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REACTION PLANE DETERMINATION USING AZIMUTHAL CORRELATIONS

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We present a new technique for the determination of the reaction plane in heavy-ion reactions at incident energies ranging from the Fermi energy up to $\approx$ 100 MeV/nucleon. Our technique exploits the azimuthal correlation between light particles and the reaction plane. For illustrative purposes, azimuthal distributions of particles with respect to the reaction plane are extracted from data taken by the MSU 4π Array for 35 MeV /nucleon Ar+V collisions. The influences of momentum conservation and detection biases on the reaction plane determination are discussed, and the accuracy of the new technique is evaluated.

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I. INTRODUCTION

The study of collective motion in heavy-ion collisions can potentially constrain the equation of state of nuclear matter,[1,2,3] one of the major goals of nuclear physics. The presence of collective motion can be inferred from the distribution of reaction products with respect to the entrance channel reaction plane using observables such as the in-plane transverse momentum and azimuthal distributions. The first such studies were performed at beam energies of a few hundred MeV/nucleon, and the observed transverse collective motion, or "flow", was interpreted as a hydrodynamical side-splash due to the compression of nuclear matter.[4,5]

Researchers using the techniques[6] developed to study flow in relativistic collisions have recently observed signatures of a similar transverse collective motion in collisions at beam energies as low as 35 MeV/nucleon.[7,8] In our initial studies,[7] we found transverse collective motion in the reaction plane to be somewhat weak in 35 MeV/nucleon Ar+V collisions when compared with flow observed at relativistic beam energies.[5] Since the reaction plane determination technique we used relied on the presence of flow, this led to a correspondingly large dispersion between the found and true reaction planes. However, results from two-particle correlation studies[9] have indicated that particle emission is strongly enhanced in the reaction plane for some systems, perhaps due to rotational collective motion. In order to improve the accuracy of the reaction plane determination for such systems, we have developed a new technique for finding the reaction plane that exploits the existence of this in-plane enhancement. Studies of collective motion using the new technique have been published previously.[3,10,11] In this paper we present the details of this new technique and compare it to previously established techniques. We show that our technique locates the reaction plane more accurately than the global transverse momentum analysis[6] for 35 MeV/nucleon Ar+V collisions. The influence of momentum conservation and the effects of finite detector granularity on the reaction plane calculations are evaluated. We also assess the accuracy of the reaction planes determined using the new technique.

The word "flow" is used in this paper to describe the orientation of the momentum distribution in the reaction plane. It should not be taken as an assumption of a hydrodynamical side-splash, which has been invoked to explain flow in relativistic heavy-ion reactions.[4] The transverse collective motion found in collisions at beam energies of a few tens of MeV/nucleon is thought to be due to attractive mean-field deflection.[12,13,14]

The data we present in this paper was taken with the MSU 4π Array.[15] A beam of 35 MeV/nucleon ^{40}Ar produced by the MSU K500 cyclotron was focused on a ^{51}V foil. At the time, the array consisted of 215 charged particle detectors covering a solid angle equal to 85% of 4π sr. The data consists predominantly of hydrogen and helium ions, with a small contribution from $Z \geq 3$ only at the most forward angles. Full details of the charge and energy acceptance of the array are contained in ref. 7.

The reaction plane is defined geometrically by the momentum vector of the projectile and the center of the target. The impact vector $\vec{b}$, which joins the centers of the projectile and target at their closest approach, also lies within this plane. These geometrical relationships are shown schematically in fig. 1 for the center of mass (c.m.) frame of reference. When collective flow is present, it is useful to distinguish between the two sides of the reaction plane (as divided by the beam axis) which contain the forward going and backward going sections of the particle flow. These two regions of the reaction plane are shown in fig. 2; they will be referred to as the forward flow side and the backward flow side respectively. As a convention, azimuthal angles of particles with respect to the reaction plane will always be measured from the forward flow slide. It is important to note that attractive and repulsive flow cannot be distinguished from one another using the techniques described in this paper since they both led to similar final distributions of light particles. (The sketches in fig. 2 should not be taken as a detailed description of the momentum distributions produced in these collisions, but as a simplification of the collective motion in the mid-rapidity region.)

II. PREVIOUSLY ESTABLISHED TECHNIQUES

At beam energies just above the Coulomb barrier, the reaction plane can be inferred from the azimuthal angles of fission fragments emitted from the rotating compound system.[12,16] This technique is limited to heavy systems for which fission is a likely exit channel. Since the cross section for fission fragment production in violent collisions decreases dramatically for beam energies above $\approx$ 40 MeV/nucleon,[17] this technique is also limited to a narrow beam energy regime. On the other hand, light fragments are produced in violent collisions for all heavy-ion systems at beam energies above the Coulomb barrier, and thus can potentially be used to determine the reaction plane for a broader range of systems.

Two techniques are commonly used to determine the reaction plane from the observed distribution of light fragments produced in heavy-ion collisions: the sphericity tensor method[18] and transverse momentum analysis.[6] Both techniques exploit the presence of transverse momentum flow in the reaction plane on an event by event basis. These methods have been applied successfully in the analysis of collisions at beam energies ranging from around 100 MeV/nucleon to more than 1 GeV/nucleon.[5,19] We will describe each method in turn, and then discuss the difficulties encountered in applying them to collisions at beam energies below 100 MeV/nucleon.

In the sphericity tensor method, the shape of the event in momentum space is determined by diagonalizing the the flow tensor F_{ij}:

$$F_{ij} = \sum_{\nu=1}^{N} w_\nu p_i(\nu) p_j(\nu), \tag{1}$$

where N is the number of particles in an event, $p_i(\nu)$ are the cartesian components of the momentum of particle ν, and w_ν is a weighting factor associated with that particle, typically $\frac{1}{2m_\nu}$. The reaction plane is taken as the plane defined by the beam axis and the major axis of the resulting flow ellipsoid.

In transverse momentum analysis, a vector $\vec{Q}$ is constructed from the transverse momenta of particles in an event:

$$\vec{Q} = \sum_{\nu=1}^{N} w_\nu \vec{p}_\nu^{\perp}, \tag{2}$$

where the weight w_ν is chosen to be positive for particles emitted in the forward c.m. hemisphere and negative for particles emitted in the backward hemisphere. The reaction plane is defined by the beam axis and $\vec{Q}$. The absolute value of w_ν is chosen to provide the maximum sensitivity to the reaction plane. In practice this is usually achieved by randomly dividing events into two sub-events, calculating $\vec{Q}$ for each sub-event, and selecting w_ν to minimize the difference between the resulting reaction planes. Typical values for $|w_\nu|$ are 1.0[5,6] and m_ν,[7] the mass of particle ν.

Both the sphericity and the transverse momentum techniques depend on the existence of collective flow in the reaction plane. The transverse momentum and sphericity tensor approaches have recently been shown to be equally sensitive to the reaction plane.[20] Transverse momentum analysis of 35 MeV/nucleon Ar+V data taken with the MSU 4π Array, however, has shown comparatively weak flow for this system[7], making the application of these techniques problematic at these beam energies. However, during our preliminary analysis using the transverse momentum technique, we found a substantial enhancement of particle emission in the reaction plane[10] that can be exploited to more accurately determine this plane.

III. PRELIMINARY ANALYSIS

The weighting used in eq. 2 for the preliminary analysis with the transverse momentum technique was $| w_\nu | = m_\nu$. Only events with at least three particles were analyzed in order to enhance the accuracy of the reaction plane determination. The azimuthal distribution was smoothed along the ϕ axis to remove structure due to the azimuthal granularity of the 4π Array.

We display in fig. 3 a contour map of $\frac{dN^2}{dy\,d\phi}$ as a function of ϕ and rapidity. N is the number of particles detected, ϕ is the azimuthal angle around the beam axis measured with respect to the forward flow side of the reaction plane, and y is the parallel rapidity of the detected particle in the lab frame of reference. Enhanced emission in the reaction plane is indicated by the simultaneous peaks on both the forward flow side ($\phi = 0°, 360°$) and the backward flow side ($\phi = 180°$) of the reaction plane at and above the c.m. rapidity. A similar in-plane enhancement was observed by Tsang et al.[21] in heavy-ion reactions with U targets in which the reaction plane was determined using the azimuthal angles of fission fragments.

This pattern of enhancement is different from that expected from strong flow, i.e.

a peak on the forward flow side at $y > y_{c.m.}$ and a peak on the backwards flow side for $y < y_{c.m.}$.[5] Since flow is weak, the sphericity tensor method would lead to small angles between the major axis of the flow ellipsoid and the beam axis, yielding a correspondingly imprecise reaction plane determination. Transverse momentum analysis would also have difficulty with this emission pattern since the vector sum of the transverse momenta would tend towards zero for the forward and backward rapidity regions in the center of mass. The observed weakness of flow and strength of in-plane emission for this system led us to develop a new technique for determining the reaction plane that could exploit the in-plane enhancement.

Note that the transverse momentum analysis was able to locate the plane well enough to show the in-plane enhancement observed in fig. 3 because flow is present in the data, even though this method of plotting the data does not make it apparent.[7] In addition, the in-plane enhancement causes the observed sample of the transverse momentum distribution contains stronger fluctuations parallel to the reaction plane than perpendicular to the plane. The sensitivity of the vector $\vec{Q}$ to these fluctuations helps to align the vector with the reaction plane.

IV. THE AZIMUTHAL CORRELATION METHOD

In essence, the new technique consists of finding the plane that aligns best with the enhancement plane. We proceed by first projecting the event on the p^x–p^y plane, taking the p^z axis to coincide with the beam axis. The projection of the reaction plane is taken as a line in the p^x–p^y plane passing through the origin (beam axis) with slope a. The deviation of the particles in the event from the reaction plane, D^2, is parameterized by the sum of the perpendicular squared distances d_ν^2 between the line and the particles in the p^x–p^y plane as shown in fig. 4. The slope corresponds to the tangent of the azimuthal angle of the reaction plane measured from the p^x axis, labeled ϕ_{rp} in the figure. The deviation D^2 as a function of the slope of the reaction plane's projection onto the p^x–p^y plane is:

$$D^2 = \sum_{\nu=1}^{N}[d_\nu^2] = \sum_{\nu=1}^{N}[(p_\nu^x)^2 + (p_\nu^y)^2 - \frac{(p_\nu^x + p_\nu^y a)^2}{1+a^2}]. \tag{3}$$

The sums in the equation are taken over the particles in the event. The condition that the derivative of D^2 with respect to a vanish produces a quadratic equation whose roots,

$$a = \frac{Y2 - X2 \pm \sqrt{(X2-Y2)^2 + 4(XY)^2}}{2 \times XY} \tag{4}$$

$$X2 = \sum_{\nu=1}^{N}(p_\nu^x)^2$$

$$Y2 = \sum_{\nu=1}^{N}(p_\nu^y)^2$$

$$XY = \sum_{\nu=1}^{N}(p_\nu^x p_\nu^y)$$

can be used to determine a. Substituting these roots into the original equation allows one to pick the root that minimizes D^2 and hence maximizes the in-plane enhancement. Since the technique exploits the correlation between the azimuthal angle of the reaction plane and the azimuthal angles of the particles produced in the collision, it will be referred to as the "azimuthal correlation" method.

The transverse momentum and sphericity tensor techniques not only provide the azimuthal angle of the reaction plane but also determine the forward flow side as defined in fig. 2. The forward half of the momentum flow ellipsoid lies on the forward flow side of the reaction plane, and the vector $\vec{Q}$ points toward this side. The new angular correlation method, however, does not distinguish between the two sides of the reaction plane, and therefore this method must be supplemented by one of the two previous techniques in order to study transverse momentum flow. We employ the transverse momentum $\vec{Q}$ vector to determine which side of the rection plane —already found using azimuthal correlations— contains the forward flow component. Thus, the procedure for determining the reaction plane consists of two distinct operations: 1) finding the plane by azimuthal correlations and 2) choosing the forward flow side from $\vec{Q}$. Each operation is expected to have its own efficiency.

In fig. 4, the azimuthal angle that a particle of interest (POI) makes with the forward flow side of the reaction plane is labeled ϕ. If this angle is to be studied, then the POI must be left out of the sums in the previous equations for all three techniques. Otherwise, a spurious autocorrelation between the POI and the reaction plane will result. Leaving out the POI reduces the number of particles used in finding the reaction plane and hence slightly reduces the accuracy of the reaction plane determination. The strength of this effect can be gauged by measuring the difference between the azimuthal angle of the reaction plane calculated using the whole event and the plane calculated while leaving out the POI. The distribution of these differences is shown in fig. 5 for the Ar+V data. There is a sharp peak at 0° difference, indicating that the reduction in the accuracy due to leaving out the POI is small in this case. A very small peak also appears at 180°, corresponding to a slight probability for $\vec{Q}$ to switch sides of the reaction plane when the POI is removed. If the reaction plane determination is strongly influenced by the presence of one particle, such as a heavy projectile fragment,[8] then leaving out that particle may have a much stronger effect on the accuracy of the reaction plane determination. In general, the possible variations in the accuracy of the reaction plane determination must be taken into account when comparing the azimuthal distributions of different types of POIs.

We have evaluated different weighting schemes in the azimuthal correlation method by substituting $\omega_\nu k^i_\nu$ for p^i_ν in equation 4, where ω_ν is a weighting factor assigned to particle ν and k^i_ν is the ith component of a unit vector $\hat{k}_\nu$ pointing in the $\vec{p}^{\perp}_\nu$ direction. Transverse momentum weighting ($\omega_\nu = p^{\perp}_\nu$) recovers equation 4. Other possibilities explored were no weighting ($\omega_\nu = 1$), mass weighting ($\omega_\nu = m_\nu$), and center of mass polar angle weighting ($\omega_\nu = \sin\theta_{c.m.}$). The latter choice was motivated by the observation that the in-plane enhancement is stronger near $y = y_{c.m.}$. The results of the four weighting choices are shown in fig. 6 which displays the distribution of azimuthal emission angles of POIs relative to the reaction plane for the Ar+V data. The distribution was smoothed over ϕ to remove structure due to the azimuthal granularity of the detector array. Transverse momentum weighting exhibits the strongest peaking in the reaction plane ($\phi = 180°$) and the deepest valleys perpendicular to the reaction plane ($\phi = 90°$, $270°$) and must therefore provide the most accurate reaction plane determination for this system.

Momentum conservation in the emission of the POI creates a bias in the reaction plane determination. This is because the remaining particles in the event recoil away from the POI, creating an axis in the p^x–p^y plane which can be mistaken for the reaction plane. Detector energy thresholds can modify this effect. In the case of the 4π Array, most of the detected particles are going forward in the c.m. frame of reference, so the momentum conservation effect causes the forward flow side of the reaction plane to be preferentially found $\approx 180°$ away from the POI. Momentum conservation can therefore be thought of as an effective anti-autocorrelation. Even if particles are emitted in random directions in their source's frame of reference, an enhancement in the reaction plane will be observed in the azimuthal distributions because of momentum conservation.

This effect is illustrated in fig. 7 for a simulation in which nucleons were emitted from mass 10 source with a temperature of 10 MeV. In the first panel, the source emitted nucleons isotropically with no momentum conservation constraints, resulting in a flat azimuthal distribution. In the second panel, the emission was again isotropic, but with the constraint that the total momentum of the event (including the POI) be identical with the initial source momentum. As a result of this constraint, the azimuthal distribution shows enhanced emission in the reaction plane. This spurious enhancement is due to the correlation between the momentum of the POI and the momenta of the remaining particles, biasing the reaction plane determination toward the plane containing the POI.

Spurious effects in reaction plane calculations due to momentum conservation have been handled in a variety of ways.[19] For simplicity, we have attempted to cancel the effect of momentum conservation by boosting the transverse momenta of the particles. As each particle of interest was chosen, the observed transverse momenta of the remaining particles were boosted toward the POI before using them to find the reaction plane.[7] We assumed that the whole system, $m_{proj.} + m_{targ.}$, shared the recoil momentum. This should be a good approximation, since our multiplicity requirements for reaction plane determination restricts us to near-central collisions in which the emitting region should encompass nearly the whole of the projectile and target. This assumption gives a boost velocity

$$\vec{V}^{\perp}_{boost} = \frac{\vec{p}^{\perp}_{POI}}{m_{total} - m_{POI}}, \qquad (5)$$

where $\vec{p}^{\perp}_{POI}$ is the transverse component of the momentum of the POI, m_{total} is the total mass of the the system, and m_{POI} is the mass of the particle of interest. This was the most conservative approach since it resulted in the minimum correction. The velocities parallel to the beam axis were not modified since they play no role in the reaction plane determination. When this boost was applied to the data created for the middle panel of fig. 7, it restored the symmetry to the azimuthal distribution, as shown in the bottom panel. In one of our previous papers,[7] we also found that this boost approximately restored the expected mid-rapidity symmetry to the average in-plane transverse momentum distribution in both 35 MeV/nucleon Ar+V collisions and 50 MeV/nucleon C+C collisions.

V. CHECKS OF THE NEW TECHNIQUE

It is important to verify that the trends of the observed azimuthal distributions are not created by biases in the reaction plane determination or detector acceptance. This can be checked by analyzing simulated distributions that have been passed through a filter containing the properties of the detection system and comparing the results with unfiltered distributions. The efficiency of this approach, however, depends on the accuracy of the physics in the simulation of the data and the degree to which an exact replica of the detector acceptance can be created. An alternate approach is to generate events using observed particles taken from separate events. Since the created events contain little or no physics correlations, analysis of these events should produce flat azimuthal distributions if there are no biases due to detector acceptance or in the reaction plane determination. These events will be referred to as "randomized events" because they are created by randomly choosing particles from different observed events. This approach is particularly convincing since the created events precisely reflect the actual acceptance of the detector array.

Our procedure for generating randomized events was as follows. An observed event with multiplicity m was used as the starting point and one particle was chosen at random from each of the subsequent m events. These random particles were taken as a new event and analyzed in the same fashion as a real event. This process was repeated for each observed event, generating a multiplicity distribution for the randomized events that was identical to the observed multiplicity distribution. Only events that could have met the analysis criteria; i.e. multiplicity cuts, energy cuts, etc., were used in the generation of randomized events, guaranteeing that the randomized events contained all the biases that were present in the real events.

Results of the analysis of randomized events are presented in fig. 8 for the Ar+V data. The azimuthal distribution of events created following the procedure outlined above are displayed in the top panel. The isotropy of the distribution indicates that there are no biases in the technique. There is, however, a bias in the detection system, that can be seen if we examine our procedure for generating random events more closely. Using the procedure outlined above, when particles are chosen from different events, it is possible for the same detector to be multiply hit within a single created event. Since each detector can register only one particle per event, this possibility should be avoided by constraining the choice of particles. The azimuthal distribution of particles from randomized events without multiple hits is shown in the lower panel. The dip at 0° is due to this new constraint and reflects the finite azimuthal granularity of the detector array. This dip occurs because it is difficult to place a POI close to the found reaction plane since some of the detectors at azimuthal angles near the reaction plane must already have been hit. This "repulsion" from the reaction plane should be borne in mind when interpreting observed azimuthal distributions; for example, it gives rise to the dips at 0° in figs. 3 and 6. Since the effect is strongest for the most forward detectors, restricting consideration to the middle and lower rapidity regions limits the dip's impact on the analysis of collective motion.[10]

VI. COMPARISIONS WITH THE TRANSVERSE MOMENTUM TECHNIQUE

We have compared the efficiency of the new azimuthal correlation technique for finding the reaction plane with the standard transverse momentum analysis of Danielewicz and Odyniec. The distribution of differences between the azimuthal angles of reaction planes found using the two techniques on Ar+V data, shown in fig. 9, indicates strong agreement between the two methods. Since transverse momentum analysis utilizes the presence of flow to find the reaction plane, this implies that the flow and the general in-plane enhancement are coplanar. This correspondence may break down at beam energies above 100 Mev/nucleon where compressional effects can lead to enhanced emission *out* of the reaction plane[22].

Although the two techniques provide similar reaction planes, our tests show that the azimuthal correlation method is slightly more accurate for Ar+V at 35 MeV/nucleon. In the azimuthal distribution of the POIs from Ar+V data shown in fig. 10, the azimuthal corelation technique provides a somewhat stronger enhancement in the reaction plane than the transverse momentum method, implying a more accurate reaction plane determination.

VII. ACCURACY

The accuracy of the reaction plane determination can be estimated by studying the difference between reaction planes found for single events using different sets of particles. If the difference between two planes found for the same event is small, then the planes must have been well determined. This procedure has been previously employed to measure the accuracy of the transverse momentum technique in the following manner[6]. First, each event was divided into two sub-events by randomly picking particles from the original event. Then the difference between the reaction planes for the two sub-events was calculated for each original event, creating a distribution of differences which peaked at 0° with a width σ. This width was found to be related to the width of the distribution of found reaction planes around the true reaction planes, σ_o, by

$$\sigma_o = \frac{1}{2}\sigma. \tag{6}$$

The factor of $\frac{1}{2}$ arises from a factor of $\frac{1}{\sqrt{2}}$ due to doubling the multiplicity of particles used to find the reaction plane in going from sub-events to whole events, and a factor of $\frac{1}{\sqrt{2}}$ in going from differences between two found planes to the deviation of one found plane. In this way, the accuracy of the reaction plane determination, σ_o was deduced from the observable σ. We have investigated the accuracy of our new technique in a similar manner, comparing reaction planes for sub-events.

As a simplifying approximation, we chose to represent the differences between reaction planes of sub-events by a Gaussian distribution with width σ. Since our data show an enhancement in the reaction plane on both sides of the beam axis (fig. 3), we expect the distribution of differences between reaction planes of sub-events to peak at both 0° and 180°. These two directions in the reaction plane are nearly indistinguishable if transverse momentum flow is weak. Thus we represent the distribution of differences between sub-events by two equal width Gaussians of different heights centered at 0° and 180°. If $\phi_{rp}(1)$ and $\phi_{rp}(2)$ are the azimuthal angles of the reaction planes found for sub-events, then the distribution of differences between sub-events labeled 1 and 2 (D_{12}) is given by

$$D_{12}(\phi = |\phi_{rp}(1) - \phi_{rp}(2)|) \propto (R\, e^{-\frac{\phi^2}{2\sigma^2}} + e^{-\frac{(\phi-180^\circ)^2}{2\sigma^2}}), \tag{7}$$

where R is the ratio of the area under of the two peaks.

The widths σ are constrained to be equal because they are both reflect the accuracy of the first step in the azimuthal correlation technique, finding the reaction plane. The second step, finding the forward flow side of the reaction plane using $\vec{Q}$ (eq. 2), distinguishes between the 0° and 180° sides of the reaction plane. Thus, the relative areas of the two Gaussians reflect how well the orientation of the reaction plane was established, and how much flow was present. If flow is strong, then $R \gg 1$ and D_{12} peaks primarily at 0°, as is the case at higher beam energies. For our system, however, $R \approx 1$ since flow is weak.

From the observable σ we can estimate the difference between found and true reaction planes. We also took this distribution to be approximated by a Gaussian, so that the distribution of differences D_{ft} between the found and true planes is

$$D_{ft}(\phi = |\phi_{rp}(found) - \phi_{rp}(true)|) \propto (e^{-\frac{\phi^2}{2\sigma_o^2}} + e^{-\frac{(\phi-180^\circ)^2}{2\sigma_o^2}}), \tag{8}$$

where the width σ_o is the accuracy of the reaction plane determination. The accuracy σ_o can be related to the width σ observed from the differences between sub-events. Since the distribution of differences between planes of sub-events involves the accuracy σ_o twice, the width σ of the resulting Gaussian (7) is given by:

$$\sigma(2m) = \sqrt{[\sigma_o(m)]^2 + [\sigma_o(m)]^2} = \sqrt{2}\sigma_o(m), \tag{9}$$

where $\sigma(m)$ represents the width for multiplicity m. In order to measure the accuracy of reaction plane determination for events of multiplicity m, we must divide events of multiplicity $2 \times m$ in two, and measure the width of the distribution of differences between the two sub-events. This procedure will allow the multiplicity dependence of σ_o to be taken into account.

Note that eqs. 7 and 8 are only approximations of Gaussians centered at 0° and 180° since they are not periodic. The approximations are good for widths less than $\approx 70°$; if the widths are larger then the approximations can be improved by adding more Gaussians at $-180°$, 360°, etc.

In fig. 11 we display the distribution of differences between reaction planes found for sub-events using the Ar+V data. The curves in the figure are due to fits using eq. 7, and the resulting widths σ_o are shown in fig. 12 as a function of the multiplicity of the sub-events. Since the average multiplicity of the events which passed our selection criteria was ≈ 6, we conclude from fig. 12 that the uncertainty in our reaction plane determination is $\approx 50°$ for 35 MeV/nucleon Ar+V collision. This should be considered an upper limit on the width, however, since multiple hit exclusion effects artificially broaden the distribution of differences between the reaction planes of sub-events. Just as in our previous discussion of the effect of excluding multiple hits on the azimuthal distribution of POIs, it is difficult to place the reaction plane of the second sub-event near that of the first since some of the detectors in the first azimuthal plane must have already been hit. Furthermore, it should also be noted that the high multiplicity events used to generate the sub-events may result from collisions more central than the average multiplicity 6 event. This may account for the flatness of the σ_o curve at the higher Ar+V multiplicities.

We have also fit the distribution of differences between the azimuthal angles of reaction planes for sub-events from simulations generated with an azimuthal distribution

$$\frac{dN}{d\phi} \propto 1 + \cos^2 \phi, \tag{10}$$

similar to that observed in the data. The results of these fits are shown in fig. 12. Note that for the simulation, σ_o is not proportional to $\frac{1}{\sqrt{m}}$, so eq. 6, used to account for the multiplicity dependence in the transverse momentum technique, is not applicable to the angular correlation method.

The validity of the assumptions used in extracting the efficiency of the reaction plane determination can be tested for simulated data, since in that case the true reaction plane is known and the accuracy can be measured directly. In fig. 13, the upper panel displays the Gaussian fit to the difference between reaction planes of simulated multiplicity 5 sub-events. The solid line in the lower panel is a prediction for the dispersion of found reaction planes around the true reaction plane using eqs. 8 and 9. The data points show the actual dispersion for simulated multiplicity 5 events, which is in good agreement with the prediction.

VIII. CONCLUSIONS

We have developed a new technique for determining the reaction plane from the distribution of light charged particles created in heavy-ion collisions. It is applicable to systems in which there is a substantial enhancement of particle emission in the reaction plane, such as 35 MeV/nucleon Ar+V. This in-plane enhancement is exploited in the new technique by locating the reaction plane that best aligns with the enhancement plane. For the above system, even though the average multiplicity of charged particles detected by the MSU 4π Array is only ≈ 6, the reaction plane is sufficiently well determined to allow the various modes of collective motion to be described.[3,10] A method for minimizing the effects of momentum conservation on observables calculated relative to the reaction plane has also been presented. In addition, we have detailed our program of tests for biases produced by our detector array, which should prove useful to others working on reaction plane determination. In particular, we belive that the comparison with randomized events is crucial for reliable interpretation of data from such arrays.

There are now three techniques currently being used to determine the reaction plane for collisions at beam energies from the Fermi energy to $\approx$ 100 MeV/nucleon. For systems in which transverse collective motion is strong, such as 50 MeV C+C,[7] and systems in which squeeze-out is possible due to the dominance of repulsive compressional forces, such as La+La and Nb+Nb with $E_{beam} \geq 60$ MeV/nucleon,[23,24] the well known approach of Danielewicz and Odyniec is appropriate. When fission is a likely exit channel for the compound system formed in the collision, the reaction plane can be found from the azimuthal angles of the fission fragments.[21,16] The azimuthal correlation method presented here allows the reaction plane to be determined when fission is not a likely exit channel and transverse collective motion is weak, but the mean field interaction is primarily attractive. Thus the new technique extends the range of systems that may be investigated for the presence of collective motion.

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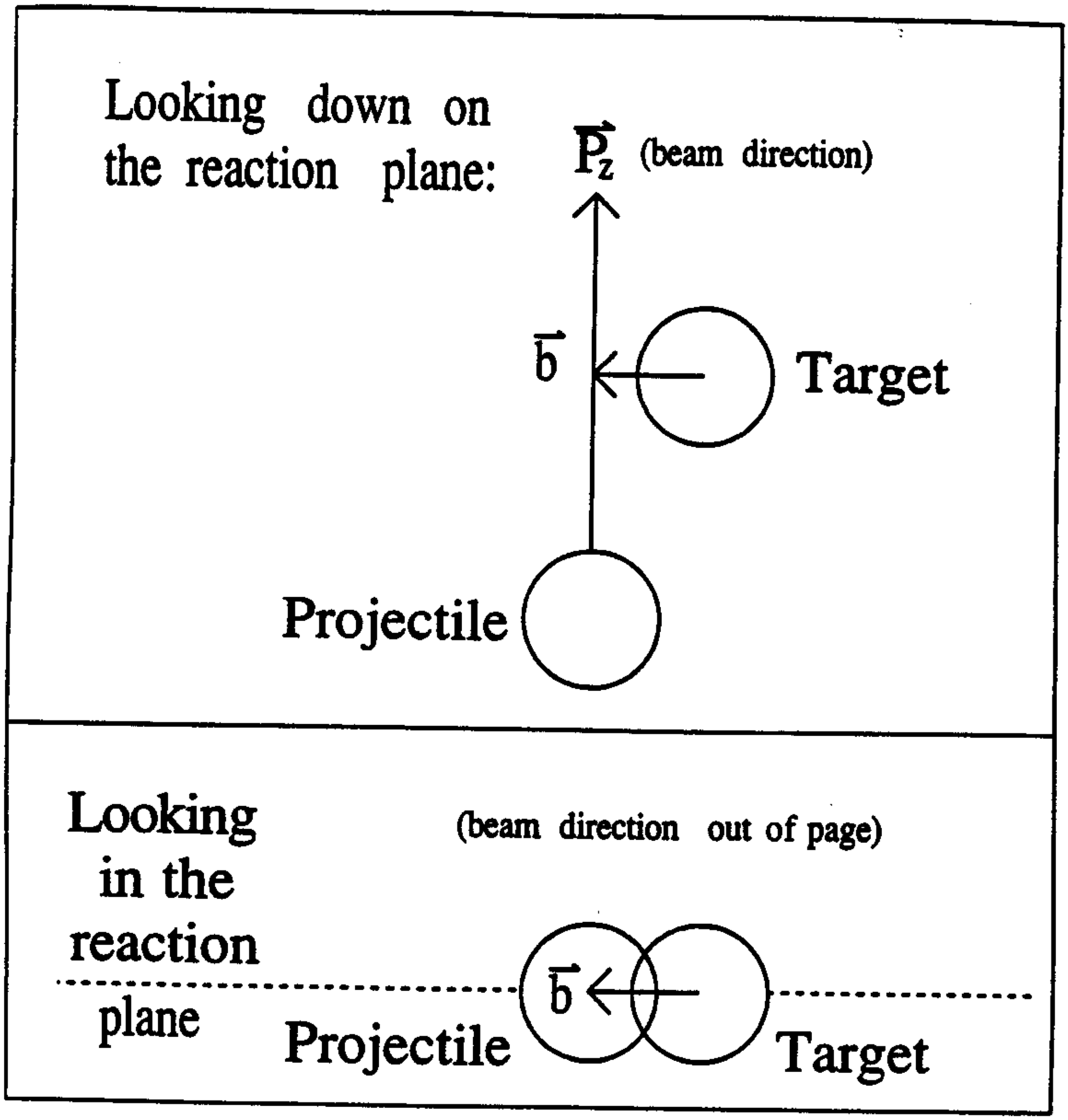

FIG. 1. A geometrical description of the reaction plane.

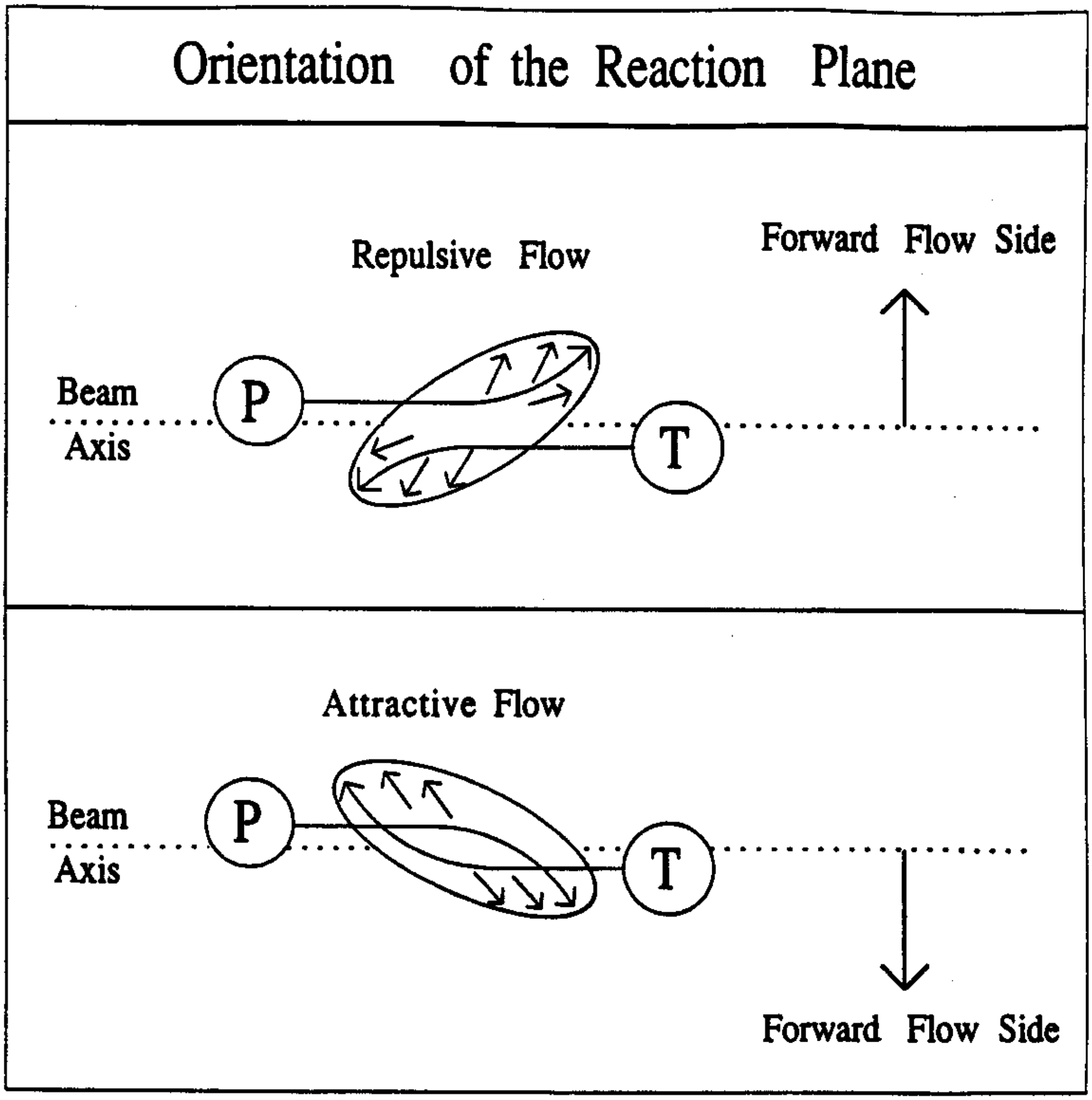

FIG. 2. The definition of the forward flow side of the reaction plane is shown for both attractive and repulsive scattering. The perspective of the figure is looking down on the reaction plane. In the top panel, the forward flow side is the area above the dotted line (beam axis), in the bottom panel it is the area below the dotted line.

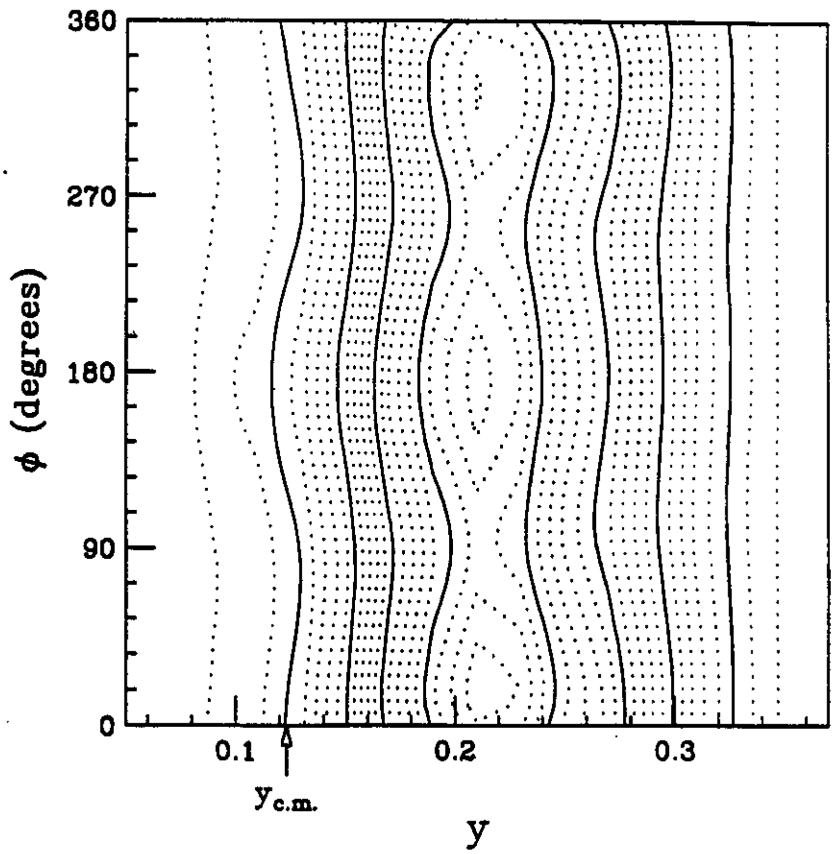

FIG. 3. The cross section is shown for charged particles from 35 MeV/nucleon Ar+V collisions. The azimuthal angle (around the beam axis) with respect to the forward flow side of the reaction plane is labeled ϕ, and y is the lab rapidity.

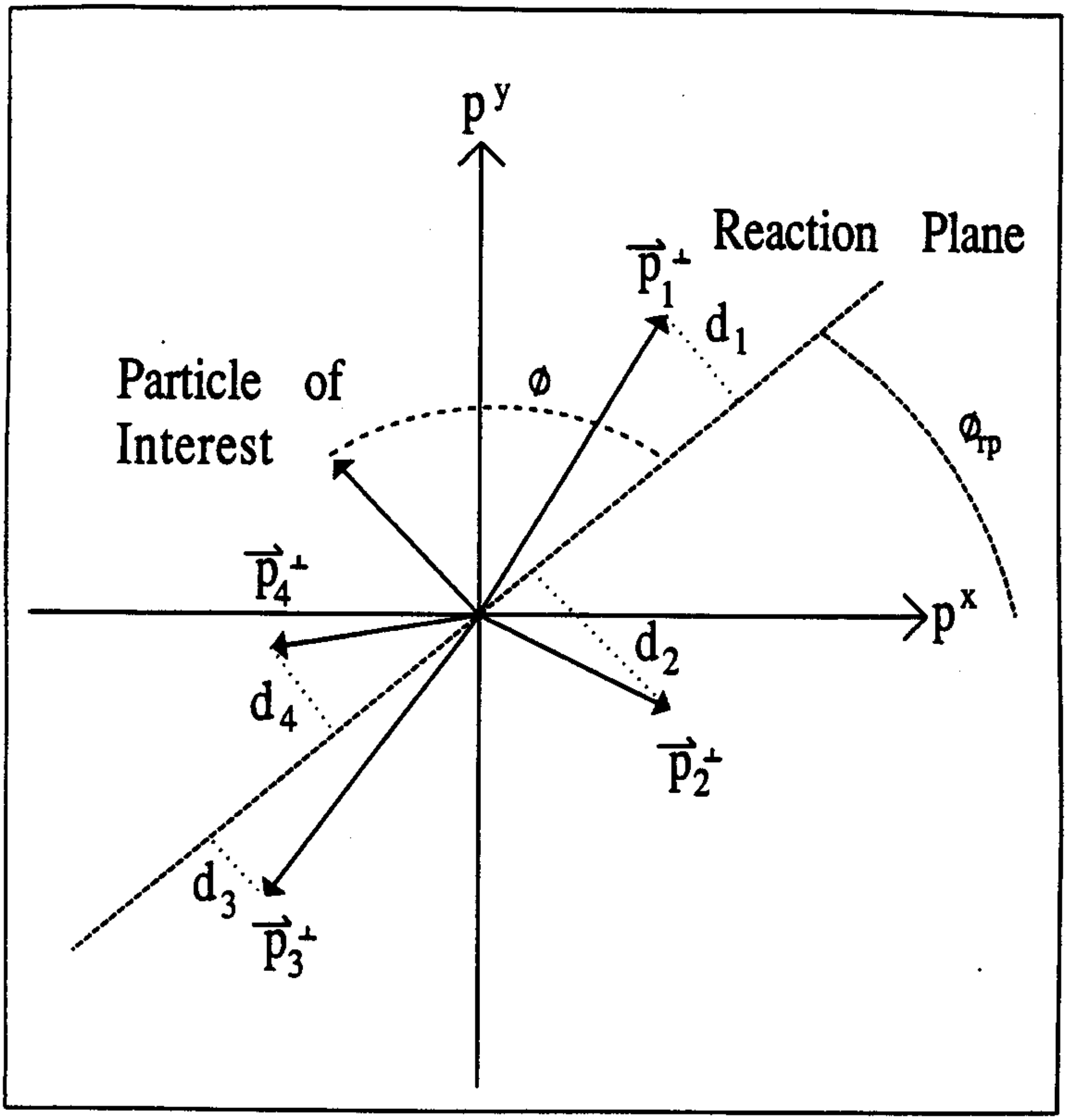

FIG. 4. The quantities used in finding the reaction plane for an event projected on the p^x–p^y plane. The angle of a particle of interest with respect to the forward flow side of the reaction plane is labeled ϕ. Note that an additional technique must be used to choose which side of the reaction plane is the forward flow side, as explained in the text.

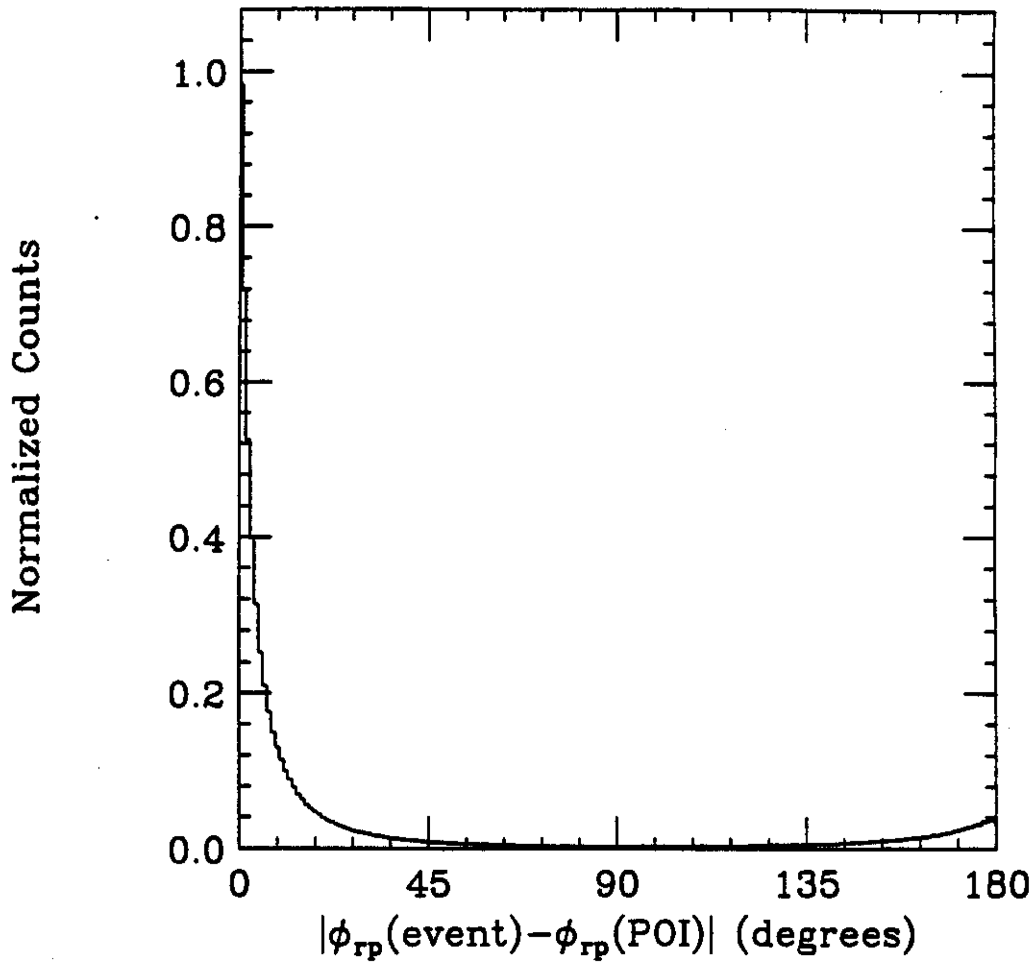

FIG. 5. The azimuthal distribution of differences between reaction planes found for the entire events ($\phi_{rp}(event)$) and reaction planes found leaving out a particle of interest ($\phi_{rp}(POI)$).

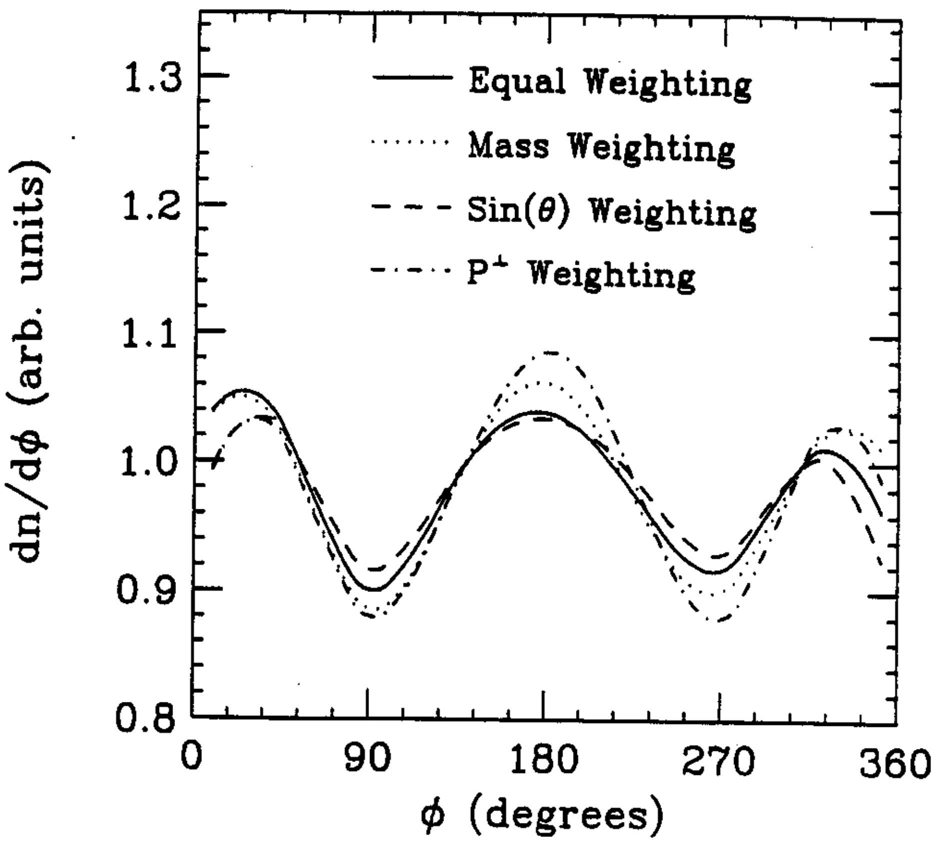

FIG. 6. Comparison of azimuthal distributions of particles of interest with respect to the reaction plane using different weightings in determining the plane. Transverse momentum weighting supplies the most accurate reaction plane determination.

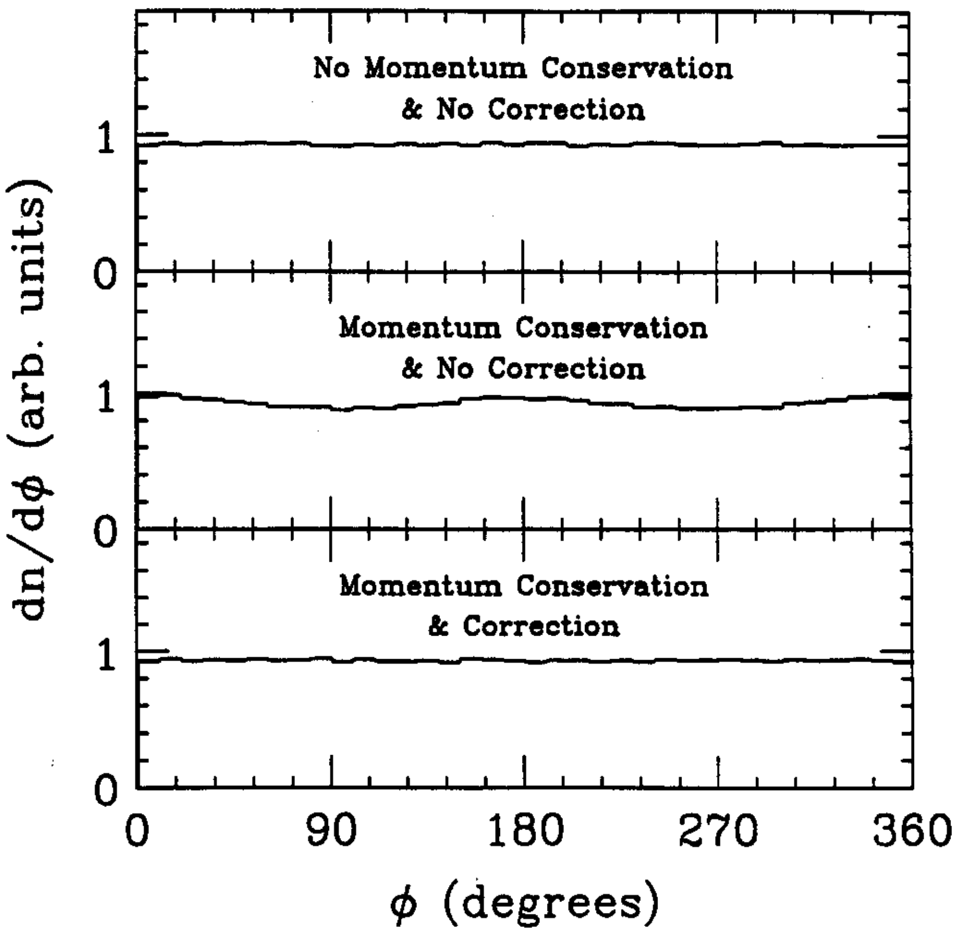

FIG. 7. The effects of momentum conservation are investigated using simulated events. In the top panel, momentum conservation is turned off and an isotropic distribution results. Turning on momentum conservation, middle panel, produces an in-plane enhancement, but the correction restores isotropy as shown in the bottom panel.

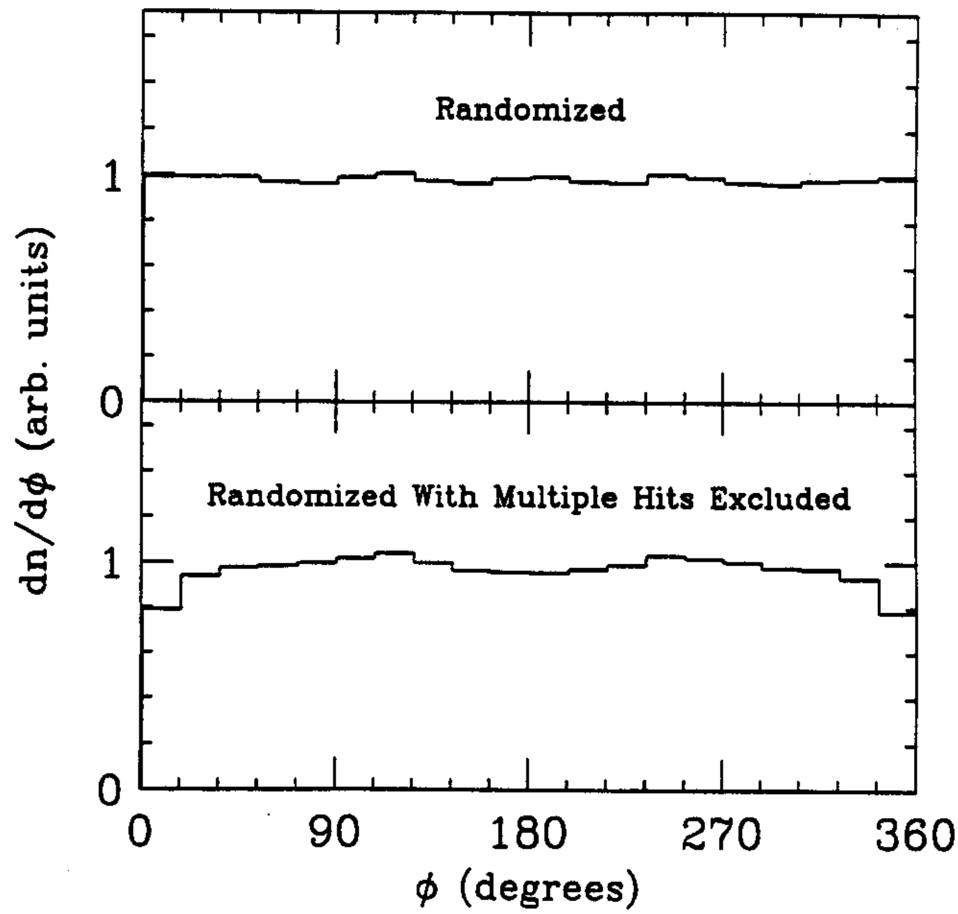

FIG. 8. The panels of this figure show the search for any azimuthal anisotropies due to detector bias in the analysis of randomized events, as described in the text.

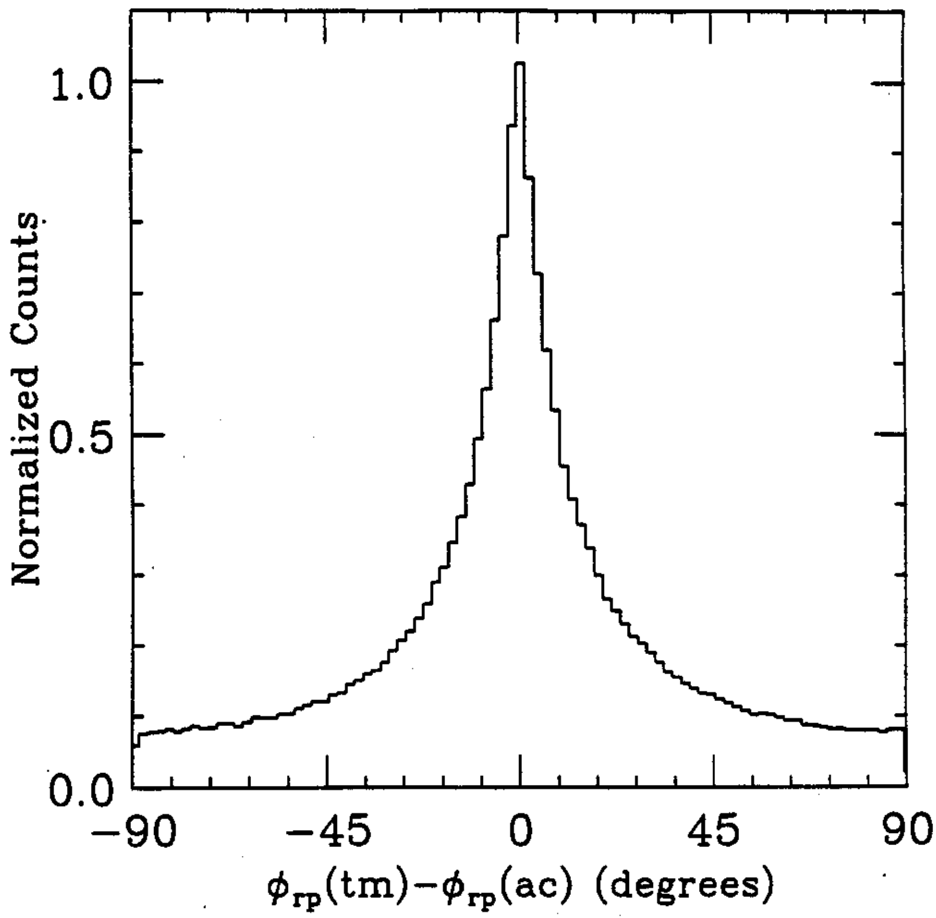

FIG. 9. The distribution of azimuthal angle differences between reaction planes found using the transverse momentum analysis ($\phi_{rp}(tm)$) and the azimuthal correlation technique ($\phi_{rp}(ac)$) is shown for 35 MeV/nucleon Ar+V events.

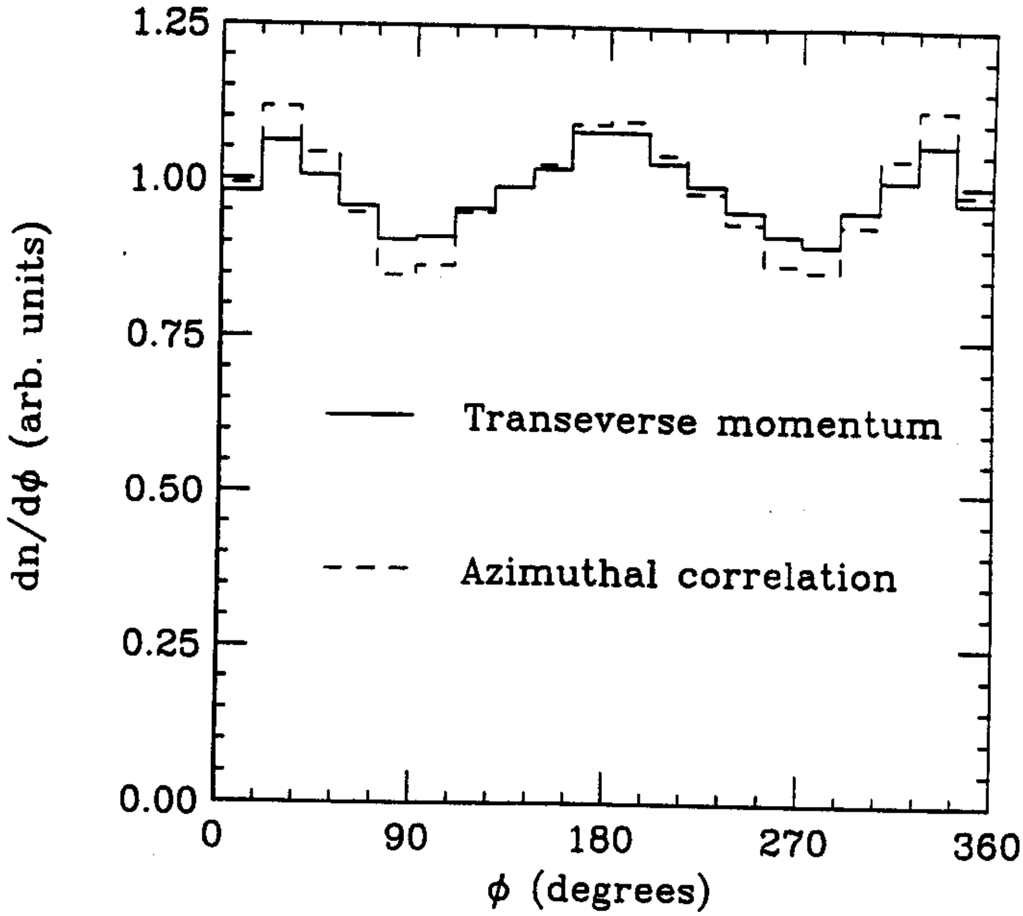

FIG. 10. A comparison of the azimuthal angle of POIs with respect to the reaction planes found using two different techniques.

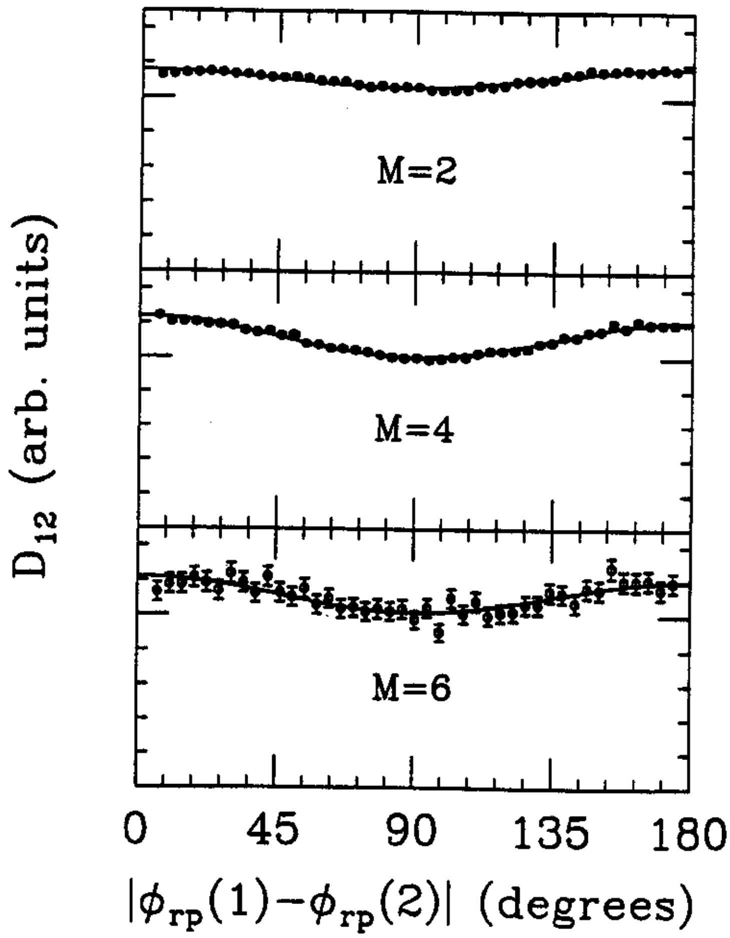

FIG. 11. The distribution of differences, D_{12}, between azimuthal angles of reaction planes found for sub-events of multiplicity 2, 4, and 6 is shown for Ar+V data. The lines are due to the Gaussian fits described in the text.

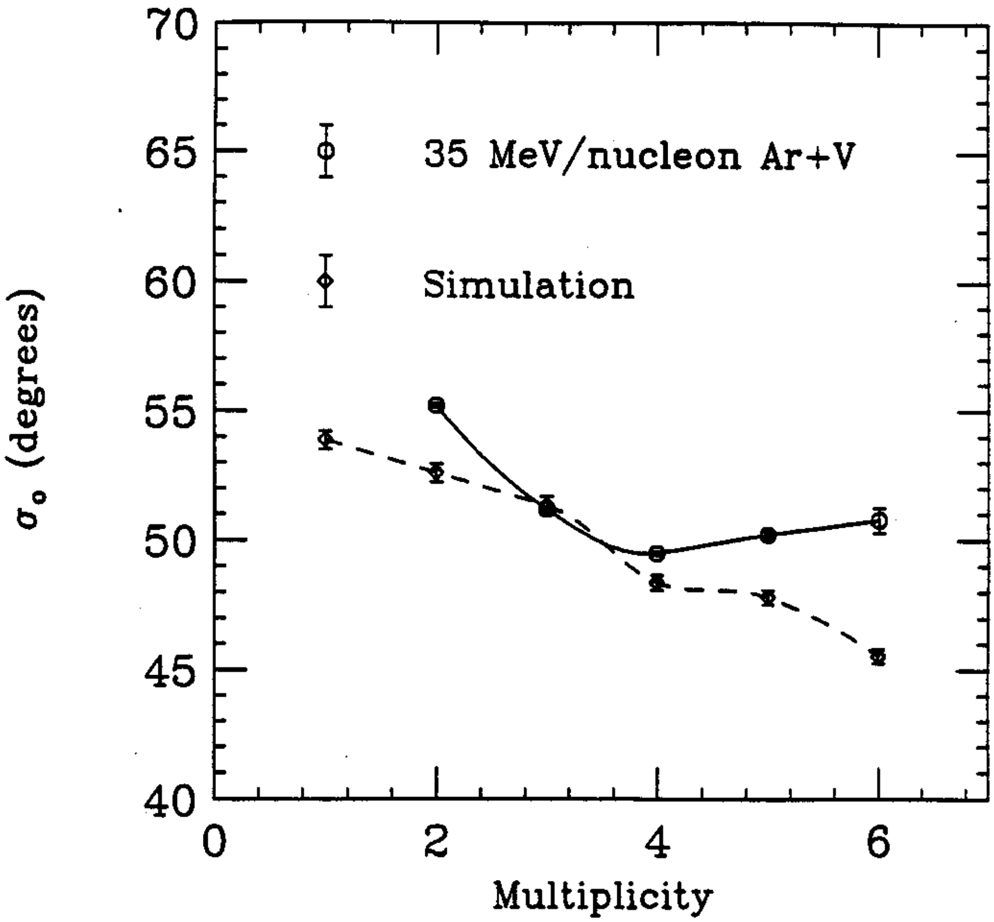

FIG. 12. The extracted widths (σ_o) from Gaussian fits using eqs. 7 and 9 are shown as a function of multiplicity.

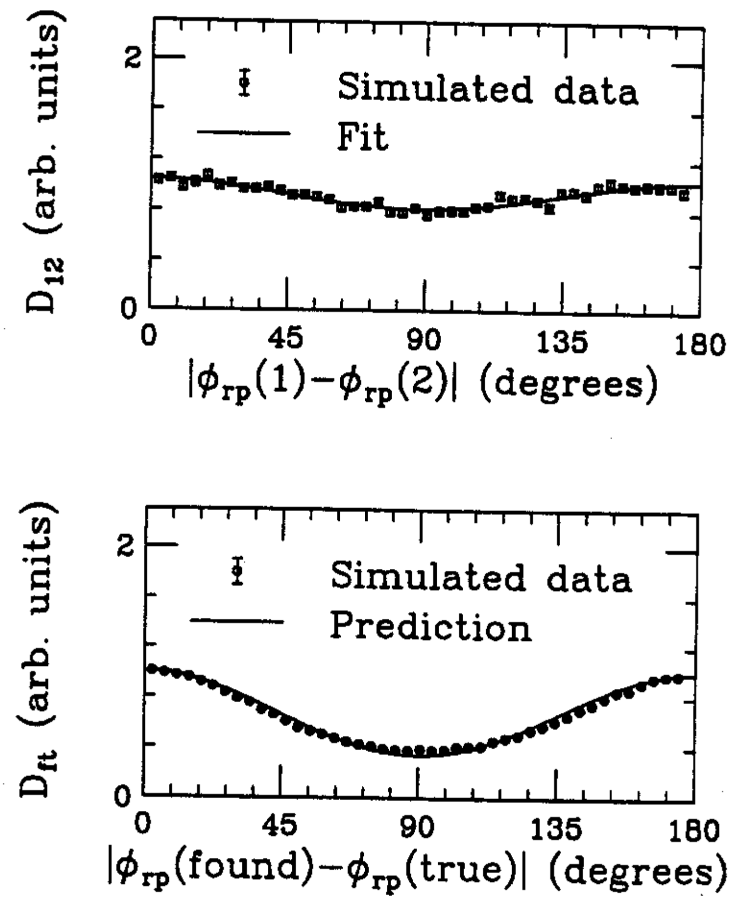

FIG. 13. An exploration of the accuracy of the reaction plane determination for simulated events. The panels are described in the text.