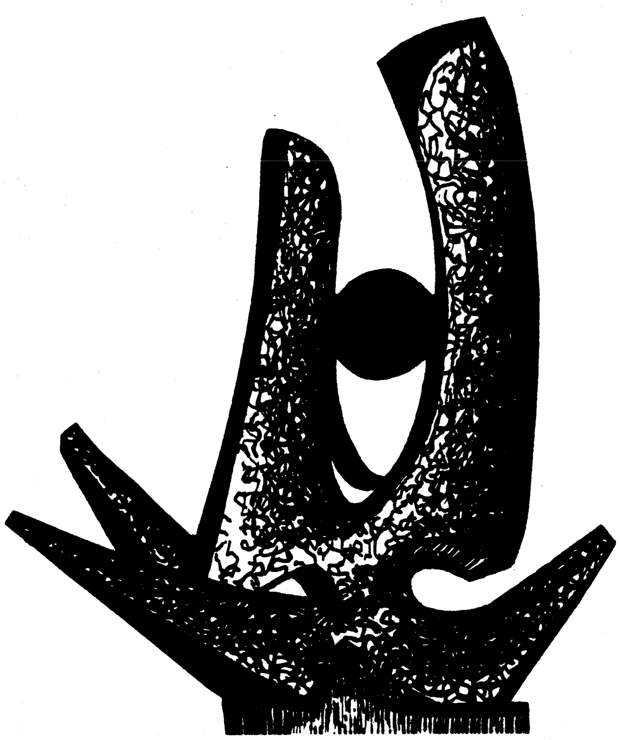
MICHIGAN STATE UNIVERSITY

CYCLOTRON LABORATORY

CALCULATIONS FOR THE DOUBLE-BETA DECAY OF ^{48}Ca

B. ALEX BROWN



FEBRUARY 1985

MSUCL-510

Invited talk presented at the International Symposium on Nuclear Shell Models, Drexel University, October 31 - November 3, 1984. MSUCL-510
Feb. 1985

Calculations for the double-beta decay of ^{48}Ca

B. Alex Brown
Cyclotron Laboratory,
Michigan State University,
E. Lansing MI 48824.

ABSTRACT

New calculations for the $^{48}Ca \rightarrow {}^{48}Ti$ double-beta decay are presented. Various matrix elements for two-neutrino and neutrinoless decay are investigated in terms of sums over intermediate states in 48Sc and 46Ca. Because of strong cancellations in the sum over intermediate states, approximations such as closure cannot be used. We obtain a half-life of 0.9×10^{20}y for the standard two-neutrino decay. If the neutrino has a Majorana mass of 31 eV, we predict a half-life of 50×10^{20}y for the neutrinoless decay.

I Introduction

In my talk at this meeting in honor of Igal Talmi I summarized three topics on the theme of nuclear structure from the viewpoint of neutron induced reactions with which I have recently been concerned. The first of these dealt with what we can learn from the (n,p) reaction at medium energy where the $\beta+$ component of the Gamow-Teller strength function can be studied. The second topic of double-beta decay followed from the first in the sense that this process can be regarded as a coherent summation over the $\beta-$ Gamow-Teller excitation of the neutron-rich nucleus times the $\beta+$ Gamow-Teller excitation of the proton-rich nucleus. The third topic dealt with the comparison of the results for the E2 matrix elements in 34S obtained by Alarcon et al. (Ref 1) from (n,n') and (p,p') inelastic scattering experiments with the predictions from sd shell-model calculations (Ref 2, Ref 3 and Ref 4).

Because of space limitations and because the first and third of these topics have recently appeared to some extent in the literature, I have decided to just briefly mention them in this paper and then go into some detail concerning some new calculations for the double-beta decay of 48Ca. The conclusion from the E2 matrix element comparison is that the transition to the second 2^+ state in 34S is predominantly isoscalar in nature and not isovector as predicted both from the Chung-Wildenthal (Ref 2) and "USD" (Ref 3) wave functions. The implication of this is still unclear (Ref 5).

The main point concerning $\beta+$ was that the total strength for this process turns out to be much more sensitive than $\beta0$ (i.e. Gamow-Teller type excitations in the same nucleus) or $\beta-$ to the structure of the target wave function. For several sd-shell nuclei we have calculated the total $\beta+$ strength in two types of model spaces: (A) the full sd basis (Ref 3) and (B) a basis truncation analogous to what is usually assumed for heavier nuclei, namely 0p0h+1p1h relative to a closed $d_{5/2}$ shell. We find that the total $\beta+$ strength for basis (A) is quenched by a factor of up to six when compared to basis (B). The implication of this for some fp shell nuclei is discussed in Ref 5 and Ref 6. For some nuclei, such as those discussed by Ian Towner at this meeting, some fraction of the $\beta+$ strength can be studied in beta decay. However, it would be important to try to study the total strength in the (n,p) reaction, in particular, for those nuclei such as 48Ti involved in double-beta decay.

The study of neutrinoless double-beta decay in nuclei is important for setting limits on the mass and right-handed couplings of the Majorana-type neutrino (Ref 7, Ref 8 and Ref 9). At first glance the decay of 48Ca is very favorable because of its large decay energy (Q=4.271 MeV) relative to other cases and good experimental limits have been obtained for the decay half-life for both neutrinoless and two-neutrino decay (Ref 10). However, it has become apparent that the nuclear structure matrix elements involved in the decay are quite hindered relative to simple single-particle estimates (Ref 8, Ref 11, Ref 12 and Ref 13). It is important to examine how reliably one can calculate these matrix elements, and we will look at several aspects of this problem and try to arrive at some "best" theoretical values. We note that the single beta decay of 48Ca to 48Sc is allowed (Q=0.281 MeV) but highly forbidden. Warburton (Ref 14) has calculated a partial half-life of 7.6×10^{20}y for the unique-fourth-forbidden decay to the 5^+ state at 0.131 MeV and has estimated much longer partial half-lives for the non-unique decays to the 4^+ and 6^+ states.

II Nuclear Structure Aspects of the ^{48}Ca Double-Beta Decay

The Fermi-Type Matrix Element

Double-beta decay may proceed via Fermi or Gamow-Teller type operators (Ref 9). For nuclei with good isospin, the Fermi type operator leads only to the double analogue state at 17.38 MeV (Ref 15) in 48Ti of the T=4 48Ca ground state, and the transition to the T=2 48Ti ground state is forbidden. We have calculated an isospin-mixing matrix element of 26 keV between the T=2 and T=4 states in 48Ti based on the isospin-nonconserving Hamiltonian used in Ref 16 to describe the displacement energy differences between analogue states in this mass region. [This was in fact an extension of the work initiated by Sherr and Talmi (Ref 17).] Using pertubation theory, we find M_F = $<0^+(Ti)|| (1/2) \Sigma_{i,j} t_-(i) t_-(j) ||0^+(Ca)>$ = 0.005. M_F is small compared to most of the Gamow-Teller type matrix elements discussed below and can be neglected.

Cancellations Within the $f_{7/2}$ Shell Model

Within the closure approximation (Ref 9), the Gamow-Teller type double-beta matrix elements for two-electron decay have the general form

$$M(DBD) = <0^+(Ti)|| O(DBD) F(DBD,r) ||0^+(Ca)> \qquad (1)$$

where $O(DBD) = (1/2) O_1(GT-)\cdot O_2(GT-)$

and $O(GT-) = \Sigma_k \sigma(k) t_-(k)$

$r = |\underline{r}(1)-\underline{r}(2)|$ and $t_-|n> = |p>$. We will consider the matrix elements for two-neutrino (2n), light neutrinoless (L), and heavy neutrinoless (H) decay with

$$F(2n,r) = 1$$
$$F(L,r) = 1/r$$
$$F(H,r) = \delta(r) \qquad (2)$$

F(L,r)=1/r is probably a good approximation to a more complicated expression used in Ref 9. For convenience we can relate these to triply reduced matrix elements (R) and tensor coupled operators (T) by defining

$$R(DBD) = <0^+(Ti)||| T(DBD) ||0^+(Ca)> \qquad (3)$$

where for the decay of 48Ca, $M(DBD)=[(3)^{1/2}/2]$x 3J(4,2,2,-4,2,2)xR(DBD) = 0.2887 R(DBD) and where T is given by a tensor cross product of two Gamow-Teller tensor operators coupled to total spin P=0 and total isospin Q=2:

$$T(DBD) = (1/2) [T_1(GT) \otimes T_2(GT)]^{(P,Q)} \qquad (4)$$

where $T(GT) = \Sigma_k \sigma(k) \tau(k)$

These reduced matrix elements R can be evaluated with shell-model codes such as OXBASH (Ref 18) by expanding the matrix element in terms of a sum over products of two-body transition densities (TBTD) times two-body matrix elements (TBME) (Ref 19 and Ref 20)

$$R(DBD) = \sum_{\substack{j_1,j_2,j_3,j_4 \\ J_{12},J_{34},T_{12},T_{34}}} TBTD \times TBME \qquad (5)$$

where $TBTD = [(2P+1)(2Q+1)]^{-1/2}$

$$\times \langle J_f,T_f ||| [A^+(J_{12},T_{34}) \otimes A^{\sim}(J_{34},T_{34})]^{P,Q} ||| J_i,T_i \rangle$$

and $TBME = \langle j_1,j_2,J_{12},T_{12} ||| T(DBD) ||| j_3,j_4,J_{34},T_{34} \rangle$.

A^+ and $A^{\sim}$ are the two-particle creation and annihilation operators

$$A^+(J_{12},T_{12}) = -[a^+(j_1) \otimes a^+(j_2)]^{J_{12},T_{12}}/[1+\delta_{j1,j2}]^{1/2}$$

$$A^{\sim}(J_{34},T_{34}) = +[a^{\sim}(j_3) \otimes a^{\sim}(j_4)]^{J_{34},T_{34}}/[1+\delta_{j3,j4}]^{1/2}.$$

In our case, since P=0, $J_{12}=J_{34}$; and since Q=2, $T_{12}=T_{34}=1$. The (antisymmetric) two-body matrix element can be evaluated with harmonic-oscillator radial wave functions by using the Talmi-Moshinsky transformation (Ref 21). We will use $\hbar\omega = 45A^{-1/3}-25A^{-2/3}$ MeV.

We consider in Table 1 the decay of 48Ca within the $f_{7/2}$ shell-model space using the TBTD calculated with the "48SC" wave functions of Ref 22. The results are very similar to those obtained with the "MBZ" (Ref 23) interaction in Ref 11.

Table 1: TBTD expansion for the $f_{7/2}$ matrix elements.

J_{12}	TBTD	(2n)	(L)	(H)
		TBME		
0	1.214	3.32	1.05	0.0479
2	-0.572	6.01	1.40	0.0255
4	0.021	3.64	0.79	0.0168
6	0.000	-3.99	-0.66	0.0101
		R(OBD)		
		0.67	0.48	0.0440

Note that the products for J_{12}=0 and 2 dominate. For 2n DBD these two terms tend to cancel due to the change in sign of the TBTD; however, the cancellation is smaller for the 0n DBD involving a light neutrino, and there is little cancellation for the 0n decay involving a heavy neutrino.

The reason for the cancellation in the case of 2n DBD has been explained in terms of a K-selection rule in the Nilson model (Ref 11). Some additional insight can be obtained by expanding the TBTD in terms of sums over the spins (J_o) and isospins (T_o) of the parent states of nuclei with (A-2) nucleons:

$$\begin{aligned} TBTD = \sum_{J_o,T_o} & (-1)^{J_f+P+J_o+J_{12}+T_f+Q+T_o+T_{12}} \\ & \times 6J(J_i,J_f,P,J_{12},J_{34},J_o)\ 6J(T_i,T_f,Q,T_{12},T_{34},T_o) \\ & \times [(2J_i+1)(2J_f+1)(2T_i+1)(2T_f+1)]^{1/2} \\ & \times TNSA(J_f,T_f)\ TNSA(J_i,T_i) \end{aligned} \quad (6)$$

The TNSA are the two-nucleon spectroscopic amplitudes defined by

$$TNSA(J_i,T_i) = [(2J_i+1)(2T_i+1)]^{-1/2} \times \langle J_i,T_i|||A^+(J_{12},T_{12})|||J_o,T_o\rangle$$

$$TNSA(J_f,T_f) = [(2J_f+1)(2T_f+1)]^{-1/2} \times \langle J_f,T_f|||A^+(J_{34},T_{34})|||J_o,T_o\rangle$$

The (A-2) core nucleus in our example is 46Ca and in the $f_{7/2}$ model space there are just three states to consider, those with (J_o,T_o) = (0,3), (2,3), (4,3) and (6,3). We must have $J_o=J_{12}=J_{34}$ (for $J_i=J_f=0$). The constants in the first three lines of Eq. (6) give $1.134/(2J_o+1)^{1/2}$. The 48Ca->46Ca TNSA for these four parent states are calculated to be 1, 2.236, 3, and 3.606, respectively, while the 48Ti->46Ca TNSA are 1.070, -0.504, 0.019 and 0.0007, respectively. It is now easy to obtain the TBTD given in Table 1.

The opposite signs for the TBTD with $J_o=0$ and $J_o=2$ are thus related to the fact that the relative TNSA products leading to the 0^+ and 2^+ states in 46Ca have opposite signs. The 48Ca TNSA are trivial to understand since, in the $f_{7/2}$ shell model, 48Ca has a closed shell of neutrons, and the two-nucleon spectroscopic factors ($TNSA^2$) are just equal to the $(2J_o+1)$ sum rule for two-nucleon pickup. The 48Ti TNSA are less easy to "guess" but are related to the fact that the 48Ti wave function has a two-proton particle two-neutron hole configuration. Since the double-beta decay can be directly related to sums over two-nucleon spectroscopic amplitudes, a study of two-nucleon transfer reactions may provide some important insight into this aspect of double-beta decay.

Beyond the $f_{7/2}$ Shell Model

We can also consider an expansion for M(DBD) involving intermediate states in the 48Sc nucleus. In general, we must sum over all intermediate spins. However, since F(r)=1 for the two-neutrino decay, we need only consider intermediate states in Sc with $J = 1^+$ for this mode, and we can write

$$M(2n) = (1/2)\ \Sigma_m\ M(2n,m) \qquad (7)$$

where $M(2n,m) = M(GT,f,m)\ M(GT,i,m) \qquad (8)$

and $M(GT,k,m) = \langle 0^+(k) || O(GT) || 1^+(m) \rangle \qquad (9)$

The Gamow-Teller transition probability for the transition k->m is given by

$$B(GT,k\to m) = M^2(GT,k,m)/(2J_k+1) \qquad (10)$$

These B(GT) obey the sum rule B(GT-)-B(GT+)=3(N-Z).

In the $f_{7/2}$ shell model there is just one intermediate 1^+ state and the DBD could be described diagramatically as in Fig.1a below. O(GT) acting on the initial state (Ca) can also go to the $f_{5/2}$ proton configuration as in Fig.1c. This transition is the dominant component of the "giant" GT resonance seen in the 48Ca(p,n) reaction (Ref 24 and Ref 25). O(GT) acting on the final state (Ti) can also create the pattern in Fig.1b.

Fig. 1: Schematic diagram for Ca<->Ti double-beta decay.

```
          p  n          p  n          p  n
         |  |  |       |  |  |       |  |  |       f5/2
(a)      |XX|00|  ->   | X| 0|  <-   |  |  |       f7/2

         |  |  |       |  | X|                     f5/2
(b)      |XX|00|  ->   | X|00|                     f7/2

                       | X|  |       |  |  |       f5/2
(c)                    |  | 0|  <-   |  |  |       f7/2

           Ti            Sc            Ca
```

Obviously, only (a) can connect Ti and Ca. However, the intermediate states may be mixed. If only (b) and (c) are mixed and the physical states are $|x\rangle = \alpha|b\rangle + \beta|c\rangle$ and

$|y\rangle = \beta|b\rangle - \alpha|c\rangle$ then 2M(2n) is given by

$$\langle f||O(GT)||x\rangle\langle x||O(GT)||i\rangle + \langle f||O(GT)||y\rangle\langle y||O(GT)||i\rangle =$$

$$\alpha\beta \langle f||O(GT)||b\rangle\langle c||O(GT)||i\rangle - \alpha\beta \langle f||O(GT||b\rangle\langle c||O(GT)||i\rangle$$

We find two routes each non-zero which cancel. More realistically, all three intermediate states mix, and these also mix with the particle states involving the $p_{3/2}$ and $p_{1/2}$ orbits causing fragmentation. In turn, all of this will be further fragmented by mixing with 2p-2h ... configurations. This pattern of cancellation and fragmentation is in fact found in all of our calculations and can be seen also in Fig. 1 of Ref 13. Since in the realistic situation the cancellation is not exact and since the $f_{7/2} \rightarrow f_{7/2}$ contribution is small, it is important to consider the entire fp shell basis in the double-beta decay calculation.

In Table 2 we give the DBD matrix elements calculated with Eq. (5) in a 0p0h-1p1h basis relative to a 48Ca closed shell [e.g. the $(f_{7/2})^8$ + $(f_{7/2})^7(f_{5/2},p_{3/2},p_{1/2})$ configurations] obtained with several recently developed empirical effective interactions. We consider: (1) the interaction of van Hees (VH) from Ref 26, (2) a modified version of this interaction (MVH) obtained by adding 2 MeV onto the $f_{5/2}$ single-particle energy (Ref 25), (3) the delta+Yukawa+monopole interaction of Yokoyama (YO) (Ref 27), and (4) the interaction of Mooy (MO) (Ref 28). For the MVH interaction we also give the results obtained in the 0p0h-2p2h model space.

Table 2: Double-beta decay matrix elements calculated with various model spaces and interactions. We also give the summed B(GT) strengths.

model space	inter-actio	M(2n)	M(L)	M(H)	ΣB(GT,f)	ΣB(GT,i)
0p0h	48SC	0.194	0.140	0.0127	0.015	10.3
0p0h-1p1h	MVH	-0.056	0.053	0.0095	2.09	22.3
	VH	-0.091	0.045	0.0094	1.94	22.3
	YO	-0.048	0.072	0.0109	2.54	22.3
	MO	-0.226	0.007	0.0082	1.73	22.3
0p0h	MVH	0.249	0.146	0.0124	0.024	10.3
0p0h-1p1h	MVH	-0.056	0.053	0.0095	2.09	22.3
0p0h-2p2h	MVH	0.234	0.163	0.0155	1.77	21.5

This comparison shows the sensitivity of the DBD matrix elements to this aspect of the calculation. However, this does not necessarily mean that the results are uncertain by this amount. We may be able to prefer one interaction over another based on how well it reproduces the experimental energy levels and Gamow-Teller strength functions for nuclei in this mass region. Our investigation of this aspect is at the moment incomplete, but we prefer the "MVH" interaction based primarily on the comparison of the experimental and calculated 48Ca(p,n) Gamow-Teller strength function (Ref 25).

M(2n) is very model-space and interaction dependent and even changes sign when the 1p1h configurations are added. In contrast, the value for M(H) is fairly model independent. The model dependence in M(2n) can be partly understood by looking at the sum in Eq. (7) as a function of excitation energy. As in Fig. 1 of (Ref 13), we find in general that the states just above the first state up to about 6 MeV in excitation (region b) add constructively to the contribution from the first state at 2.52 MeV and that the states above about 6 MeV (region c) add destructively. The breakdown in terms of these three contributions is given in Table 3. The states in region (b) constitute the relatively strong $f_{7/2} \rightarrow f_{5/2}$ transition for the $\beta+$ strength (Fig.1b above) and are relatively weak on the $\beta-$ side. The reverse is true for the states in region (c).

Table 3: M(2n) broken down over regions of excitation energy in 48Sc. Region (a) consists of the 1st state only. We also give the B(GT) strength to the lowest state at 2.52 MeV (region a).

model space	inter-action	M(2n) region (a)	M(2n) region (b)	M(2n) region (c)	B(GT,f) region (a)	B(GT,i) region (a)
0p0h-1p1h	MVH	0.171	0.551	-0.778	0.060	1.95
	VH	0.179	0.394	-0.664	0.117	1.09
	YO	0.243	0.253	-0.544	0.056	4.20
	MO	0.101	0.391	-0.718	0.035	1.16
0p0h	MVH	0.249	0	0	0.024	10.3
0p0h-1p1h	MVH	0.171	0.551	-0.778	0.060	1.95
0p0h-2p2h	MVH	0.045	0.743	-0.549	0.076	0.11

How Good is the Closure Approximation?

The double-beta decay matrix elements considered in Eq.

(1) are actually approximations to more complex equations involving explicit sums over intermediate states weighted by energy denominators (Ref 9). For two-neutrino decay we should consider the matrix element

$$B(2n) = \Sigma_m [Ex(m) + \Delta M + T_o/2]^{-1} M(2n,m) \quad (11)$$

where Ex(m) is the excitation energy of the 1^+ state in 48Sc, ΔM=-0.281 is the atomic mass difference and T_o =4.271 MeV is the kinetic energy release. [We assume that this equation is a good approximation to the more exact expression given by Eq. 1 in Ref 13]. The closure approximation consists in using an average excitation energy <Ex> in the denominator and taking this outside the summation:

$$B^c(2n) = [\langle Ex\rangle + \Delta M + T_o/2]^{-1} \Sigma_m M(2n,m) \quad (12)$$

There are many ways in which <Ex> could be estimated, but a common method (Ref 9) is to use the β- strength function $M^2(GT,i,m)$ [see Eq. (9)]:

$$\Sigma_m [Ex(m) + \Delta M + T_o/2]^{-1} M^2(GT,i,m)$$

$$= [\langle Ex\rangle + \Delta M + T_o/2]^{-1} \Sigma_m M^2(GT,i,m) \quad (13)$$

Table 4: Comparison of the quantities B(2n) with those obtained in the closure approximation $B^c(2n)$. energy in 48Sc. We also give the summed B(GT) strengths in region (a+b) (see caption to Table 3).

model space	inter- action	B(2n)	$B^c(2n)$	<Ex>	ΣB(GT,f) -- region	ΣB(GT,i) (a+b) --
0p0h-1p1h	MVH	0.036	-0.006	7.86	1.87	3.39
	VH	0.036	-0.009	7.69	1.82	2.13
	YO	0.043	-0.006	6.48	2.44	6.27
	MO	0.012	-0.022	8.22	1.60	2.11
0p0h	MVH	0.057	0.057	2.52	0.024	10.3
0p0h-1p1h	MVH	0.036	-0.006	7.86	1.87	3.39
0p0h-2p2h	MVH	0.059	0.025	7.81	1.47	4.51

In Table 4 the quantities B(2n) and $B^c(2n)$ are compared. If the closure approximation is valid, they should be about equal. This is certainly not the case, and there is even a change in sign between the two quantities due to the cancellations as a function of excitation energy discussed in connection with Table 3. We agree with Tsuboi et al. (Ref 13) on this point. However, it is encouraging to note that there is less model dependence in the more exact quantity B(2n) than there was in M(2n) [or B^c].

For the neutrinoless double-beta decay there is an additional term of order $\hbar c/R$ in the energy denominator which arises from the fact that one of the neutrinos is confined to the region inside the nuclear radius R. Since this number of about 50 MeV is large compared to <Ex>, the closure approximation is better for neutrinoless decay.

Going to the Full fp Shell-Model Space

Up to now we have considered calculations in an fp model space which included up to 2p2h excitation beyond a $f_{7/2}$ closed shell for 48Ca. The largest (J,T) dimensions in this space are 291 for the 0^+ T=2 states. We might consider going up to 3p3h or 4p4h, but calculations in the full fp space (up to 8p8h), where the largest (J,T) dimension is 10872 for the 0^+ T=2 states, is presently impossible. Also, it is important to keep in mind that the effective interactions we use are designed to reproduce energy levels in the 0p0h+1p1h space and that a different interaction might be needed for the full space calculation.

Full space calculations are possible, however, for the sd shell. It is instructive to consider the situation for the nucleus is the sd shell which is most closely analogous to 48Ca, namely 22O, which in the simplest shell model has a $d_{5/2}$ closed-shell configuation for neutrons. In Table 5 we give the results of our calculations for the double-beta decay of 22O obtained with the "USD" interaction of Wildenthal (Ref 3).

Table 5: Calculated double-beta decay matrix elements for 22O->22Ne in the sd shell-model space. For B(2n) and B^c(2n) we use $\Delta M+T_o/2$=1.854 MeV. We also give the summed B(GT) strengths in region (a+b).

model space	B(2n)	B^c(2n)	<Ex>	M(L)	M(H)	ΣB(GT,f) region	ΣB(GT,i) (a+b)
0p0h	0.092	0.092	2.52	0.246	0.0181	0.056	6.6
0p0h-1p1h	0.026	-0.012	9.01	0.047	0.0118	0.88	1.48
0p0h-2p2h	0.039	0.011	8.17	0.169	0.0199	0.50	3.68
full sd	0.019	0.0008	9.65	0.135	0.0198	0.28	1.48

The trends in the matrix elements seen in Table 5 on going from the 0p0h to the 0p0h-2p2h model space are very similar to those obtained for the 48Ca matrix elements (see Tables 2 and 4). Therefore, it seems reasonable to use the results of Table 5 to extrapolate the MVH matrix elements for 48Ca to the full fp space. Our algorithm for extrapolation is to multiply the fp shell 0p0h-1p1h MVH values by the ratio of the full-sd over the 0p0h-1p1h values given in Table 5. The results are given in the first line of Table 6.

Our preference for the MVH interaction comes from comparing the B(GT,i) values given in the above Tables with the experimental values obtained from the 48Ca(p,n) reaction (Ref 25 and Ref 29): 1.31 for region (a) (the state at 2.52 MeV), 2.07 for region (a+b) (the 1^+ states below the IAS), and 9.9-16.2 for the entire spectrum up to 30 MeV in excitation (excluding the T=4 state at 16.8 MeV). The range of values for the entire spectrum depends on whether one includes just the discreet states below 14.5 MeV in excitation (9.9) or adds part of the L=0 continuum (16.2) [see Table I in Ref 25]. The ratio of experiment over theory is 0.67, 0.61 and 0.43-0.71, respectively. The quenching appears rather uniform in agreement with the systematics obtained for Gamow decays in the sd shell (Ref 30). In terms of this comparison, the second best choice would be the MO interaction of Mooy. However, more comparisons to B(GT) values and energy levels should be made to gain confidence in the reliability of the interactions. In addition, it would be important to have an experimental check on the the 48Ti β+ strength which might be obtained with the (n,p) or (t,3He) reactions. The theoretical predictions are given by B(GT,f) in Tables 2, 3 and 4 above.

Beyond the fp Shell

We should consider the effects of higher-order configuration mixing, delta-isobar admixtures and mesonic exchange currents. These effects (primarily the first two) are responsible for the quenching in the GT strength observed above and in other nuclei (Ref 31). Empirically this quenching is very constant as a function of mass (Ref 32) and transition strength (Ref 30) with a typical value of 0.60 for the B(GT) values. At a minimum, this quenching should be included in B(2n), and we will also assume it applies to M(L) and M(H) as in the second line of Table 6. The role of these corrections in a more quantitative sense is still a matter of some controversy (Ref 9).

Predictions for the ^{48}Ca Double-Beta Decay Half-Life

The relationship between the matrix elements considered above and the partial half-lives have the form (Ref 9 and Ref 13)

$$T_{1/2}(2n) = C(2n)/[B(2n)]^2 \qquad (14)$$

and $T_{1/2}(0n) = C(L)/[m^2 M(L)]^2$ (15)

where m is the mass of the light neutino. (For simplicity, we assume that the right-handed neutrino couplings vanish.) The statistical and phase-space factor $C(2n)=2.3\times10^{16}$y deduced from Ref 13 is consistent with Ref 9. From Ref 9 we obtain $C(L)=420\times10^{20}(eV)^2$y. For our comparison in Table 6, we take the value m=31 eV suggested from a recent measurement of the end-point spectrum of the beta decay of tritium (Ref 33). We will not discuss the neutrinoless decay involving heavy (H) virtual neutrinos here. The comparison with experiment in Table 6 suggests that it would be interesting to push these experimental limits an order of magnitude further.

Table 6: Final results for the ^{48}Ca two-neutrino (2n) and neutrinoless (0n) double-beta decay.

Matrix elements	B(2n)	M(L)	M(H)
Extrapolated fp	0.026	0.152	0.0159
Extrapolated fp times 0.60 quenching	0.016	0.091	0.0096

Partial Half-life	$T_{1/2}(2n)$	$T_{1/2}(0n)$	in units of 10^{20}y
Calculated	0.9	53*	
Experimental	>0.36	>20	(Ref 10)

* Assuming 31 eV for the light Majorana neutrino mass and vanishing right-handed couplings.

Acknowledgments: This work was supported in part by the National Science Foundation under grant no. PHY-83-12245.

References:

1. R. Alarcon, D. Wang, J. Rapaport, R. T. Kouzes, W. H. Moore and B. A. Brown, to be published in Phys. Rev. C Rapid Communications (Feb. 1985).

2. B. A. Brown, B. H. Wildenthal, W. Chung, S. E. Massen, M. Bernas, A. M. Bernstein, R. Mickimen, V. R. Brown and V. A. Madsen, Phys. Rev. C26, 2247 (1982).

3. B. H. Wildenthal, Lectures presented at the International School on Nuclear Physics; 7th course: Mesons, Quarks and Nuclear Excitations, Erice-Trapany-Sicily, 6-18 April 1983, Progress in Particle and Nuclear Physics, Vol 11, p.5.

4. B. A. Brown and B. H. Wildenthal, unpublished.

5. B. A. Brown, Proceedings of the Burr-Oak meeting on Neutron Nucleus Collisions - Probe of Nuclear Structure, Sept. 1984, to be published.

6. J. Rapaport et al., Nucl. Phys. A427, 332 (1984).

7. H. Primakoff and S. P. Rosen, Ann. Rev. Nucl. Part. Sci. 31, 145 (1981).

8. W. C. Haxton, G. J. Stephenson and D. Strottman, Phys. Rev. Lett. 47, 153 (1981) and Phys. Rev. D25, 2360 (1982).

9. W. C. Haxton and G. J. Stephenson preprint submitted to Progress in Particle and Nuclear Physics.

10. R. K. Bardin, P. J. Gollon, J. D. Ullman and C. S. Wu, Nucl. Phys. A158, 337 (1970).

11. L. Zamick and N. Auerbach, Phys. Rev. C26, 2185 (1982).

12. L. D. Skouras and J. D. Vergados, Phys. Rev. C28, 2122 (1983) and J. D. Vergados, Phys. Rev. C13, 865 (1976).

13. T. Tsuboi, K. Muto and H. Horie, Phys. Lett. 143B, 293 (1984).

14. E. K. Warburton, preprint submitted to Phys. Rev. C. (1985).

15. R. T. Kouzes, P. Kutt, D. Mueller and R. Sherr, Nucl. Phys. A309, 329 (1978).

16. B. A. Brown and R. Sherr, Nucl. Phys. A322, 61 (1979).

17. R. Sherr and I. Talmi, Phys. Lett. 56B, 212 (1975).

18. W. M. D. Rae, A. Etchegoyen, N. S. Godwin and B. A. Brown, The Oxford-Buenos Aires Shell Model Code, unpublished.

19. P. J. Brussaard and P. W. M. Glaudemans, Shell Model Applications in Nuclear Spectroscopy (North Holland, Amsterdam, 1977).

20. B. A. Brown, W. A. Richter and N. S. Godwin, Phys. Rev. Lett. 45, 1681 (1980).

21. I. Talmi, Helv. Phys. Acta 25, 185 (1952) and M. Moshinsky, Nucl. Phys. 13, 104 (1959).

22. W. Kutschera, B. A. Brown and K. Ogawa, Rivista del Nuovo Cimento, Serie 3, Vol. 1, N. 12, 1978.

23. J. D. McCullen, B. F. Bayman and L. Zamick, Phys. Rev. 134B, 515 (1964).

24. B. D. Anderson, J. N. Knudson, P. C. Tandy, J. W. Watson, R. Madey and C. C. Foster, Phys. Rev. Lett. 45, 699 (1980).

25. B. D. Anderson, T. Chittrakarn, A. R. Baldwin, C. Lebo, R. Madey, J. W. Watson, B. A. Brown and C. C. Foster, submitted to Phys. Rev. C.

26. A. G. M. van Hees and P. W. M. Glaudemans, Z. Phys. A303, 267 (1981).

27. A. Yokoyama, T. Oda and H. Horie, Phys. Lett. 99B, 5 (1981) and private communication.

28. R. B. M. Mooy and P. W. M. Glaudemans, Z. Phys. A312, 59 (1983) and private communication.

29. B. D. Anderson, private communication.

30. B. A. Brown and B. H. Wildenthal, Phys. Rev. C28, 2397 (1983) and submitted to Atomic and Nuclear Data Tables.

31. I. S. Towner and F. C. Khanna, Nucl. Phys. A399, 334 (1983).

32. C. D. Goodman et al., Phys. Rev. Lett. 44, 1755 (1980) and C. D. Goodman, Proceedings of the International Conference on Nuclear Physics (Tipografia Compositori, Bologna) edited by B. Blasi and R. A. Ricci, Vol. 2 (1983), p. 165.

33. S. Boris et al., paper presented at HEP 83, Brighton (July, 1983).