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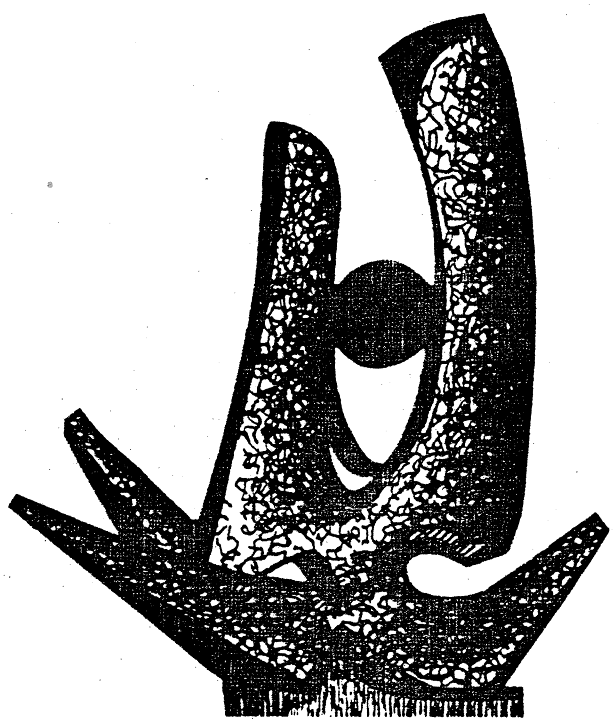

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Double Beta Decay in the Interacting Boson Approximation

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Abstract

In order to include the effects of nuclear deformation in the calculation of the nuclear many body matrix element involved in the double beta decay process, we have performed a calculation in the Interacting Boson Approximation. An explicit expression for the boson image of the 2β transition operator is given. Results for the $^{128,130}Te \rightarrow Xe$ decays are presented. The calculation indicates that the nuclear deformation strongly hinders the double beta decay.

Double beta decay is an interesting process since it probes the breakdown of lepton number conservation due to the finite mass of the Majorana neutrino or possible right-handed weak lepton current[1]. This can be determined from the ratio of the 0 and 2 neutrino decay width. However to extract evidence on the rates of the two possible decay processes from nuclear measurements, the nuclear many body matrix element involved has to be known accurately. A major complication in the calculation of the nuclear double beta decay matrix element is that the g.s.-g.s. decay (which is the only allowed decay in most cases for Q value reasons) represents only a few percent of the total sum rule of strength. Extensive calculations of the nuclear matrix element have been done by Haxton et al[2] in terms of the nuclear shell model.

For medium heavy nuclei there appears to be a large difference between the theoreticaly calculated nuclear matrix element and experiment[2]. Part of this discrepancy could arise from the fact that in the shell model calculations, for technical reasons, it is assumed that the nuclei are very close to spherical. For medium heavy nuclei this need not to be a good approximation. In ref 3 the effect of deformation has been studied using the Nilsson model. This calculation indicates that for well deformed nuclei the double beta decay rate vanishes. This indicates that the influence of deformation has a strong influence on the double beta decay rate. To investigate the effects of nuclear deformation we have therefore repeated the calculation in the framework of the Interacting Boson Approximation (IBA)[4].

In this letter we will limit ourselves to the calculation of two neutrino double beta decay. Further more, since we want to compare the results with those of the shell-model calculations presented in ref 2, we will make the same kind of approximations. In particular we will make use of the closure approximation to eliminate the explicit summation over intermediate states which occurs in the exact operator. In addition in the microscopic calculation of the coefficients in the boson image of the double beta decay operator will be limited to a single major shell only. In a forth coming longer paper the validity of some of these approximations will be addressed in more detail.

The operator describing the two neutrino double beta decay process can to a good approximation be written as a double Gamow-Teller operator[1]. Using the closure approximation to carry out the summation over intermediate states, the operator can be written as[1]

$$O^{F}_{2\beta} = \frac{1}{2} \sum_{i,k} (\vec{\sigma}(i)\ t_{+}(i)) \cdot (\vec{\sigma}(k)\ t_{+}(k)) \qquad (1)$$

Where t_+ is the isospin raising operator. In eq(1) we have limited ourselves to the description of L=0 decay.

Introducing the shell model single particle (s.p) creation (annihilation) operators $a^{\dagger}_{j}(a_{j})$, we may rewrite the operator $O^{F}_{2\beta}$ as

$$O^{F}_{2\beta} = \frac{-1}{2} \sum_{j_\pi j'_\pi j_\nu j'_\nu I} (2I+1) \left((a^{\dagger}_{j_\pi} \times a^{\dagger}_{j'_\pi})^{(I)}_{M} \times (\tilde{a}_{j_\nu} \times \tilde{a}_{j'_\nu})^{(I)}_{M} \right)^{(0)} G^{(I)}_{j_\pi j'_\pi j_\nu j'_\nu} \qquad (2)$$

where

$$\tilde{a}_{jm} = (-)^{j-m} a_{j-m}, \text{ and} \qquad (3)$$

$$G^{(I)}_{j_1 j_2 j_3 j_4} = 6(-)^{j_1+j_3+\ell+\ell'+I}\ \hat{j}_1 \hat{j}_2 \hat{j}_3 \hat{j}_4 \begin{Bmatrix} j_1 & j_2 & I \\ j_4 & j_3 & 1 \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & j_1 & \ell \\ j_3 & \frac{1}{2} & 1 \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & j_2 & \ell' \\ j_4 & \frac{1}{2} & 1 \end{Bmatrix}$$

where $\hat{j} = \sqrt{2j+1}$, and the curly brackets denote the usual 6j symbol.

In the IBA[4] model the structure of the low lying collective states in medium heavy nuclei are described in terms of a system of mutually interacting bosons. Each boson corresponds to a collective pair of neutrons or protons in either an J=0 (s-boson) or an J=2 (d-boson) state[5]. This equivalence relation insures for example, that in the description of a given nucleus, the number of neutron and proton bosons a is strictly conserved.

In the nuclei for which there exists experimental data on the double beta decay process the protons are in the beginning of the valence shell while for the neutrons the valence shell is more than half filled. This implies that in the conventional[5] IBA picture of the bosons the protons bosons correspond to particle pairs, while the neutrons correspond to hole pairs. In the double beta decay process a proton pair is created and two neutron particles are annihilated, or equivalently, a neutron hole pair is created. In the boson space therefore the operator creates both a neutron and a proton boson, and in lowest order in the boson operators, the boson image of the L=0 double beta operator can be written as

$$O^{B}_{2\beta} = A(s^{\dagger}_{\pi}\cdot s^{\dagger}_{\nu}) + B(d^{\dagger}_{\pi}\cdot d^{\dagger}_{\nu}) + C(s^{\dagger}_{\pi}s^{\dagger}_{\pi}s^{\dagger}_{\nu}s^{\dagger}_{\nu}\tilde{d}_{\pi}\cdot\tilde{d}_{\nu})/(Z_{\pi}Z_{\nu}) + E_{\pi}(s^{\dagger}_{\pi}s^{\dagger}_{\pi}\tilde{d}_{\pi}\cdot d^{\dagger}_{\nu})/Z_{\pi} + E_{\nu}(s^{\dagger}_{\nu}s^{\dagger}_{\nu}d^{\dagger}_{\pi}\cdot\tilde{d}_{\nu})/Z_{\nu} \tag{4}$$

where parameters, A to E have been introduced. The order of the operator in this context is defined by the number of d-boson creation and annihilation operators. The normalization factors Z are defined according to

$$Z_{\rho} = \sqrt{N_{\rho}(N_{\rho}-1)}\ , \quad \rho=\pi,\nu \quad , \tag{5}$$

and serve only for convenience.

These parameters in eq (4) can be determined from a microscopic model following the OAI[5] procedure, in which matrix elements between equivalent states in the boson and fermion space are equated. In the conventional microscopic picture behind the IBA model, mentioned above, a state with n_d d-bosons corresponds[5] to a state in the fermion space with (generalized) seniority[6] $v=2n_d$. The first term in eq(4) thus corresponds to the part of the double beta operator which keeps the seniority fixed, for neutrons and protons separately, while the second term corresponds to the part in which the total seniority is increased by four units. In the fermion space the operator can also decrease the seniority by four units. In the image one therefore expects also a term which annihilates two d-bosons. This term must have the form of the third term in eq 4, where the s-boson operators can be seen as to only take care of fermion conservation. In spite of its complicated appearance it thus still is part of the lowest order boson operator. An equivalent argument also applies to the last two terms in eq 4.

The microscopic calculation of the coefficients in the boson image of the double beta decay operator is based on the generalized seniority model[7],which in turn is based on the shell model. General details of the calculations involved are given in ref 7. As interaction we have chosen a surface delta force (strength A') with an enhanced quadrupole component (enhancement factor F2)[7] which should account for core polarizition effects. The parameters were obtained from a best fit calculation to excitation energies in semi-closed shell nuclei and are given in table 1. The structure of the S pair is determined by the structure of the ground-state and that of the D pair by the 2_1^+ in the semi closed shell nuclei.

The coefficients which enter in the operator of the double β decay, can now be determined by equating[5] the matrix elements of the operator in the fermion space between states build up from S-pairs only and states containing at most one neutron and one proton D-pair, as calculated in the generalized seniority model[7]. These matrix elements can be equated to those of the operator eq(4) between states with an equivalent number of s- and d-bosons. The calculated values for the nuclei in the question are given in table 2. In the calculation of the coefficients of the operator (4) the contributions from the different shell model configurations that constitute the S and D pair state, almost all add up constructively. This makes that the calculated results are relatively insensitive on the details of the shell model interaction that is used in constructing the microscopic structure of the bosons, at least as long as the quasi particle energies and the energies of the 2^+ and 4^+ levels in the semi closed shell nuclei are reproduced in the right bulk part.

The IBA model wave functions for the Te and Xe isotopes are calculated using the parameters given in ref 8 and 9. Using the thus obtained wave functions the matrix elements of the operator eq(4) was calculated, and the results are given in table 3, where they are also compared with the results of Haxton[2]. From the values given in the table in can be seen that the decay rate is dominated by the first term in the operator 4, partially due to the fact that $|A| > |B|$ and partially to the fact that the boson matrix element for the first term in (4) is larger than that of the second. Since B has the opposite sign of A it does have the effect of decreasing the double beta decay matrix element. The contribution coming from the last three terms of (4) is of only minor importance (of the order of a few percent) and could have been safely neglected. The microscopic reason for this is closely related to the fact that the proton bosons are particle like, while the neutrons are hole like.

The limiting case of a well deformed nucleus is described in the IBA model by the SU(3) limit. In this limit the number of d- bosons in the ground-state is twice that of s-bosons. Taking the values for A and B as calculated for Te, this results in an essentially vanishing double beta decay rate. This is in agreement with the results obtained from Nilsson model calculations[3]. Also in the limit of an extreme shell-model nucleus, like ^{48}Ca, a link with the IBA model can be made. In this specific case the A and B coefficients are of the same order of magnitude (1.28 respectively - 1.04). Using the Cohen and Kurath wave functions for ^{48}Ti the equivalent IBA wave function can be constructed and the matrix elements of the boson operators for the first two terms in eq (4) can be calculated, giving values of 0.905 and 0.953 respectively. Also in this case one thus has an almost complete destructive interference, in agreement with the results given in ref 2. The picture now emerges that for near closed shell nuclei the nuclear matrix element vanishes. In going away from the closed shell the A term takes over in importance since the number of s-bosons in the ground state increases, while the number of d-bosons stays approximately constant in the vibrational regime. When reaching the point where deformation sets in, the trend is reversed, and the matrix element decreases again to zero, because of an increased cancellation between the A and B terms. The Xe and Te nuclei are just somewhere in the vibrational region where the number of s-bosons is appreaciable, but the deformation still small. As a result the calculated double beta decay matrix element is relatively large.

In the calculation of the coefficients we have limited ourselves to a single major shell, in order not to have the complication of spurious center of mass motion. In the calculation of A the contributions of the different valence orbits all have the same sign. One therefore might expect that extension of the model space will increase the value for A. The calculations presented in ref 3 support this finding. For the other coefficients the picture is somewhat more complex, however still the contributions from the different components of the D-pair state appear to add up mostly coherently. This would imply that extending the model space might not affect the double beta decay rate too much since the contribution related to the d-boson give a destructive contribution . The effect of core polarization (or equivalently model space extension) on B is therefore not clear. Also degrees of freedom outside the IBA model could have an influence, The G-pair state in this respect will be most important. In forthcoming longer paper we intend to investigate these points in some more detail.

In conclusion it has been shown that the IBA model provides a comprehensive framework for the calculation of double beta decay. Since it is relatively simple it can be used to calculate the effects of the corrections mentioned above.

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Table 1. Interaction parameters (see text), used to determine the microscopic structure of the s- and d-bosons in the generalized seniority calculation.

isotopes	A'	F2	single particle energies				
			$0g_{\frac{7}{2}}$	$1d_{\frac{5}{2}}$	$1d_{\frac{3}{2}}$	$2s_{\frac{1}{2}}$	$0h_{\frac{11}{2}}$
N=82	0.20	1.5	0.0	0.96	2.69	2.99	2.76
Z=50	0.18	1.9	-2.3	-2.80	-.30	-.80	0.10

Table 2. The calculated values for the coefficients in the boson image of the double beta decay operator, eq (4). Also the expectation value of the boson operators multiplying these coefficients is given.

Decays	A	B	C	E_π	E_ν
$^{128}Te \rightarrow ^{128}Xe$	.848	-.402	-.007	.041	.145
boson m.e.	2.20	1.18	1.19	0.19	0.33
$^{130}Te \rightarrow ^{130}Xe$	.900	-.436	-.005	.039	.117
boson m.e.	2.20	1.12	0.49	0.05	0.06

Table 3. The calculated and experimental double Gamow-Teller matrix elements

Decays	$\lvert M_{GT} \rvert_{IBA}$	$\lvert M_{GT} \rvert^{2}_{Haxton}$	$\lvert M_{GT} \rvert^{*}_{exp}$
$^{128}Te \rightarrow ^{128}Xe$	1.44	1.47	0.185-0.230
$^{130}Te \rightarrow ^{130}Xe$	1.49	1.48	0.104-0.129

*Taken from Ref.2