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FROM THE SINGLE-PARTICLE DENSITY TO FLUCTUATION PHENOMENA IN COLLISION THEORY

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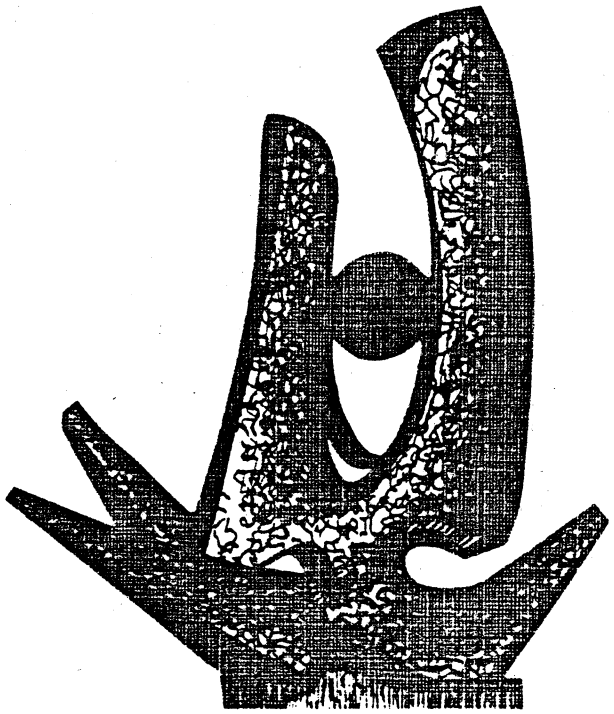

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From the single-particle density to fluctuation phenomena in collision theory

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Abstract

A modification of the Vlasov-Boltzmann theory is proposed to allow calculation of fluctuation phenomena in heavy ion reactions. The numerical solution of the equation of motion would require about the same computational effort as in a present method for solving the Vlasov-Boltzmann equation, but a calculation would generate an individual event rather than a global probability distribution of the single-particle density.

The Vlasov-Boltzmann equation with the Uehling-Uhlenbeck form of the collision integral provides a numerically tractable theory of the single particle density f(p,r) in nuclear collisions. The theory has been successfully applied to one-body observables such as the inclusive particle spectra [1] and global momentum transport [2]. However, the theory is completely unrealistic in its predictions of fluctuation phenomena. For examples, strong correlations are observed between emitted particles that are not contained in the inhomogeneities of the single particle density. The collision may fragment the nuclei in diverse ways, but the description of the system by f(p,r) alone permits only a single channel to be present in the final state. Here I propose a modification of the Vlasov-Boltzmann approach that I anticipate will overcome these deficiencies.

I first recall the theoretical foundation of the Vlasov-Boltzmann approach. The starting point in many-particle quantum theory is the representation of a many-fermion wavefunction by a Slater determinant. This leads directly to the time-dependent Hartree-Fock equation of motion, which can be reduced to the Vlasov equation without violent approximation. Of course, a realistic theory at intermediate energy requires that many Slater determinants be mixed. If many determinants are degenerate, time-dependent perturbation theory or fancier methods can be applied to derive the Uehling-Uhlenbeck collision integral as a term to be added to the Vlasov equation.

Unfortunately, the theory carries only limited information because the one-body density is an average over all the Slater determinants in the wavefunction. However, the averaging is not really necessary because the different Slater determinants are essentially incoherent. The one-body part of the Hamiltonian has vanishing matrix elements between different determinants because collisions change the wavefunction of two particles

simultaneously. The two-particle interaction is coherent, but its effects are already mostly taken into account by the collisional mixing of determinants. One can imagine situations where the phase relation between determinants is important, for example when pairing affects the dynamics or when correlations associated with low collective states are studied. But these effects are insignificant at energies much above the ground state, and certainly at the intermediate energy regime studied here.

In the absence of coherence between different parts of the wavefunction, one may solve the equation of motion for each portion separately. My proposed modification of the one-body dynamics is to treat the collisions as branching points in the density evolution rather than as a continuous source function in the equation of motion. This idea can be readily incorporated into the numerical scheme that is used to solve the Boltzmann equation in refs. [1] and [2]. There the equation of motion is solved using test particles to represent the single-particle phase space density. The number of test particles needed to represent one physical particle, denoted by N below, is 50 to 100 in practice. The collision integral is presently treated by allowing the test particles to collide with cross sections reduced from the physical values by 1/N.

In the new approach, the collisions between test particles would be candidates for changing the phase space density, but the change would only be made with a probability 1/N. Thus different possible changes of the initial Slater determinant would be represented in a stochastic way. The different way of treating collisions is sketched in Fig. 1, contrasting the approximation to the continuous physics in the Boltzmann equation with the discontinuous jumps of the probabilistic treatment.

The main problem in implementing this approach is specifying the final state as a Slater determinant. Quantum mechanically, a Slater determinant is concisely defined by the density matrix equation

$$\rho^2 = \rho \tag{1}$$

In the Wigner representation used for the Vlasov equation, eq. (1) is rather involved. In the same spirit used to construct a classical solution of the Vlasov equation we make may treat f semiclassically and use the formula

$$f^2 = f$$

This implies the intuitive result that f=0 or 1. Thus, when a collision occurs, the test particles should be moved in a way to preserve f=0 or 1 in the final state. The number of test particles that should be moved is 2N, corresponding to the change of two single particle states. The particles that are moved should obviously be contiguous to the test particles of the successful collision, but the details of how the phase space boundaries should be chosen remains to be worked out. Energy and momentum conservation, and compactness of the phase space boundaries should provide sufficient limitation. The change in test particle distributions is depicted in Fig. 2.

This model would be applicable to a broad range of phenomena that are now poorly understood. For example, the distribution of masses in energetic heavy ion collisions have been interpreted in a number of ad hoc models which invoke such concepts as thermodynamic phase transitions [3], statistical equilibrium [4], and percolation [5]. Even approaches based on

time-dependent Hartree-Fock dynamics have up to now required an ad hoc assumption to introduce the fluctuations[6,7]. The model proposed here would certainly predict multiparticle fragmentation with final state clusters of various sizes. Another observable that would be interesting to study is the correlation between energetic particles emitted from the reaction. The experimental data show a large enhancement of oppositely directed particles, which is ascribed to the effects of momentum conservation in two-particle collisions[8]. The Boltzmann equation has only the weak correlation between particles associated with an anisotropy of the single-particle distribution function. The proposed model has collisionally induced correlations which respect momentum conservation on a two-particle level, and so would allow the testing of the empirical description on a microscopic level.

Of course, it is far from clear that the physics of this approach contains enough fluctuation to explain the empirical observations. The semiclassical reduction of TDHF to the Vlasov equation is adequate for treating many integrated quantities, but it might be the case that the quantum mechanics shows up more insistently in the fluctuations.

The computational effort that would be required is quite large. Because the numerical evolution of the equation of motion generates an event rather that a probability, many runs must be examined to get an overall picture of the final state. Still, it should be possible to get some insight from a small set of runs about the effectiveness of the collisional mechanism in creating the fluctuations. But in any case, the problem requires the power of the present day supercomputers.

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Figure Captions

Fig. 1. Schematic plot of the cumulative number of test particle collisions in the numerical evolution of the distribution function. The thin line depicts the Vlasov-Boltzmann algorithm, which approximates a continuous rate by a large number of test particles. In the proposed stochastic collision model, depicted by the thick line, 2N test particles would be moved together.

Fig. 2. Schematic plot of test particle distribution in momentum space, showing the effect of a single collision. In (a) a test particle is moved in momentum space as shown by the arrow. In the proposed model, if that collision were successful, a physical particle would be moved, as represented by the finite areas of momentum space in (b).

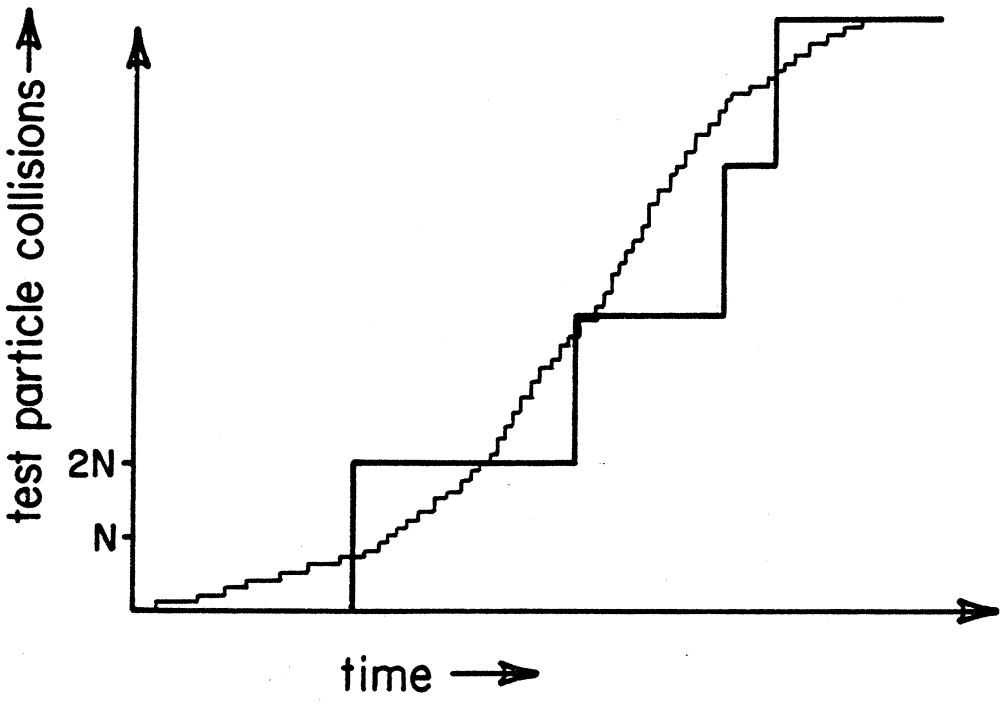

FIGURE 1

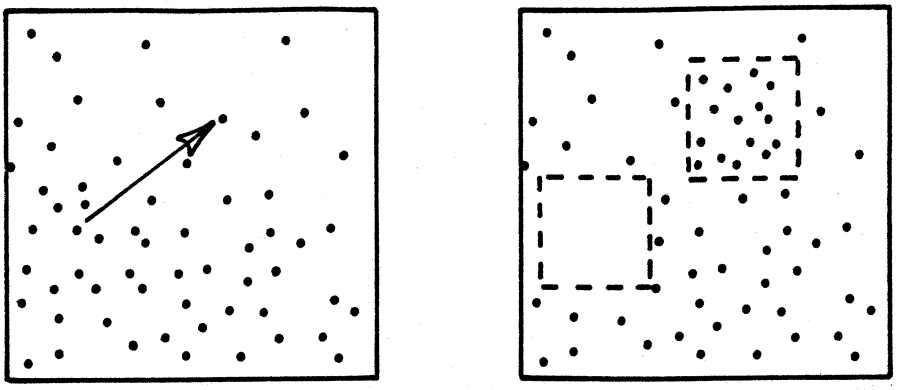

FIGURE 2