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CRITICAL PION OPACITY

WOLFGANG BAUER

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Wolfgang Bauer

National Superconducting Cyclotron Laboratory
and
Department of Physics and Astronomy
Michigan State University
East Lansing, MI 48824-1321, USA

Abstract: The new effect of critical pion opacity is proposed. It is a reduction of the emission probability for central-rapidity pions in the vicinity of the critical point of the nuclear "liquid-gas" phase diagram. We show in a strongly simplified model how this effect can arise, and we discuss how it could be measured.

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The classical phenomenon of critical opalescence is an increased scattering of electromagnetic radiation near to the critical point of a substance.[1] A particularly instructive example is found in Ref. 2 for the case of cyclohexane-aniline. Far away from T_c, there is no appreciable light scattering. As T_c is approached, however, the sample takes on a milky appearance. This is due to the scattering of light off the density fluctuations.

The density fluctuations are usually characterized by introducing the density-density correlation function

$$C(\vec{r}-\vec{r}') = \langle \rho(\vec{r})\rho(\vec{r}')\rangle - \rho^2. \tag{1}$$

Its relation to the isothermal compressibility is given by

$$\int d\vec{r}\, C(\vec{r}) = \rho \frac{\kappa_T}{\kappa_T^0}, \tag{2}$$

where the isothermal compressibility is defined as $\kappa_T = -V^{-1} \cdot (\partial V/\partial P)_{T,N}$ and its value for the ideal gas is $\kappa_T^0 = (\rho k_B T)^{-1}$. Since κ_T diverges at the critical point, the range of the correlation function has to increase near the critical point. Sufficiently close to T_c, the correlation length becomes as large as the wavelength of light. In this case, the density fluctuations scatter light strongly. This leads to the phenomenon of critical opalescence which was first explained in the classical theory of Ornstein and Zernicke.[3] The scattering intensity is given by

$$I(\vec{q}) = \frac{I^0(\vec{q})}{\rho} \int d\vec{r}\, e^{-i\vec{q}\vec{r}}\, C(\vec{r}), \tag{3}$$

where $\vec{q}$ is the momentum transfer, and $I^0(\vec{q})$ is the intensity for noninteracting particles.

We expect nuclear matter to have a phase diagram similar to a Van der Waals gas, because the nuclear interaction exhibits short range repulsion and long range attraction.[4,5] Consequently, we should expect infinite nuclear matter to be able to go through a phase transition from the liquid to the gas phase. This phase transition is expected to be of first order, terminating in a second order phase transition at the critical point. It should be stressed here that there is no firm experimental evidence for the existence of a second order phase transition and a corresponding critical point as of yet. However, it is hoped that with the new heavy ion accelerators and new detector arrays covering almost the full solid angle this interesting point of the nuclear phase diagram will be studied in detail. In a previous publication, we have pointed out methods to look for the critical point, to treat the finite size effects and to determine the critical exponents.[6]

In this paper, we propose a new effect that should occur as we approach the critical point of nuclear matter from above as a function of the order parameter. It is the decrease of transparency of nuclear matter near the critical point for pions traversing it, and we have therefore dubbed it "Critical Pion Opacity".

We start out by noting that pion absorption is (at least) a two-nucleon process. The absorption of a pion on one nucleon is forbidden for simple kinematical reasons. The dominant channel for pion absorption is the so-called Delta process, in which a nucleon is excited to the isospin 3/2 Delta resonance state by the incoming pion. This Δ can then interact with a second nucleon via $\Delta + N \to N + N$ to complete the process of the pion absorption. This process has to take place at short inter particle distances between the two nucleons of the order of[7]

$$d = \frac{\hbar}{\sqrt{M_N M_\pi}} \approx 0.5 \text{ fm}. \tag{4}$$

Thus the pion absorption cross section exhibits a dependence of $\sigma_{abs} \propto \rho^2$. In Ref. 7, it is pointed out that there may also be sizable contributions to pion absorption from three or four nucleon processes, such as the double-isobar process. In any case, the pion absorption cross section has a density dependence of $\sigma_{abs} \propto \rho^\lambda$, with $\lambda \geq 2$, and is therefore sensitive to the fluctuations in the density. If we could irradiate nuclear matter at its critical point with pions, we should see the effects of the density fluctuations in the pion absorption cross section. The following simple illustrative model is meant to demonstrate this effect.

We consider a system of nuclear matter at an average density of $\langle\rho\rangle = \rho_0/\alpha$ contained in a sphere of radius $R = r_0\, A^{\frac{1}{3}}\, \alpha^{\frac{1}{3}}$ where A is the total number of baryons, the radius constant r_0 is given by 1.2 fm, and ρ_0 is the nuclear matter density. The constant α is in the range 2–4, such that the average density is at the critical value.

We take two limiting cases: One which contains no fluctuations in the density, $\rho(\vec{r}) = \rho_0/\alpha$, and one which contains maximum fluctuation, $\rho(r, \Omega_i) = \rho_0$, where Ω_i is an arbitrary solid angle with the constraint $\sum_i \Omega_i = 4\pi/\alpha$, and 0 otherwise (for the sake of analytical simplicity). In the homogeneous case, we obtain for the average absorption cross section for a pion escaping from the center of the distribution ($\lambda = 2$)

$$\begin{aligned} \sigma_{\mathbf{hom}} &\propto \frac{1}{4\pi} \int_0^{4\pi} d\Omega \int_0^R dr \left(\frac{\rho_0}{\alpha}\right)^2 \\ &= r_0 A^{\frac{1}{3}} \rho_0^2 \alpha^{-\frac{5}{3}}, \end{aligned} \tag{5}$$

whereas the case with fluctuations yields

$$\begin{aligned} \sigma_{\mathbf{inhom}} &\propto \frac{1}{4\pi} \int_0^{4\pi} d\Omega \int_0^R dr \rho^2(r, \Omega) \\ &= r_0 A^{\frac{1}{3}} \rho_0^2 \alpha^{-\frac{2}{3}}. \end{aligned} \tag{6}$$

We therefore obtain the result $\sigma_{\mathbf{inhom}}/\sigma_{\mathbf{hom}} = \alpha$, an increase in the absorption cross section by a factor of 2–4. For a general value of $\lambda \geq 2$, we obtain $\sigma_{\mathbf{inhom}}/\sigma_{\mathbf{hom}} = \alpha^{\lambda-1}$.

We now discuss if and how this effect could be observed in experiment. It is generally believed that there are in principle two ways in which the nuclear matter critical point can be reached. One would be in very asymmetric systems (proton + heavy target), in which for sufficiently high bombarding energies ($E_{\mathrm{beam}} > 5$ GeV) the spectator matter could be heated up to the temperature necessary to reach the critical point. Some indications that this process may indeed take place can be found in the work of the Purdue group[8] who used ultra-high energy protons to fragment Xenon and Krypton targets. The other possible way to reach the critical point would be the central collision of symmetric heavy ion systems with $E_{\mathrm{beam}} \approx 100$ MeV/nucleon. Here the participants are at excitation energies of around 25 MeV/nucleon, if complete stopping is assumed. For sufficiently heavy systems of a combined mass greater then approximately 200 nucleons, this is still a reasonable assumption at 100 MeV/nucleon.[9] This second way is preferred in our case, because of the simpler determination of the deposited excitation energy,[9] and because in this case thermalization predominantly proceeds via elastic nucleon-nucleon collisions.

During the compression phase of this heavy ion collision, densities of 1.5 to 2 ρ_0 are reached and some pions are produced. The overall excitation function for these "subthreshold" pions is well understood in this beam energy domain.[10] Even though there is a large amount of absorption during this high density phase of the collision, some pions escape from it. To experience the effect of density fluctuations near the critical point, the pions have to stay in the baryon matter long enough for it to expand to the critical point. We have estimated the time interval for this expansion in a simple manner.

We consider the case of isothermal expansion from a compressed spherically symmetric state at a temperature $T \approx T_c$. The equation to solve for the expansion is then

$$\frac{d}{dt}\vec{P} = \oint \mathcal{P} d\vec{s}, \tag{7}$$

where $d\vec{s}$ is a surface element of the sphere in which the baryons are contained, and $\mathcal{P}$ is the pressure. To obtain quantitative estimates for the expansion time, we chose the equation of state of Ref. 11,

$$\mathcal{P}(\rho, T) = \frac{1}{9} K \frac{\rho^2}{\rho_0} \left(\frac{\rho}{\rho_0} - 1 \right) + \frac{1}{6} b^2 m \rho^{1/3} T^2 \tag{8}$$

with b=1.809 for nuclear matter, the saturation density ρ_0=0.15 fm^{-3}, and the compressibility K=210 MeV. $m = M_n/\hbar^2 = 0.024$ MeV^{-1}fm^{-2} is the nucleon mass.

From this equation of state, the critical point is found from the solution of $\partial\mathcal{P}(\rho, T)/\partial\rho = 0$ and $\partial^2\mathcal{P}(\rho, T)/\partial\rho^2 = 0$. This results in a critical density of $\rho_c = \frac{5}{12}\rho_0$ and a critical temperature of $T_c = 0.326\rho_0^{1/3}\sqrt{K/m}$=16.2 MeV. Other, more realistic, equations of state[14,15] yield similar predictions for T_c and ρ_c (within about 10-20%). The above parametrization is sufficient for our purposes since we are only interested in an estimate for the expansion time interval. However, it turns out that the expansion dynamics generated by this set of equations is similar to much more elaborate models[12] and is close to the one predicted by non-equilibrium transport theories.[13]

With the dimensionless variables $\overline{\rho} = \rho(t)/\rho_0$ and $\overline{T} = T/T_c$, we obtain then

$$\mathcal{P}(\overline{\rho},\overline{T}) = (3.5\,(\overline{\rho}^3 - \overline{\rho}^2) + 1.825\,\overline{\rho}^{\frac{1}{3}}\,\overline{T}^2)\ \mathrm{MeV\,fm^{-3}}. \tag{9}$$

Instead of solving the full set of hydrodynamic equations, we obtain a simple estimate by computing the force on a circle which is obtained by cutting through the center of the nucleus. This leads to a simple second order differential equation initial value problem for the radius, $R(t)$ of the sphere containing the baryons,

$$\frac{d^2}{dt^2}R(t) = \frac{16\pi}{3\,A\,M_n}R^2(t)\mathcal{P}(\overline{\rho},\overline{T}). \tag{10}$$

We solve this differential equation under the assumption of isentropic expansion.[16] For the equation of state above, the interaction between the baryons was assumed to be a represented by a mean field potential, U, which is only dependent on the density, but not not the temperature. In this case, the entropy is a homogeneous function of $\tilde{\mu}/T$ of order 0 ($\tilde{\mu} = \mu - \partial U(\rho)/\partial\rho$). Since the entropy is conserved during the expansion, the ratio of $\tilde{\mu}$ and T has to be a constant as well. This leads to the well-known relation[17]

$$\overline{\rho}\,\overline{T}^{-3/2} = \mathrm{const}. \tag{11}$$

which we use to calculate $\overline{T}$.

In Fig. 1, we show the solution of this differential equation for $A = 200$ nucleons. Displayed are the the average density, temperature and pressure and the velocity and radius of the surface of the expanding system. The dashed lines correspond to an initial compression of $\rho(t=0) = 2\,\rho_0$ and therefore $R(t=0) = 5.4$ fm. The solid lines represent the case of an initial density of $\rho(t=0) = \rho_0$, corresponding to $R(t=0) = 6.8$ fm. In both cases, $\dot{R}(t=0) = v(t=0) = 0$. The dotted line in the upper part of the figure indicates the critical density for this equation of state, $\rho_{cr} = \frac{5}{12}\rho_0$.

In this calculation, the critical density is reached after $t_c = 12.2$ fm/c for $\rho(t=0) = 2\,\rho_0$ and $t_c = 15.5$ fm/c for $\rho(t=0) = \rho_0$. If we use $A = 300$ nucleons instead, the corresponding times are $t_c = 13.9$ fm/c and 17.7 fm/c for $\rho(t=0) = 2\,\rho_0$ and ρ_0, respectively.

For $A = 200$, a pion which was created at $\rho = 2\rho_0$, $t = 0$, $R = 0$, can therefore have a maximum velocity of $\beta \approx 0.75$ in the baryon center of mass frame, if it is required to be within the baryon sphere at $t = t_{cr}$. This corresponds to a kinetic energy of $T^{\pi}_{cr} = (\gamma - 1)M_{\pi} \approx 70$ MeV. For the $A = 300$, $\rho(t{=}0) = \rho_0$ initial condition, we obtain $T^{\pi}_{cr} \approx 33$ MeV. Other authors, who have solved the full set of hydrodynamic equations, report similar numbers.[18,19] The time to reach the critical point is about a factor of 0.3 to 0.5 of the monopole vibration time.

Pions having less kinetic energy in the c.m. frame than the critical value will on average experience this effect, whereas the faster pions should not. The way in which we propose to test our predictions is linked to this observation. We suggest to measure the ratio

$$\mathcal{R}_\pi(E_{\rm beam}) = \frac{\sigma_\pi(T^\pi < T^\pi_{\rm cr})}{\sigma_\pi(T^\pi > T^\pi_{\rm cr})} \tag{12}$$

for central collisions of symmetric heavy ion systems. We predict that this ratio will have a dip at the beam energy which leads to an expansion which goes through the critical point in the nuclear matter phase diagram. This dip should be superimposed on an otherwise smoothly rising function $\mathcal{R}_\pi(E_{\rm beam})$. The background rise is due to the available phase space volume which increases with beam energy. Since the absorption cross section for low energy pions in nuclear matter is small compared to the high energy ones, we should actually be able to probe the interior of the expanding baryon system with the above ratio. The detected high energy pions are believed to be predominantly produced in the surface region of the baryon system. High energy pions produced in the interior are quickly reabsorbed because of the high absorption cross section for them.[20] Therefore the division by the cross section for the high energy pions can serve to remove much of the steep beam energy dependence of $\mathcal{R}_{pi}$ in this "subthreshold" pion production region, since the high energy pions are not subject to the effect of the critical pion opacity.

The exact width and depth of this postulated dip is not predictable within the simple framework developed here. However, from finite size scaling we know that the width of the dip as a function of the beam energy should decrease with increasing mass of the baryonic system. We also expect the magnitude of the drop in cross section to be about a factor of 2–4, as outlined above. The experimental detection of this effect would indicate exciting new nuclear many-body physics.

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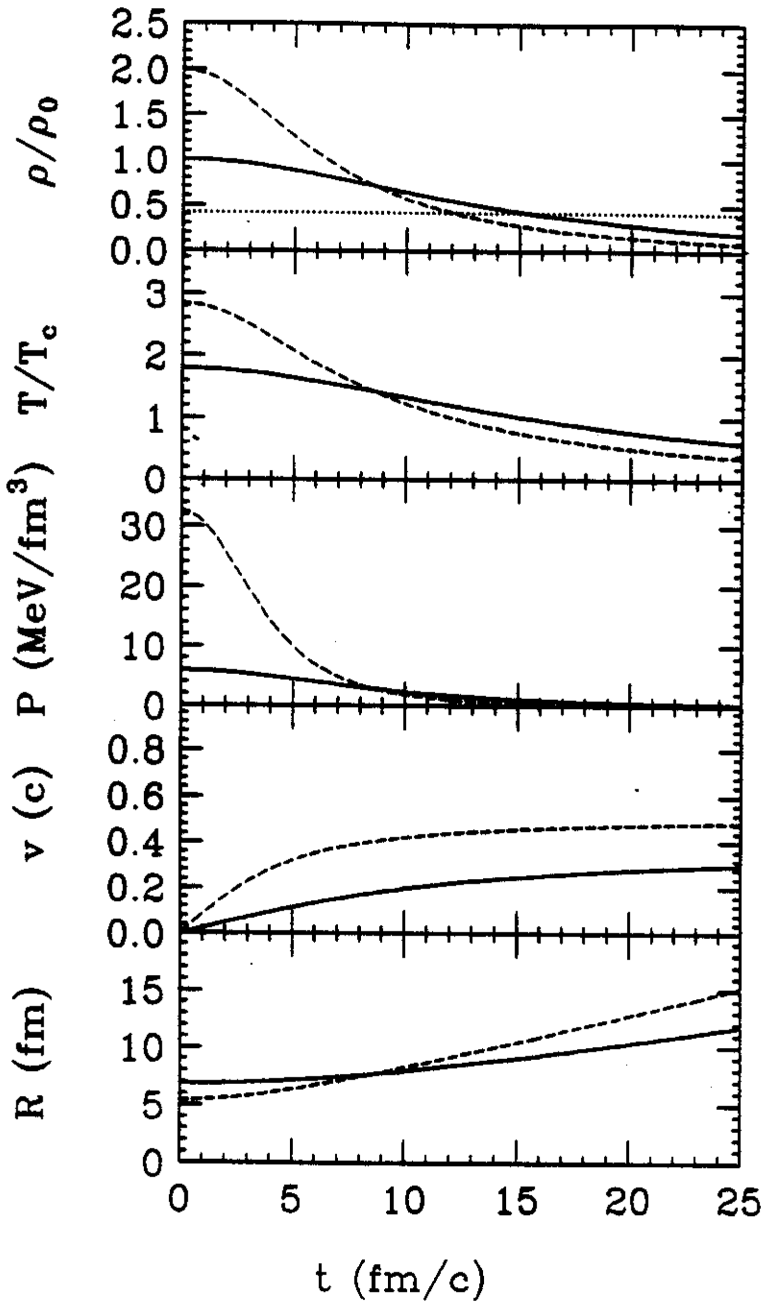
Figure Captions

Fig. 1 Time evolution of the density, temperature, pressure, and the velocity and radius of the surface of the expanding baryon system, as obtained from the numerical solution of the differential equation 10. The dashed lines correspond to an initial compression of $\rho(t=0) = 2\,\rho_0$, and the solid lines are for $\rho(t=0) = \rho_0$. The dotted line in the upper part of the figure indicates the critical density for the equation of state used.



ρ/ρ_0
T/T_c
P (MeV/fm^3)
v (c)
R (fm)
t (fm/c)