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RELATIVISTIC TRANSPORT THEORY FOR HADRONIC MATTER

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MSUCL-752

DECEMBER 1990

MSUCL-752
December, 1990

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Abstract

We derive coupled equations of motion for the density matrices for nucleons, A resonances, and π mesons, as well as for the pion-baryon interaction vertex function for the description of nuclear reactions at intermediate energies. We start from an effective hadronic Lagrangian density with minimal coupling between baryons and mesons. By truncating at the level of threebody *correlations* and using the G-matrix method to solve the equations of motion for the two-body correlation functions, a closed equation of motion for the one-body density matrices is obtained. A subsequent Wiier transformation then leads to a tractable set of relativistic **transport equations for interacting nucleons,** deltas, and pions.

PACS: 25.70.-z

*This work was supported in part by the National Science Foundation under Grant PHY-8906116, the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, and the National Nature Science Foundation of China.

1 Introduction

In nuclear collisions of energy up to around one GeV per nucleon, nuclear matter at high density and high temperature can be formed transiently. In such systems baryon excitations and mesonic degrees of freedom play a significant role, whereas the pressure is well below what is required to dissolve the hadrons into deconfined quarks and gluons. Relativistic quantum hadrodynamics is the appropriate tool for describing these dynamical processes. Aspects of particular interest are both of microscopic nature, such as the in-medium hadron-hadron cross sections and the dispersion relation of the pion in hot matter, and macroscopic, such as the nuclear equation of state, the transport properties (*e.g.* viscosity and heat conductivity), and the collective motion during the decompression phase.

Considerable amounts of experimental data have been accumulated during the past decade by recording the products resulting from relativistic[1] heavy-ion collisions, such as nucleons, light and heavy nuclear fragments, and pions and kaons (see, for example, refs. [1, 2]). Yet, many of the quantitative interpretations of these data remain rather uncertain, as the properties of the hot and dense matter extracted by comparing the experimental observations with theoretical calculations vary considerably with the specific model employed. Some properties extracted are even mutually conflicting. One example for this is the numerical value of the nuclear compressibility which varies greatly depending on the model assumptions employed [3].

The most successful models, in terms of reproducing a variety of the experimental observables in intermediate-energy nuclear collisions, are the Boltzmann-Uhling-Uehlingbeck model (*BUU*) [4], molecular dynamics [5], and quantum-correlation dynamics [6, 7]. In these models the dynamics is restricted to the baryonic level, and the mesonic degrees of freedom enter only via the potentials, although meson production has been treated in a perturbative manner in the subthreshold regions [8, 9].

Another approach which has been successfully applied to high-energy nucleus-nucleus collisions is to treat all nucleons as essentially free particles interacting with each other with their free nucleon-nucleon cross sections. Intranuclear cascade models [10, 11, 12, 13] are based on this approach. Here, pion production and reabsorption is included into the dynamical process through the formation and decay of Δ resonances. These models were able to calculate properly the overall features of nuclear equilibration [12] and pion production [11, 14] in heavy-ion collisions with beam energies around 1 GeV per nucleon. At this energy about half of the nucleon-nucleon collisions are inelastic, mainly through pion production. Although they have some shortcomings when quantitaively compared to experimental data, the intranuclear cascade models have been remarkably successful.

However, in this energy range the long range nucleon-nucleon interactions are still sufficientlt significant that the particles are not free but moving in a varying mean field. Recent computer simulations of relativistic heavy-ion collisions[15, 16] have extended the original *BUU* model to contain pion production and reabsorption in

[1] We use the term "relativistic" to characterize reactions in which the kinetic energy of the projectile is comparable to its rest mass (*i.e.* $y_{beam} \approx 1$), leaving the term "ultrarelativistic" for reactions where the projectile energy is much larger than its rest mass (*i.e.* $y_{beam} \gg 1$).

the dynamical process. They indicate that it is important to include the mesonic degrees of freedom explicitly, while keeping the mean field, in order to explain the dilepton production data [17] and quantitative aspects of pion spectra such as the two-temperature shape observed at the BEVALAC [18].

Nevertheless, a complete set of transport equations which govern the dynamical process in hadronic matter is not available. With this situation in mind, several groups have set out to provide a derivation of such transport equations [19, 20, 21, 22]. These attempts, however, are still at an early stage and a complete numerical realization is not available as of yet.

It is the purpose of this paper to present the derivation of a set of kinetic equations for nucleons, deltas, and pions, which are the main constituents of the hadronic matter formed in relativistic nuclear collisions. These equations reflect the physics of relativistic nuclear collisions in an instructive manner. Moreover, the approximate solution of the equations is possible with present computers, though time-consuming. Our derivation is rather similar to the approach taken in ref. [23, 24], but we go beyond that work by including both Δ resonance and dynamical pions, which are expected to be significant at relativistic energies. The framework for describing nuclear reactions is extended from the baryon dynamics level to the hadron dynamics level.

First, in section 2, we construct the Hamiltonian for the hadronic matter, starting from the effective Lagrangian density containing free fields of nucleons, deltas, σ- ,ω- and π-mesons, as well as the minimum coupling between them. In section 3 we reduce the dynamics to the baryonic level, by treating the pions as contributing to the potentials only, and we derive the equations of motion for the density matrix and the correlation functions for nucleons and deltas. Then, in section 4, we extend the baryon dynamics to hadron dynamics by introducing an independent dynamical pion field, and we obtain equations of motion for the density matrix of nucleons, deltas, and pions, as well as the pion-baryon interaction vertex function. Subsequently, in section 5, we make Wigner transformations of these equations, in order to obtain a set of transport equations for the phase-space distribution functions for baryons and pions. These equations contain a Vlasov term of the usual form and several collision terms, in analogy with the standard *BUU* equations. Finally, we summarize and give an outlook in section 6. In order to facilitate the presentation, a number of detailed derivations have been relegated to the appendices.

2 Model for hadronic matter

This section introduces the model description of hadronic matter in terms of interacting baryonic and mesonic fields. Throughout the developments, we employ units in which $\hbar$ and c are unity.

2.1 Model Lagrangian

For nuclear collisions at beam energies of up to around one GeV per nucleon, the main baryonic excitation is the $\Delta(1236)$ resonance, the higher resonances having negligible excitation functions. Therefore, a first step towards a complete description of hadronic matter should include nucleons and Δ resonances, in addition to π, σ and ω mesons. The ρ meson need only be included when electromagnetic probes are considered. Accordingly, we adopt a model hadronic Lagrangian density involving the baryon fields $N(x)$ and $\Delta^\nu(x)$, the meson fields $\boldsymbol{\pi}(x)$, $\sigma(x)$, and $\omega^\mu(x)$, and their interactions in the minimal coupling scheme commonly used for relativistic hadronic systems [23, 24],

$$\mathcal{L}(x) = \mathcal{L}^0(x) + \mathcal{L}^{\rm int}(x) , \tag{1}$$

where $\mathcal{L}^0$ and $\mathcal{L}^{\rm int}$ are the free-field and interaction Lagrangian densities, respectively. We have used x to denote the Minkowski four-vector $(t, \mathbf{r})$. Moreover, the free Lagrangian density is

$$\begin{aligned}
\mathcal{L}^0(x) &= \overline{N}(x)(i\gamma^\mu\partial_\mu - m_N)N(x) + \overline{\Delta}_\nu(x)(i\gamma^\mu\partial_\mu - M_\Delta)\Delta^\nu(x) \\
&+ \frac{1}{2}[\partial_\mu\boldsymbol{\pi}(x)\cdot\partial^\mu\boldsymbol{\pi}(x) - m_\pi^2\boldsymbol{\pi}(x)\cdot\boldsymbol{\pi}(x)] \\
&+ \frac{1}{2}[\partial_\mu\sigma(x)\partial^\mu\sigma(x) - m_\sigma^2\sigma^2(x)] \\
&- \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}m_\omega^2\omega_\mu(x)\omega^\mu(x) ,
\end{aligned} \tag{2}$$

and the interaction Lagrangian density is

$$\begin{aligned}
\mathcal{L}^{\rm int}(x) &= -ig_{\pi NN}\overline{N}(x)\gamma_5\boldsymbol{\tau}N(x)\cdot\boldsymbol{\pi}(x) + g_{\sigma NN}\overline{N}(x)N(x)\sigma(x) \\
&- g_{\omega NN}\overline{N}(x)\gamma^\mu N(x)\omega_\mu(x) \\
&+ g_{\pi N\Delta}[\overline{\Delta}_\mu(x)\boldsymbol{T}N(x)\cdot\partial^\mu\boldsymbol{\pi}(x) + \overline{N}(x)\boldsymbol{T}^\dagger\Delta^\mu(x)\cdot\partial_\mu\boldsymbol{\pi}(x)] \\
&- ig_{\pi\Delta\Delta}\overline{\Delta}_\mu(x)\gamma_5\boldsymbol{T}\Delta^\mu(x)\cdot\boldsymbol{\pi}(x) + g_{\sigma\Delta\Delta}\overline{\Delta}_\mu(x)\Delta^\mu(x)\sigma(x) \\
&- g_{\omega\Delta\Delta}\overline{\Delta}_\mu(x)\gamma^\nu\Delta^\mu\omega_\nu(x) ,
\end{aligned} \tag{3}$$

with $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$. The nucleon field $N(x)$ is an isospinor,

$$N(x) = \begin{pmatrix} \psi_p(x) \\ \psi_n(x) \end{pmatrix} , \; \overline{N}(x) = \begin{pmatrix} \psi_p^\dagger(x)\gamma_0 , & \psi_n^\dagger(x)\gamma_0 \end{pmatrix} , \tag{4}$$

while $\psi_p(x)$ and $\psi_n(x)$ are proton and neutron spinors in Minkowski space,

$$\psi_{p(n)}(x) = \begin{pmatrix} u_1(x) \\ u_2(x) \\ u_3(x) \\ u_4(x) \end{pmatrix} , \; \psi_{p(n)}^\dagger(x) = \begin{pmatrix} u_1^\dagger(x), & u_2^\dagger(x), & u_3^\dagger(x), & u_4^\dagger(x) \end{pmatrix} . \tag{5}$$

The Δ field is described by the Rarita-Schwinger formalism[26] as a four-vector whose components $\Delta_\mu(x)$ obey the Dirac equation

$$(i\gamma^\nu\partial_\nu - M_\Delta)\Delta_\mu(x) = 0 , \tag{6}$$

when no interactions are present. Each component of $\Delta_\mu(x)$ is an isospinor,

$$\Delta_\mu = \begin{pmatrix} \Delta_\mu^{++}(x) \\ \Delta_\mu^{+}(x) \\ \Delta_\mu^{0}(x) \\ \Delta_\mu^{-}(x) \end{pmatrix}, \tag{7}$$

where the four components represent the four charge states of the Δ resonance.

The pion field $\boldsymbol{\pi}(x)$ is an isovector and a Minkowski pseudo-scalar,

$$\boldsymbol{\pi}(x) = \begin{pmatrix} \pi^{+}(x) \\ \pi^{0}(x) \\ \pi^{-}(x) \end{pmatrix}. \tag{8}$$

Furthermore, the sigma field $\sigma(x)$ is a scalar in both Minkowski and isospin space, whereas the omega field $\omega_\mu(x)$ is a Minkowski vector and an isoscalar.

It is convenient to employ the isospin generators $\boldsymbol{\tau}$, $\boldsymbol{T}$ and $\boldsymbol{t}$ which act on the isospinor $N(x)$, the isospinor $\Delta(x)$, and the isovector $\boldsymbol{\pi}(x)$, respectively. They satisfy

$$\boldsymbol{T} = \boldsymbol{t} \oplus \frac{1}{2}\boldsymbol{\tau} \tag{9}$$

and

$$\tau_3 = \begin{cases} 1 & \text{for } p \\ -1 & \text{for } n \end{cases}, \tag{10}$$

$$t_3 = \begin{cases} 1 & \text{for } \pi^+ \\ 0 & \text{for } \pi^0 \\ -1 & \text{for } \pi^- \end{cases}, \tag{11}$$

It is also convenient to employ the isospin transition operator $\boldsymbol{\mathcal{T}} = (\mathcal{T}_+, \mathcal{T}_0, \mathcal{T}_-)$, as in refs. [27, 28]. The matrix representation of $\mathcal{T}_\mu$, μ=1, 0, -1, can be obtained from the following equation,

$$(\mathcal{T})_{M_T m_\tau} = \sum_{k=-1}^{1} \langle \frac{3}{2} M_T | 1 k \frac{1}{2} m_\tau \rangle (t^k)^*, \tag{12}$$

where $t^{\pm 1} = (1, \pm i, 0)/\sqrt{2}$ and $t^0 = (0,0,1)$, and only $k = M_T - m_\tau$ contributes. The above two expressions imply

$$\mathcal{T}_+ = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{6}} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \ \mathcal{T}_- = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{\sqrt{6}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{pmatrix}, \ \mathcal{T}_0 = \begin{pmatrix} 0 & 0 \\ \sqrt{\frac{2}{3}} & 0 \\ 0 & \sqrt{\frac{2}{3}} \\ 0 & 0 \end{pmatrix}. \tag{13}$$

In the following, we use the notation $\boldsymbol{\mathcal{T}}^\dagger \cdot \boldsymbol{\pi} = \mathcal{T}_\mu^\dagger \pi_\mu$ and $\mathcal{T}_\mu^\dagger = \mathcal{T}_{-\mu}^T$.

Employing the above matrix representations we obtain

$$\mathcal{T}N\partial^\mu\boldsymbol{\pi}(x) = \begin{pmatrix} \frac{1}{\sqrt{2}}\psi_p\partial^\mu\pi^+ \\ \frac{1}{\sqrt{6}}\psi_n\partial^\mu\pi^+ + \sqrt{\frac{2}{3}}\psi_p\partial^\mu\pi^0 \\ \sqrt{\frac{2}{3}}\psi_n\partial^\mu\pi^0 + \frac{1}{\sqrt{6}}\psi_p\partial^\mu\pi^- \\ \frac{1}{\sqrt{2}}\psi_n\partial^\mu\pi^- \end{pmatrix} \propto \begin{pmatrix} \Delta^{\mu++} \\ \Delta^{\mu+} \\ \Delta^{\mu 0} \\ \Delta^{\mu-} \end{pmatrix} , \tag{14}$$

and

$$\mathcal{T}^\dagger\Delta^\mu\partial_\mu\boldsymbol{\pi}(x) = \begin{pmatrix} \frac{1}{\sqrt{6}}\Delta^{\mu 0}\partial_\mu\pi^+ + \sqrt{\frac{2}{3}}\Delta^{\mu+}\partial_\mu\pi^0 + \frac{1}{\sqrt{2}}\Delta^{\mu++}\partial_\mu\pi^- \\ \frac{1}{\sqrt{2}}\Delta^{\mu-}\partial_\mu\pi^+ + \sqrt{\frac{2}{3}}\Delta^{\mu 0}\partial_\mu\pi^0 + \frac{1}{\sqrt{6}}\Delta^{\mu+}\partial_\mu\pi^- \end{pmatrix} \propto \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} . \tag{15}$$

These relations bring out the role of the isospin transition operator $\mathcal{T}$ in converting the isospin and show how the interactions containing $\mathcal{T}N\partial^\mu\boldsymbol{\pi}$ and its Hermitean conjugate in equation (3) cause $N \leftrightarrow \Delta$ transitions.

2.2 Hilbert space for hadron fields

The Hilbert space for hadronic matter, $\mathcal{H}^H$ consists of the direct product of irreducible representations of both the Lorentz group G^Λ and the isospin group G^T. Thus the dynamical group G^H and it's representation space for hadronic matter can be expressed as

$$G^H = G^\Lambda \otimes G^T \ , \ \mathcal{H}^H = \mathcal{H}^\Lambda \otimes \mathcal{H}^T \ . \tag{16}$$

Here the representation space $\mathcal{H}^\Lambda$ for G^Λ and $\mathcal{H}^T$ for G^T consist of several irreducible representations respectively, *i.e.*

$$\mathcal{H}^\Lambda = \mathcal{H}_0^\Lambda \oplus \mathcal{H}_1^\Lambda \oplus \mathcal{H}_{\frac{1}{2}}^\Lambda \oplus \mathcal{H}_{\hat{0}}^\Lambda \ , \tag{17}$$

$$\mathcal{H}^T = \mathcal{H}_0^T \oplus \mathcal{H}_1^T \oplus \mathcal{H}_{\frac{1}{2}}^T \oplus \mathcal{H}_{\frac{3}{2}}^T \ . \tag{18}$$

Corresponding to each irreducible subspace a projection operator can be introduced such that

$$I^\Lambda = P_0^\Lambda \oplus P_1^\Lambda \oplus P_{\frac{1}{2}}^\Lambda \oplus P_{\hat{0}}^\Lambda \ , \tag{19}$$

$$I^T = P_0^T \oplus P_1^T \oplus P_{\frac{1}{2}}^T \oplus P_{\frac{3}{2}}^T \ , \tag{20}$$

$$I^H = I^\Lambda \otimes I^T \ , \tag{21}$$

where I^λ and I^T are the identity elements in the Lorentz and isospin groups, and the subscripts 0, $\hat{0}$, and 1 denote scalar, pseudoscalar, and vector, and $\frac{1}{2}$ and $\frac{3}{2}$ denote spinors, respectively. The characteristic subspaces for each kind of hadron considered in this work are listed in table 1. Using the above notations we can define the trace operation as

$$\mathrm{Tr}(\mathcal{H}^H) = \sum_{\Lambda m_\lambda} \sum_{T m_T} \int d\boldsymbol{r} \ , \tag{22}$$

namely the trace taken in the space $\mathcal{H}^H$ is the summation over the internal space $\mathcal{H}^\Lambda$ and $\mathcal{H}^T$, as well as an integration over coordinate space. Furthermore, the summation in space $\mathcal{H}^\Lambda$ and $\mathcal{H}^T$ can be expanded as

$$\sum_{\Lambda m_\Lambda} = \sum_{\Lambda=0,m_\Lambda} P_0^\Lambda + \sum_{\Lambda=1,m_\Lambda} P_1^\Lambda + \sum_{\Lambda=\frac{1}{2},m_\Lambda} P_{\frac{1}{2}}^\Lambda + \sum_{\Lambda=\hat{0},m_\Lambda} P_{\hat{0}}^\Lambda , \tag{23}$$

and

$$\sum_{T m_T} = \sum_{T=0,m_T} P_0^\Lambda + \sum_{T=1,m_T} P_1^T + \sum_{T=\frac{1}{2},m_T} P_{\frac{1}{2}}^T + \sum_{T=\frac{3}{2},m_T} P_{\frac{3}{2}}^T . \tag{24}$$

2.3 Equations of motion for the hadron fields

The equations of motion for the hadron fields can be obtained from the Lagrangian density given in equations (1-3) by means of the Euler-Lagrange equations. The result is

$$\begin{aligned}(i\gamma^\mu\partial_\mu - m_N)N(x) &= ig_{\pi NN}\boldsymbol{\pi}(x)\cdot\gamma_5\boldsymbol{\tau}N(x) - g_{\sigma NN}\sigma(x)N(x) \\ &+ g_{\omega NN}\omega_\mu(x)\gamma^\mu N(x) - g_{\pi N\Delta}\boldsymbol{\mathcal{T}}^\dagger\Delta^\mu(x)\cdot\partial_\mu\boldsymbol{\pi}(x) ,\end{aligned} \tag{25}$$

$$\begin{aligned}(i\gamma^\mu\partial_\mu - M_\Delta)\Delta^\mu(x) &= ig_{\pi\Delta\Delta}\boldsymbol{\pi}(x)\cdot\gamma_5\boldsymbol{T}\Delta^\mu(x) - g_{\sigma\Delta\Delta}\sigma(x)\Delta^\mu(x) \\ &+ g_{\omega\Delta\Delta}\omega_\nu(x)\gamma^\nu\Delta^\mu(x) - g_{\pi N\Delta}\partial^\mu\boldsymbol{\pi}(x)\cdot\boldsymbol{\mathcal{T}}N(x) ,\end{aligned} \tag{26}$$

$$(\partial^\mu\partial_\mu + m_\sigma^2)\sigma(x) = g_{\sigma NN}\overline{N}(x)N(x) + g_{\sigma\Delta\Delta}\overline{\Delta}_\mu(x)\Delta^\mu(x) , \tag{27}$$

$$\begin{aligned}(\partial_\mu\partial^\mu + m_\pi^2)\boldsymbol{\pi}(x) &= -ig_{\pi NN}\overline{N}(x)\gamma_5\boldsymbol{\tau}N(x) - ig_{\pi\Delta\Delta}\overline{\Delta}_\mu(x)\gamma_5\boldsymbol{T}\Delta^\mu(x) \\ &- g_{\pi N\Delta}[\partial^\mu(\overline{\Delta}_\mu(x)\boldsymbol{\mathcal{T}}N(x)) + \partial_\mu(\overline{N}(x)\boldsymbol{\mathcal{T}}^\dagger\Delta^\mu(x))] ,\end{aligned} \tag{28}$$

$$\partial^\mu F_{\mu\nu}^\omega + m_\omega^2\omega_\nu(x) = g_{\omega NN}\overline{N}(x)\gamma_\nu N(x) + g_{\omega\Delta\Delta}\overline{\Delta}_\mu(x)\gamma_\nu\Delta^\mu(x) . \tag{29}$$

The above set of equations form a complete closed set of equations for the evolution of the various hadronic fields considered. However, their solution is not within reach, even with modern computers, and so considerable simplification is called for, as discussed below.

2.4 Green's functions for the meson fields

The main role of the meson fields in the Lagrangian (1) is to mediate the strong interaction between the baryons. This is strictly true for the fields $\sigma(x)$ and $\omega_\mu(x)$ which have no manifestations in terms of real physical particles. Therefore, these fields can be regarded as representing virtual mesons and they can be eliminated in exchange for effective potentials acting among the baryons. However, the pion field plays a dual role: not only can the pion mediate interactions (and in this role it acts as a virtual meson, similarly to σ and ω) but it can also be manifesed as a real physical particle that can be observed. Thus, the pion field is somewhat akin to the electromagnetic field: the transverse component of the electromagnetic field represents real photons while the longitudinal component mediates the Coulomb interaction between charged particles.

Our goal is to eliminate all virtual meson fields, leaving only the real pion, in addition to the baryons, since real particles are amenable to numerical simualtion. Of course, such a separation can not be made in an exact manner, because the pion has a finite mass, in contradistinction to the photon in *QED*. Nevertheless, a useful approximate treatment can be made, as we shall now describe.

The first step is to eliminate the virtual meson fields. This can be accomplished by means of the Green's function technique. The Green's functions for the mesons are introduced through the following inhomogeneous field equations,

$$(\partial^\mu\partial_\mu + m_\pi^2)G^\pi(x-x') = \delta(x-x') \ , \tag{30}$$

$$(\partial^\mu\partial_\mu + m_\sigma^2)G^\sigma(x-x') = \delta(x-x') \ , \tag{31}$$

$$(\partial^\kappa\partial_\kappa g^{\mu\nu} - \partial^\mu\partial^\nu + m_\omega^2 g^{\mu\nu})G^\omega_{\mu\nu}(x-x') = \delta(x-x') \ . \tag{32}$$

The formal solutions can be written as [7]

$$G^\pi(x-x') = \int e^{-iq(x-x')}G^\pi(q)\ d^4q \ , \tag{33}$$

$$G^\sigma(x-x') = \int e^{-iq(x-x')}G^\sigma(q)\ d^4q \ , \tag{34}$$

$$G^\omega_{\mu\nu}(x-x') = \int e^{-iq(x-x')}G^\omega_{\mu\nu}(q)\ d^4q \ , \tag{35}$$

where the momentum representation of the Green's functions are

$$G^\pi(q) = (q^2+m_\pi^2)^{-1} \ , \tag{36}$$

$$G^\sigma(q) = (q^2+m_\sigma^2)^{-1} \ , \tag{37}$$

$$G^\omega_{\mu\nu}(q) = (-g_{\mu\nu} + q_\mu q_\nu/m_\omega^2)/(q^2+m_\omega^2) \ , \tag{38}$$

with the tensor $g_{\mu\nu}$ expressing the Minkowski metric. It is understood that appropriate form factors should be implemented in the above Green's functions before numerical calculation. By using of the Green's functions defined above, one can derive expressions for the virtual mesons from eqs. (27-29) in terms of the baryon fields,

$$\sigma(x) = g_{\sigma NN}\{G^\sigma * (\overline{N}N)\}(x) + g_{\sigma\Delta\Delta}\{G^\sigma * (\overline{\Delta}_\mu\Delta^\mu)\}(x) \ , \tag{39}$$

$$\begin{aligned}\pi(x) &= -ig_{\pi NN}\{G^\pi * (\overline{N}\gamma_5\boldsymbol{\tau}N)\}(x) - ig_{\pi\Delta\Delta}\{G^\pi * \partial_\nu(\overline{\Delta}_\mu\gamma^\nu\gamma_5\boldsymbol{T}\ \Delta^\mu)\}(x) \\ &- g_{\pi N\Delta}\{G^\pi * [\partial^\mu(\overline{\Delta}_\mu\boldsymbol{T}N) + \partial_\mu(\overline{N}\boldsymbol{T}^\dagger\Delta^\mu)]\}(x) \ ,\end{aligned} \tag{40}$$

$$\omega_\nu(x) = g_{\omega NN}\{G^\omega_{\nu\mu} * (\overline{N}\gamma^\mu N)\}(x) + g_{\omega\Delta\Delta}\{G^\omega_{\nu\mu} * (\overline{\Delta}_\lambda\gamma^\mu\Delta^\lambda)\}(x) \ , \tag{41}$$

where the products should be understood as convolutions, *i.e.*

$$\{G * F\}(x) = \int G(x-x')F(x')\, d^4x' \,. \qquad (42)$$

The relations (39-41) express the meson fields as functionals of the baryon fields $N(x)$ and $\Delta(x)$.

Furthermore, exploiting the fact that differentiation commutes with convolution, *i.e.*

$$\{G*(\partial_\nu F)\}(x) = \int G(x-x')\frac{\partial F(x')}{\partial x'_\nu} d^4x' = \int \frac{\partial G(x-x')}{\partial x_\nu} F(x') d^4x' = \{\partial_\nu(G*F)\}(x)\,, \qquad (43)$$

we can rewrite the pion field (40) on the form

$$\begin{aligned} \pi(x) \;=\; & -ig_{\pi NN}\{G^\pi * (\overline{N}\gamma_5 \boldsymbol{\tau} N)\}(x) - ig_{\pi\Delta\Delta}\{\partial_\nu G^\pi * (\overline{\Delta}_\mu \gamma^\nu \gamma_5 \boldsymbol{T}\Delta^\mu)\}(x) \\ & -g_{\pi N\Delta}[\{\partial^\mu G^\pi * (\overline{\Delta}_\mu \boldsymbol{T} N)\}(x) + \{\partial_\mu G^\pi * (\overline{N}\boldsymbol{T}^\dagger \Delta^\mu)\}(x)] \,. \qquad (44) \end{aligned}$$

2.5 Elimination of the virtual meson fields

To the extent that the mesons act only to mediate the nuclear interactions, they can be eliminated from the equations of motion for the baryon fields, leaving an effective baryon-baryon interaction. This can readily be accomplished by inserting the results (39-41) into the original equations (25) and (26) for the baryons. The resulting coupled equations then descibe the system entirely in terms of nucleons and Δ resonances. However, at energies for which the Δ plays a role, the pion production cross section is also considerable. Therefore, in order to obtain a physically reasonable picture, it is necessary to include also real pions in the treatment.

Towards this end, we introduce a separate pion field $\boldsymbol{\pi}^r(x)$, meant to represent physical pions in the same manner as $N(x)$ and $\Delta(x)$ are supposed to represent real baryons. The eq. (28) is then retained as the equation of motion for for $\boldsymbol{\pi}^r(x)$ and, moreover, this particle is assumed to couple to the baryons in the same manner as the virtual pion field. (It may eventually be desirable to readjust the coupling constants assocaited with the real pion.) This approach leads to the following equations of motion for the nucleons,

$$\begin{aligned} & (i\gamma^\mu \partial_\mu - m_N)N(x) \qquad (45) \\ & = g_{\pi NN}^2 \{G^\pi * (\overline{N}\gamma_5 \boldsymbol{\tau} N)\}(x) \cdot \boldsymbol{\tau}\gamma_5 N(x) \\ & + g_{\pi NN} g_{\pi\Delta\Delta} \{G^\pi * (\overline{\Delta}_\mu \gamma_5 \boldsymbol{T}\Delta^\mu\}(x) \cdot \boldsymbol{\tau}\gamma_5 N(x) \\ & + ig_{\pi NN} g_{\pi N\Delta} [\{G^{\pi\mu} * (\overline{\Delta}_\mu \boldsymbol{T} N)\}(x) + \{G^{\pi\mu} * (\overline{N}\boldsymbol{T}^\dagger \Delta^\mu)\}(x)] \cdot \boldsymbol{\tau}\gamma_5 N(x) \\ & + g_{\pi N\Delta} \{G^\pi_\mu * [ig_{\pi NN}\overline{N}\gamma_5 \boldsymbol{\tau} N + ig_{\pi\Delta\Delta}\overline{\Delta}_\nu \gamma_5 \boldsymbol{T}\Delta^\nu]\}(x) \cdot \boldsymbol{T}^\dagger \Delta^\mu(x) \\ & + g_{\pi N\Delta}^2 [\{G^{\pi\nu}_\mu * (\overline{\Delta}_\nu \boldsymbol{T} N)\}(x) + \{G^\pi_{\mu\nu} * (\overline{N}\boldsymbol{T}^\dagger \Delta^\nu)\}(x)] \cdot \boldsymbol{T}^\dagger \Delta^\mu(x) \\ & - [g_{\sigma NN}^2 \{G^\sigma * (\overline{N}N)\}(x) + g_{\sigma NN} g_{\sigma\Delta\Delta} \{G^\sigma * (\overline{\Delta}_\mu \Delta^\mu)\}(x)] N(x) \\ & + [g_{\omega NN}^2 \{G^\omega_{\nu\mu} * (\overline{N}\gamma^\mu N)\}(x) + g_{\omega NN} g_{\omega\Delta\Delta} \{G^\omega_{\nu\mu} * (\overline{\Delta}_\lambda \gamma^\mu \Delta^\lambda)\}(x)]\gamma^\nu N(x) \\ & + ig_{\pi NN}\boldsymbol{\pi}^r(x) \cdot \gamma_5 \boldsymbol{\tau} N(x) - g_{\pi N\Delta}\partial_\mu \boldsymbol{\pi}^r(x) \cdot \boldsymbol{T}^\dagger \Delta^\mu(x) \,, \end{aligned}$$

for the Δ resonances,

$$
\begin{aligned}
&(i\gamma^\nu\partial_\nu - M_\Delta)\Delta^\mu(x) \\
&= g_{\pi\Delta\Delta}[g_{\pi NN}\{G^\pi * (\overline{N}\gamma_5\boldsymbol{\tau}N)\}(x) + g_{\pi\Delta\Delta}\{G^\pi * (\overline{\Delta}_\nu\gamma_5\Delta^\nu)\}(x) \\
&+ig_{\pi N\Delta}(\{G^{\pi\nu} * (\overline{\Delta}_\nu\boldsymbol{T}N)\}(x) + \{G^\pi_\nu * (\overline{N}\boldsymbol{T}^\dagger\Delta^\nu)\}(x))]\cdot\boldsymbol{T}\gamma_5\Delta^\mu(x) \\
&+g_{\pi N\Delta}[ig_{\pi NN}\{G^{\pi\mu} * (\overline{N}\gamma_5\boldsymbol{\tau}N)\}(x) + ig_{\pi\Delta\Delta}\{G^{\pi\mu} * (\overline{\Delta}_\nu\gamma_5\boldsymbol{T}\Delta^\nu)\}(x) \\
&+g_{\pi N\Delta}(\{G^{\pi\mu\nu} * (\overline{\Delta}_\nu\boldsymbol{T}N)\}(x) + \{G^{\pi\nu}_\nu * (\overline{N}\boldsymbol{T}^\dagger\Delta^\nu)\}(x))]\cdot\boldsymbol{T}N(x) \\
&-g_{\sigma\Delta\Delta}[g_{\sigma NN}\{G^\sigma * (\overline{N}N)\}(x) + g_{\sigma\Delta\Delta}\{G^\sigma * (\overline{\Delta}_\nu\Delta^\nu)\}(x)]\Delta^\mu(x) \\
&+g_{\omega\Delta\Delta}[g_{\omega NN}\{G^\omega_{\nu\kappa} * (\overline{N}\gamma^\kappa N)\}(x) + g_{\omega\Delta\Delta}\{G^\omega_{\nu\kappa} * (\overline{\Delta}_\lambda\gamma^\kappa\Delta^\lambda)\}(x)]\gamma^\nu\Delta^\mu(x) \\
&+ig_{\pi\Delta\Delta}\boldsymbol{\pi}^r(x)\cdot\gamma_5\boldsymbol{T}\Delta^\mu(x) - g_{\pi N\Delta}\partial^\mu\boldsymbol{\pi}^r(x)\cdot\boldsymbol{T}N(x)\,,
\end{aligned}
\tag{46}
$$

and for the real pion,

$$
\begin{aligned}
&(\partial^\mu\partial_\mu + m_\pi^2)\boldsymbol{\pi}^r(x) \\
&= -ig_{\pi NN}\overline{N}(x)\gamma_5\boldsymbol{\tau}N(x) - ig_{\pi\Delta\Delta}\overline{\Delta}_\mu(x)\gamma_5\boldsymbol{T}\Delta^\mu(x) \\
&-g_{\pi N\Delta}\{\partial^\mu[\overline{\Delta}_\mu(x)\boldsymbol{T}N(x)] + \partial_\mu[\overline{N}(x)\boldsymbol{T}^\dagger\Delta^\mu(x)]\}\,.
\end{aligned}
\tag{47}
$$

In the equations for the baryons the following notations have been used,

$$
G^\pi_\mu(x) = \frac{\partial G^\pi(x)}{\partial x_\mu} = \partial_\mu G^\pi(x)\,,\; G^{\pi\mu}(x) = \frac{\partial G^\pi(x)}{\partial x^\mu} = \partial^\mu G^\pi(x)\,, \tag{48}
$$

$$
G^\pi_{\mu\nu}(x) = \partial_\mu\partial_\nu G^\pi(x)\,,\; G^{\pi\nu}_\mu(x) = \partial_\mu\partial^\nu G^\pi(x)\,. \tag{49}
$$

The equation for the dynamical pion has the same form as eq. (28). From now on, the superscript for the real pion field will be dropped, since no other meson appears explicitly anymore.

These three coupled equations (45,46,47) represent our first approximation to the general equations of motion (25-29). The advantage of eliminating the virtual meson fields is that they are hard to treat numerically, whereas the real particles can be treated approximately by simulating their semi-classical phase-space distribution, as is done in the usual *BUU* treatment. Yet, further simplifications are needed to bring the equations on a sufficiently tractable form, as discussed below. We note that if the pion field $\boldsymbol{\pi}^r(x)$ is omitted in the above equations, then the equations reduce to the set mentioned above involving nucleons and deltas only.

2.6 Instantaneous meson exchange approximation

Although the equations of motion (45,46,47) are relativistically covariant, they are non-local in space and time. This is the price paid for the elimination of virtual meson fields. Since we wish to formulate a transport theory within the density-matrix framework, non-locality in time is inconvenient. Fortunately, meson retardation effects are not significant in the energy range considered here. Consequently, the instantaneous meson exchange approximation can be made to establish time locality and thus make the equations of motion amenable to the density-matrix treatment [6, 7, 23, 24, 29].

For the σ and ω mesons this approximation seems to be well justified. The pion, however, has a smaller mass and correspondingly longer Compton wavelength of around 1.4 fm. In this case our approximation has to be used with caution. At normal nuclear matter density, nucleons are spaced about 2 fm apart, only slightly farther than the pion Compton wave length. However, numerical studies of Blättel *et al.*[25] show that the instantaneous meson exchange approximation leads only to a negligible modification of the time evolution of the baryon phase space distribution up to at least several GeV per nucleon. The instantaneous meson exchange approximation can be expressed as

$$G(x-x') \approx G(\boldsymbol{r}-\boldsymbol{r}',t)\delta(t-t'). \tag{50}$$

With this approximation, an effective Hamiltonian for hadronic matter can be constructed as

$$\hat{H}^h_{\text{eff}} = \hat{H}^b_{\text{eff}} + \hat{H}^\pi + \hat{V}^{b\pi} \; . \tag{51}$$

The effective baryon Hamiltonian $\hat{H}^b_{\text{eff}}$, the Hamiltonian for the real pion $\hat{H}^\pi$, and the baryon-pion interaction $\hat{V}^{b\pi}$ are

$$\begin{aligned} \hat{H}^b_{\text{eff}} &= \int \psi^\dagger(x)\hat{E}(x)\psi(x)d\boldsymbol{r} && (52)\\ &+ \frac{1}{2}\int \psi^\dagger(x_1)\psi^\dagger(x_2)\hat{V}(x_1-x_2)\psi(x_2)\psi(x_1)d\boldsymbol{r}_1 d\boldsymbol{r}_2 \; , \\ \hat{H}^\pi &= \frac{1}{2}\int [\dot{\boldsymbol{\pi}}(x)\cdot\dot{\boldsymbol{\pi}}(x) + \nabla\boldsymbol{\pi}(x)\cdot\nabla\boldsymbol{\pi}(x) + m_\pi^2\boldsymbol{\pi}(x)\cdot\boldsymbol{\pi}(x)]d\boldsymbol{r} \; , && (53) \end{aligned}$$

and

$$\hat{V}^{b\pi} = \int \psi^\dagger(x)\hat{U}^\pi(x)\psi(x)d\boldsymbol{r} \; , \tag{54}$$

where

$$\psi(x) = \begin{pmatrix} \psi_N(x) \\ \psi_\Delta(x) \end{pmatrix} = \begin{pmatrix} N(x) \\ \Delta(x) \end{pmatrix} \tag{55}$$

and

$$\psi(x_2)\psi(x_1) = \begin{pmatrix} N(x_2)N(x_1) \\ N(x_2)\Delta(x_1) \\ \Delta(x_2)N(x_1) \\ \Delta(x_2)\Delta(x_1) \end{pmatrix} . \tag{56}$$

The energy operator $\hat{E}$ can be written as

$$\hat{E}(x) = \begin{pmatrix} \hat{E}_N(x) & 0 \\ 0 & \hat{E}_\Delta(x) \end{pmatrix} , \tag{57}$$

where

$$\hat{E}_N(x) = \alpha_i(-i\partial^i) + \gamma_0 m_N \; , \tag{58}$$

$$\hat{E}_\Delta(x) = \alpha_i(-i\partial^i) + \gamma_0 M_\Delta \; , \tag{59}$$

and $\alpha_i = \gamma_0\gamma_i$ and $\alpha_o = \gamma_0\gamma_0 = 1$. Furthermore, $\hat{U}^\pi$ and $\hat{V}(x_1-x_2)$ are defined as follows,

$$\hat{U}^\pi(x) = \hat{\boldsymbol{U}}_\pi \cdot \boldsymbol{\pi}(x) \; , \tag{60}$$

with

$$\hat{U}_\pi = \begin{pmatrix} \hat{U}_{NN} & \hat{U}_{N\Delta} \\ \hat{U}_{\Delta N} & \hat{U}_{\Delta\Delta} \end{pmatrix}, \tag{61}$$

$$\hat{U}_{NN} = ig_{\pi NN}\gamma_0\gamma_5\boldsymbol{\tau} \ , \ \hat{U}_{N\Delta} = -g_{\pi N\Delta}\gamma_0\boldsymbol{T}^\dagger\partial_\nu \ , \tag{62}$$

$$\hat{U}_{\Delta N} = -g_{\pi N\Delta}\gamma_0\boldsymbol{T}\partial^\nu \ , \ \hat{U}_{\Delta\Delta} = ig_{\pi\Delta\Delta}\gamma_0\gamma_5\boldsymbol{T}\partial_{\mu\nu} \ , \tag{63}$$

and $\hat{V}(x_1 - x_2) = (\hat{v}_{ij,kl})$, $i,j,k,l = N,\Delta$. The explicit expressions for the matrix elements $\hat{v}_{ij,kl}$ are lengthy and given in Appendix A. Keeping in mind that $\hat{\boldsymbol{U}}_\pi$ is a matrix of isovectors, we can omit the boldface in the following derivations without causing any confusion.

The equations of motion given in (45,46,47) can be derived from the effective hadronic Hamiltonian under the instantaneous meson exchange approximation by virtue of the following Heisenberg equations,

$$i\frac{\partial N}{\partial t} = [\hat{H}^h_{\text{eff}}, N] \ , \tag{64}$$

$$i\frac{\partial \Delta^\mu}{\partial t} = [\hat{H}^h_{\text{eff}}, \Delta^\mu] \ , \tag{65}$$

$$i\frac{\partial \boldsymbol{\pi}}{\partial t} = [\hat{H}^h_{\text{eff}}, \boldsymbol{\pi}] \ . \tag{66}$$

The salient features of the hadronic Hamiltanion (eq. (51)) and the above Heisenberg equations are 1) they have structure similar to those of non-relativistic quantum many-body theory, and, as we shall see below, 2) they can be cast into density-matrix form, which, when augmented with correlation dynamics, is suitable for a non-perturbative treatment of quantum many-body problems.

3 Density matrix treatment

In this section we shall reformulate the baryon dynamics in terms of the density matrix. This is basically an extension of the previous work [23, 24] to treat the Δ resonances in a more practical way. As shown in ref. [23, 24], the introduction of correlation dynamics suggests a reasonable truncation scheme which is essentially non-perturbative and can go beyond the mean field approximation and provide an approach to calculate the two-body collision terms as well. Within the two-body correlation approximation, it is possible to obtain a set of coupled equations of motion for the one-body density matrix and the two-body correlations for baryons which are kinetic equations for the baryons in configuration space. An extension of the baryon dynamics to hadron dynamics will be given in the next section.

3.1 Density operators

The n-body baryon density operator $\hat{\rho}_n$ is defined as

$$\hat{\rho}_n(1,2,\cdots n;1',2',\cdots n') = \psi^\dagger(1')\psi^\dagger(2')\cdots\psi^\dagger(n')\psi(n)\cdots\psi(1), \tag{67}$$

with

$$n = x_n = (t, \boldsymbol{r}_n, m_n) \ , \ n' = x'_n = (t, \boldsymbol{r}'_n, m'_n) \ , \tag{68}$$

where m_n is the spin-isospin quantum number and we have used that $t' = t$ due to the instantaneous approximation (50). The n-body density matrix is defined as the expectation value of $\hat{\rho}_n$, namely

$$\rho_n(1,2,\cdots n;1',2',\cdots n') = \langle \hat{\rho}_n(1,2,\cdots n;1',2'\cdots n') \rangle \ . \tag{69}$$

The baryon number operator is defined as

$$\hat{N} = \mathrm{Tr}_1 \psi^\dagger(1)\psi(1) = \mathrm{Tr}_1 N^\dagger(1) N(1) + \mathrm{Tr}_1 \Delta^\dagger_\mu(1)\Delta^\mu(1) \ . \tag{70}$$

It is straightforward to show that

$$[\hat{N}, \psi(1)] = -\psi(1) \ , \ [\hat{N}, \psi^\dagger(1)] = \psi^\dagger(1) \ , \ \hat{N}^\dagger = \hat{N} \ . \tag{71}$$

From eqs. (67,70,71) we obtain the following reduction relations

$$\hat{\rho}_n = \frac{1}{\hat{N} - n} \mathrm{Tr}_{(n+1)} \hat{\rho}_{n+1} = \mathrm{Tr}_{(n+1)} \hat{\rho}_{n+1} \frac{1}{\hat{N} - n} \ , \tag{72}$$

which is a generalization of a formula used recently in ref. [30].

3.2 Equations of motion for ρ_n

By using of the basic anticommutation relation among ψ and $\psi^\dagger$, it is straightforward, though tedious, to obtain equations of motion for the reduced baryon density operators (the operator version of the *BBGKY* hierarchy, see Appendix B). It reads

$$i\frac{\partial \hat{\rho}_n}{\partial t} = [\hat{\rho}_n, \hat{H}^b_{\mathrm{eff}}] = [\hat{H}(n), \hat{\rho}_n] + \mathrm{Tr}_{(n+1)}[\hat{V}(n+1), \hat{\rho}_{n+1}] \ , \tag{73}$$

where

$$\begin{aligned}
\hat{H}(n) &= \sum_{i=1}^{n} \hat{E}(i) + \sum_{i<j}^{n} \hat{v}(i,j) \ , & (74)\\
\hat{V}(n+1) &= \sum_{i=1}^{n} \hat{v}(i, n+1) \ , & (75)\\
\left[\hat{H}(n), \hat{\rho}_n\right] &= \hat{H}(1,2,\cdots n)\hat{\rho}_n(1,2,\cdots n;1',2',\cdots n') & (76)\\
&\quad - \hat{\rho}_n(1,2,\cdots n;1',2',\cdots n')\hat{H}(1',2',\cdots n') \ .
\end{aligned}$$

Taking the expectation value of eq. (73), we obtain the following *BBGKY* hierarchy for the reduced baryon density matrices,

$$i\frac{\partial \rho_n}{\partial t} = [\hat{H}(n), \rho_n] + \mathrm{Tr}_{(n+1)}[\hat{V}(n+1), \rho_{n+1}] \ , \tag{77}$$

In particular, the equations of motion for $\rho \equiv \rho_1$ and ρ_2 are given by

$$\begin{aligned}
i\frac{\partial \rho}{\partial t} &= [\hat{E}, \rho] + \mathrm{Tr}_2[\hat{v}(1,2), \rho_2] \ , & (78)\\
i\frac{\partial \rho_2}{\partial t} &= [\hat{E}(1) + \hat{E}(2) + \hat{v}(1,2), \rho_2] + \mathrm{Tr}_3[\hat{v}(1,3) + \hat{v}(2,3), \rho_3] \ . & (79)
\end{aligned}$$

3.3 Dynamics of the many-body correlations

To obtain correlation dynamics from the *BBGKY* hierarchy, *i.e.* eq. (77), the key step is to separate out many-body correlations from the reduced density matrices. This can be realized by a non-linear transformation [6],

$$\begin{aligned} &\rho_n(1,2,\cdots,n;1',2',\cdots,n') \\ = \; & AS_n \sum_{p=1}^{n-1} \rho_{n-p}(1,2,\cdots,n-p;1',2',\cdots,(n-p)') \\ &\times \rho_p(n-p+1,\cdots,n;(n-p+1)',\cdots,n') + C_n(1,2,\cdots,n;1',2',\cdots,n') \, , \end{aligned} \tag{80}$$

where the operation AS_n should be understood as follows. The operator A denotes the antisymmetrization operation among those of the variables $(1,2,\cdots,n)$ that refer to identical particles. (Thus, labels referring to nucleons are antisymmetrized separately from those referring to deltas.) Furthermore, the subsequent operation by S symmetrizes among variable pairs $(1,1'),\cdots,(n,n')$ of identical particles. The combined operation AS_n then acts among n particles and the repeated terms should be omitted. Thus, for the one-particle density we have $\rho \equiv \rho_1 = C_1$, while the two-particle density ρ_2 is given by

$$\rho_2(1,2;1',2') = AS_2\rho(1;1')\rho(2;2') + C_2(1,2;1',2') \, . \tag{81}$$

Inserting eq. (80) into eq. (77), we obtain the equation of motion for the n-body correlation function, after somewhat lengthy manipulations, [6]

$$\begin{aligned} i\frac{\partial C_n}{\partial t} &= [\hat{H}(n), C_n] + AS_n \sum_{p=1}^{n-1} [\hat{v}(n), \rho_{n-p}\rho_p]_L \\ &+ \sum_{p=1}^{n} \mathrm{Tr}_{(n+1)}[\hat{V}(n+1), AS_{(n+1)}\rho_{n-p+1}\rho_p + C_{n+1}]_L \, . \end{aligned} \tag{82}$$

Here

$$\hat{H}(n) = \hat{T}(n) + \hat{v}(n) \, , \tag{83}$$

$$\hat{T}(n) = \sum_{i=1}^{n} \hat{E}(i) \, , \tag{84}$$

$$\hat{v}(n) = \sum_{j}^{n} \sum_{i<j}^{n} \hat{v}(i,j) \, , \tag{85}$$

$$\hat{V}(n+1) = \sum_{i=1}^{n} \hat{v}(i,n+1) \, , \tag{86}$$

and $[\cdot]_L$ means linked terms in which any multi-variable functions can not be factorized according to particle variables[6]. eqs. (82) are the basic equations for the correlation dynamics of the baryons. The advantage of (82) over (77) is that the former provides a natural truncation scheme according to the order of correlations and the resulting equations of motion are essentially non-linear.

In nuclear reaction studies, only the lowest-order equations are practically significant. They read (using that $C_1 = \rho$)

$$\begin{aligned} i\frac{\partial \rho}{\partial t} &= [\hat{E}, \rho] + \mathrm{Tr}_2[\hat{v}(1,2), AS_2\rho(1)\rho(2)] + \mathrm{Tr}_2[\hat{v}(1,2), \hat{C}_2(1,2)] \ , \\ i\frac{\partial C_2}{\partial t} &= [\hat{E}(1) + \hat{E}(2) + \hat{v}(1,2), \hat{C}_2] + [\hat{v}(1,2), AS_2\rho(1)\rho(2)] \\ &+ \mathrm{Tr}_3[\hat{v}(1,3) + \hat{v}(2,3), AS_3(\rho(1)\rho(2)\rho(3) + \rho(1)C_2(2,3)) + C_3]_L \ . \end{aligned} \quad (87)$$

3.4 Conservation laws

In the construction of any kinetic equations, it is crucial to check whether or not the equations obtained after a number of approximations still obey the conservation laws. Particularly important is conservation of baryon number, energy, linear momentum, and angular momentum. Baryon number conservation is easily verified, since

$$i\frac{\partial \hat{N}}{\partial t} = [\hat{N}, \hat{H}^h_{\mathrm{eff}}] = 0 \quad (88)$$

readily implies

$$\dot{B} = \langle \mathrm{Tr}\frac{\partial \hat{N}}{\partial t} \rangle = \frac{d}{dt}\mathrm{Tr}\rho = 0 \ . \quad (89)$$

The energy of the baryonic system is defined as

$$E_b = \langle \mathrm{Tr} H^b_{\mathrm{eff}} \rangle \ , \quad (90)$$

so we immediately obtain

$$\dot{E}_b = \frac{d}{dt}\langle \mathrm{Tr} H^b_{\mathrm{eff}} \rangle = -i\langle \mathrm{Tr}[H^b_{\mathrm{eff}}, H^b_{\mathrm{eff}}] \rangle = 0 \ . \quad (91)$$

The linear momentum is defined as

$$\boldsymbol{P} = \mathrm{Tr}_1(-i\boldsymbol{\nabla}_1 \rho_1) \ . \quad (92)$$

Thus $\boldsymbol{P}$ is also conserved,

$$\begin{aligned} \dot{\boldsymbol{P}} &= \mathrm{Tr}_1(-i\boldsymbol{\nabla}_1 \dot{\rho}(1)) \\ &= \mathrm{Tr}_1(-i\boldsymbol{\nabla}_1[\hat{E}, \rho]) + \mathrm{Tr}_1(-i\boldsymbol{\nabla}_1 \mathrm{Tr}_2[\hat{v}(1,2), \rho_2]) \\ &= \mathrm{Tr}_1\rho[-i\boldsymbol{\nabla}_1, \hat{E}(1)] + \frac{1}{2}\mathrm{Tr}_{(1,2)}\rho_2[(-i\boldsymbol{\nabla}_1 - i\boldsymbol{\nabla}_2), \hat{v}(1,2)] = 0 \ . \end{aligned} \quad (93)$$

Finally, in a similar way, angular momentum conservation can be shown as in appendix D.

3.5 Truncation schemes

The most common approximations in quantum many-body theory can be obtained within the lowest order truncations from eq. (82). The simplest truncation approximation assumes that all the many-body correlations vanish, $C_2, C_3, \cdots = 0$, and leads to the mean-field approximation. The next-order truncations, namely $C_2 \neq 0,\ C_3, C_4, \cdots = 0$ and $C_2, C_3 \neq 0, C_4, C_5, \cdots = 0$ lead to two-body and three-body correlation dynamics, whose stationary versions reduce to the Brueckner and Faddeev approximations, respectively.[6]. In the following we restrict ourselves to the mean field approximation and to two-body correlation dynamics, which are the only truncation schemes that are practically tractable with present computers.

3.5.1 Mean-field approximation

In the simplest truncation scheme, $C_{n>1} = 0$, we have for the one-body density matrix

$$i\frac{\partial \rho}{\partial t} = [\hat{E}, \rho] + \mathrm{Tr}_2[\hat{v}(1,2), AS_2\rho(1)\rho(2)] = [\hat{h}, \rho] \ , \tag{94}$$

where the mean-field Hamiltonian satisfies

$$\begin{aligned} \hat{h}\rho &= \hat{E}\rho + \hat{U}_{HF}\rho \ , & (95)\\ \hat{U}_{HF}\rho(1) &= \mathrm{Tr}_2\hat{v}(1,2)AS_2\rho(1)\rho(2) \ , & (96)\\ \hat{v}(1,2) &= \sum_{ij} v_{ij}(x_1 - x_2)\hat{\Gamma}_i(1)\hat{\Gamma}_j(2) \ , \ i,j = N, \Delta \ . & (97) \end{aligned}$$

This is the density-matrix form of the time-dependent Hartree-Fock-Dirac equation for baryons.

For the stationary case, we have

$$\frac{\partial \rho}{\partial t} = [\hat{h}, \rho] = 0 \ , \tag{98}$$

which indicates that $\hat{h}$ and ρ have common eigensolutions. In its eigenrepresentation, $\rho(11')$ can be written as

$$\rho(11') = P_N \sum_\alpha n_\alpha^N u_\alpha(1) u_\alpha^\dagger(1') + P_\Delta \sum_{\alpha\mu} n_\alpha^\Delta v_{\alpha\mu}(1) v_{\alpha\mu}^\dagger(1') \ , \tag{99}$$

where P_N and P_Δ are projection operators onto the nucleon and Δ subspace, respectively, and n_α^N and n_α^Δ are the occupation numbers of different eigenstates. Furthermore, u_α and $v_{\alpha\mu}$ are the eigensolutions of the nucleon Hamiltonian $\hat{h}_N$ and Δ Hamiltonian $\hat{h}_\Delta$, respectively,

$$\begin{aligned} \hat{h}_N u_\alpha &= e_\alpha^N u_\alpha \ , \ \hat{h}_N = \hat{E}_N + \hat{U}_{HF}^N \ , & (100)\\ \hat{h}_\Delta v_{\alpha\mu} &= e_\alpha^\Delta v_{\alpha\mu} \ , \ \hat{h}_\Delta = \hat{E}_\Delta + \hat{U}_{HF}^\Delta \ . & (101) \end{aligned}$$

The mean fields for N and Δ are given by

$$\begin{aligned}\hat{U}^{N}_{HF}u_\alpha(x) &= \sum_{ij}\int v_{ij}(x-x')d^{n}_{j}(x')d\boldsymbol{r}'\,\hat{\Gamma}_i u_\alpha(x) \\ &- \sum_{ij}\int v_{ij}(x-x')\sum_{\beta} n^{N}_{\beta} e^{j}_{\alpha\beta}(x')u_\beta(x)d\boldsymbol{r}' \ , \end{aligned} \tag{102}$$

$$\begin{aligned}\hat{U}^{\Delta}_{HF}v_{\alpha\mu}(x) &= \sum_{ij}\int v_{ij}(x-x')d^{\Delta}_{j}(x')d\boldsymbol{r}'\,\hat{\Gamma}_i v_{\alpha\mu}(x) \\ &- \sum_{ij}\int v_{ij}(x-x')\sum_{\beta\nu} e^{j}_{\alpha\mu}(x')_{\beta\nu}d\boldsymbol{r}'\hat{\Gamma}_i v_{\beta\nu}(x) \ , \end{aligned} \tag{103}$$

where

$$d^{n}_{j}(x') = \sum_{\alpha} n^{N}_{\alpha}\mathrm{Tr}(\hat{\Gamma}_j u_\alpha(x')u^{\dagger}_{\alpha}(x')) + \sum_{\alpha\mu} n^{\Delta}_{\alpha}\mathrm{Tr}(\hat{\Gamma}_j v_{\alpha\mu}(x')v^{\dagger}_{\alpha\mu}(x')) \ , \tag{104}$$

$$d^{\Delta}_{j}(x') = \sum_{\alpha\mu} n^{\Delta}_{\alpha}\mathrm{Tr}(\hat{\Gamma}_j v_{\alpha\mu}(x')v^{\dagger}_{\alpha\mu}(x')) + \sum_{\alpha} n^{N}_{\alpha}\mathrm{Tr}(\hat{\Gamma}_j u_\alpha(x')u^{\dagger}_{\alpha}(x')) \ , \tag{105}$$

$$e^{j}_{\alpha\beta}(x') = \mathrm{Tr}u^{\dagger}_{\beta}(x')\hat{\Gamma}_j u_\alpha(x') \ , \tag{106}$$

$$e^{j}_{\alpha\mu}(x')_{\beta\nu} = \mathrm{Tr}v^{\dagger}_{\beta\nu}(x')\hat{\Gamma}_j v_{\alpha\mu}(x') \ . \tag{107}$$

The potential $v_{ij}(x_1 - x_2)$ and the vertex operators $\hat{\Gamma}$ are given in table 2.

3.5.2 Two-body correlation dynamics

The next level of truncation, $C_{n>2} = 0$, goes beyond the mean-field approximation and admits two-body correlation dynamics. The associated equations of motion for the one-body density matrix and the two-body correlations are

$$\begin{aligned} i\frac{\partial\rho}{\partial t} &= [\hat{E},\rho] + \mathrm{Tr}_2[\hat{v}(1,2), AS_2\rho(1)\rho(2) + C_2(1,2)] \ , \end{aligned} \tag{108}$$

$$\begin{aligned} i\frac{\partial C_2(1,2)}{\partial t} &= [\hat{E}(1)+\hat{E}(2)+\hat{v}(1,2), C_2(1,2)] + [\hat{v}(1,2), AS_2\rho(1)\rho(2)] \\ &+ \mathrm{Tr}_3[\hat{v}(1,3)+\hat{v}(2,3), AS_3(\rho(1)\rho(2)\rho(3) + \rho C_2(1,2))]_L \ . \end{aligned} \tag{109}$$

These two coupled equations are non-linear and closed. Under suitable approximations, these equations reduce to time-dependent G-matrix theory, whose stationary analog is Brueckner G-matrix theory[31]. With the local-density approximation, a quantum version of the BUU equation can be derived for the baryon phase-space distribution functions from the above two equations[23, 24]. Below we shall extend these equations to include real pions and pion-baryon interactions, so that a description for the hadron dynamics is achieved.

4 Inclusion of pions

With the experience of formulating the baryon dynamics within the density-matrix framework, we now proceed to formulate the full correlation dynamics for hadronic matter. To describe hadronic matter, besides many-body correlations for baryons and pions, we also need the irreducible pion-baryon interaction vertices. Since we only have experience to handle correlations, it is not known how to derive a set of equations of motion for the irreducible vertices in the context of density matrix formalism. To reach the above goal we should extend the density matrix formalism further. Most likely one should combine the density matrix formalism with Green's function technique. In the following we take a less ambitious approach and extend the baryon dynamics to hadron dynamics in the lowest-order approximation in the calculation of pion-baryon interaction vertices.

4.1 Equations of motion

It is easy to extend eq. (73) to include the pion-baryon interactions. Using $H^h_{\rm eff}$ as the total Hamiltonian, we obtain the equation of motion for the n-body density operator,

$$i\frac{\partial\hat{\rho}_n}{\partial t} = [\hat{H}^h(n), \hat{\rho}_n] + {\rm Tr}_{(n+1)}[\hat{V}(n+1), \hat{\rho}_{n+1}] \,. \tag{110}$$

Here $\hat{H}^h(n)$ now also contains the pion-baryon interactions,

$$\hat{H}^h(n) = \sum_{i=1}^{n}[\hat{E}(i) + \hat{U}^\pi(i)] + \sum_{i<j}^{n}\hat{v}(i,j) = \hat{H}(n) + \sum_{i=1}^{n}\hat{U}^\pi(i) \,, \tag{111}$$

with $\hat{U}^\pi(x)$ given by eqs. (60-63) and $\hat{H}(n)$ by eqs. (74).

The equation of motion for ρ_n can then be obtained by taking the expectation value of the operator $\hat{\rho}_n$,

$$i\frac{\partial\rho_n}{\partial t} = [\hat{H}(n), \rho_n] + {\rm Tr}_{(n+1)}[\hat{V}(n+1), \rho_{n+1}] + \langle[\sum_{i=1}^{n}\hat{U}^\pi(i), \hat{\rho}_n]\rangle \,. \tag{112}$$

Since $\hat{U}^\pi(x)$ contains the pion fields, the last term in the above equation depends on the quantity

$$\langle\hat{\rho}_n(x_1, x_2, \cdots x_n; x_1', x_2', \cdots x_n')\pi(y)\rangle \,, \tag{113}$$

which contain the irreducible vertices associated with the pion-baryon interactions. At the present, we are not able to treat these in general. Nevertheless, we do find a way to include the lowest-order vertex, which suffices for our present purpose.

Truncating eq. (112) at the second order, *i.e.* assuming $C_{n>2} = 0$, we obtain

$$\begin{aligned} i\frac{\partial\rho}{\partial t} &= [\hat{E}, \rho] + {\rm Tr}_2[\hat{v}(1,2), \rho_2] + [\hat{U}_\pi, \Gamma] \,, & (114)\\ i\frac{\partial\rho_2}{\partial t} &= [\hat{E}(1) + \hat{E}(2) + \hat{v}(1,2), \rho_2] + {\rm Tr}_3[\hat{v}(1,3) + \hat{v}(2,3), \rho_3] \\ &+ \langle[\hat{U}^\pi(1) + \hat{U}^\pi(2), \hat{\rho}_2]\rangle \,, & (115)\end{aligned}$$

where the vertex function $\Gamma(x,y,x')$ is an isovector and it is defined as

$$\Gamma(x,y,x') = \langle\hat{\Gamma}(x,y,x')\rangle , \tag{116}$$
$$\hat{\Gamma}(x,y,x') = \psi^\dagger(x')\pi(y)\psi(x) . \tag{117}$$

Since we have assumed that $C_{n>2}=0$, we have

$$\rho_2 = AS_2(\rho\rho)+C_2 , \tag{118}$$
$$\rho_3 = AS_3(\rho\rho\rho+\rho C_2) , \tag{119}$$

and, furthermore,

$$\begin{aligned}\langle\hat{U}^\pi(x_1)\hat{\rho}_2(x_1,x_2;x_1',x_2')\rangle &= \langle\hat{U}^\pi(x_1)AS_2\hat{\rho}(x_1,x_1')\hat{\rho}(x_2,x_2')\rangle \\ &= AS_2\hat{U}_\pi(y)\Gamma(x_1,y,x_1')\rho(x_2,x_2')|_{y=x_1} ,\end{aligned} \tag{120}$$

$$\langle\hat{U}^\pi(x_2)\hat{\rho}_2(x_1,x_2;x_1',x_2')\rangle = AS_2\rho(x_1,x_1')\hat{U}_\pi(y)\Gamma(x_2,y,x_2')|_{y=x_2} , \tag{121}$$
$$\langle\hat{\rho}_2(x_1,x_2;x_1',x_2')\hat{U}^\pi(x_1')\rangle = AS_2\Gamma(x_1,y,x_1')\hat{U}_\pi(y)\rho(x_2,x_2')|_{y=x_1'} , \tag{122}$$
$$\langle\hat{\rho}_2(x_1,x_2;x_1',x_2')\hat{U}^\pi(x_2')\rangle = AS_2\rho(x_1,x_1')\Gamma(x_2,y,x_2')\hat{U}_\pi(y)|_{y=x_2'} , \tag{123}$$
$$\langle[\hat{U}^\pi(1)+\hat{U}^\pi(2),\hat{\rho}_2]\rangle = [\hat{U}_\pi(1),AS_2\Gamma(1)\rho(2)]+[\hat{U}_\pi(2),AS_2\rho(1)\Gamma(2)] . \tag{124}$$

With the truncations expressed above in eqs. (118-124), the equations of motion, eqs. (114,115) become

$$i\frac{\partial\rho}{\partial t} = [\hat{E}+\hat{U}_{HF},\rho]+Tr[\hat{v}(1,2),C_2]+[\hat{U}_\pi,\Gamma] , \tag{125}$$

$$\begin{aligned}i\frac{\partial C_2}{\partial t} &= [\hat{E}(1)+\hat{E}(2)+\hat{v}(1,2),C_2]+[\hat{v}(1,2),AS_2\rho\rho] \\ &+ \mathrm{Tr}_3[\hat{v}(1,3)+\hat{v}(2,3),AS_3(\rho\rho\rho+\rho C_2)]_L ,\end{aligned} \tag{126}$$

where $\hat{U}_{HF}$ is the hadronic mean field Hamiltonian of eqs. (102,103) $\hat{U}_\pi$ is given by eqs. (60-63) and

$$[\hat{U}_\pi,\Gamma] = \hat{U}_\pi(y)\Gamma(x,y,x')|_{y=x}-\Gamma(x,y,x')\hat{U}_\pi(y)|_{y=x'} . \tag{127}$$

The above eqs. (125,126) show that, within the two-body correlation approximation, the inclusion of the dynamical pion field affects only the equation of motion for the one-body density ρ, but not the equation for the correlation function C_2. This is expected because the dynamical pions enter only at the level of three-body correlations, which have been neglected in eq. (126). Eqs. (125,126) are not closed, however, but couple to the pion fields through the vertex function $\Gamma(x,y,x')$. In order to close the equations of motion, it is necessary to determine the equations of motion for the density matrix of the pion and for the pion-baryon interaction vertex function Γ.

4.2 The pion density matrix

We start from eq. (47) which can be rewritten as

$$(\partial^\mu\partial_\mu + m_\pi^2)\boldsymbol{\pi}(x) = -\hat{\boldsymbol{u}}\hat{\rho}(xx) \ , \tag{128}$$

where

$$\hat{\boldsymbol{u}} = \begin{pmatrix} \hat{\boldsymbol{u}}_{NN} & \hat{\boldsymbol{u}}_{N\Delta} \\ \hat{\boldsymbol{u}}_{\Delta N} & \hat{\boldsymbol{u}}_{\Delta\Delta} \end{pmatrix} = \begin{pmatrix} \hat{\boldsymbol{U}}_{NN} & -\hat{\boldsymbol{U}}_{N\Delta} \\ -\hat{\boldsymbol{U}}_{\Delta N} & \hat{\boldsymbol{U}}_{\Delta\Delta} \end{pmatrix} , \tag{129}$$

with

$$\hat{\boldsymbol{u}}_{NN} = ig_{\pi NN}\gamma_0\gamma_5\boldsymbol{\tau} \ , \ \hat{\boldsymbol{u}}_{N\Delta} = g_{\pi N\Delta}\gamma_0\boldsymbol{T}^\dagger\partial_\mu \ , \tag{130}$$

$$\hat{\boldsymbol{u}}_{\Delta N} = g_{\pi N\Delta}\gamma_0\boldsymbol{T}\partial^\mu \ , \ \hat{\boldsymbol{u}}_{\Delta\Delta} = ig_{\pi\Delta\Delta}\gamma_0\gamma_5\boldsymbol{T}\delta_{\mu\nu} \ . \tag{131}$$

Similarly to $\hat{\boldsymbol{U}}$, $\hat{\boldsymbol{u}}$ is a matrix of isovectors, and we omit the boldface in the following. Eq. (128) is the Klein-Gordon equation for the pion field, which contains the second-order time derivative. Therefore this equation is not a convenient starting point for the derivation of the equation of motion for the pion density matrix. In order to linearize the equation, we find the following identity useful,

$$\begin{aligned} \partial_\mu\partial^\mu + m_\pi^2 &= \frac{\partial^2}{\partial t^2} - \nabla^2 + m_\pi^2 \\ &= -(i\frac{\partial}{\partial t} + \sqrt{-\nabla^2 + m_\pi^2})(i\frac{\partial}{\partial t} - \sqrt{-\nabla^2 + m_\pi^2}) \ . \end{aligned} \tag{132}$$

If this relation is combined with the free-particle approximation to the Klein-Gordon operator, namely

$$i\frac{\partial}{\partial t} \approx \sqrt{-\nabla^2 + m_\pi^2} = \hat{E}_\pi \ , \tag{133}$$

the Klein-Gordon equation for the pion can be approximated by the following two equations,

$$i\dot{\boldsymbol{\pi}}(x) = \hat{E}_\pi\boldsymbol{\pi}(x) + \frac{1}{2\hat{E}_\pi}\hat{u}\hat{\rho}(xx) \ , \tag{134}$$

$$\ddot{\boldsymbol{\pi}} = -\hat{E}_\pi^2\boldsymbol{\pi}(x) - \hat{u}\hat{\rho}(xx) \ . \tag{135}$$

Furthermore, we note that eqs. (132-134) imply that the pion field satisfies the relation

$$\hat{E}_\pi(x)\boldsymbol{\pi}(x) = -\boldsymbol{\pi}(x)\hat{E}_\pi(x) \ . \tag{136}$$

The pion density operator is defined as

$$\hat{\rho}_\pi(x;x') = \boldsymbol{\pi}(x')\cdot\boldsymbol{\pi}(x) \ . \tag{137}$$

The associated equation of motion for $\hat{\rho}_\pi$ is readily obtained from eqs. (134,135),

$$\begin{aligned} i\frac{\partial\hat{\rho}_\pi(x;x')}{\partial t} &= [\hat{E}_\pi, \hat{\rho}_\pi] + [\frac{1}{2\hat{E}_\pi}\hat{u}, \hat{\Gamma}] \\ &= \hat{E}_\pi(x)\hat{\rho}_\pi(x;x') - \hat{\rho}_\pi(x;x')\hat{E}_\pi(x') \\ &+ \frac{1}{2\hat{E}_\pi(x)}\hat{u}(x)\hat{\Gamma}(x,x',x) - \hat{\Gamma}(x',x,x')\hat{u}(x')\frac{1}{2\hat{E}_\pi(x')} \ . \end{aligned} \tag{138}$$

The density matrix for the pion is the expectation value of $\hat{\rho}_\pi$, and so it obeys the following equation of motion,

$$i\frac{\partial \rho_\pi}{\partial t} = [\hat{E}, \rho_\pi] + \frac{1}{2}[\frac{1}{\hat{E}_\pi}\hat{u}, \Gamma] . \qquad (139)$$

With the approximation in eq. (133), $\hat{H}^\pi$ can be written as

$$\hat{H}^\pi = \int \hat{E}_\pi^2(x)\hat{\rho}_\pi(x;x')|_{x'=x} d\boldsymbol{r} . \qquad (140)$$

Moreover, the operators for the number of pions and their momentum are given by

$$\hat{N}_\pi = \int \hat{E}_\pi(x)\hat{\rho}_\pi(x;x')|_{x'=x} d\boldsymbol{r} , \qquad (141)$$

$$\hat{\boldsymbol{P}} = \int (-i\boldsymbol{\nabla}_x \hat{E}_\pi(x)\hat{\rho}_\pi(x;x')|_{x=x'}) d\boldsymbol{r} , \qquad (142)$$

respectively.

The Wigner function for the pions, $f_\pi(\boldsymbol{r},\boldsymbol{p},t)$, can be obtained from the density matrix $\rho_\pi(x;x')$ by means of a Fourier transformation (see eq. (169) in sect. 5.2.1),

$$\int \mathrm{Tr}\ \rho_\pi(\boldsymbol{r}+\frac{\boldsymbol{s}}{2};\boldsymbol{r}-\frac{\boldsymbol{s}}{2})\ e^{-i\boldsymbol{p}\cdot\boldsymbol{s}} d\boldsymbol{s} = \frac{f_\pi(\boldsymbol{r},\boldsymbol{p},t)}{E_\pi(p)} . \qquad (143)$$

If the pion distribution is uniform in space, we have $f_\pi(\boldsymbol{r},\boldsymbol{p},t) = f_\pi(\boldsymbol{p},t)$.

To illustrate how the physical quantities can be calculated from the pion density matrix, we give the pion number, momentum, and energy for a uniform system,

$$\begin{aligned} N_\pi &= \langle \mathrm{Tr}\hat{N}_\pi \rangle = \int \mathrm{Tr}\hat{E}_\pi(x)\rho_\pi(x;x')|_{x'=x} d\boldsymbol{r} \qquad (144)\\ &= \int \frac{1}{(2\pi)^3} f_\pi(\boldsymbol{p}) d\boldsymbol{p} d\boldsymbol{r} = \sum_{\boldsymbol{p}} \langle a_\pi^\dagger(\boldsymbol{p}) a_\pi(\boldsymbol{p})\rangle , \end{aligned}$$

$$\begin{aligned} \boldsymbol{P} &= \langle \mathrm{Tr}\hat{\boldsymbol{P}} \rangle = \mathrm{Tr}\int (-i\boldsymbol{\nabla}_x \hat{E}_\pi(x)\rho_\pi(x;x')|_{x'=x}) d\boldsymbol{r} \qquad (145)\\ &= \int \frac{1}{(2\pi)^3} \boldsymbol{p} f_\pi(\boldsymbol{p}) d\boldsymbol{p} d\boldsymbol{r} = \sum_{\boldsymbol{p}} \boldsymbol{p}\ \langle a_\pi^\dagger(\boldsymbol{p}) a_\pi(\boldsymbol{p})\rangle , \end{aligned}$$

$$\begin{aligned} E_\pi &= \langle \mathrm{Tr}\hat{H}^\pi \rangle = \int \hat{E}_\pi(x)^2 \frac{1}{(2\pi)^3} \rho_\pi(x;x')|_{x'=x} \qquad (146)\\ &= \int \frac{1}{(2\pi)^3} E_\pi(p) f_\pi(\boldsymbol{p}) d\boldsymbol{p} d\boldsymbol{r} = \sum_{\boldsymbol{p}} E_\pi(p) \langle a_\pi^\dagger(\boldsymbol{p}) a_\pi(\boldsymbol{p})\rangle , \end{aligned}$$

where

$$\langle a_\pi^\dagger(\boldsymbol{p}) a_\pi(\boldsymbol{p})\rangle = \int f_\pi(\boldsymbol{r},\boldsymbol{p}) d\boldsymbol{r} \qquad (147)$$

is the distribution of the pions in momentum space.

4.3 The pion-baryon interaction vertex

The pion-baryon interaction vertex operator $\hat{\Gamma}$ was defined in eq. (117) . Using the Heisenberg equation, we obtain

$$
\begin{aligned}
& i\frac{\partial\hat{\Gamma}(x,y,x')}{\partial t} \qquad (148)\\
= & (\hat{E}(x)+\hat{U}^{\pi}(x))\hat{\Gamma}(x,y,x')-\hat{\Gamma}(x,y,x')(\hat{E}(x')+\hat{U}^{\pi}(x'))+\mathrm{Tr}_2[\hat{v}(1,2),\hat{\rho}_2\hat{\pi}(y)]\\
+ & \hat{E}_{\pi}(y)\hat{\Gamma}(x,y,x')+\frac{1}{2\hat{E}_{\pi}(y)}\hat{\rho}(x;x')\hat{u}(y)\hat{\rho}(y;y)\\
\approx & (\hat{E}(x)+\hat{U}_{HF}(x)+\hat{U}^{\pi}(x))\hat{\Gamma}(x,y,x')-\hat{\Gamma}(x,y,x')(\hat{E}(x')+\hat{U}_{HF}(x')+\hat{U}^{\pi}(x'))\\
+ & \hat{E}_{\pi}(y)\hat{\Gamma}(x,y,x')+\frac{1}{2\hat{E}_{\pi}(y)}\hat{\rho}(x;x')\hat{u}(y)\hat{\rho}(y;y)\,. \qquad (149)
\end{aligned}
$$

In deriving this equation, we have invoked the equation of motion for the pion field expressed in eq. (134) and the two-body truncation approximation. The solution of this equation is given in Appendix C in detail. Up to second order in $\hat{U}^{\pi}$(x), the interaction vertex reads

$$
\begin{aligned}
\hat{\Gamma}(x,y,x') = & -i\pi\delta(\hat{h}(x)+\hat{E}_{\pi}(y)-\hat{h}(x'))[\frac{1}{\hat{E}_{\pi}(y)}+\frac{1}{2}(\hat{U}^{\pi}(x)-\hat{U}^{\pi}(x'))\frac{1}{\hat{E}_{\pi}(y)}\\
+ & \frac{1}{8}(\hat{U}^{\pi}(x)-\hat{U}^{\pi}(x'))^2\frac{1}{\hat{E}_{\pi}(y)^3}]\hat{\rho}(x;x')\hat{u}(y)\hat{\rho}(y;y)\,. \qquad (150)
\end{aligned}
$$

This second order approximation for $\hat{\Gamma}$ will be used in the evaluation of the collision terms. Since we are neglecting the two-body correlations between pions and baryons, the equation of motion for the pion-baryon interaction vertex can then be reduced to

$$
\begin{aligned}
i\frac{\partial\Gamma(x,y,x')}{\partial t} = & [\hat{h}(x),\Gamma(x,y,x')]\\
+ & \hat{U}_{\pi}(x)\rho_{\pi}(y;x)\rho(x;x')-\rho(x;x')(\rho_{\pi}(x';y)\hat{U}_{\pi}(x')) \qquad (151)\\
+ & \hat{E}_{\pi}(y)\Gamma(x,y,x')+\frac{1}{2\hat{E}_{\pi}(y)}(AS_2\rho(x;y)\rho(y;x')+C_2(xy;yx'))\,.
\end{aligned}
$$

4.4 Comparison with other approaches

Within the two-body correlation approximation, we have derived the equations of motion for the baryon density matrix (eq. (125)), the baryonic two-body correlation function (eq. (126)), the pion density matrix (eq. (139)), and the pion-baryon interaction vertex (eq. (151)). As shown in Appendix D, these equations obey the conservation laws. The equations are coupled, closed, and non-linear. Their numerical solution is beyond the capacity of present computers. The major difficulty is associated with the equation of motion for the baryonic two-body correlation function, which depends on thirteen variables (two phase-space points and time). To make these equations tractable, further approximations are needed. In the following section, the G-matrix approximation will be used to solve the equation of motion for the density matrix (eq. (125)), and the two-body correlation dynamics is reduced to the one-body level.

A different approach towards the relativistic transport theory for hadrons has been taken by Siemens *et al.*[19]. The relationship between the two treatments is summarized in table 3. The major difference is that we assume that $\langle \boldsymbol{\pi}(x) \rangle = \mathbf{o}$, which leads to the three-boson vertex $\Gamma_{\alpha\beta\gamma}$=0. We also neglect correlations between a pion and a baryon as well as that among pions, which result in $U_{\alpha\beta,ab} = 0$ and $U_{\alpha\beta,\alpha'\beta'} = 0$. For nuclear reactions no real pions exist initially. Then $\langle \boldsymbol{\pi}(0) \rangle = \mathbf{o}$ is a good approximation, and the parity conservation guarantees that $\langle \boldsymbol{\pi}(x) \rangle$ vanishes during the reaction course. In intermediate-energy reactions, the density of dynamical pions is much smaller than that of baryons. The correlations among pions and that between the pion and baryon are therefore much smaller than that among baryons. Two- and more-pion resonances are in principle a source for strong pion-pion correlations, but they are not included in our theoretical approach. It is worth noting that the present treatment of pions hinders us from applying the theory to the problem of pion condensation, where the non-vanishing $\langle \boldsymbol{\pi}(x) \rangle$ is essential. However, we can treat any statistical mixture of real pions and that of the baryon and pion.

We also note the recent work by the authors of ref. [20]. Starting with a Lagrangian density similar to ours, and treating σ and ω in the mean field approximation, they derive the equations of motion for nucleons and deltas from the Kadanoff-Baym equations[32] with Martin-Schwinger-Keldysh Green's function formalism[33]. These equations are solved numerically in the one-dimensional shock wave approximation for studying the relaxation processes and pion production in intermediate energy heavy-ion collisions. This work is discussed further in section 6.

5 Transport equations for hadronic matter

In this section we first introduce the G-matrix method to solve the equation of motion for the two-body correlation function C_2 and thus reduce the two-body problem to the one-body problem. Subsequently, we perform the Wigner transformation of the kinetic equations obtained in the previous section to obtain a set of transport equations for the phase-space distribution functions of baryons and pions. These equations can be used directly to describe the transport phenomena of hadronic matter in intermediate-energy heavy-ion collisions by tractable extensions of the numerical techniques presently used to study nuclear transport phenomena.[16]

5.1 The baryon-baryon collision integral

By proceeding in a manner similar to ref. [23, 31] it is possible to rewrite the equation of motion for C_2 as

$$i\frac{\partial C_2}{\partial t} = [\hat{h}(1) + \hat{h}(2), C_2] + \hat{\theta}_{12}\hat{v}\rho_2 - \rho_2\hat{v}\hat{\theta}_{12} \,, \tag{152}$$

where

$$\hat{h}(i) \;=\; \hat{E}(i) + \hat{U}(i) \,, \tag{153}$$

$$
\begin{aligned}
\hat{U}(i) &= \mathrm{Tr}_{3=3'}\hat{v}(i3)(1-\hat{P}_{i3})\rho(33') , &(154)\\
\hat{\theta}_{12} &= 1-\mathrm{Tr}_{3=3'}(\hat{P}_{13}+\hat{P}_{23})\rho(33') . &(155)
\end{aligned}
$$

Here the operator $\hat{P}_{ij}$ interchanges the coordinates of the identical particles i and j; for non-identical particles it is zero. Within the same approximation as in ref. [23, 31], eq. (152) leads to a time-dependent G-matrix formalism as described below,

$$
\rho_2 = \hat{\Omega}\rho_{20}\hat{\Omega}^{\dagger} , \qquad (156)
$$

where

$$
\begin{aligned}
\rho_{20} &= AS_2\rho(1)\rho(2) , &(157)\\
\hat{\Omega} &= 1+\hat{g}_{12}(E)\hat{\theta}_{12}\hat{G}(E) , &(158)\\
\hat{\Omega}^{\dagger} &= 1+\hat{G}^{\dagger}(E)\hat{\theta}_{12}\hat{g}^{\dagger}_{12}(E) , &(159)\\
\hat{g}_{12}(E) &= [E-\hat{h}(1)-\hat{h}(2)+i\varepsilon]^{-1} . &(160)
\end{aligned}
$$

The $\hat{G}$-matrix obeys the equation

$$
\hat{G}(E) = \hat{v}+\hat{v}\hat{g}_{12}(E)\hat{\theta}_{12}\hat{G}(E) = \hat{v}\hat{\Omega} , \qquad (161)
$$

which is the operator version of the usual integral equation. From the above relations we obtain

$$
\begin{aligned}
&\mathrm{Tr}_2[\hat{v}\rho_2-\rho_2\hat{v}] = Tr_2[\hat{v},\rho_2] \qquad (162)\\
=\ &\mathrm{Tr}_2[\hat{G}\rho_{20}(\hat{G}^{\dagger}\hat{\theta}_{12}\hat{g}^{\dagger}_{12}+1)-(1+\hat{g}_{12}\hat{\theta}_{12}\hat{G})\rho_{20}\hat{G}^{\dagger}]\\
=\ &\mathrm{Tr}_2[\hat{G}\rho_{20}-\rho_{20}\hat{G}^{\dagger}]+\mathrm{Tr}_2[\hat{G}\rho_{20}\hat{G}^{\dagger}\hat{\theta}_{12}\hat{g}^{\dagger}_{12}-\hat{g}_{12}\hat{\theta}_{12}\hat{G}\rho_{20}\hat{G}^{\dagger}]\\
=\ &[\hat{U}_{HF}(G),\rho]+I_{bb} ,
\end{aligned}
$$

where the mean field $\hat{U}_{HF}$ and the baryon-baryon collision term I_{bb} are defined in terms of the $\hat{G}$-matrix as

$$
\hat{U}_{HF}\rho = \mathrm{Tr}_2\mathrm{Re}(\hat{G}\rho_{20}) \qquad (163)
$$

and

$$
\begin{aligned}
I_{bb} &= -i\mathrm{Tr}_2[i\hat{G}^{\dagger}\hat{\theta}_{12}\hat{G}\mathrm{Im}(\hat{g}_{12})\rho_{20}-i\rho_{20}\mathrm{Im}(\hat{g}^{\dagger}_{12})\hat{G}^{\dagger}\hat{\theta}_{12}\hat{G}\\
&+ \hat{G}\rho_{20}\hat{G}^{\dagger}\hat{\theta}_{12}\hat{g}^{\dagger}_{12}-\hat{g}_{12}\hat{\theta}_{12}\hat{G}\rho_{20}\hat{G}^{\dagger}] . \qquad (164)
\end{aligned}
$$

Inserting eq. (162) into eq. (108) or (125) for the equation of motion of ρ, we obtain an equation of motion for the one-body baryon density matrix,

$$
i\frac{\partial\rho}{\partial t} = [\hat{U}_{HF}(G),\rho]+I^{b}_{bb}+I^{b}_{b\pi} , \qquad (165)
$$

where the baryon-pion collision term $I^{b}_{b\pi}$ is given by

$$
I^{b}_{b\pi} = [\hat{U}_{\pi},\Gamma] . \qquad (166)
$$

Comparing the above two equations with the equation of motion for the pion density matrix in eq. (139), the latter one can be rewritten as

$$i\frac{\partial \rho_\pi}{\partial t} = [\hat{E}_\pi, \rho_\pi] + I^\pi_{b\pi} \ , \tag{167}$$

where

$$I^\pi_{b\pi} = \frac{1}{2}[\frac{1}{\hat{E}_\pi}\hat{u}, \Gamma] \ . \tag{168}$$

The above eqs. (164-168) are the basic equations to be used in the following subsection to derive a set of transport equations for hadronic matter.

5.2 Transport equations

It should in principle be possible to solve equations (164-168) numerically by, for example, expansion on a basis of *TDHF* wave functions. This was done by Tohyama *et al.*[34] for the nucleonic systems. This approach, however, is severely limited by the available computing resources and has only lent itself to very few exploratory studies and comparisons with experimental observables.

A more tractable procedure is to introduce the Wigner transformation of the density matrix. Within a semiclassical approximation, numerical solutions of the equations of motion for the Wigner transform can be obtained by utilizing the test-particle method. For a purely baryonic system this approach was introduced on a mean field level by Wong[35] and later utilized to study nuclear transport phenomena by Bertsch *et al.*[36].

We follow this latter approach and perform the Wigner transformation of our equations of motion in this section.

5.2.1 Wigner transformation

The Wigner transformations for the baryon and pion density matrices are

$$\hat{f}_b(\boldsymbol{r}, \boldsymbol{p}, t) = \int \rho(\boldsymbol{r} + \frac{\boldsymbol{s}}{2}, m; \boldsymbol{r} - \frac{\boldsymbol{s}}{2}, m') \ e^{-i\boldsymbol{p}\cdot\boldsymbol{s}} d\boldsymbol{s} \ , \tag{169}$$

$$\hat{f}_\pi(\boldsymbol{r}, \boldsymbol{k}, t) = \int \rho_\pi(\boldsymbol{r} + \frac{\boldsymbol{s}}{2}, m; \boldsymbol{r} - \frac{\boldsymbol{s}}{2}, m') \ e^{-i\boldsymbol{p}\cdot\boldsymbol{s}} d\boldsymbol{s} \ , \tag{170}$$

where the caret is used to remind of the fact that these quantities are still matrices with respect to the spin-isospin labels m and m'. They can be expanded as

$$\hat{f}_b(\boldsymbol{r}, \boldsymbol{p}, t) = \sum_{\alpha\alpha'} f_{\alpha\alpha'}(xp) \left(\frac{M^*_\alpha M^*_{\alpha'}}{E^*_\alpha(p) E^*_{\alpha'}(p)}\right)^{1/2} u^\dagger_{\alpha'}(\Pi) u_\alpha(\Pi) \ , \tag{171}$$

$$\hat{f}_\pi(\boldsymbol{r}, \boldsymbol{k}, t) = \sum_{\beta\beta'} f_{\beta\beta'}(xk) \left(\frac{1}{E_\beta(k) E_{\beta'}(k)}\right)^{1/2} v^\dagger_{\beta'}(k) v_\beta(k) \ , \tag{172}$$

where the summation is taken over all possible single particle states. Here v_β is the isospinor of the pion, $u_\alpha(\Pi)$ is the spin-isospinor of the baryon which has the effective mass M^*_α and the momentum Π in accordance with the fact that the baryons are moving in scalar and vector fields. Furtehrmore, $\alpha = (b, m_s, m_t)$ is used to specify quantum numbers of the baryon, where $b = N$ or Δ, and m_s/m_t is the spin/isospin of the baryon. These spinors satisfy the following orthonormal conditions

$$\langle u_\alpha(\Pi)|u_{\alpha'}(\Pi)\rangle = \delta_{\alpha\alpha'} E^*_\alpha(\Pi)/M^*_\alpha \,, \tag{173}$$

and

$$\langle v_\beta(k)|v_{\beta'}(k)\rangle = \delta_{\beta\beta'} \,. \tag{174}$$

In the above equations we have used $p = (E, \boldsymbol{P})$ for the four-momentum of baryons and $k = (E_\pi, \boldsymbol{k})$ for that of pions. The effective momentum and mass of the baryon are related to the vector and scalar fields through

$$\Pi_\mu = p_\mu - U_\mu \,, \tag{175}$$

$$M^*_\alpha = M_\alpha + U_s \,, \tag{176}$$

and the energy of the baryon in the nuclear medium is given by

$$E^*_\alpha = (\Pi_i \Pi^i + M^{*2}_\alpha)^{1/2} \,. \tag{177}$$

As shown in ref. [23], in the equal-time limit the above Wigner transformation in three-dimensional space is consistent with the Wigner transformation of the corresponding one-body Green's function in four-dimensional space, as given by de Groot[37].

5.2.2 The Vlasov terms

In order to bring out the physics of the kinetic equations for the hadron density matrices more clearly, it is instructive to compare their form with the standard BUU equation. For this purpose, it is useful to recast our equations of motion on the form

$$i\frac{\partial \rho}{\partial t} - [\hat{E} + \hat{U}_{HF}, \rho] = I^b_{bb} + I^b_{b\pi} \,, \tag{178}$$

$$i\frac{\partial \rho_\pi}{\partial t} - [\hat{E}^\pi, \rho_\pi] = I^\pi_{b\pi} \,. \tag{179}$$

The left-hand sides of these two equations are the Vlasov terms, corresponding to the collisionless one-body propagation. These terms will be rewritten in the usual form below. The mean field U_{HF} in these equations has been assumed to only contain scalar and vector components. It can therefore be decomposed as

$$\hat{U}_{HF}(G) = -\alpha_\mu U^\mu(x) + \gamma_0 U_s(x) \,, \tag{180}$$

with $\alpha_\mu = (\gamma_0\gamma_i, \gamma_0\gamma_0) = (\alpha_i, 1)$. Considering only the diagonal elements of ρ and ρ_π in isospin space, as is usually done, we may now perform a Wigner transformation of eqs. (178,179) and subsequently take the trace in spin space. Employing the semi-classical limit for the mean-field terms, the following equations of motion for the baryon and pion phase space distributions are then obtained, with the Vlasov terms in an explicit form (see Appendix E for the derivation)

$$\begin{aligned}\frac{\partial f_b(xp)}{\partial t}+\frac{\Pi^i}{E_b^*(p)}\nabla_i^x f_b(xp) &- \frac{\Pi^\mu}{E_b^*(p)}\nabla_i^x U_\mu(x)\nabla_p^i f_b(xp)+\frac{M_b^*}{E_b^*(p)}\nabla_i^x U_s\nabla_p^i f_b(xp)\\ &= I_{bb}^b(xp)+I_{b\pi}^b(xp) \qquad (181)\end{aligned}$$

for the particular state b of the baryon. For (any charge state of) the pion we have

$$\frac{\partial f_\pi(xk)}{\partial t}+\frac{\boldsymbol{k}\cdot\boldsymbol{\nabla}^x}{E_\pi(k)}f_\pi(xk)=I_{b\pi}^\pi(xk)\ , \qquad (182)$$

where the collision terms are given by

$$I_{bb}^b(xp)=-i\int \mathrm{Tr} I_{bb}^b(x,1;x',1')\ \mathrm{e}^{-i\boldsymbol{p}\cdot\boldsymbol{r}}d\boldsymbol{r}\ , \qquad (183)$$

$$I_{b\pi}^b(xp)=-i\int \mathrm{Tr} I_{b\pi}^b(x,1;x',1')\ \mathrm{e}^{-i\boldsymbol{p}\cdot\boldsymbol{r}}d\boldsymbol{r}\ , \qquad (184)$$

and

$$I_{b\pi}^\pi(xk)=-i\int \mathrm{Tr} I_{b\pi}^\pi(x,1;x',1')\ \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}d\boldsymbol{r}\ . \qquad (185)$$

It is easy to show that the Vlasov terms are relativistically covariant [23, 24].

5.2.3 The baryon collision terms

The collision term I_{bb}^b represent the rate of change of the baryon phase-space distribution function as a result of baryon-baryon collisions. To calculate the collision term, we use the spin-isospinor u_α to represent a baryon; they satisfy the orthonormality relation eq. (173). In this notation, the different N and Δ charge states can be considered as identical particles with different intrinsic quantum numbers α. The two-body density matrix ρ_{20}=AS$\rho(1)\rho(2)$ can then be antisymmetrized even between N and Δ. The interactions used here does not contain the exchange term between N and Δ, and so the exchange term of the matrix elements of the interaction between N and Δ automatically vanishes, due to the above orthonormality relations. Therefore, the spurious terms in ρ_{20} due to the antisymmetrization between N and Δ have no contribution to the collision terms I_{bb}^b. With this in mind, the calculation of I_{bb}^b is straightforward, although lengthy. The final result is

$$\begin{aligned}I_{bb}^b(xp) &= \frac{\pi}{(2\pi)^9}\sum_{\alpha_1\alpha_2\alpha_3,m_s^b}\int\int\int d\boldsymbol{p}_1 d\boldsymbol{p}_2 d\boldsymbol{p}_3\frac{M_b^*M_{\alpha_1}^*M_{\alpha_2}^*M_{\alpha_3}^*}{E_b^*E_{\alpha_1}^*E_{\alpha_2}^*E_{\alpha_3}^*}\\ &\cdot\ \delta(E_b^*(p)+E_{\alpha_1}^*(p_1)-E_{\alpha_2}^*(p_2)-E_{\alpha_3}^*(p_3))\delta(\boldsymbol{p}+\boldsymbol{p}_1-\boldsymbol{p}_2-\boldsymbol{p}_3)\\ &\cdot\ \langle\langle p\alpha_b p_1\alpha_1|\hat{G}|p_2\alpha_2 p_3\alpha_3\rangle\rangle \qquad (186)\\ &\cdot\ [\langle\langle p_2\alpha_2 p_3\alpha_3|\hat{G}|p\alpha_b p_1\alpha_1\rangle\rangle-\langle\langle p_2\alpha_2 p_3\alpha_3|\hat{G}|p_1\alpha_1 p\alpha_b\rangle\rangle]\\ &\cdot\ [f_{\alpha_2}(xp_2)f_{\alpha_3}(xp_3)\overline{f}_{\alpha_1}(xp_1)\overline{f}_b(xp)-\overline{f}_{\alpha_2}(xp_2)\overline{f}_{\alpha_3}(xp_3)f_{\alpha_1}(xp_1)f_b(xp)]\ ,\end{aligned}$$

where

$$\langle\langle p\alpha_b p_1\alpha_1|\hat{G}|p_2\alpha_2 p_3\alpha_3\rangle\rangle = \tag{187}$$
$$\int \langle u_\alpha(p)u_{\alpha_1}(p_1)|\hat{G}|u_{\alpha_2}(p_2)u_{\alpha_3}(p_3)\rangle e^{-i[(\boldsymbol{p}-\boldsymbol{p}_1)-(\boldsymbol{p}_2-\boldsymbol{p}_3)]\cdot\boldsymbol{r}/2} d\boldsymbol{r}.$$

The collision term I^b_{bb} respects the Pauli exclusion principle as shown in the appearance of $\overline{f}_\alpha(xp) = 1 - f_\alpha(xp)$ and $\overline{f}_b(xp) = 1 - f_b(xp)$. The effective interaction $\hat{v}$, and hence $\hat{G}$, contains the $p \leftrightarrow n$ and $N \leftrightarrow \Delta$ transition operators, and therefore all the possible collision processes are included in this collision integral. The above baryon collsion term is of the same form as the NN collision term appearing in the standard BUU equations, but generalized to accommodate the four Δ states of the baryon.

5.2.4 The baryon-pion collision terms

In this section we calculate the rate of change of the hadron phase-space distribution function due to baryon-pion interactions. Let us first consider $I^b_{b\pi}$. It is given by $I^b_{b\pi}(1,1') = \langle \hat{I}^b_{b\pi}(11')\rangle$, where the collision operator is

$$\begin{aligned}\hat{I}^b_{b\pi}(11') &= \hat{U}_\pi(x)\hat{\Gamma}(x,x,x') - \Gamma(x,x',x')\hat{U}_\pi(x') \\ &= -i\pi\delta(\hat{h}(x) + \hat{E}_\pi(x) - \hat{h}(x'))[(\frac{1}{\hat{E}_\pi(y)} + \frac{1}{2}(\hat{U}^\pi(x) - \hat{U}^\pi(x'))\left(\frac{1}{\hat{E}_\pi(y)}\right)^2 \\ &+ \frac{1}{8}(\hat{U}^\pi(x) - \hat{U}^\pi(x'))^2\left(\frac{1}{\hat{E}_\pi(y)}\right)^3)\hat{U}_\pi(y)\hat{\rho}(xx')\hat{u}(y)\hat{\rho}(yy) - (h.c.)]_{y=x}\,.\end{aligned} \tag{188}$$

Taking the average of the collision operator is tedious and the details of this process can be found in Appendix F. As in the BUU equation, the collision terms can be separated into gain terms and loss terms. (The same is true for $I^b_{b\pi}$.) The physical processes represented by the gain and loss terms of $I^b_{b\pi}$ are shown in fig. 1. Explicitly,

$$I^b_{b\pi}(\boldsymbol{r},\boldsymbol{p},t) = I^b_{\text{gain}}(xp) - I^b_{\text{loss}}(xp)\,. \tag{189}$$

The gain term is given by

$$\begin{aligned}I^b_{\text{gain}}(xp) &= \\ &\frac{\pi}{8(2\pi)^6}\sum_{\pi\alpha' m^b_s}\int\int\frac{M^*_b M^*_{\alpha'}}{E^*_b(p)E^*_{\alpha'}(p')}\frac{\langle u_{\alpha'p'}|\hat{u}(p')\underline{\hat{u}}(k)\hat{u}(p)|u_{\alpha p}\rangle\cdot\langle u_{\alpha p}|\underline{\hat{u}}(k)|u_{\alpha'p'}\rangle}{E^4_\pi(k)} \\ &\cdot\ [\overline{f}_\pi(xk)f_{\alpha'}(xp')\overline{f}_b(xp)\delta(E^*_b(p) + E_\pi(k) - E^*_{\alpha'}(p'))\delta(\boldsymbol{p}' - \boldsymbol{k} - \boldsymbol{p}) \\ &+ f_\pi(xk)f_{\alpha'}(xp')\overline{f}_b(xp)\delta(E^*_b(p) - E_\pi(k) - E^*_{\alpha'}(p'))\delta(\boldsymbol{p}' + \boldsymbol{k} - \boldsymbol{p})]\ d\boldsymbol{p}'d\boldsymbol{k}\,,\end{aligned} \tag{190}$$

while the loss term is

$$\begin{aligned}I^b_{\text{loss}}(xp) &= \\ &\frac{\pi}{8(2\pi)^6}\sum_{\pi\alpha' m^b_s}\int\int\frac{M^*_b M^*_{\alpha'}}{E^*_b(p)E^*_{\alpha'}(p')}\frac{\langle u_{\alpha'p'}|\hat{u}(p')\underline{\hat{u}}(k)\hat{u}(p)|u_{\alpha p}\rangle\cdot\langle u_{\alpha p}|\underline{\hat{u}}(k)|u_{\alpha'p'}\rangle}{E^4_\pi(k)} \\ &\cdot\ [f_\pi(xk)\overline{f}_{\alpha'}(xp')f_b(xp)\delta(E^*_b(p) + E_\pi(k) - E^*_{\alpha'}(p'))\delta(\boldsymbol{p}' - \boldsymbol{k} - \boldsymbol{p}) \\ &+ \overline{f}_\pi(xk)\overline{f}_{\alpha'}(xp')f_b(xp)\delta(E^*_b(p) - E_\pi(k) - E^*_{\alpha'}(p'))\delta(\boldsymbol{p}' + \boldsymbol{k} - \boldsymbol{p})]\ d\boldsymbol{p}'d\boldsymbol{k}\,.\end{aligned} \tag{191}$$

Where $\overline{f}_\pi(xk) = 1 + f_\pi(xk)$ and the subscript π has been used to specify the isospin of the pion. These are the general expressions and the matrix elements in these equations assure that only physical processes can happen. For example, only when b specifies a nucleon do the first terms in these two equations contribute, while only when b specifies a Δ will the second terms contribute. Each of the matrix elements is a vector in spin-isospin space, as dictated by the underlined quantities, and the dot signifies a contraction with respect to these labels.

We now turn to the collision term $I^\pi_{b\pi}$. It is given by $I^\pi_{b\pi} = \langle \hat{I}^\pi_{b\pi} \rangle$, where the collision operator is

$$i\hat{I}^\pi_{b\pi}(x;x') = \frac{1}{2}\left\{ \frac{1}{\hat{E}_\pi(x)}\hat{u}(x)\hat{\Gamma}(x,x',x) - \hat{\Gamma}(x',x,x')\hat{u}(x')\frac{1}{\hat{E}_\pi(x')} \right\} . \tag{192}$$

It can also be separated into gain terms and loss terms,

$$I^\pi_{b\pi}(\boldsymbol{r},\boldsymbol{k},t) = I^\pi_{\rm gain}(xk) - I^\pi_{\rm loss}(xk) . \tag{193}$$

The physical processes represented by the gain and loss term have been depicted in fig. 2. In Appendix F we derive the explicit expressions as

$$\begin{aligned} I^\pi_{\rm gain}(xk) &= \frac{\pi}{16(2\pi)^6}\sum_{\alpha\alpha'}\int\int \frac{M^*_\alpha M^*_{\alpha'}}{E^*_\alpha(p)E^*_{\alpha'}(p')} \\ &\cdot \frac{\langle u_{\alpha' p'}|\underline{\hat{u}}(k)\hat{u}(p+p')^2|u_{\alpha p}\rangle \cdot \langle u_{\alpha p}|\underline{\hat{u}}(k)|u_{\alpha' p'}\rangle}{E^4_\pi(k)} \\ &\cdot \delta(E^*_{\alpha'}(p') - E_\pi(k) - E_\alpha(p))\ \delta(\boldsymbol{p}'-\boldsymbol{p}-\boldsymbol{k}) \\ &\cdot \overline{f}_\pi(xk) f_{\alpha'}(xp')\overline{f}_\alpha(xp)\ d\boldsymbol{p}d\boldsymbol{p}' , \end{aligned} \tag{194}$$

and

$$\begin{aligned} I^\pi_{\rm loss}(xk) &= \frac{\pi}{16(2\pi)^6}\sum_{\alpha\alpha'}\int\int \frac{M^*_\alpha M^*_{\alpha'}}{E^*_\alpha(p)E^*_{\alpha'}(p')} \\ &\cdot \frac{\langle u_{\alpha' p'}|\underline{\hat{u}}(k)\hat{u}(p+p')^2|u_{\alpha p}\rangle \cdot \langle u_{\alpha p}|\underline{\hat{u}}(k)|u_{\alpha' p'}\rangle}{E^4_\pi(k)} \\ &\cdot \delta(E^*_{\alpha'}(p') - E_\pi(k) - E_\alpha(p))\ \delta(\boldsymbol{p}'-\boldsymbol{p}-\boldsymbol{k}) \\ &\cdot f_\pi(xk) f_\alpha(xp)\overline{f}_{\alpha'}(xp')\ d\boldsymbol{p}d\boldsymbol{p}'. \end{aligned} \tag{195}$$

It should be noted that the fermion suppression factors and the boson enhancement factors are included in these collision terms automatically. We have now given a complete set of transport equations which govern the dynamical evolution of the hadronic matter.

6 Summary and Outlook

In the previous section, we have given the semiclassical equations of motion, eqs. (181,182), for the phase-space distributions $f_b(xp)$ and $f_\pi(xp)$ for baryons (nucleons and deltas) and π mesons, respectively. The collision terms appearing in these equations of motion are presented in eq. (186) for baryon-baryon collisions, and in eqs. (190,191) and (194,195) for pion-baryon collisions. Together, these equations form a complete set of coupled transport eqautions for nucleons, Δ resonances, and π mesons, including all many-particle correlations up to and including the two-body level. Three-body and higher correlations are neglected. This limits the applicability of our theory to heavy-ion collisions up to only a few GeV per nucleon. For higher energies, three- and more-particle effects are expected to play a more significant role [38].

We have started from a relativistic field theoretical Lagrangian of the Walecka type, including σ, ω and π mesons. By integrating over the degrees of freedom of the virtual mesons, we were able to obtain mean-field terms for the baryon interactions. However, the pion is treated as a real particle. In this way, we are able to incorporate the formation and decay of Δ resonances.

Even though our theoretical framework fully utilizes relativistic kinematics, and although we start from a fully covariant Lagrangian, our final results do not include the true relativistic effects of retardation.

This is because we made use of the instanteneous meson exchange approximation, eq. (50), replacing the Green's function $G(x - x')$ by $G(\mathbf{r} - \mathbf{r}', t)\delta(t - t')$. This was done to remove the non-locality in time introduced by the elimination of the virtual meson fields.

A similar substitution has previously been utilized for the derivation of a relativistic transport equation for nucleons by starting from a Walecka-type Lagrangian including nucleons, σ and ω mesons [6, 7, 23, 24, 29]. In numerical tests [25], it was found that the use of equal-time Green's functions only leads to negligible changes in the time evolution of the baryon phase-space distributions up to beam energies of a few GeV per nucleon. These studies, however, did not include dynamical pions. For pions, the instantaneous meson exchange approximation is more questionable because of the smaller mass and correspondingly bigger Compton wave length of the pion. Numerical studies analoguous to the ones presented in ref. [25], but using our equations, will have to be undertaken in the future to determine the practical validity of our approximations. One should, in any case, keep in mind that because of the instantaneous meson exchange approximation our relativistic transport theory for hadronic matter is not fully covariant.

Our transport equations include in principle the possibility for a changed dispersion relation of pions in nuclear matter and for a density dependence of the baryon-baryon and baryon-meson cross sections in nuclear matter. Ko *et al.*[39, 40, 41, 42] have also derived a relativistic transport theory for baryons by starting from a Walecka-type field theory Lagrangian including cubic and quartic terms in the scalar meson mean field potential. In this way, they attempt to mimic the effects of the exchange term, vacuum polarization, and particle-hole polarization, which are not contained in the mean field approximation used by them, but which are important to reproduce the nucleon effective mass and the nuclear compressibility. They study the effect of a changed meson propagator in nuclear matter and find an increase of the nucleon-nucleon cross section due to this effect, not unlike the one originally proposed by Brown *et al.*[43]. Ko and collaborators employ Green's function techniques for the derivation of their collision terms, but only take into account the imaginary part of the second-order nucleon self energy. They do not consider the real part which should in principle contribute to the nucleon mean field.

In a program similar to ours, the GSI theory group [20, 44, 45, 46] has derived coupled transport equations for nucleons, deltas, and π mesons by starting from a Walecka-type Lagrangian. Different from our approach, they do not utilize the hierachy of correlation functions, and derive the collision terms in the first Born approximation. In a simplified reaction geometry, they solve their equations and demonstrate the importance of the momentum-dependence of the nuclear mean field interaction as obtained from a relativistic theory. In the infinite-matter approximation, they study the influence of the elastic and inelastic baryon-baryon collisions on the equilibration of nuclear matter by using their equations [44]. In another investigation [45], they study pion production within a "fireball" expansion model. By solving their kinetic equations in this geometry, they find a large sensitivity to the medium effects, which are caused by using the self-consistent dressed mean field propagators for the baryons. A change of the effective mass of the nucleon by 30% results in a factor of two difference in the reabsorbtion of pions and deltas in their calculations.

Yet another approach to deriving coupled Boltzmann-like transport equations for hadrons has been taken by Davis and Perry [47]. They obtain relativistic Boltzmann equations with medium dependent collision terms by starting from a Lagrangian for spin-half fermions and spin-zero bosons with Yukawa coupling. They find that only under the condition that the boson field remain in local equilibrium throughout the duration of the nuclear collision can the dynamics be described in terms of the fermion distribution function alone.

The advantage of the transport equations as derived by us is that one can represent the phase-space distribution functions for nucleons, deltas and pions by quasiparticle distributions for the different species. With this, one should be able to extend the powerful simulation techniques developed for the non-relativistic case of the dynamical simulation of the phase-space distribution function of nucleons in heavy-ion collisions to the relativistic coupled problem for nucleons, deltas and pions.

In parallel with the present work, we have developped a method of solution for our equations that is based on the quasiparticle techniques. Here, we solve the collision integrals statistically via a Monte Carlo simulation method. The phase-space occupation factors for the final state of the fermions, $1 - f_\alpha(xp)$, are treated via a Monte-Carlo rejection method. Since the computation of all possible final-state phase-space occupation factors is a very computer intensive task, we have developped a technique to store the six-dimensional phase-space occupation probability at every time step on a lattice [48]. For the implementaion of this technique, we have to, however, make use of the locality in time and the mass-shell condition for the energy. The final state phase-space occupation probability factors for bosons, $1 + f_\pi(xp)$, cannot be treated by conventional rejection methods, because the possible range of values of this function is not between 0 and 1. However, it is possible to introduce a cutoff, F, such $F > \max(1 + f_\pi(xp))$ for all coordinate values (x, p) during the course of the nuclear collision. By multiplying the interaction matrix element by F and dividing $(1 + f_\pi(xp))/F$ one can use the conventional rejection technique on this scaled occupation probability factor.

For the present scope of possible applications, the pion multiplicity experimentally found in central heavy-ion collisions is too small for us to expect a sizeable effect of the Bose-Einstein statistics on the average pion multiplicity. What can, however, be expected is a (small) redistribution of pions in their available phase space, which could lead to the development of bigger fluctuations in the pion distribution function than the ones expected from naive kinetic considerations.

We have conducted a first exploratory study of pion production physics with the aid of a simplified numerical implementation of our kinetic equations discussed above [16]. By substituting the experimentally measured baryon-baryon and baryon-meson cross sections in the Born approximation for the interaction matrix elements contained in our kinetic equations, we were able to numerically investigate the phenomenon of the "two-temperature" pion energy spectra observed at the BEVALAC [18].

Our numerical studies are so far only in an infant stage. In future research, we have to include the effects of a possible change of the pion dispersion relation. This possibility is contained in our equations, but so far not realized in our computer program. We have to also test numerically up to which energy one can neglect the retardation effects, which were not included in the present derivation. In all likelihood this can only be done in some restricted geometry, and details will have to be worked out in the future.

The availability of high-quality heavy-ion beams of energies up to one GeV per nucleon over the entire mass range should be possible at GSI in the near future. Our approach, as well as related ones discussed in this last chapter and in the introduction, aim at formulating a framework for the theoretical understanding of the nuclear physics phenomena (expected and unexpected) in this energy range. But only an interative process of interaction between theory and experiment will enable the development of a consistent relativistic many-body theory with predictive power in this field.

Acknowledgements

This work was supported in part by the National Science Foundation under Grant PHY-8906116, the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, and the National Nature Science Foundation of China.

A The effective baryon-baryon interactions

$$\begin{aligned}\hat{v}_{NN,NN}(x_1 - x_2) \;=\;& g^2_{\pi NN} G_\pi(x_1 - x_2)\gamma_0(2)\gamma_5(2)\gamma_0(1)\gamma_5(1)\boldsymbol{\tau}(2)\cdot\boldsymbol{\tau}(1) \\ & - g^2_{\sigma NN} G_\sigma(x_1 - x_2)\gamma_0(2)\gamma_0(1) \\ & + g^2_{\omega NN} G^\omega_{\nu\mu}(x_1 - x_2)\alpha^\mu(2)\alpha^\nu(1) \qquad \text{(A.1)}\end{aligned}$$

$$\hat{v}_{NN,\Delta N}(x_1 - x_2) = i g_{\pi N\Delta} g_{\pi NN} G_{\pi\mu}(x_1 - x_2)\gamma_0(2)\gamma_5(2)\gamma_0(1)\boldsymbol{\tau}(2)\cdot\boldsymbol{\mathcal{T}}^\dagger(1) \qquad \text{(A.2)}$$

$$\hat{v}_{NN,N\Delta}(x_1 - x_2) = ig_{\pi N\Delta} g_{\pi NN} G_{\pi\mu}(x_1 - x_2)\gamma_0(1)\gamma_5(1)\gamma_0(2)\boldsymbol{\tau}(2) \cdot \boldsymbol{T}^\dagger(1) \quad \text{(A.3)}$$

$$\hat{v}_{NN,\Delta\Delta}(x_1 - x_2) = g^2_{\pi N\Delta} G_{\pi\mu\nu}(x_1 - x_2)\gamma_0(2)\gamma_0(1)\boldsymbol{T}^\dagger(2) \cdot \boldsymbol{T}^\dagger(1) \quad \text{(A.4)}$$

$$\hat{v}_{N\Delta,NN}(x_1 - x_2) = ig_{\pi NN} g_{\pi N\Delta} G^\mu_\pi(x_1 - x_2)\gamma_0(2)\gamma_0(1)\gamma_5(1)\boldsymbol{T}(2) \cdot \boldsymbol{\tau}(1) \quad \text{(A.5)}$$

$$\hat{v}_{N\Delta,\Delta N}(x_1 - x_2) = g^2_{\pi N\Delta} G^\nu_{\pi\mu}(x_1 - x_2)\gamma_0(2)\gamma_0(1)\boldsymbol{T}(2) \cdot \boldsymbol{T}^\dagger(1) \quad \text{(A.6)}$$

$$\hat{v}_{N\Delta,N\Delta}(x_1 - x_2) = g_{\pi NN} g_{\pi\Delta\Delta} G_\pi(x_1 - x_2)\gamma_0(2)\gamma_5(2)\gamma_0(1)\gamma_5(1)\boldsymbol{T}(2) \cdot \boldsymbol{\tau}(1) \quad \text{(A.7)}$$

$$\hat{v}_{N\Delta,\Delta\Delta}(x_1 - x_2) = ig_{\pi N\Delta} g_{\pi\Delta\Delta} G_{\pi\nu}(x_1 - x_2)\gamma_0(2)\gamma_5(2)\gamma_0(1)\boldsymbol{T}(2) \cdot \boldsymbol{T}^\dagger(1) \quad \text{(A.8)}$$

$$\hat{v}_{\Delta N,NN}(x_1 - x_2) = ig_{\pi N\Delta} g_{\pi NN} G^\mu_\pi(x_1 - x_2)\gamma_0(2)\gamma_5(2)\gamma_0(1)\boldsymbol{\tau}(2) \cdot \boldsymbol{T}(1) \quad \text{(A.9)}$$

$$\hat{v}_{\Delta N,N\Delta}(x_1 - x_2) = g^2_{\pi N\Delta} G^\mu_{\pi\nu}(x_1 - x_2)\gamma_0(2)\gamma_0(1)\boldsymbol{T}^\dagger(2) \cdot \boldsymbol{T}(1) \quad \text{(A.10)}$$

$$\hat{v}_{\Delta N,\Delta N}(x_1 - x_2) = g_{\pi\Delta\Delta} g_{\pi NN} G_\pi(x_1 - x_2)\gamma_0(2)\gamma_5(2)\gamma_0(1)\gamma_5(1)\boldsymbol{\tau}(2) \cdot \boldsymbol{T}(1) \quad \text{(A.11)}$$

$$\hat{v}_{\Delta N,\Delta\Delta}(x_1 - x_2) = ig_{\pi\Delta\Delta} g_{\pi N\Delta} G_{\pi\nu}(x_1 - x_2)\gamma_0(2)\gamma_0(1)\gamma_5(1)\boldsymbol{T}^\dagger(2) \cdot \boldsymbol{T}(1) \quad \text{(A.12)}$$

$$\begin{aligned}\hat{v}_{\Delta\Delta,\Delta\Delta}(x_1 - x_2) &= g^2_{\pi\Delta\Delta} G_\pi(x_1 - x_2)\gamma_0(2)\gamma_5(2)\gamma_0(1)\gamma_5(1)\boldsymbol{T}(2) \cdot \boldsymbol{T}(1) \\ &\quad -g^2_{\sigma\Delta\Delta} G_\sigma(x_1 - x_2)\gamma_0(2)\gamma_0(1) \\ &\quad g^2_{\omega\Delta\Delta} G^\omega_{\mu\nu}(x_1 - x_2)\alpha^\nu(2)\alpha^\mu(1) \end{aligned} \quad \text{(A.13)}$$

$$\hat{v}_{\Delta\Delta,NN}(x_1 - x_2) = g^2_{\pi N\Delta} G^{\mu\nu}_\pi(x_1 - x_2)\gamma_0(2)\gamma_0(1)\boldsymbol{T}(2) \cdot \boldsymbol{T}(1) \quad \text{(A.14)}$$

$$\hat{v}_{\Delta\Delta,\Delta N}(x_1 - x_2) = ig_{\pi\Delta\Delta} g_{\pi N\Delta} G^\nu_\pi(x_1 - x_2)\gamma_0(2)\gamma_0(1)\gamma_5(1)\boldsymbol{T}(2) \cdot \boldsymbol{T}(1) \quad \text{(A.15)}$$

$$\hat{v}_{\Delta\Delta,N\Delta}(x_1 - x_2) = ig_{\pi\Delta\Delta} g_{\pi N\Delta} G^\nu_\pi(x_1 - x_2)\gamma_0(1)\gamma_0(2)\gamma_5(2)\boldsymbol{T}(2) \cdot \boldsymbol{T}(1) \quad \text{(A.16)}$$

B Derivation of the equation of motion for $\hat{\rho}_n$

From the basic anticommutation relations

$$\begin{aligned} \left\{\psi(x), \psi^\dagger(x')\right\}_{t'=t} &= \delta(\boldsymbol{r} - \boldsymbol{r}')\,, && \text{(B.1)} \\ \{\psi(x), \psi(x')\} &= \left\{\psi^\dagger(x), \psi^\dagger(x')\right\} = 0\,, && \text{(B.2)} \end{aligned}$$

we obtain the following relations,

$$\begin{aligned} & [\hat{\rho}_n, \psi^\dagger(x)\hat{E}(x)\psi(x)] \\ =\; & \psi^\dagger(1')\psi^\dagger(2')\cdots\psi^\dagger(n')\psi(n)\cdots\psi(2)\psi(1)\psi^\dagger(x)\hat{E}(x)\psi(x) \\ -\; & \psi^\dagger(x)\hat{E}(x)\psi(x)\psi^\dagger(1')\psi^\dagger(2')\cdots\psi^\dagger(n')\psi(n)\cdots\psi(2)\psi(1) \\ =\; & \sum_{i=1}^{n}\int \psi^\dagger(1')\psi^\dagger(2')\cdots\psi^\dagger(n')\psi(n)\cdots\hat{E}(x)\psi(x)\psi(i-1) \\ & \cdots\psi(2)\psi(1)\delta(i-x)d\boldsymbol{x} \\ -\; & \sum_{i'=1}^{n}\int \psi^\dagger(1')\psi^\dagger(2')\cdots\psi^\dagger(x)\hat{E}(x)\psi^\dagger((i+1)') \\ & \cdots\psi^\dagger(n')\psi(n)\cdots\psi(2)\psi(1)\delta(i'-x)d\boldsymbol{x} \\ =\; & \sum_{i=1}^{n}\hat{E}(i)\hat{\rho}_n(12\cdots n;1'2'\cdots n') - \hat{\rho}_n(12\cdots n;1'2'\cdots n')\sum_{i'=1'}^{n'}\hat{E}(i') \end{aligned} \quad \text{(B.3)}$$

and

$$
\begin{aligned}
& [\hat{\rho}_n, \frac{1}{2}\psi^\dagger(x_1)\psi^\dagger(x_2)\hat{v}(x_1-x_2)\psi(x_2)\psi(x_1)] \\
= & \sum_{i<j}\int \psi^\dagger(1')\cdots\psi^\dagger(n')\psi(n)\hat{v}(x_1-x_2)\delta(i-x_2)\psi(x_2) \\
& \cdots\delta(j-x_1)\psi(x_1)\cdots\psi(2)\psi(1)d\boldsymbol{x}_1 d\boldsymbol{x}_2 \\
- & \sum_{i'<j'}\int \psi^\dagger(1')\cdots\delta(i'-x_1)\psi^\dagger(x_1)\cdots\delta(j'-x_2)\psi^\dagger(x_2)\hat{v}(x_1-x_2) \\
& \cdots\psi^\dagger(n')\psi(n)\cdots\psi(2)\psi(1)d\boldsymbol{x}_1 d\boldsymbol{x}_2 \\
+ & \sum_{i=1}^{n}\int \psi^\dagger(1')\cdots\psi^\dagger(n')\psi^\dagger(x_1)\psi(x_1)\hat{v}(x_1-x_2)\delta(i-x_2) \\
& \cdot\psi(n)\cdots\psi(x_2)\psi(x_1)d\boldsymbol{x}_1 d\boldsymbol{x}_2 \\
- & \sum_{i'=1}\int \psi^\dagger(1')\cdots\psi^\dagger(x_1)\cdots\psi^\dagger(n')\psi^\dagger(x_2)\hat{v}(x_1-x_2)\psi(x_2) \\
& \cdot\psi(n)\cdots\psi(1)\delta(i'-x_1)d\boldsymbol{x}_1 d\boldsymbol{x}_2 \\
= & \sum_{i<j}^{n}\hat{v}(i,j)\hat{\rho}_n - \hat{\rho}_n\sum_{i'<j'}^{n}\hat{v}(i',j') \\
+ & \sum_{i=1}^{n}\mathrm{Tr}_{(n+1)}\hat{v}(i,n+1)\hat{\rho}_{n+1}(12\cdots n,n+1;1'2'\cdots n'n+1) \\
- & \sum_{i'=1}^{n}\mathrm{Tr}_{(n+1)}\hat{\rho}_{n+1}(12\cdots n,n+1;1'2'\cdots n',n+1)\hat{v}(i',n+1)
\end{aligned}
\qquad \text{(B.4)}
$$

combing the above results, we arrive at the equation of motion for the reduced density matrix,

$$
i\frac{\partial\hat{\rho}_n}{\partial t} = [\hat{\rho}_n, \hat{H}^b_{\text{eff}}] = [\hat{H}(n), \hat{\rho}_n] + \mathrm{Tr}_{(n+1)}[\hat{V}(n+1), \hat{\rho}_{n+1}] \qquad \text{(B.5)}
$$

where

$$
\hat{H}(n) = \sum_{i=1}^{n}\hat{E}(i) + \sum_{i<j}^{n}\hat{v}(i,j) \qquad \text{(B.6)}
$$

and

$$
\hat{V}(n+1) = \sum_{i=1}^{n}\hat{v}(i,n+1)\ . \qquad \text{(B.7)}
$$

C Solution for the vertex operator $\hat{\Gamma}$

The equation of motion for the vertex operator $\hat{\Gamma}$ is

$$
\begin{aligned}
i\frac{\partial\hat{\Gamma}(x,y,x')}{\partial t} = & [\hat{E}(x) + \hat{U}_{HF}(x) + \hat{U}^{\pi}(x)]\hat{\Gamma}(x,y,x') \\
- & \hat{\Gamma}(x,y,x')[\hat{E}(x') + \hat{U}_{HF}(x') + \hat{U}^{\pi}(x')] \\
+ & \hat{E}_{\pi}(y)\hat{\Gamma}(x,y,x') + \frac{1}{2\hat{E}_{\pi}(y)}\hat{\rho}(xx')\hat{u}(y)\hat{\rho}(yy)\ .
\end{aligned}
\qquad \text{(C.1)}
$$

Let

$$\begin{aligned}\hat{\Gamma}(x,y,x') &= e^{-i\hat{E}_\pi(y)t} T e^{-i\int_{-\infty}^{t}(\hat{E}(x)+\hat{U}_{HF}(x)+\hat{U}^\pi(x))_\tau d\tau}\Gamma_0(x,y,x') \\ &\cdot \; T e^{i\int_{-\infty}^{t}(\hat{E}(x')+\hat{U}_{HF}(x')+\hat{U}^\pi(x'))_\tau d\tau}, \end{aligned} \tag{C.2}$$

where $\hat{\Gamma}_0(x,y,x')$ obeys

$$\begin{aligned} i\frac{\partial\hat{\Gamma}_0(x,y,x')}{\partial t} &= e^{i\hat{E}_\pi(y)t} T e^{i\int_{-\infty}^{t}(\hat{E}(x)+\hat{U}_{HF}(x)+\hat{U}^\pi(x))_\tau d\tau} \\ &\cdot \; \frac{1}{2\hat{E}_\pi(y)}\hat{\rho}(xx')\hat{u}(y)\hat{\rho}(yy) \\ &\cdot \; T e^{-i\int_{-\infty}^{t}(\hat{E}(x')+\hat{U}_{HF}(x')+\hat{U}^\pi(x'))_\tau d\tau} . \end{aligned} \tag{C.3}$$

The solution for $\hat{\Gamma}_0$ is

$$\begin{aligned} \hat{\Gamma}_0(x,y,x') &= -i\int_{-\infty}^{t} dt' e^{i\hat{E}_\pi t'} T e^{i\int_{-\infty}^{t'}(\hat{E}(x)+\hat{U}_{HF}(x)+\hat{U}^\pi(x))_\tau d\tau} \\ &\cdot \; [\frac{1}{2\hat{E}_\pi(y)}\hat{\rho}(xx')\hat{u}(y)\hat{\rho}(yy)]_{t'} \\ &\cdot \; T e^{-i\int_{-\infty}^{t}(\hat{E}(x')+\hat{U}_{HF}(x')+\hat{U}^\pi(x'))_\tau d\tau} . \end{aligned} \tag{C.4}$$

From eqs. (C.2) and (C.4) we then obtain

$$\begin{aligned} \hat{\Gamma}(x,y,x') &= -i\int_{-\infty}^{t} dt' e^{-i\hat{E}_\pi(y)(t-t')} T e^{-i\int_{t'}^{t}(\hat{E}(x)+\hat{U}_{HF}(x)+\hat{U}^\pi(x))_\tau d\tau} \\ &\cdot \; [\frac{1}{2\hat{E}_\pi(y)}\hat{\rho}(xx')\hat{u}(y)\hat{\rho}(yy)]_{t'} T e^{i\int_{t'}^{t}(\hat{E}(x')+\hat{U}_{HF}(x')+\hat{U}^\pi(x'))_\tau d\tau} \end{aligned} \tag{C.5}$$

Invoking the Markovian approximation, we may replace $\hat{F}(\tau)$ and $\hat{F}(t')$ in the time integration by $\hat{F}(t)$. We then find

$$\begin{aligned} \hat{\Gamma}(x,y,x') &\approx -i\int_0^\infty d\tau e^{-i\hat{E}_\pi(y)\tau} e^{-i(\hat{h}(x)-\hat{h}(x'))_t\tau} \\ &\cdot \; e^{-i(\hat{U}^\pi(x)-\hat{U}^\pi(x'))_t\tau}\left[\frac{1}{2\hat{E}_\pi(y)}\hat{\rho}(xx')\hat{u}(y)\hat{\rho}(yy)\right]_t \\ &= -i\int_0^\infty d\tau\left[e^{-i(\hat{U}^\pi(x)-\hat{U}^\pi(x'))_t} i\frac{\delta}{\delta\hat{E}_\pi(y)} e^{-i[\hat{h}(x)-\hat{h}(x')+\hat{E}_\pi(y)]_t\tau}\right] \\ &\cdot \; \left[\frac{1}{2\hat{E}_\pi(y)}\hat{\rho}(xx')\hat{u}(y)\hat{\rho}(yy)\right]_t \\ &= -i\pi\left[e^{-i(\hat{U}^\pi(x)-\hat{U}^\pi(x'))} i\frac{\delta}{\delta\hat{E}_\pi(y)}\delta(\hat{h}(x)+\hat{E}_\pi(y)-\hat{h}(x'))\right]_t \\ &\cdot \; \left[\frac{1}{2\hat{E}_\pi(y)}\hat{\rho}(xx')\hat{u}(y)\hat{\rho}(yy)\right]_t . \end{aligned} \tag{C.6}$$

Expanding the difference $(\hat{U}^\pi(x) - \hat{U}^\pi(x'))$ up to second order, we finally obtain the following expression for the vertex operator,

$$\begin{aligned}
\hat{\Gamma}(x,y,x') &= -i\pi \Big[\delta(\hat{h}(x) + \hat{E}_\pi(y) - \hat{h}(x')) \\
&+ (\hat{U}^\pi(x) - \hat{U}^\pi(x'))\frac{\delta}{\delta \hat{E}_\pi(y)}\delta(\hat{h}(x) + \hat{E}_\pi(y) - \hat{h}(x')) \\
&+ \frac{1}{2}(\hat{U}^\pi(x) - \hat{U}^\pi(x'))^2\frac{\delta^2}{\delta^2 \hat{E}_\pi(y)}\delta(\hat{h}(x) + \hat{E}_\pi(y) - \hat{h}(x') + \cdots\Big] \\
&\cdot \frac{1}{2\hat{E}_\pi(y)}\hat{\rho}(xx')\hat{u}(y)\hat{\rho}(yy) \\
&= -i\pi\delta(\hat{h}(x) + \hat{E}_\pi(y) - \hat{h}(x'))\Big[\frac{1}{\hat{E}_\pi(y)} \\
&+ \frac{1}{2}[\hat{U}^\pi(x) - \hat{U}^\pi(x')]\frac{1}{(\hat{E}_\pi(y))^2} \\
&+ \frac{1}{8}[\hat{U}^\pi(x) - \hat{U}^\pi(x')]^2\frac{1}{(\hat{E}_\pi(y))^3} + \cdots\Big]\hat{\rho}(xx')\hat{u}(y)\hat{\rho}(yy) \ ,
\end{aligned} \tag{C.7}$$

where

$$\hat{h}(x) = \hat{E}(x) + \hat{U}_{HF}(x) \tag{C.8}$$

is the effective one-particle Hamiltonian.

D Conservation laws for the hadronic system

To facilitate the discussion of conservation laws for the hadronic system, we express $\hat{H}^{\text{eff}}$ in terms of $\hat{\rho}, \hat{\rho}_2, \hat{\rho}_\pi$ and $\hat{\Gamma}$. With the approximation expressed in eq. (133), we have

$$\begin{aligned}
\hat{H}^h_{\text{eff}} &= \int \left[\hat{E}(x)\hat{\rho}(xx')\right]_{x=x'} dx \\
&+ \frac{1}{2}\int [\hat{v}(x_1,x_2)\hat{\rho}_2(x_1x_2, x_1'x_2')]_{x_1'=x_1, x_2'=x_2} dx_1 dx_2 \\
&+ \int \left[\hat{E}^2_\pi(x)\hat{\rho}_\pi(xx')\right]_{x'=x} dx \\
&+ \int \left[\hat{U}_\pi(y)\hat{\Gamma}(x,y,x)\right]_{y=x} dx \ .
\end{aligned} \tag{D.1}$$

Since $[\hat{N}, \hat{H}^h_{\text{eff}}] = 0$, baryon number conservation is evident,

$$\dot{B} = \langle \frac{\partial \hat{N}}{\partial t} \rangle = \langle -i[\hat{N}, \hat{H}^h_{\text{eff}}] \rangle = 0 \ . \tag{D.2}$$

The conservation of the total momentum is also readily verified. First note that the operator representing the total linear momentum is given by

$$\hat{\boldsymbol{P}} = \text{Tr}\int \left[-i\boldsymbol{\nabla}_x[\hat{\rho}(xx') + \hat{E}_\pi(x)\hat{\rho}_\pi(xx')]\right]_{x=x'} dx \ , \tag{D.3}$$

and the corresponding expectation value is $P = \langle \hat{P} \rangle$. Because of the translation invariance, namely $[\hat{P}, H^h_{\rm eff}] = 0$, we then find

$$\begin{aligned} \dot{P} &= \langle \mathrm{Tr} \int \left[-i\nabla_x [\dot{\hat{\rho}}(xx') + \hat{E}_\pi(x) \dot{\hat{\rho}}_\pi(xx')] \right]_{x=x'} dx \rangle \\ &= \langle \mathrm{Tr} \int \left[-i[-i\nabla_x(\hat{\rho}(xx') + \hat{E}_\pi(x)\hat{\rho}_\pi(xx')), \hat{H}^h_{\rm eff}] \right]_{x=x'} dx \rangle \\ &= \langle -i[\hat{P}, H^h_{\rm eff}] \rangle = \mathrm{o}\,. \end{aligned} \tag{D.4}$$

The verification of energy conservation requires more detailed consideration. The total energy of the hadronic system is $E = \langle \mathrm{Tr} \hat{H}^h_{\rm eff} \rangle$. The time derivative of E is

$$\begin{aligned} i\dot{E} &= i \int \mathrm{Tr}[\hat{E}(x)\dot{\rho}(xx')]_{x=x'} dx \\ &\quad + \frac{i}{2} \int \mathrm{Tr}[\hat{v}(x_1, x_2)\dot{\rho}_2(x_1x_2x_1'x_2')]_{x_1=x_1', x_2=x_2'} dx_1 dx_2 \\ &\quad + i \int \mathrm{Tr}[\hat{E}^2_\pi(x)\dot{\rho}_\pi(xx')]_{x=x'} dx \\ &\quad + i \int \mathrm{Tr}[\hat{U}_\pi(y)\dot{\Gamma}(x,y,x)]_{y=x} dx \\ &= i \left(\int \mathrm{Tr}[\hat{E}(x)\dot{\rho}(xx')]_{x=x'} dx \right. \\ &\quad \left. + \frac{1}{2} \int \mathrm{Tr}[\hat{v}(x_1,x_2)\dot{\rho}_2(x_1x_2, x_1'x_2')]_{x_1'=x_1, x_2=x_2'} dx_1 dx_2 \right)_{\pi(x)=0} \\ &\quad + \int \mathrm{Tr} \left[\hat{E}(x)[\hat{U}_\pi(y), \Gamma(x,y,x')] \right]_{x'=x, y=x} dx \\ &\quad + \frac{1}{2} \int \mathrm{Tr} \left[\langle \hat{v}(x_1,x_2)[\hat{U}^\pi(1) + \hat{U}^\pi(2), \hat{\rho}_2(x_1x_2, x_1'x_2')] \rangle \right]_{x'=x, x=y} dx_1 dx_2 \\ &\quad + \int \mathrm{Tr} \left[\hat{E}^2_\pi(x)([\hat{E}_\pi, \rho_\pi] + \frac{1}{2}[\frac{1}{\hat{E}_\pi}\hat{u}, \Gamma]) \right]_{x=x', x=y} dx \\ &\quad + \int \mathrm{Tr}[\hat{U}_\pi(y)\langle [\hat{E} + \hat{U}_{HF} + \hat{U}^\pi, \hat{\Gamma}] \\ &\quad + \hat{E}_\pi(y)\hat{\Gamma}(x,y,x') + \frac{1}{2\hat{E}_\pi(y)} \hat{\rho}(xx')\hat{u}(y)\hat{\rho}(yy) \rangle]_{x'=x, x=y} dx\,. \end{aligned} \tag{D.5}$$

In the above equation, we sometimes used density operators and vertex operators, sometimes use their averages, depending on which is more convenient. In ref. [6] we have proved that for fermion fields, truncation to the second order preserves energy conservation. Therefore, the first two terms in the r.h.s. of eq. (D.5) vanish, since the proof in ref.[6] is applicable to it. Hence the energy is conserved if the remaining terms cancel. With the truncation scheme of the two-body correlations, we have

$$\begin{aligned} &\frac{1}{2} \int \mathrm{Tr} \left[\langle \hat{v}(x_1,x_2)[\hat{U}^\pi(1) + \hat{U}^\pi(2), \hat{\rho}_2(x_1x_2x_1'x_2')] \rangle \right]_{x_1=x_1', x_2=x_2'} dx_1 dx_2 \\ &= \int \mathrm{Tr} \left[\langle [\hat{U}^\pi(1)\hat{\rho}(x_1x_1')\hat{U}_{HF}(x_1') - \hat{U}_{HF}(x_1)\hat{\rho}(x_1x_1')\hat{U}^\pi(1')] \rangle \right]_{x_1=x_1'} dx_1 \\ &= \int \mathrm{Tr} \left[\hat{U}_\pi(y)[\hat{U}_{HF}, \Gamma(x,y,x')] \right]_{x'=x, y=x} dx\,. \end{aligned} \tag{D.6}$$

By using the approximation $\dot{\pi} \simeq -i\hat{E}_\pi \pi$, and noticing that $\overrightarrow{\hat{E}_\pi}(x) = -\overleftarrow{\hat{E}_\pi}(x)$, we find

$$
\begin{aligned}
&\frac{1}{2}\int \mathrm{Tr}\left[\hat{E}_\pi^2(x)[\frac{1}{\hat{E}_\pi}\hat{u},\Gamma]\right]_{x'=x,y=x} dx \\
= &\int \mathrm{Tr}\left[\hat{E}_\pi(x)\hat{u}(x)\Gamma(x,y,x)\right]_{x=y} dx,
\end{aligned}
\tag{D.7}
$$

and

$$
\begin{aligned}
&-\int \mathrm{Tr}\left[\langle \hat{U}_\pi(y)\hat{E}_\pi(y)\hat{\Gamma}(x,y,x')\rangle\right]_{x'=x,x=y} dx \\
= &-\int \mathrm{Tr}\left[\hat{E}_\pi(x)\hat{U}_\pi(y)\Gamma(x,y,x)\right]_{x=y} dx \\
= &-\int \mathrm{Tr}\left[\hat{E}_\pi(x)\hat{u}(x)\Gamma(x,y,x)\right]_{x=y} dx \,.
\end{aligned}
\tag{D.8}
$$

Where the explicit expressions of $\hat{U}_\pi(y)$ and $\hat{u}(x)$ have been used. In the derivation of the transport equations for hadronic matter, we assumed that $\langle \boldsymbol{\pi} \rangle = \mathbf{o}$, which, in combining with the two-body truncation approximation, leads to

$$
\begin{aligned}
&\int \mathrm{Tr}\left[\hat{U}_\pi(y)\langle[\hat{U}^\pi,\hat{\Gamma}]\rangle\right]_{x'=x,y=x} dx \\
= &\int \mathrm{Tr}\left[\hat{U}_\pi(y)[\langle\hat{U}^\pi\rangle,\Gamma]\right]_{x'=x,y=x} dx \\
= &\int \mathrm{Tr}\left[\hat{U}_\pi(y)[\hat{U}_\pi(x)\langle\pi(x)\rangle,\Gamma(x,y,x')]\right]_{x'=x,x=y} dx = 0 \,.
\end{aligned}
\tag{D.9}
$$

It is obvious that

$$
\int \mathrm{Tr}\left[\rho_\pi[\hat{E}_\pi^2,\hat{E}_\pi]\right]_{x'=x} dx = \int \mathrm{Tr}\left[\rho_\pi[\hat{E}^2,\hat{E}_\pi]\right]_{x'=x} dx = 0 \,. \tag{D.10}
$$

Due to the Pauli blocking, we would have $\psi^\dagger(x)\psi^\dagger(x) = \psi(x)\psi(x) = 0$, which leads to

$$
\begin{aligned}
&\int \mathrm{Tr}\left[\hat{U}_\pi(y)\langle\frac{1}{2\hat{E}_\pi(y)}\hat{\rho}(xx')\hat{u}(y)\hat{\rho}(yy)\rangle\right]_{x'=x,x=y} dx \qquad \text{(D.11)} \\
= &\frac{1}{2}\int \mathrm{Tr}\left[\langle\hat{U}_\pi(y)\frac{1}{\hat{E}_\pi(y)}\hat{u}(y)\psi^\dagger(y)\psi^\dagger(x)\psi(x)\psi(y)\rangle\right]_{y\to x} dx = 0 \,.
\end{aligned}
$$

Combining all the above results, it follows that $\dot{E} = 0$. Therefore, within the two-body truncation approximation for the hadronic fields and the approximations listed above for the pion fields, the total energy of the hadronic matter is conserved.

Now we consider the conservation of total angular momentum. The angular momentum operators for baryons and pions are

$$
\hat{\boldsymbol{L}}_b(x) = \boldsymbol{r} \times \boldsymbol{p} + \hat{\boldsymbol{s}} \,, \quad \hat{\boldsymbol{L}}_\pi(x) = \boldsymbol{r} \times \boldsymbol{p} \,. \tag{D.12}
$$

The total angular momentum of the hadronic system is

$$\boldsymbol{L} = \mathrm{Tr}\int [\hat{\boldsymbol{L}}_b(x)\rho(xx')]_{x'=x}d\boldsymbol{x} + \mathrm{Tr}\int [\hat{\boldsymbol{L}}_\pi(x)\hat{E}_\pi(x)\rho_\pi(xx')]_{x'=x}d\boldsymbol{x}\ , \tag{D.13}$$

and so the time derivative of $\boldsymbol{L}$ is

$$i\dot{\boldsymbol{L}} = \mathrm{Tr}\int [\hat{\boldsymbol{L}}_b(x)\dot{\rho}(xx')_{x'=x}d\boldsymbol{x} + \mathrm{Tr}\int [\hat{\boldsymbol{L}}_\pi(x)\hat{E}_\pi(x)\dot{\rho}_\pi(xx')]_{x'=x}d\boldsymbol{x}\ . \tag{D.14}$$

By using equations of motion for ρ and ρ_π, we then find

$$\begin{aligned} i\dot{\boldsymbol{L}} &= \mathrm{Tr}_1\int ([\hat{E},\hat{\boldsymbol{L}}_b\rho])|_{x'=x}d\boldsymbol{x} \\ &+ \mathrm{Tr}_{(1,2)}\frac{1}{2}\int ([\hat{v}(1,2),(\hat{\boldsymbol{L}}_b(1)+\hat{\boldsymbol{L}}_b(2))\rho_2])|_{x_1'=x_1,x_2'=x_2}d\boldsymbol{x}_1 d\boldsymbol{x}_2 \\ &+ \mathrm{Tr}\int [\hat{U}_\pi(x)\hat{\boldsymbol{L}}_b(x)\Gamma(x,x,x') - \hat{\boldsymbol{L}}_b\Gamma(x,x',x')\hat{U}_\pi(x')]|_{x'=x}d\boldsymbol{x} \\ &+ \mathrm{Tr}\int (E_\pi(x)[\hat{E}_\pi,\hat{\boldsymbol{L}}_\pi\rho_\pi])|_{x'=x}d\boldsymbol{x} + \int \hat{\boldsymbol{L}}_\pi(x)\mathrm{Tr}[\frac{1}{2\hat{E}_\pi}\hat{u},\Gamma]_{x'=x}d\boldsymbol{x} \\ &= \mathbf{o}\ . \end{aligned} \tag{D.15}$$

In the above derivations, the following rotational invariances have been used

$$[\hat{\boldsymbol{L}}_b,\hat{E}] = \mathbf{o}\ ,\ [\hat{\boldsymbol{L}}_\pi,\hat{E}_\pi] = \mathbf{o}\ , \tag{D.16}$$

$$[\hat{\boldsymbol{L}}_b(1)+\hat{\boldsymbol{L}}_b(2),\hat{v}(1,2)] = \mathbf{o}\ , \tag{D.17}$$

$$[\hat{\boldsymbol{L}}_\pi,\frac{1}{2\hat{E}_\pi}\hat{u}] = \mathbf{o}\ , \tag{D.18}$$

and

$$\hat{\boldsymbol{L}}_b(x)\hat{U}_\pi(x)\Gamma(x,x,x') = \hat{U}_\pi(x)\hat{\boldsymbol{L}}_b(x)\Gamma(x,x,x')\ . \tag{D.19}$$

E Derivation of the Vlasov terms

To perform Wigner transformations, we use the following coordinate and gradient operator transformations

$$\boldsymbol{x} = (\boldsymbol{x}_1 + \boldsymbol{x}'_1)/2,\ \boldsymbol{r} = \boldsymbol{x}'_1 - \boldsymbol{x}_1\ , \tag{E.1}$$

$$\nabla_{x_1} = \frac{1}{2}\nabla_x - \nabla_r\ , \tag{E.2}$$

$$\nabla_{x_{1'}} = \frac{1}{2}\nabla_x + \nabla_r\ , \tag{E.3}$$

Also we use the notation $\nabla_i^r = \nabla_x^i = \partial/\partial r_i$. After a Wigner transformation, the Vlasov terms of eq. (178) become

$$\begin{aligned} \dot{\hat{f}}_b(xp,11') &+ \frac{1}{2}[\alpha_i(1)\nabla_x^i\hat{f}_b(xp,11') + \nabla_x^i\hat{f}_b(xp,11')\alpha_i(1')] \\ &- ip^i[\alpha_i(1)\hat{f}_b(xp11') - \hat{f}_b(xp11')\alpha_i(1')] \\ &+ iM[\gamma_0(1)\hat{f}_b(xp11') - \hat{f}_b(xp11')\gamma_0(1')] \\ &+ \frac{1}{2}([-\alpha_i(1)\nabla_x^j U^i - \nabla_x^j U^0 + \gamma_0(1')\nabla_x^j U_s]\nabla_{pj}\hat{f}_b(xp11') \\ &+ \nabla_{pj}\hat{f}_b(xp11')[-\alpha_i(1')\nabla_x^j U^i - \nabla_x^j U^0 + \gamma_0(1')\nabla_x^j U_s])\ . \end{aligned} \tag{E.4}$$

Taking trace over spin and noticing that

$$\langle u_\alpha|\alpha_i|u_\alpha\rangle = v_i = \Pi_i/M^* , \tag{E.5}$$

$$\langle u_\alpha|\gamma_0|u_\alpha\rangle = 1 , \tag{E.6}$$

we obtain the Vlasov terms for the baryons as

$$\dot{f}_b + \frac{\Pi^i}{E_b^*}\nabla_i^x f_b(xp) - \frac{\Pi^\mu}{E_b^*}\nabla_i^x U_\mu(x)\nabla_p^i f_b(xp) + \frac{M_b^*}{E_b^*}\nabla_i^x U_s(x)\nabla_p^i f_b(xp) . \tag{E.7}$$

For the pion field we have, according to the approximation (133),

$$\hat{E}_\pi(x_1) - \hat{E}_\pi(x_1') = \frac{1}{2E_\pi(k)}(\hat{E}_\pi^2(x_1) - \hat{E}_\pi^2(x_1')) = \frac{1}{E_\pi(k)}\boldsymbol{\nabla}_x \cdot \boldsymbol{\nabla}_r . \tag{E.8}$$

With this approximation, the Vlasov terms for the pions follow as

$$\dot{f}_\pi(xk) + \frac{\boldsymbol{k}}{E_\pi(k)} \cdot \boldsymbol{\nabla}^x f_\pi(xk) . \tag{E.9}$$

F Derivation of $I_{b\pi}^b$ and $I_{b\pi}^\pi$

To calculate $I_{b\pi}^b$ and $I_{b\pi}^\pi$, we need to use the explicit expressions for $\hat{U}^\pi(x)$, $\hat{U}_\pi(x)$, and $\hat{u}(x)$, *i.e.* eqs. (60), (61) and (129). The explicit expressions for $\hat{\rho}(xx')$ is

$$\hat{\rho}(xx') = \psi^\dagger(x')\psi(x) , \tag{F.1}$$

where

$$\psi^\dagger(x) = \sum_{\alpha p}\left(\frac{M_\alpha^*}{E_\alpha^*(p)}\right)^{1/2} a_{\alpha p}^\dagger u_{\alpha p}^\dagger e^{i\boldsymbol{p}\cdot\boldsymbol{x}} , \tag{F.2}$$

and

$$\psi(x) = \sum_{\alpha p}\left(\frac{M_\alpha^*}{E_\alpha^*(p)}\right)^{1/2} a_{\alpha p} u_{\alpha p} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} . \tag{F.3}$$

The explicit expression for $\rho_\pi(xx')$ is

$$\rho_\pi(xx') = \boldsymbol{\pi}(x') \cdot \boldsymbol{\pi}(x) \tag{F.4}$$

where

$$\boldsymbol{\pi}(x) = \sum_{\pi k}\left(\frac{1}{2E_\pi(k)}\right)^{1/2}[b_{\pi k}e^{i\boldsymbol{k}\cdot\boldsymbol{x}-iE_\pi(k)t} + b_{\pi k}^\dagger e^{-i\boldsymbol{k}\cdot\boldsymbol{x}+iE_\pi(k)t}] , \tag{F.5}$$

where the creation and annihilation operators $b_{\pi k}^\dagger$ and $b_{\pi k}$ are isovectors. In the above expansions, only the positive energy components are included.

Before proceeding to calculate the collision terms due to pion-baryon collisions, we list the approximations to be used in the following

$$\begin{aligned}\langle a_{\alpha'p'}^\dagger a_{\alpha p} a_{\alpha_1'p_1'}^\dagger a_{\alpha_1 p_1}\rangle &\approx f_\alpha(p) f_{\alpha_1}(p_1)\delta_{\alpha\alpha'}\delta_{pp'}\delta_{\alpha_1\alpha_1'}\delta_{p_1p_1'} \\ &+ f_{\alpha'}(p')(1 - f_\alpha(p))\delta_{\alpha\alpha_1'}\delta_{pp_1'}\delta_{\alpha'\alpha_1}\delta_{p'p_1} ,\end{aligned} \tag{F.6}$$

$$\langle b^\dagger_{\pi k} b_{\pi' k'}\rangle \approx f_\pi(k)\delta_{\pi\pi'}\delta_{kk'} \ , \tag{F.7}$$

$$\langle b_{\pi k} b^\dagger_{\pi' k'}\rangle \approx (1 + f_\pi(k))\delta_{\pi\pi'}\delta_{kk'} \ , \tag{F.8}$$

$$\langle b_{\pi k} b_{\pi' k'}\rangle = \langle b^\dagger_{\pi k} b^\dagger_{\pi' k'}\rangle = \langle b_{\pi k}\rangle = \langle b^\dagger_{\pi k}\rangle = 0 \ . \tag{F.9}$$

Now we are ready to calculate the collision terms, first for $I^b_{b\pi}(xp)$. The collision operator can be written as

$$\hat{I}^b_{b\pi}(xx') = \hat{I}^L_{0\pi} + \hat{I}^L_{1\pi} + \hat{I}^L_{2\pi} - \hat{I}^R_{0\pi} - \hat{I}^R_{1\pi} - \hat{I}^R_{2\pi} \ , \tag{F.10}$$

where $\hat{I}^L$ and $\hat{I}^R$ are Hermitian conjugate of each other. The zero-order terms in $\pi(x)$ can be expressed as

$$\begin{aligned} & \hat{I}^L_{0\pi}(xx') \\ = & -\pi\hat{\rho}(xx')\delta(\overleftarrow{\hat{h}}(x) + \hat{E}_\pi(x) - \overleftarrow{\hat{h}}(x'))\hat{U}_\pi(\frac{\vec{\partial}}{\partial x})\frac{1}{\hat{E}_\pi(x)}\hat{u}(\frac{\vec{\partial}}{\partial x})\hat{\rho}(xx') \\ = & \ \pi \sum_{\alpha' p', \alpha p} \sum_{\alpha'_1 p'_1, \alpha_1 p_1} [u^\dagger_{\alpha' p'}\underline{\hat{u}}(p'_1 - p)u_{\alpha p}] \cdot [\hat{u}_{\alpha'_1 p'_1}\underline{\hat{u}}(p'_1 - p)u_{\alpha_1 p_1}] \\ \cdot & \ \frac{1}{E_\pi(p'_1 - p_1)}[\frac{M^*_\alpha M^*_{\alpha'} M^*_{\alpha_1} M^*_{\alpha'_1}}{E^*_\alpha(p)E^*_{\alpha'}(p')E^*_{\alpha_1}(p_1)E^*_{\alpha'_1}(p'_1)}]^{1/2} \\ \cdot & \ \delta(E^*_\alpha(p) + E_\pi(p'_1 - p_1) - E^*_{\alpha'}(p')) \\ \cdot & \ a^\dagger_{\alpha' p'} a_{\alpha p} a^\dagger_{\alpha'_1 p'_1} a_{\alpha_1 p_1} e^{ip'x' - ipx + ip'_1 x - ip_1 x} \ . \end{aligned} \tag{F.11}$$

The expectation value of this quantity is

$$\begin{aligned} I^L_{0\pi}(xx') \ = & \ \langle \hat{I}^L_{0\pi}(xx')\rangle = \pi \sum_{\alpha' p', \alpha p} \frac{M^*_\alpha M^*_{\alpha'}}{E^*_\alpha(p)E^*_\alpha(p')}(u^\dagger_{\alpha' p'}\underline{\hat{u}}(p - p')u_{\alpha p}) \\ \cdot & \ (u^\dagger_{\alpha p}\underline{\hat{u}}(p - p')u_{\alpha' p'})\frac{1}{E_\pi(p - p')} \\ \cdot & \ \delta(E^*_\alpha(p) + E_\pi(p - p') - E^*_{\alpha'}(p')) \\ \cdot & \ f_{\alpha'}(p')(1 - f_\alpha(p))e^{ip'(x' - x)} \end{aligned} \tag{F.12}$$

In phase space it has the form

$$\begin{aligned} I^L_{0\pi}(xp) \ = & \ \pi \sum_{\alpha' p'} \frac{M^*_\alpha M^*_{\alpha'}}{E^*_\alpha(p)E^*_{\alpha'}(p')}\langle u_{\alpha' p'}|\underline{\hat{u}}(p - p')|u_{\alpha p}\rangle \cdot \langle u_{\alpha p}|\underline{\hat{u}}(p - p')|u_{\alpha' p'}\rangle \\ \cdot & \ \frac{1}{E_\pi(p - p')}\delta(E^*_\alpha(p) + E_\pi(p - p') - E^*_{\alpha'}(p')) \\ \cdot & \ \delta(\mathbf{p} - \mathbf{p}')f_{\alpha'}(p')(1 - f_\alpha(p)) \ . \end{aligned} \tag{F.13}$$

To go further, we notice that the interaction matrix $\hat{u}$ (eq. 129) contains differential operator ∂_μ in its off-diagonal elements, namely the $N \leftrightarrow \Delta$ transition matrix elements. Correspondingly, it has terms linear in p and terms independent of p in momentum space. For the terms linear in p, $\underline{\hat{u}}(p - p')$ and therefore $I^L_{0\pi}$ must vanish in accordance with $\delta(\boldsymbol{p}-\boldsymbol{p}')$. Since $E^*_\alpha(p)+E_\pi(0)-E^*_{\alpha'}(p)\neq 0$ with $\alpha=N$ or Δ, the terms independent of p, namely $\langle u_{\Delta'p'}|\underline{\hat{u}}(1)|u_{\Delta p}\rangle$ and $\langle u_{N'p'}|\underline{\hat{u}}(1)|u_{Np}\rangle$, also vanish in accordance with the constraint $\delta(E^*_\alpha(p)+E_\pi(0)-E^*_{\alpha'}(p))$. We then have $I^L_{0\pi}(xp)=0$, and $I^R_{0\pi}(xp)=0$. Because $\langle b_{\pi k}\rangle$ and $\langle b^\dagger_{\pi k}\rangle$ are zero, the linear terms in the $\boldsymbol{\pi}$ field vanish identically, $I^L_{1\pi}(xp)=I^R_{1\pi}(xp)=0$. Therefore, only bilinear terms in $\boldsymbol{\pi}(x)$ contribute to $I^b_{b\pi}$. Formally

$$\hat{I}^b_{b\pi}=\hat{I}^L_{2\pi}-\hat{I}^R_{2\pi}\ , \tag{F.14}$$

where

$$\begin{aligned}
&\hat{I}^L_{2\pi}(xx')\\
=\ & -\frac{\pi}{8}[\hat{U}^\pi(x)-\hat{U}^\pi(x')]^2\delta(\hat{h}(x)+\hat{E}_\pi(x)-\hat{h}(x'))\\
\cdot\ & \hat{U}_\pi(\frac{\overleftarrow{\partial}}{\partial x})\hat{\rho}(xx')\,(\hat{E}_\pi(\overrightarrow{x}))^{-3}\,\hat{u}(\frac{\overrightarrow{\partial}}{\partial x})\hat{\rho}(xx)\\
=\ & -\frac{\pi}{8}[\hat{U}^\pi(x)-\hat{U}^\pi(x')]^2\delta(\hat{h}(x)+\hat{E}_\pi(x)-\hat{h}(x'))\\
\cdot\ & \underline{\hat{u}}(\frac{\overleftarrow{\partial}}{\partial x})\hat{\rho}(xx')\,(\hat{E}_\pi(\overrightarrow{x}))^{-3}\cdot\underline{\hat{u}}(\frac{\overrightarrow{\partial}}{\partial x})\hat{\rho}(xx)\\
=\ & \frac{\pi}{8}\sum_{\pi k,\pi'k'}\ \sum_{\alpha p,\alpha'p'}\ \sum_{\alpha_1p_1,\alpha_1'p_1'}\left(\frac{M^*_\alpha M^*_{\alpha'}M^*_{\alpha_1}M^*_{\alpha_1'}}{4E_\pi(k)E_{\pi'}(k')E^*_\alpha(p)E^*_{\alpha'}(p')E^*_{\alpha_1}(p_1)E^*_{\alpha_1'}(p_1')}\right)^{1/2}\\
\cdot\ & \delta(E^*_\alpha(p)+E_\pi(k)-E^*_{\alpha'}(p'))a^\dagger_{\alpha'p'}a_{\alpha p}a^\dagger_{\alpha_1'p_1'}a_{\alpha_1p_1}\\
\cdot\ & e^{ip'x'-ipx+i(p_1'-p_1)x}(u^\dagger_{\alpha_1'p_1'}\underline{\hat{u}}(p_1'-p_1)u_{\alpha_1p_1})\frac{1}{E^3_\pi(p_1'-p_1)}\\
\cdot\ & [(b_{\pi k}e^{ikx-iE_\pi(k)t}+b^\dagger_{\pi k}e^{-ikx+iE_\pi(k)t})\\
\cdot\ & (b_{\pi'k'}e^{ik'x-iE_{\pi'}(k')t}+b^\dagger_{\pi'k'}e^{-ik'x+iE_{\pi'}(k')t})\cdot(u^\dagger_{\alpha'p'}\underline{\hat{u}}(k-k')\hat{u}(p)^2u_{\alpha p})\\
+\ & (u^\dagger_{\alpha'p'}\hat{u}(p')^2\underline{\hat{u}}(0)u_{\alpha p})(b_{\pi k}e^{ikx'-iE_\pi(k)t}+b^\dagger_{\pi k}e^{-ikx'+iE_\pi(k)t})\\
\cdot\ & (b_{\pi'k'}e^{ik'x'-iE_{\pi'}(k')t}+b^\dagger_{\pi'k'}e^{-ik'x'+iE_{\pi'}(k')t})\\
+\ & 2(b_{\pi k}e^{ikx-iE_\pi(k)t}+b^\dagger_{\pi k}e^{-ikx+iE_\pi(k)t})\\
\cdot\ & (b_{\pi'k'}e^{ik'x'-iE_{\pi'}(k')t}+b^\dagger_{\pi'k'}e^{-ik'x'-iE_{\pi'}(k')t})(u^\dagger_{\alpha'p'}\hat{u}(p')\underline{\hat{u}}(k)\hat{u}(p)u_{\alpha p})]\ ,
\end{aligned} \tag{F.15}$$

and analogously for $\hat{I}^R_{2\pi}$. The expectation value of $\hat{I}^L_{2\pi}$ is then

$$\begin{aligned}
&I^L_{2\pi}(xx')=\langle\hat{I}^L_{2\pi}(xx')\rangle\\
=\ & \frac{\pi}{8}\sum_{\pi k}\sum_{\alpha p,\alpha'p'}\frac{M^*_\alpha M^*_{\alpha'}}{2E_\pi(k)E^*_\alpha(p)E^*_{\alpha'}(p')}\frac{1}{E^3_\pi[(p'-p)}(u^\dagger_{\alpha p}\underline{\hat{u}}(p'-p)u_{\alpha'p'})\\
\cdot\ & \delta(E^*_\alpha(p)+E_\pi(k)-E^*_{\alpha'}(p'))f_{\alpha'}(p')(1-f_\alpha(p))e^{ip'(x'-x)}\\
\cdot\ & \Big([f_\pi(k)+(1+f_\pi(k))][u^\dagger_{\alpha'p'}(\hat{u}(p')^2\underline{\hat{u}}(0)+\underline{\hat{u}}(0)\hat{u}(p)^2)u_{\alpha p}]\\
&+2[f_\pi(k)e^{-ik(x-x')}+(1+f_\pi(k))e^{ik(x-x')}](u^\dagger_{\alpha'p'}\hat{u}(p')\underline{\hat{u}}(k)\hat{u}(p)u_{\alpha p})\Big)\ .
\end{aligned} \tag{F.16}$$

The baryon component of the Wigner transformation of $I^L_{2\pi}(xx')$ is therefore

$$I^L_{2\pi}(xp) = \frac{1}{(2\pi)^3}\int \mathrm{Tr}_{(b)} I^L_{2\pi}(xx') e^{-i\boldsymbol{p}\cdot\boldsymbol{r}}\, d\boldsymbol{r} \tag{F.17}$$
$$\begin{aligned}
= & \frac{\pi}{8}\sum_{\pi k}\sum_{\alpha' p' p''}\frac{M_b^* M_{\alpha'}^*}{2E_\pi(k)E_b^*(p'')E_{\alpha'}^*(p')}\frac{1}{E_\pi^3(p''-p')}\langle u_{\alpha p''}|\underline{\hat{u}}(p''-p)|u_{\alpha' p'}\rangle \\
& ([f_\pi(k)+(1+f_\pi(k))]f_{\alpha'}(p')(1-f_b(p'')) \\
& \cdot\delta(E_b^*(p'')+E_\pi(k)-E_{\alpha'}^*(p'))\delta(\boldsymbol{p}'-\boldsymbol{p})\cdot\langle u_{\alpha' p'}|\hat{u}(p')^2\underline{\hat{u}}(0)+\underline{\hat{u}}(0)\hat{u}(p)^2|u_{\alpha p}\rangle \\
& +2[f_\pi(k)f_{\alpha'}(p')(1-f_b(p''))\delta(\boldsymbol{p}'+\boldsymbol{k}-\boldsymbol{p})\delta(E_b^*(p'')-E_\pi(k)-E_{\alpha'}^*(p')) \\
& +(1+f_\pi(k))f_{\alpha'}(p')(1-f_b(p''))\delta(\boldsymbol{p}'-\boldsymbol{k}-\boldsymbol{p})\delta(E_b^*(p'')+E_\pi(k)-E_{\alpha'}^*(p'))] \\
& \cdot\langle u_{\alpha' p'}|\hat{u}(p')\underline{\hat{u}}(k)\hat{u}(p)|u_{\alpha p}\rangle)\ .
\end{aligned}$$

This expression for the collision terms can be simplified, as we shall show. As discussed previously in the calculation of $I^L_{0\pi}(xp)$, the terms containing $\langle u_{\alpha' p'}|\underline{\hat{u}}(0)|u_{\alpha p}\rangle$ vanish. Moreover, $\underline{\hat{u}}(k)$ consists of terms linear in k and terms independent of k. For the first kind, $\underline{\hat{u}}(k=0)=0$. For the second kind, $\underline{\hat{u}}(k)=\underline{\hat{u}}(1)$. We also have $\langle u_{Np'}|\hat{u}(p')^2\underline{\hat{u}}(1)+\underline{\hat{u}}(1)\hat{u}(p)^2|u_{\Delta p}\rangle = 0$. Therefore only the "diagonal" terms survive, $\langle u_{\alpha p'}|\hat{u}(p')^2\underline{\hat{u}}(1)+\underline{\hat{u}}(1)\hat{u}(p)^2|u_{\alpha p}\rangle \neq 0$, with α=N or Δ. With the on-shell approximation, we also have $p''=p$, and the constraint becomes $\delta(E_b^*(p)+E_\pi(k)-E_{\alpha'}^*(p'))\delta(\boldsymbol{p}'-\boldsymbol{p})$. Therefore the terms containing $\underline{\hat{u}}(0)$ vanish. With the above conditions and approximations, the collision terms for baryons due to pion-baryon interactions can be simplified and, furthermore, they can be separated into gain terms and loss terms,

$$\begin{aligned}
& I^{\mathrm{gain}}_{B\pi}(xp) \\
= & \frac{\pi}{8}\sum_{\pi k}\sum_{\alpha' p' m_s^b}\frac{M_b^* M_{\alpha'}^*}{E_b^*(p)E_{\alpha'}^*(p')}\frac{\langle u_{\alpha' p'}|\hat{u}(p')\underline{\hat{u}}(k)\hat{u}(p)|u_{\alpha p}\rangle\cdot\langle u_{\alpha p}|\underline{\hat{u}}(k)|u_{\alpha' p'}\rangle}{E_\pi^4(k)} \\
\cdot & [f_\pi(k)f_{\alpha'}(p')(1-f_b(p))\delta(E_b^*(p)-E_\pi(k)-E_{\alpha'}^*(p'))\delta(\boldsymbol{p}'+\boldsymbol{k}-\boldsymbol{p}) \\
+ & (1+f_\pi(k))f_{\alpha'}(p')(1-f_b(p))\delta(E_b^*(p)+E_\pi(k)-E_{\alpha'}^*(p'))\delta(\boldsymbol{p}'-\boldsymbol{k}-\boldsymbol{p})]
\end{aligned} \tag{F.18}$$

and

$$\begin{aligned}
& I^{\mathrm{loss}}_{B\pi}(xp) \\
= & \frac{\pi}{8}\sum_{\pi k}\sum_{\alpha' p' m_s^b}\frac{M_b^* M_{\alpha'}^*}{E_b^*(p)E_{\alpha'}^*(p')}\frac{\langle u_{\alpha' p'}|\hat{u}(p')\underline{\hat{u}}(k)\hat{u}(p)|u_{\alpha p}\rangle\cdot\langle u_{\alpha p}|\underline{\hat{u}}(k)|u_{\alpha' p'}\rangle}{E_\pi^4(k)} \\
\cdot & [f_\pi(k)(1-f_{\alpha'}(p'))f_b(p)\delta(E_b^*(p)+E_\pi(k)-E_{\alpha'}^*(p'))\delta(\boldsymbol{p}'-\boldsymbol{k}-\boldsymbol{p}) \\
+ & (1+f_\pi(k))(1-f_{\alpha'}(p'))f_b(p)\delta(E_b^*(p)-E_\pi(k)-E_{\alpha'}^*(p'))\delta(\boldsymbol{p}'+\boldsymbol{k}-\boldsymbol{p})]\ .
\end{aligned} \tag{F.19}$$

Finally, let us calculate $I^\pi_{b\pi}$. Its operator form is

$$\hat{I}^\pi_{b\pi}(xx') = \hat{I}^\pi_{\mathrm{gain}}(xx') - \hat{I}^\pi_{\mathrm{loss}}(xx')\ , \tag{F.20}$$

where

$$\hat{I}^\pi_{\mathrm{gain}}(xx') = \frac{1}{2\hat{E}_\pi(x)}\underline{\hat{u}}(x)\hat{\Gamma}(x,x',x)\ , \tag{F.21}$$

and

$$\hat{I}^{\pi}_{\rm loss}(xx') = \hat{\Gamma}(x', x, x')\hat{\underline{u}}(x')\frac{1}{2\hat{E}_\pi(x')} \ . \tag{F.22}$$

In more detail

$$\begin{aligned} \hat{I}^{\pi}_{\rm gain}(xx') &= -\frac{\pi}{16}\frac{1}{\hat{E}_\pi(x)}\hat{\underline{u}}(x)\delta(\hat{h}(x_-) + \hat{E}_\pi(x') - \hat{h}(x_+)) \\ &\cdot \ [\hat{U}^\pi(x_-) - \hat{U}^\pi(x_+)]^2\hat{\rho}(x_-x_+)\frac{1}{\hat{E}_\pi(x')^3}\cdot\hat{\underline{u}}(x')\hat{\rho}(xx') \ , \end{aligned} \tag{F.23}$$

where $x_\pm = x \pm \epsilon$ ($\epsilon \to 0$) means that $\hat{h}(x_-), \hat{h}(x_+), \hat{U}^\pi(x_-)$, and $\hat{U}^\pi(x_+)$ should operate on $\hat{\rho}(x_-x_+)$; after having operated, $x_\pm$ should then assume the value x. Since $\hat{U}^\pi(x)$ contains the pion field $\boldsymbol{\pi}(x)$, we encounter the difficulty of calculating the expectation value $\langle\boldsymbol{\pi}(x)\cdot\boldsymbol{\pi}(x)\rangle$ for the gain term and $\langle\boldsymbol{\pi}(x')\cdot\boldsymbol{\pi}(x')\rangle$ for the loss term. Since we know that

$$\begin{aligned} &\rho_\pi(xx') = \langle\boldsymbol{\pi}(x')\cdot\boldsymbol{\pi}(x)\rangle \\ \to\ &\int\frac{1}{E_\pi(k)}f_\pi(k)d^3k \\ \to\ &\sum_k\frac{1}{2E_\pi(k)}\langle(b_k(x') + b_k^\dagger(x'))(b_k(x) + b_k^\dagger(x))\rangle \\ \to\ &\sum_k\frac{1}{2E_\pi(k)}\langle b_k^\dagger(x')b_k(x) + b_k^\dagger(x)b_k(x') + b_k(x')b_k(x) + b_k^\dagger(x')b_k^\dagger(x)\rangle \\ \to\ &\sum_k\frac{1}{E_\pi(k)}f_\pi(k) \ , \end{aligned} \tag{F.24}$$

as $x' \to x$, there is an uncertainty for the order of operators $b_k(x)$ and $b_k^\dagger(x)$. Since the gain term for the pion due to pion-baryon interactions is related to the pion production process and the loss term is related to the pion reabsorption process, to eliminate the above uncertainty one should use

$$\begin{aligned} &\langle\boldsymbol{\pi}(x)\cdot\boldsymbol{\pi}(x)\rangle \\ \to\ &\sum_k\frac{1}{2E_\pi(k)}\langle(b_k(x)b_k^\dagger(x) + b_k(x)b_k^\dagger(x) + b_k(x)b_k(x) + b_k^\dagger(x)b_k^\dagger(x)\rangle \\ =\ &\sum_k\frac{1}{E_\pi(k)}(1 + f_\pi(k)) \ , \end{aligned} \tag{F.25}$$

and

$$\begin{aligned} &\langle\boldsymbol{\pi}(x')\cdot\boldsymbol{\pi}(x')\rangle \\ \to\ &\sum_k\frac{1}{2E_\pi(k)}\langle b_k^\dagger(x)b_k(x) + b_k^\dagger(x)b_k(x) + b_k(x)b_k(x) + b_k^\dagger(x)b_k^\dagger(x)\rangle \\ =\ &\sum_k\frac{1}{E_\pi(k)}f_\pi(k) \ . \end{aligned} \tag{F.26}$$

Apart from the above exceptions, the calculation of $I^{\pi}_{B\pi}$ is similar to that of $\hat{I}^{b}_{b\pi}$. The expectation value of the gain term $I^{\pi}_{\rm gain}$ is

$$
\begin{aligned}
& I^{\pi}_{\rm gain}(xx') = \langle \hat{I}^{\pi}_{\rm gain} \rangle \\
= \; & \frac{\pi}{16} \sum_{\pi k} \sum_{\alpha p \alpha' p'} \sum_{\alpha_1 p_1 \alpha_1' p_1'} \frac{1}{E_\pi(k)} \frac{M^*_\alpha M^*_{\alpha'}}{E^*_\alpha(p) E^*_{\alpha'}(p')} \\
\cdot \; & \int dq dy \frac{e^{iq(x-y)+i(p'-p)y}}{(2\pi)^3 E_\pi(q)} \delta(E_{\alpha'}(p') - E_\pi(p_1' - p_1) - E_\alpha(p)) \\
\cdot \; & (u^\dagger_{\alpha' p'} \hat{\underline{u}}(q) \hat{u}(p'+p)^2 u_{\alpha p}) \int dq' dy' \frac{e^{iq'(x'-y')+i(p_1'-p_1)y'}}{(2\pi)^3 E^3_\pi(q')} \\
\cdot \; & (u^\dagger_{\alpha_1' p_1'} \underline{\hat{u}}(q') u_{\alpha_1 p_1})(1 + f_\pi(k)) f_{\alpha'}(p')(1 - f_\alpha(p)) \delta_{\alpha'\alpha_1} \delta_{\alpha\alpha_1'} \delta_{p' p_1} \delta_{p p_1'} \\
= \; & \frac{\pi}{16} \sum_{\pi k} \sum_{\alpha p \alpha' p'} \frac{1}{E_\pi(k)} \frac{M^*_{\alpha'}}{E^*_{\alpha'}(p') E^*_\alpha(p)} \delta(E_{\alpha'}(p') - E_\pi(p - p') - E_\alpha(p)) \\
\cdot \; & e^{i(p'-p)(x-x')} \frac{1}{E^4_\pi(p-p')} (1 + f_\pi(k)) f_{\alpha'}(p')(1 - f_\alpha(p)) \\
\cdot \; & (u^\dagger_{\alpha' p'} \hat{\underline{u}}(p - p') \hat{u}(p + p')^2 u_{\alpha p}) \cdot (u^\dagger_{\alpha p} \hat{\underline{u}}(p - p') u_{\alpha' p'}).
\end{aligned}
\tag{F.27}
$$

The gain term in phase space is then

$$
\begin{aligned}
I^{\pi}_{\rm gain}(xk) \; &= \; \int {\rm Tr} I^{\pi}_{\rm gain}(xx') e^{-i\boldsymbol{p}\cdot\boldsymbol{r}} d\boldsymbol{r} \, E_\pi(k) \\
&= \; \frac{\pi}{16} \sum_{\alpha p \alpha' p'} \frac{M^*_\alpha M^*_{\alpha'}}{E^*_\alpha(p) E^*_{\alpha'}(p')} \\
& \cdot \; \frac{\langle u_{\alpha' p'} | \hat{\underline{u}}(k) \hat{u}(p + p')^2 | u_{\alpha p} \rangle \cdot \langle u_{\alpha p} | \hat{\underline{u}}(k) | u_{\alpha' p'} \rangle}{E^4_\pi(k)} \\
& \cdot \; \delta(E^*_{\alpha'}(p') - E_\pi(k) - E_\alpha(p)) \delta(\boldsymbol{p}' - \boldsymbol{p} - \boldsymbol{k}) \\
& \cdot \; (1 + f_\pi(k)) f_{\alpha'}(p')(1 - f_\alpha(p)) \, .
\end{aligned}
\tag{F.28}
$$

The loss term can be found analogously to be

$$
\begin{aligned}
& I^{\pi}_{\rm loss}(xk) \\
= \; & \int {\rm Tr} I^{\pi}_{\rm loss}(xx') e^{-i\boldsymbol{p}\cdot\boldsymbol{r}} d\boldsymbol{r} \, E_\pi(k) \\
= \; & \frac{\pi}{16} \sum_{\alpha p \alpha' p'} \frac{M^*_\alpha M^*_{\alpha'}}{E^*_\alpha(p) E^*_{\alpha'}(p')} \\
\cdot \; & \frac{\langle u_{\alpha' p'} | \hat{\underline{u}}(k) \hat{u}(p + p')^2 | u_{\alpha p} \rangle \cdot \langle u_{\alpha p} | \hat{\underline{u}}(k) | u_{\alpha' p'} \rangle}{E^4_\pi(k)} \\
\cdot \; & \delta(E^*_{\alpha'}(p') - E_\pi(k) - E_\alpha(p)) \delta(\boldsymbol{p}' - \boldsymbol{p} - \boldsymbol{k}) \\
\cdot \; & f_\pi(k) f_\alpha(p)(1 - f_{\alpha'}(p')) \, .
\end{aligned}
\tag{F.29}
$$

In the continuous limit, changing the summation over momentum into integrations, we obtain the expressions for $I^{\pi}_{\rm gain}$ and $I^{\pi}_{\rm loss}$ as in eqs. (194) and (195).

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Table 1: Irreducible representation spaces

Type of field	Λ space	T space
Nucleon N	1/2 spinor	1/2 spinor
Delta Δ_μ	3/2 spinor	3/2 spinor
pion $\boldsymbol{\pi}$	$\hat{0}$ pseudoscalar	1 vector
sigma σ	0 scalar	0 scalar
omega ω_μ	1 vector	0 scalar

Table 2: Potential v_{ij} and the interaction vertex $\hat{\Gamma}$ for baryons

For Nucleon	$\hat{v}(1,2) = \hat{v}_{NN,NN} + \hat{v}_{N\Delta,\Delta N}$
$\hat{v}_{NN,NN}$:	$v_{ij} = g^2_{\pi NN} G^{\pi}(x_1 - x_2) - g^2_{\sigma NN} G^{\sigma}(x_1 - x_2) + g^2_{\omega NN} G^{\omega}_{\nu\mu}(x_1 - x_2)$
	$\hat{\Gamma}_i(1) = \gamma_0(1)\gamma_5(1)\hat{\tau}(1)\gamma_0(1)\alpha^{\mu}(1)$
	$\hat{\Gamma}_j(2) = \gamma_0(2)\gamma_5(2)\hat{\tau}(2)\gamma_0(2)\alpha^{\nu}(2)$
$\hat{v}_{N\Delta,\Delta N}$:	$v_{ij} = g_{\pi NN} g_{\pi\Delta\Delta} G^{\pi}(x_1 - x_2)$
	$\hat{\Gamma}_i(1) = \gamma_0(1)\gamma_5(1)\hat{\tau}(1)$
	$\hat{\Gamma}_j(2) = \gamma_0(2)\gamma_5(2)\hat{T}(2)$
For Delta	$\hat{v}(1,2) = \hat{v}_{\Delta\Delta,\Delta\Delta} + \hat{v}_{\Delta N,N\Delta}$
$\hat{v}_{\Delta\Delta,\Delta\Delta}$:	$v_{ij} = g^2_{\pi\Delta\Delta} G^{\pi}(x_1 - x_2) - g^2_{\sigma\Delta\Delta} G^{\sigma}(x_1 - x_2) + g^2_{\omega\Delta\Delta} G^{\omega}_{\mu\nu}(x_1 - x_2)$
	$\hat{\Gamma}_i(1) = \gamma_0(1)\gamma_5(1)\hat{T}(1)\gamma_0(1)\alpha^{\mu}(1)$
	$\hat{\Gamma}_j(2) = \gamma_0(2)\gamma_5(2)\hat{T}(2)\gamma_0(2)\alpha^{\mu}(2)$
$\hat{v}_{\Delta N,N\Delta}$:	$v_{ij} = g_{\pi\Delta\Delta} g_{\pi NN} G^{\pi}(x_1 - x_2)$
	$\hat{\Gamma}_i(1) = \hat{T}(1)$
	$\hat{\Gamma}_j(2) = \hat{T}(2)$

Table 3: Comparison with Siemens *et al.*

This work	Siemens *et al.*[19]	
$\rho(x\,x')$	$G_{ab}(x\,x')$	a, b, c=fermions
$\rho_{\pi}(x\,x')$	$G_{\alpha\beta}(x\,x')$	α, β, γ=bosons
$C_2(x_1x_2; x_1'x_2')$	$U_{ab,a'b'}(x_1x_2; x_1'x_2')$	
0	$U_{\alpha\beta,ab}(x_1x_1; x_1'x_2')$	
0	$U_{\alpha\beta,\alpha'\beta'}(x_1x_2; x_1'x_2')$	
$\Gamma(x, y, x')$	$\Gamma_{\alpha ab}(x, y, x')$	
0	$\Gamma_{\alpha\beta\gamma}(x_1, x_2, x_3)$	
0	$\phi(x) = \langle \pi(x) \rangle$	

The collision term $I^b_{b\pi}$

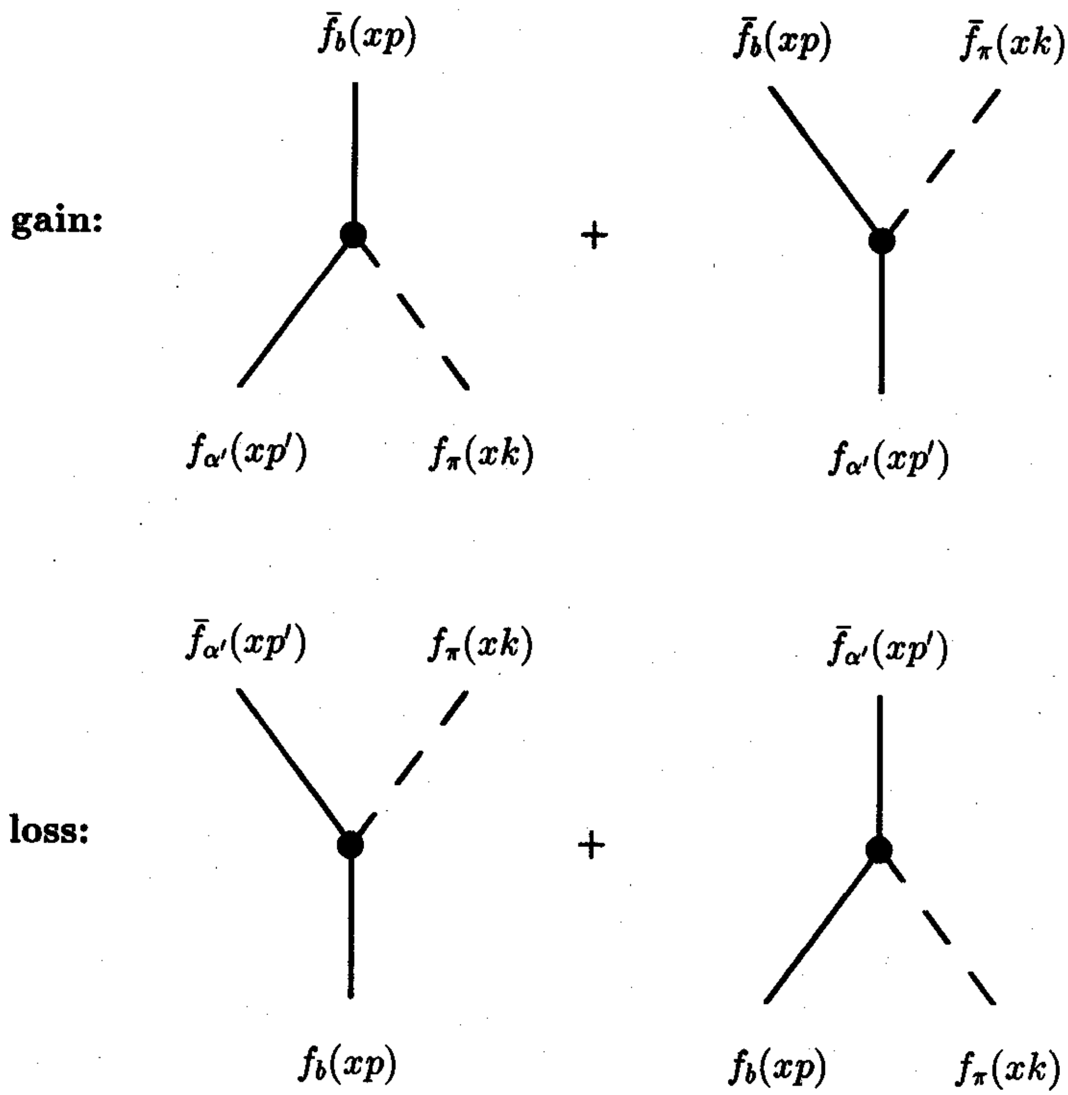

Figure 1: Diagrammatic representation of the gain and loss terms responsible for changing the baryon phase-space distribution $f_b(x,p)$ as a result of baryon-pion collisions. The terms on the left pertain to Δ resonances, $b = \Delta$, while those on the right are for nucleons, $b = N$.

The collision term $I^{\pi}_{b\pi}$

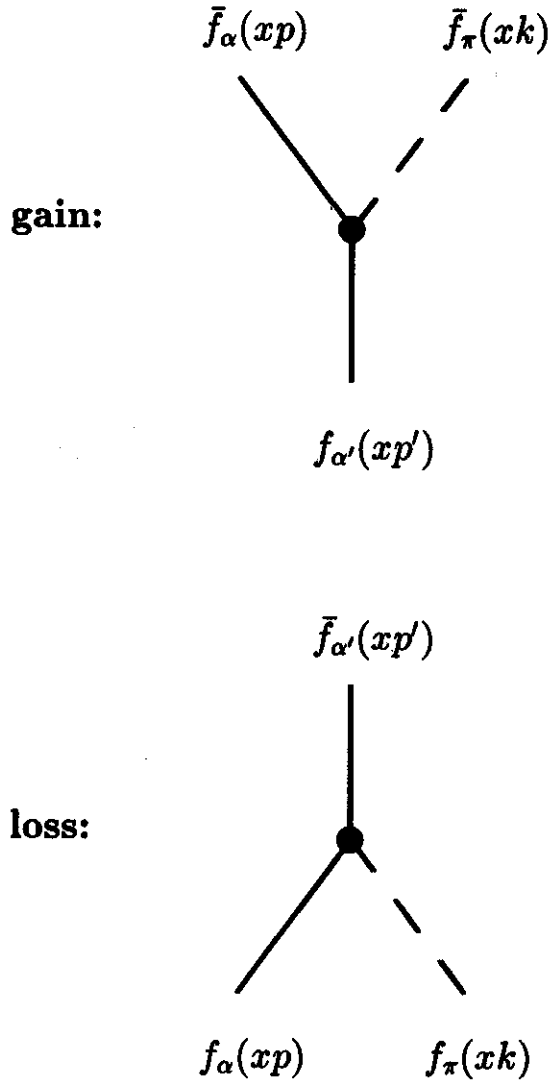

Figure 2: Diagrammatic representation of the gain and loss terms responsible for changing the pion phase-space distribution $f_{\pi}(x, k)$ as a result of baryon-pion collisions.