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COULOMB FORM FACTORS OF E2 AND E4 TRANSITIONS IN S-D SHELL NUCLEI

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Coulomb Form Factors of E2 and E4 Transitions in S-D Shell Nuclei*)

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Abstract

Coulomb form factors of E2 and E4 transitions in the s-d shell nuclei are discussed taking into account core-polarization effects due to the quadrupole and hexadecupole giant resonances, respectively. The calculation has been performed within the framework of a microscopic theory and gives remarkably good agreement with experimental form factors both in the absolute strength and the q-dependence.

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*)in collaboration with B. A. Brown

Recent high-resolution electron scattering data make it possible to obtain model independent transition densities and current densities in many nuclei over a broad region of the mass table.[1] These data provide precise and interesting information which can be used to test the validities of various nuclear models.

A microscopic model has recently been proposed by the present author and B. A. Brown [2] in order to study the core-polarization effect on the transition and current densities of single-particle configurations. This model is essentially composed of two parts. At the first stage, we calculate the single-particle wave functions and giant resonances by using the self-consistent Hartree-Fock (H-F) + random phase approximation (RPA) theory. Then, we evaluate the core-polarization effect due to these giant resonances by the particle-vibration coupling model. Let me describe our model for the calculations of the core-polarization effect due to the giant resonances. The modified single-particle wave functions is obtained by perturbation theory as follows;

$$|\tilde{\alpha}\rangle = |\alpha\rangle + \sum_{\beta,\omega_\lambda} \langle(\beta \times \omega_\lambda)\alpha|V_{ph}|\alpha\rangle[\varepsilon_\alpha-(\varepsilon_\beta-\omega_\lambda)]^{-1}|(\beta\times\omega_\lambda)\alpha\rangle \qquad (1)$$

The reduced matrix element for the one-body operator is then modified as

$$\langle\tilde{\alpha}||\hat{T}_\lambda||\tilde{\beta}\rangle = \langle\alpha||\hat{T}_\lambda||\beta\rangle + \sum_{\omega_\lambda} [2\omega_\lambda/(\varepsilon^2_{\alpha\beta} - \omega^2_\lambda)]$$
$$*\langle(\beta \times \omega_\lambda)\alpha|V_{ph}|\beta\rangle\langle\omega_\lambda||\hat{T}_\lambda||0\rangle/(2\lambda+1)^{1/2} \qquad (2)$$

where ω_λ and $\varepsilon_{\alpha\beta}=\varepsilon_\alpha-\varepsilon_\beta$ are the excitation energy of giant resonance and single-particle energy difference, respectively. The proton and neutron core-polarization charges are defined by,

$$\delta e_p = 1 - [\langle\tilde{\alpha}||\hat{T}_\lambda||\tilde{\beta}\rangle_\pi/\langle\alpha||\hat{T}_\lambda||\beta\rangle_\pi]$$
$$\delta e_n = \langle\tilde{\alpha}||\hat{T}_\lambda||\tilde{\beta}\rangle_\nu/\langle\alpha||\hat{T}_\lambda||\beta\rangle_\pi \qquad (3)$$

First, we studied the core-polarization charges for single-particle transitions. For the dominant matrix elements involved in the multi-particle sd-shell transitions, we obtained an average isoscalar E2 effective charge of $\delta e(IS)=0.34$ in good agreement with the empirical result of 0.35 ± 0.05. The calculated E2 isovector effective charge is $\delta e(IV)=0.11$ which is consistent with the analysis of ref. (5). The E4 core-polarization charges have been also calculated in the vicinity of ^{28}Si. The average calculated polarization charges are 0.45 and 0.15 for the IS and IV transitions, respectively.

A more comprehensive test of our calculations comes from a comparison with experimental electron-scattering form factor data. The coulomb form

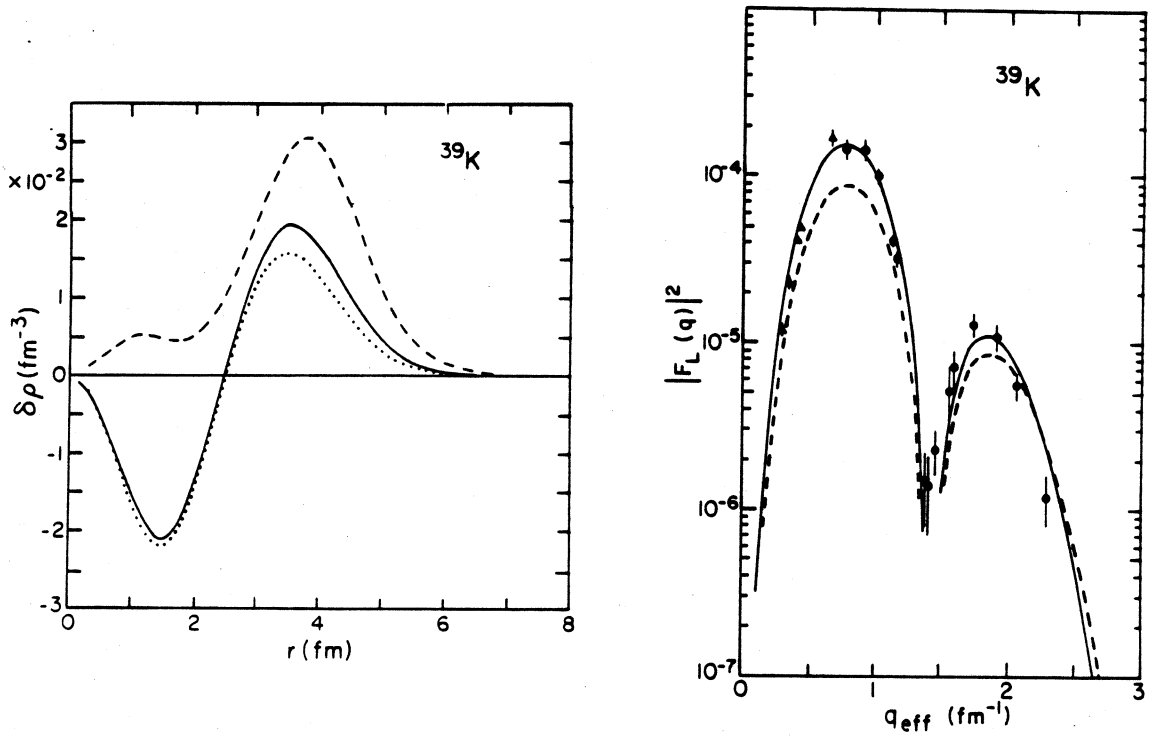

Fig.1 Transition densities and coulomb form factors for the $(1d_{3/2}^{-1} \rightarrow 2s_{1/2}^{-1})_{\pi}$ excitation at Ex=2.53 MeV in ^{39}K. The solid and dotted curves at l.h.s. correspond to the transition density with and without core polarization effects, respectively. For comparison, the transition density for the IS giant resonance is also shown by the dashed curve. We show at r.h.s. the coulomb form factors calculated using H-F single particle wave functions. The solid and dashed curves correspond to the results with and without core-polarization effects, respectively. The center of mass correction is taken into account in the harmonic oscillator model [8] and the nucleon finite size correction is incorporated in the dipole approximation [9]. The data are taken from ref. (6) (triangles) and ref. (7) (circles).

factor of the quadrupole transition $(1d_{3/2}^{-1} \rightarrow 2s_{1/2}^{-1})_\pi$ in ^{39}K is shown in Fig.(1) together with the transition density. The q-dependence of the form factor is nicely reproduced by the calculation with core-polarization. The calculation also shows quantitative agreement of the absolute magnitudes of the first peak at q=0.75 fm^{-1} and the second peak at 1.7 fm^{-1}. One of the aspects of our calculation which is important in obtaining this good agreement is the fact that the calculated H-F mean-square charge radii are in general in very good agreement with the experimental values. For example, the calculated radius for ^{40}Ca is 3.48 fm while the experimental one is 3.481 ± 0.005 fm. As is shown in Fig.(1), the transition density for the $(1d_{3/2}^{-1} \rightarrow 2s_{1/2}^{-1})_\pi$ transition is increased at the surface region and decreased slightly in the inside of the nucleus due to the coupling to the giant resonances. This change of the transition density is responsible for the larger enhancement at the first peak than the second peak in the form factor in Fig.(1).

Let us study now the core-polarization effect on many-body wave functions. The reduced one-body matrix element for the shell-model wave functions can be expressed as a linear combination of the single-particle matrix elements;

$$\langle J_f||\hat{T}_\lambda||J_i\rangle = \sum_{\alpha,\beta} C_{J_f,J_i}(\alpha,\beta)\langle\alpha||\hat{T}_\lambda||\beta\rangle \qquad (1)$$

where J_f and J_i stand for the shell-model states and $C_{J_f,J_i}(\alpha,\beta)$ are the structure factors (one-body transition densities). The modified transition matrix element for the shell-model wave function is now given by inserting $\langle\tilde{\alpha}||\hat{T}||\tilde{\beta}\rangle$ in Eq. (4);

$$\langle\tilde{J}_f||\hat{T}_\lambda||\tilde{J}_i\rangle = \sum_{\alpha,\beta} C_{J_f,J_i}(\alpha,\beta)\langle\tilde{\alpha}||\hat{T}_\lambda||\tilde{\beta}\rangle \qquad (5)$$

This effect can be regarded as a polarization of the core protons by the valence protons and neutrons through the proton-proton and proton-neutron two-body interaction.

The shell-model calculations have been performed in the full s-d shell model space with the empirical two-body matrix elements of Wildenthal.[4] The shell-model wave functions give a quite satisfactory agreement in the

excitation energies in comparison with experimental data. Nevertheless, the calculated B(E2)- and B(E4)-values are typically several times smaller than the empirical ones. The excitation energies and B(E4)-values of collective states are listed in Table (1). The strong hexadecupole transitions in ^{24}Mg, ^{27}Al and ^{28}Si are dominated by the isoscalar part and the B(E4)-values are enhanced by the core-polarization effects by a factor of 3.5. The agreement between experiment and the shell model calculations with core polarization is remarkably good.

Table (1) Excitation energies and B(E4)-values for E4 transitions in s-d shell nuclei. The calculated B(E4)-values are obtained by the s-d shell model wave functions with and without core-polarization effects. The experimental B(E4)-values are taken from ref. 10(^{24}Mg), ref. 11 (^{26}Mg), ref. 12 (^{27}Al) and ref. 13 (^{28}Si).

		Energy		B(E4) (e^2 fm^8)		
nucleus	#	theory	exp.	SM(*10^3)	SM+CP(*10^3)	exp.(*10^3)
^{24}Mg	1	4.38	4.12	0.032	0.159	(2.0 ± 0.3)
	2	5.93	6.01	12.0	40.4	(43 ± 6)
^{26}Mg	1	4.53	4.32	3.14	9.21	(4.1 ± 1.6)
	2	4.93	4.90	4.24	19.3	(15.6 ± 3.4)
	3	5.47	5.47	0.54	0.02	(0.89 ± 0.34)
	4	6.01	5.72	4.48	9.69	(6.4 ± 1.8)
^{27}Al		4.58	4.51	2.59	8.47	
^{28}Si	1	4.66	4.62	8.30	27.4	(27 ± 5)

We show the transition density and the coulomb form factor for the ($0^+ \rightarrow 4^+_1$) transition in ^{28}Si in Fig. 2. The form factor is increased by a factor of 2.5 at the maximum around q= 1.4 fm^{-1}, however, there is not much increase in the high q-region above 2.0 fm^{-1}. This change is attributed to the enhancement of the transition density at the surface region. The agreement of the calculation with the experimental data in general remarkably good for the E4 form factor. The E2 form factors of nuclei at the middle of the sd-shell might be discussed elsewhere. [15]

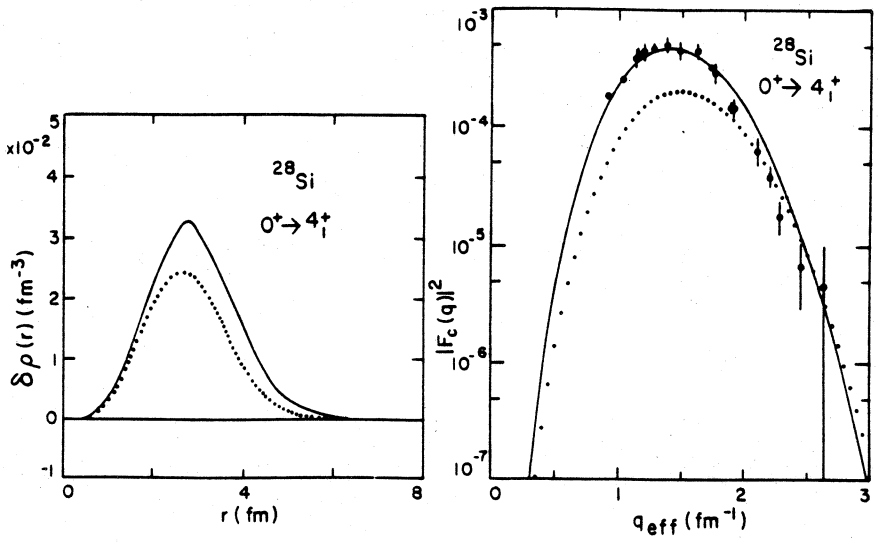

Fig.2 Transition densities and coulomb form factors for the ($0^+ \rightarrow 4_1^+$) transition in ^{28}Si. The solid and dashed curves correspond to the results with and without core-polarization effects, respectively. The data are taken from ref. (14).

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