LONGITUDINAL MOMENTUM TRANSFER AND THE NUCLEON'S MEAN FREE PATH IN MEDIUM ENERGY HEAVY ION COLLISIONS - TDHF VERSUS VLASOV-UEHLING-UHLENBECK THEORY

Jörg AICHELIN [a,b,1] and Horst STÖCKER [a,2]

[a] *National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA*

[b] *Joint Institute for Heavy Ion Research, Holifield Heavy Ion Research Facility, Oak Ridge, TN 37831, USA*

Received 24 May 1985; revised manuscript received 12 August 1985

Two pure mean field theories, the classical Vlasov- and the quantal TDHF-approach, are used to study 85 MeV/N ^{12}C induced reactions with targets from ^{12}C to ^{197}Au. Both theories predict nearly identical results: the nuclei slip through each other; for C + C about 85% of the longitudinal momentum is conserved in the surviving fragments. Inclusion of two body collisions via an Uehling–Uhlenbeck collision integral yields – in sharp contrast to results at lower energies – two clearly distinct components: only 20% of the initial momentum resides in the slipped-through fragments, which retain only 40% of the nucleons. The remaining 60% of the nucleons undergo one or more n–n scatterings and form a mid-rapidity source. These nucleons decelerate rapidly with a time constant $t = 12.5$ fm/c, corresponding to a deceleration length of $x = 2.5$ fm. The number of slipped-through nucleons decreases even further when heavier target nuclei are studied: for C + Au collisions, 97% of the projectile nucleons undergo at least one collision, transferring 80% of the total longitudinal momentum to the target-like fragment. The number of emitted uncollided projectile nucleons falls off exponentially with the thickness of the target nucleus, yielding a mean free path of the nucleon of $\lambda = 2.6$ fm.

Recent data on pion production in intermediate energy heavy ion reactions [1–3] [‡1] suggest that at energies as high as 84 MeV/nucleon carbon nuclei may be able to stop each other. This point of view seems to be supported by the successful description of these pion yields and spectra by phenomenological models which rest on the assumption of complete momentum transfer [5] [‡2] [7]. In contrast, TDHF calculations [8] which do no include n–n collisions predict transparency even for small impact parameters. The nucleon's mean free path has been estimated to exceed the diameter of the carbon nucleus (see, however, ref. [9]). The more surprising is the reasonable agreement of the inclusive proton cross sections with the predictions of the fluid dynamical model, which relies on the assumption of a short mean free path and predicts therefore nuclear stopping and rapid disintegration at these energies [10]. In the present work we study the interplay of the mean field dynamics and two body collisions and their importance for the nuclear stopping power in a self-consistent theory.

Recently several attempts have been made to study the influence of the nuclear mean field dynamics and Pauli blocking on the reaction dynamics in a microscopic theory based on the Vlasov–Uehling–Uhlenbeck (VUU) equation [11] [‡3] [15–20]. The VUU equation can be solved via a Monte Carlo pro-

[1] Permanent address: Institut für Theoretische Physik, Universität Heidelberg, D-6900 Heidelberg, Germany.

[2] Permanent address: Institut für Theoretische Physik, Universität Frankfurt, D-6000 Frankfurt am Main, Germany.

[‡1] For γ-spectra see ref. [4].

[‡2] For γ-bremsstrahlung see ref. [6].

[‡3] This method has been applied to nuclear collisions at various levels of sophistication by several authors [12]. The equivalence of the Vlasov equation and TDHF in one dimension has been demonstrated by Tang et al. [13], and Wong [14], who first employed the particle-in-cell method used in refs. [15,16] to solve the Vlasov equation.

0370-2693/85/$ 03.30 © Elsevier Science Publishers B.V. (North-Holland Physics Publishing Division)

cedure [11–20], which represents the single particle distribution function by a large number (several thousands) of test particles. In this framework, the time evolution of the single particle distribution function is given by

$$(\partial/\partial t)f + \boldsymbol{v}\cdot(\partial/\partial \boldsymbol{r})f + \boldsymbol{a}\cdot(\partial/\partial \boldsymbol{v})f$$
$$= \int \frac{d^3p_2\, d^3p_1'\, d^3p_2'}{(2\pi)^6}\, \sigma v_{12}$$
$$\times [f'f_2'(1-f_1)(1-f_2) - f_1 f_2(1-f')(1-f_2')]$$
$$\times\, \delta^3(p + p_2 - p_1' - p_2') \,. \qquad (1)$$

Vlasov's classical transport equation for a collisionless plasma with a mean potential field is obtained by setting the right-hand side of eq. (1) equal to zero [11–14]. Trajectories of the test particles in configuration and momentum space are computed by assuming that each particle moves under the influence of an acceleration term generated by the gradient of the mean field. For the density dependent potential field, $U(\rho)$, a local Skyrme interaction is used: $U(\rho) = -353\ \rho/\rho_0 + 303\ (\rho/\rho_0)^{7/6}$ MeV, with a compressibility coefficient of $K = 200$ MeV.

Two nucleons may undergo s-wave scattering if they approach each other with a minimum distance of less than $(\sigma/\pi)^{1/2}$ and if the final states are not Pauli blocked. The Pauli blocking factor is computed via the ensemble averaged density in the six-dimensional sphere around the phase space coordinates of the scattered particles. The Pauli blocking factor for each nucleon is given by $(1-f)$, and the scattering cross section is then reduced by the Uehling–Uhlenbeck factor $(1-f_1)(1-f_2)$.

A constant time-step integration routine is used to insure synchronization of the ensembles. The local gradient of the field is computed via a finite difference method. The acceleration of the test particles due to the field gradient is calculated prior to each transport step, and is assumed to be constant within each synchronization time-step. Two somewhat different numerical techniques have been developed independently to integrate the VUU equations of motion in three dimensions, which use the lagrangian (co-moving) method [17,18,20] and the eulerian (space-fixed) description [15,16], respectively. Technical details of the different approaches have been given elsewhere. In the present work we have employed the two independently developed computer codes and compared the results to check for the reliability of our calculation. We observe that the results of the lagrangian method [17,18,20] and the revised eulerian code [16,19] are in excellent agreement, which gives us confidence into our numerical procedures.

We first study the system C (85 MeV/N) + C in the mean field approximation by excluding two body collisions in the present theory, thus mimicking TDHF by solving the Vlasov equation [11–14,16,18]. These calculations are then compared to actual TDHF results obtained by using the TDHF code of Cusson et al. [8].

Fig. 1 shows – for C (85 MeV/N) + C at $b = 1$ fm – the time evolution of the density profile as obtained in the TDHF and Vlasov equation calculations. We find a very similar behavior in both the quantum mechanical and classical mean field theories: both calculations exhibit nearly identical small longitudinal and transverse momentum transfers to the slipped-through projectile and target remnants, which survive the reaction rather intact. The lack of two body collisions results in strongly forward peaked angular distributions, in sharp contrast with the data in this energy regime. About 85% of the initial longitudinal momentum is conserved in the projectile- and target-like nuclear fragments. This is in sharp contrast to calculations at lower energies [8,16], which exhibit large deflection angles as a result of the mean field dynamics.

The inclusion of the Uehling–Uhlenbeck collision integral into the Vlasov equation changes this drastically (see the right-hand side of fig. 1). In contrast to results at 25 MeV/N [16], each individual reaction can now be separated into two clearly distinct components. First, we observe again the slipped-through projectile- and target-like fragments which, however, now retain only about 40% of the nucleons and less than 20% of the initial longitudinal cm momentum. As we will see later these slipped-through residues contain mostly particles which have not scattered at all.

The second component consists of 60% of the projectile nucleons which undergo at least one n–n scattering and form a non-equilibrated mid-rapidity system, which shows an almost isotopic emission pattern. We would like to note that complete stopping of the

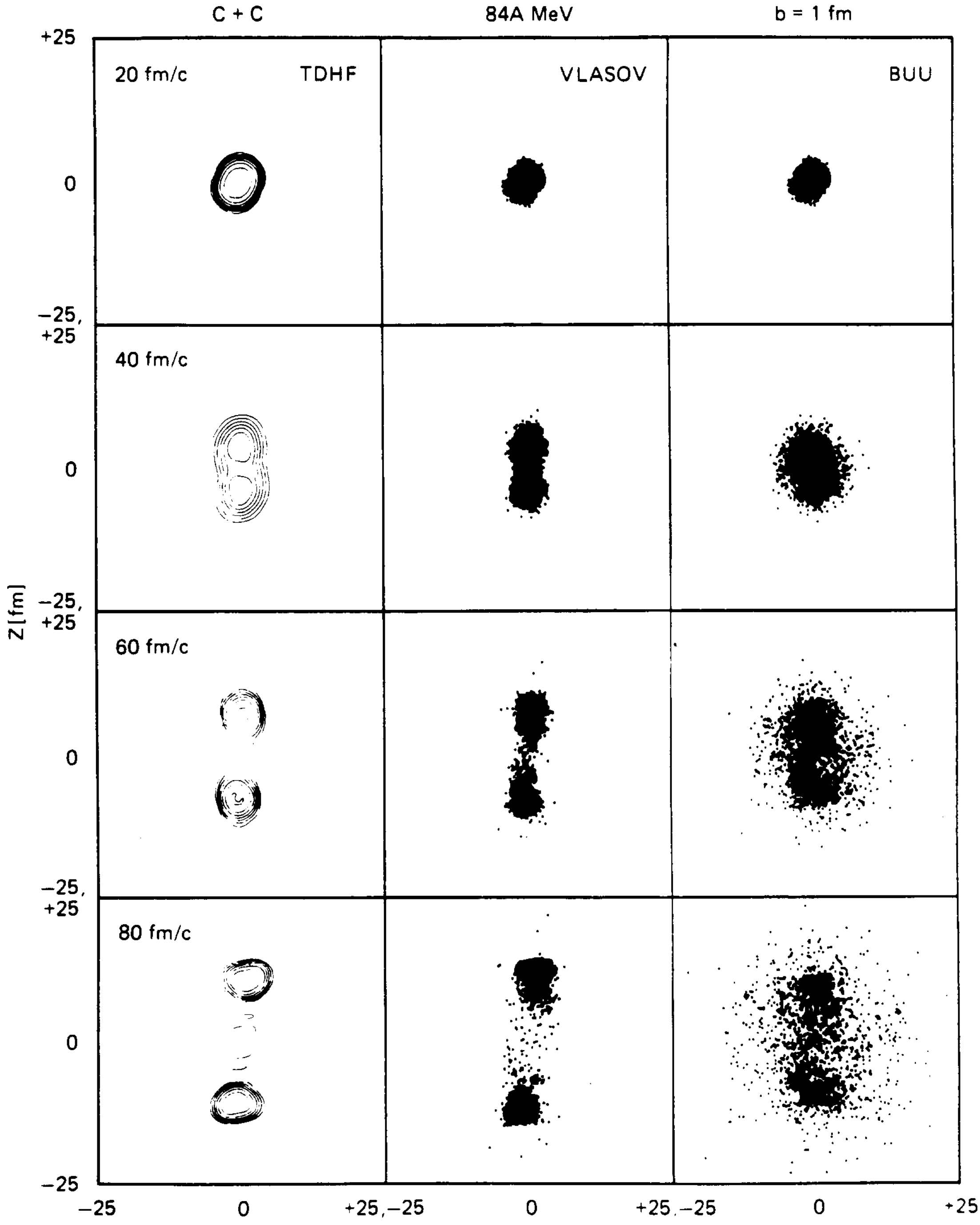

Fig. 1. Time evolution of the single particle distribution function for the reaction C (85 MeV/N) + C at b = 1 fm as predicted by the quantal mean field theory without collision term (TDHF, left-hand side column), the classical mean field theory (Vlasov's equation without collision term, center column) and the Vlasov equation with a Uehling–Uhlenbeck collision term included (right-hand side column). The transparency is evident in both mean field results without collision term. In contrast, a rapid deceleration of 60% of the incident nuclei is observed once the collision term is included into the Vlasov equation.

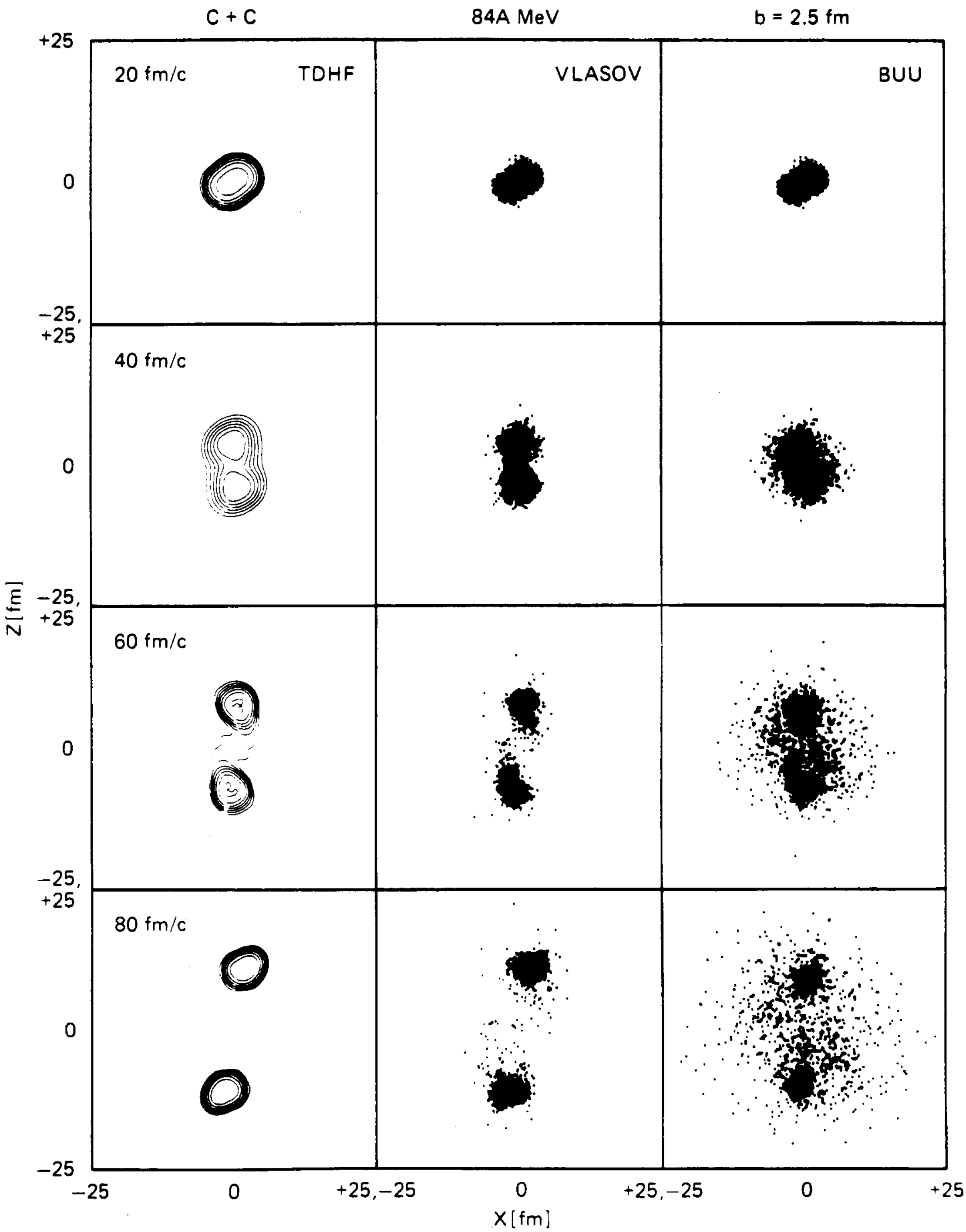

Fig. 2. Same as fig. 1 but for $b = 2.5$ fm. The two mean field calculations again agree very well, yielding negative angle scattering, while the introduction of the collision term results in a different sideways deflection angle. Stopping is not observed at these large impact parameters due to insufficient geometrical overlap, which results in too few nucleon–nucleon collisions.

projectile in the target is predicted by the VUU approach if heavier systems are studied, for bombarding energies from 90 to 800 MeV/N [18,20]. What is then the reason for the incomplete deceleration of the nuclei seen in fig. 1? First, the chances for n–n collisions to occur are smaller in C + C reactions than in a more extended, massive system. Second, we observe that a rather large fraction of the attempted n–n collisions is forbidden due to Pauli blocking of the exit channels. This will be studied in detail below.

Fig. 2 shows the same reaction as above, but at b = 2.5 fm. Negative angle scattering is observed in both pure mean field approaches, and again the classical and the quantum approach agree remarkably well. The collision term results in less inward scattering. The mid-rapidity source is much less apparent; two slightly decelerated and excited residues survive the collision. The effect of the collision term is less dramatic at these larger impact parameters due to the smaller geometrical overlap of the nuclei, which reduces the number of nucleon–nucleon collisions even more.

In fig. 3 we investigate the reaction C (85 MeV/N) + C at b = 1 fm in more detail. We display the initial and the final density profiles in coordinate and momentum space. For the display of the final state we distinguish between particles which did not undergo any collision (N_c = 0) and scattered particles with N_c > 0. The projectile-like fragments contain on the average 3–4 unscattered nucleons and about 1–2 scattered nucleons, which are trapped by the attractive mean field of the projectile-like fragment. The mid-

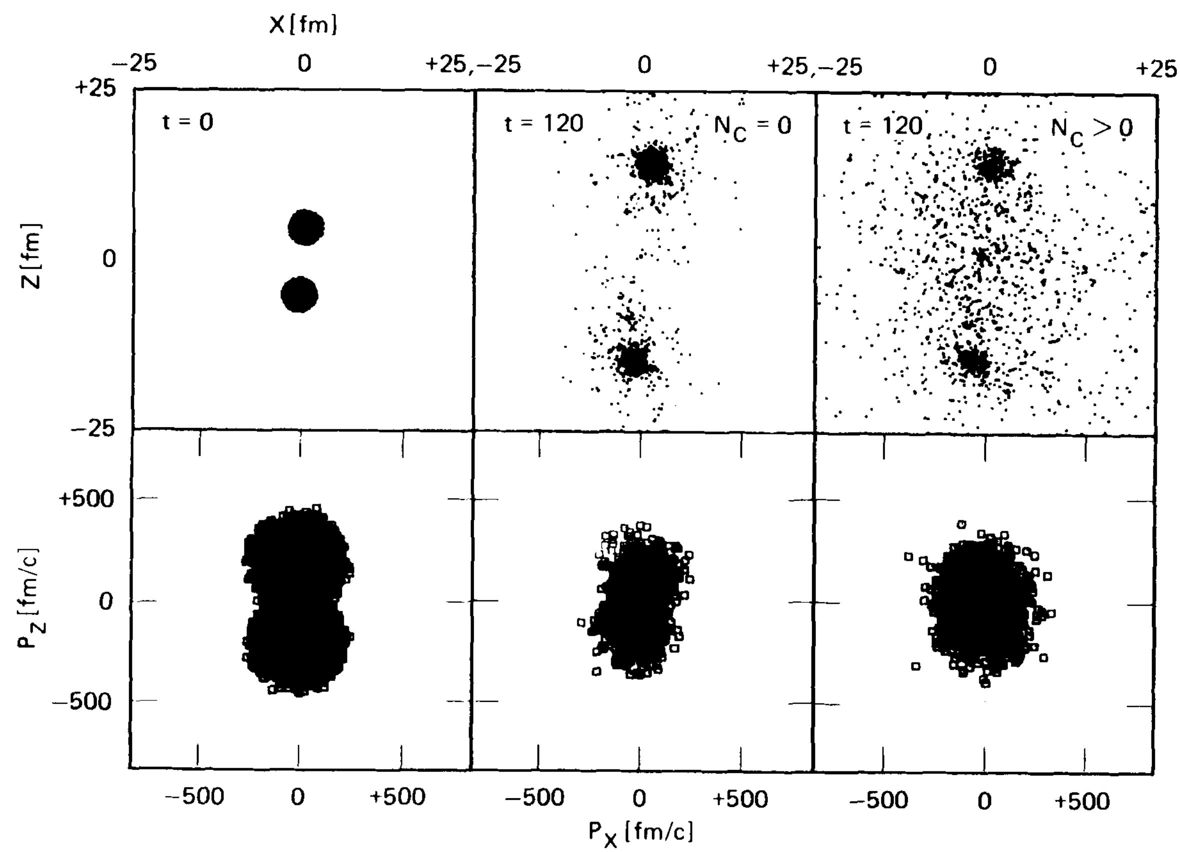

Fig. 3. Density profiles in coordinate and momentum space. The left column shows the initial distributions. The centre column show the final distributions of nucleons which did not suffer a collision. The right column displays the final distribution of the scattered particles.

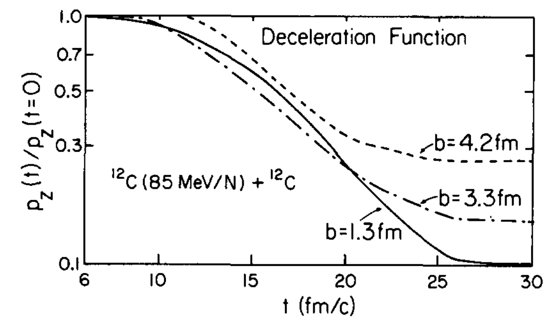

Fig. 4. Time dependence of the longitudinal momentum of nucleons which undergo at least one collision. A deceleration time of t = 12.5 fm/c reproduces the curve for b = 1.3 fm, corresponding to a stopping length of 2.5 fm.

rapidity region contains almost exclusively scattered particles. The high momentum components of the initial momentum space distribution get most effectively depleted by collisions because for them the Pauli blocking is least effective. The projectile-like fragments have densities around $\rho_0/2$. Hence their Fermi energy is lowered. A collective deceleration of these projectile-like fragments is caused by the mean field. Those particles which have undergone collisions exhibit a nearly isotropic distribution in momentum space, with some forward–backward asymmetry.

The time dependence of the average momentum of only those nucleons which have suffered at least one collision is shown in fig. 4. At the beginning of the reaction the attractive mean field leads to a small acceleration of projectile and target. As soon as two body collisions set in the average momentum decreases rapidly. Observe that nearly all (90%) of the initial longitudinal momentum is dissipated during a period of about 12.5 fm/c if central collisions, b = 1.3 fm, are selected. Even in rather peripheral reactions, b = 4.2 fm, there is about a 75% longitudinal momentum loss for particles which have undergone at least one collision. However, the number of colliding nucleons drops rapidly with increasing impact parameter. Therefore, the strongest deceleration and equilibration effects occur at small impact parameters, as expected. We find a time constant of t_B = 12.5 fm/c for the deceleration, roughly independent of the impact parameter. This corresponds to a deceleration length of x = 2.5 fm. These results are of great importance as input for phenomenological models such as the pion bremsstrahlung [5,6] and hot spot models [7]. Our microscopically calculated parameters are a factor of two larger than the values obtained by Vasak et al. [5,6] via a comparison of collective γ-bremsstrahlung to the data [3]. A microscopic treatment of the bremsstrahlung process seems necessary for a quantitative understanding of the absolute photon yields.

The dependence of the nuclear stopping power on the target mass is studied via ^{12}C (85 MeV/N) induced reactions with six different targets from ^{12}C to ^{197}Au. The number of projectile nucleons undergoing at least one collision increases from 60% for the C target to 97% for the Au target. These collisions result in a momentum transfer on the target like residue of 44 MeV/c/nucleon (66% of the maximum momentum transfer possible, 67 MeV/c/nucleon) for Ni targets to 18 MeV/c/nucleon (80% of the maximum 22 MeV/c/nucleon) for Au targets. For the heavier targets ($A_T > 50$) this goes hand in hand with an almost complete stopping of the projectile in the target. This result is similar to what has been obtained with the VUU method for ^{40}Ar + ^{197}Au at 92, 400 and 800 MeV/nucleon bombarding energy [18,20].

The number of emitted uncollided projectile nucleons, N_0, falls off exponentially as a function of the target diameter. From this fall-off we find that the mean free path of the nucleons in intermediate energy heavy ion collisions is λ = 2.6 fm. This value is larger than the mean free path estimated from classical kinetic theory, $\lambda_c = (\sigma\rho)^{-1/2}$ = 1.4 fm. This descrepancy is mainly due to the effects of the Pauli-principle, which blocks a fraction of the attempted collisions in the VUU approach in accord with the local phase space density. Other effects which contribute are the density increase in the collision (wich decreases λ) and the finite deceleration due to the mean field (which also decreases λ). Please note that for C and Al targets we observe projectile remnants surviving the interaction. Because of residual collisions within these final clusters – after the clusters have separated – we expect N_0 to be smaller than what is expected from the extrapolation from the heavier targets. This is indeed observed.

We are presently studying the dependence of the mean free path on the bombarding energy [19]. The dependence on the achieved compression and the excitation energy ("temperature") of the target is also presently being investigated [19]. This is done by going from proton induced reactions to heavy projectiles as available at GANIL and at the low energy beam line at LBL [19].

In conclusion we have studied ^{12}C induced reactions at 85 MeV/nucleon on a variety of targets using the Vlasov–Uehling–Uhlenbeck approach. We have confronted the VUU results with the pure mean field TDHF and Vlasov theories, which neglect two body collisions completely. Slight deceleration of projectile and target is predicted for both the quantal and the classical mean field approaches, which are in excellent agreement with each other. Inclusion of the collision term via the VUU approach results in two distinct components and greatly enhanced nuclear stopping power: Up to 97% of the projectile nucleons undergo at least one n–n scattering, which results in substantial linear momentum transfers to the target-like residues. Deceleration times of 12.5 fm have been obtained. The number of uncollided projectile nucleons emitted drops off exponentially with the target diameter. A nucleon mean free path of $\lambda = 2.6$ fm can be deduced from the present results.

References

[1] W. Benenson et al., Phys. Rev. Lett. 43 (1979) 683; 44 (1980) 54;
J.P. Sullivan et al., Phys. Rev. C25 (1982) 1499.
[2] T. Johansson et al., Phys. Rev. Lett. 48 (1982) 732;
V. Bernard et al., unpublished.
[3] H. Noll et al., Phys. Rev. Lett. 52 (1984) 1284;
P. Braun-Munzinger et al., Phys. Rev. Lett. 52 (1984) 255;
H. Heckwolf et al., Z. Phys. A315 (1984) 243.
[4] E. Grosse, Proc. 5th Adriatic Intern. Conf. on Nuclear Physics (Hvar, 1984), ed. N. Cindro.
[5] D. Vasak, H. Stöcker, B. Müller and W. Greiner, Phys. Lett. 93B (1980) 243;
D. Vasak, B. Müller and W. Greiner, Phys. Scr. 22 (1980) 25.
[6] D. Vasak, W. Greiner, B. Müller, T. Stahl and M. Uhlig, Nucl. Phys. A428 (1984) 291.
[7] J. Aichelin, Phys. Rev. Lett. 52 (1984) 2340.
[8] H. Stöcker, R.Y. Cusson, J.A. Maruhn and W. Greiner, Z. Phys. A294 (1980) 125; Phys. Lett. 101B (1981) 379.
[9] H. Toki and H. Stöcker, Phys. Lett. 150B (1985) 412; 152B (1985) 326.
[10] G. Buchwald, G. Graebner, J. Theis, J.A. Maruhn, W. Greiner and H. Stöcker, Phys. Rev. C28 (1983) 1119.
[11] E.A. Uehling and G.E. Uhlenbeck, Phys. Rev. 43 (1933) 552.
[12] See e.g. B. Remaud, F. Sebille, C. Gregoire and F. Scheuter, Nucl. Phys. A428 (1984) 101; to be published;
C. Gregoire and F. Scheuter, Phys. Lett. 146B (1984) 21;
W. Cassing, Proc. Bormio Meeting 1984, GSI-Preprint 84-15;
C. Toepffer and C.Y. Wong, Phys. Rev. C25 (1982) 1018.
[13] H.H.K. Tang, C.H. Dasso, H. Esbensen, R.A. Broglia and A. Winther, Phys. Lett. 101B (1981) 10.
[14] C.Y. Wong, Phys. Rev. C25 (1982) 1460.
[15] G.F. Bertsch, H. Kruse and S. Das Gupta, Phys. Rev. C29 (1984) 673.
[16] J. Aichelin and G. Bertsch, Phys. Rev. C31 (1985) 1730.
[17] H. Kruse, B.V. Jacak and H. Stöcker, Phys. Rev. Lett. 54 (1985) 289.
[18] H. Kruse, B.V. Jacak, J.J. Molitoris, G.D. Westfall and H. Stöcker, Phys. Rev. C31 (1985) 1770.
[19] J. Aichelin and H. Stöcker, to be published.
[20] J.J. Molitoris and H. Stöcker, Phys. Rev. C32 (1985) 346; Phys. Lett. 162B (1985) 47.