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ABSTRACT

Estimates of the magnitude of the pion phase space distribution function in relativistic heavy ion collisions suggest the importance of considering final state enhancement factors in a kinetic description of these collisions. We present a test particle method for simulating the Boltzmann equation for Bosons and illustrate it by way of the mean collision times in equilibrium, and the approach to equilibrium.

1. Introduction

Much attention has been devoted in recent years to isolate possible signals of a phase transition to the quark gluon plasma in relativistic heavy ion collisions.' It has become clear that a detailed understanding of the final state hadronic system is required. Here we consider the "cool pion" excess (over the p-p case) observed in heavy ion experiments at CERN and AGS.[2] Various authors have examined thus far the effects of softened pion dispersion relations;[3] collective flow;[4,5] and heavy resonance[6] and excited baryon' decays. The possibility of a non-zero chemical potential has also been pointed out by Kataja and Ruuskanen:[8] a thermal model with non-zero chemical potential reproduces the experimental' negative $k_\perp$-data for 0+ Au at 200 GeV/n if $T =$ 167 MeV and μ_π = 126 MeV. However, transverse flow tends to wash out the desired effect;[8] for such a system the pion mean free path is too short ($\lambda \sim$ 1 fm) to be compatible with freeze-out radii, $R \sim$ 7 fm;[10,11] and the corresponding rapidity distributions are too narrow.

This suggests that one should consider non-equilibrium features via a quantum kinetic equation with realistic initial conditions. An example of such an approach is the bosonic Boltzmann equation (BBE). In the next section we shall demonstrate the need for quantum statistics, i.e. an "induced radiation," or "feedback," effect. In sections 3 and 4 we introduce a test particle method for solving the BBE.

2. The phase space distribution function in heavy ion collisions

To estimate the magnitude of $f(\vec{r},\vec{k},t)$ and motivate the necessity of considering quantum statistics, we shall consider first a Boltzmann cascade simulation (no final state enhancement; no mean field) of the mesonic component for 200 GeV/n O + Au collisions. The initial conditions for the system are obviously important in making this determination.

For ^{16}O + Au collisions at 200 GeV/n the number of final state negatives per event is ~ 150.[12] Assuming a contribution of $\sim 10\%$ from K^- and β^-, this implies ~ 400 pions in the final state. If the mesons are produced according to their statistical weight, the number of primary particles is therefore ~ 250. In accordance with the baryon–free nature of the model, we choose a Bjorken–like initial scenario.[13] Therefore the space–time rapidity η is given by the fluid rapidity variable y, which is given a Gaussian distribution with $\sigma_y = 1.3$ to reproduce roughly the data. The local momentum distribution is chosen to be fully equilibrated at a temperature of 200 MeV. For central collisions, it is further natural to assume that the transverse position distribution is given by an oxygen density profile.

The evolution of the distribution function may now be simulated using the standard test particle method for the classical Boltzmann equation.[14] Only particles whose proper time exceeds the initial value of $\tau_0 = 1$ fm are considered. Below $s \sim 1$ GeV the π–π elastic scattering cross–section is taken from experimental parameterizations,[15] while all other channels allowed by conservation laws are incorporated using the Hauser–Feshbach formalism.

To investigate the question of final state enhancement, we wish to compute within this microscopic cascade framework the average value of the phase space distribution $\langle f^{col}(t) \rangle$ into which pions are scattered as a result of collisions or decays. To facilitate such a calculation the pion distribution function at fixed fireball time t is first parameterized as

$$f(\vec{r},\vec{k},t) = \frac{(2\pi)^3}{g}\,\frac{1}{t\,m_\perp}\,\frac{\cosh^2\eta}{\cosh\,y}\,\frac{d^6N(t)}{dy\,dk_\perp^2\,d\eta\,dr_\perp^2}\,, \qquad (1)$$

$$\frac{d^6N(t)}{dy\,dk_\perp^2\,d\eta\,dr_\perp^2} = N_0\,\mathrm{e}^{-(y-\eta)^2/2\sigma_y^2-\eta^2/2\sigma_\eta^2-(\vec{r}_\perp-\vec{v}_\perp t)^2/2\sigma_r^2}\,\frac{m_\perp}{2\pi}\sum_{n=1}^{\infty}K_1(n\beta\,m_\perp)\,. \qquad (2)$$

Here, K_1 is a McDonald function and $\vec{r}_\perp, \vec{k}_\perp, m_\perp$ are the transverse position, momentum and mass, respectively. The five parameters N_0, σ_y, σ_η, σ_r and β are functions of t. The functional form of Eq. 2 is motivated partly by physical considerations, but also allows for reasonable fits without resorting to multidimensional fitting. Fig. 1 shows the parameter values obtained, as a function of the fireball time.

For each collision or decay at time t with a final state pion, Eqs. 1 and 2 now permit an evaluation of the average distribution function $\langle f^{col}(t) \rangle$ for the final state of that pion. In Fig. 2 we show this quantity for all pions (solid line), and for pions in the rapidity interval -0.5 to 0.5 (dashed curve), as a function of time. For

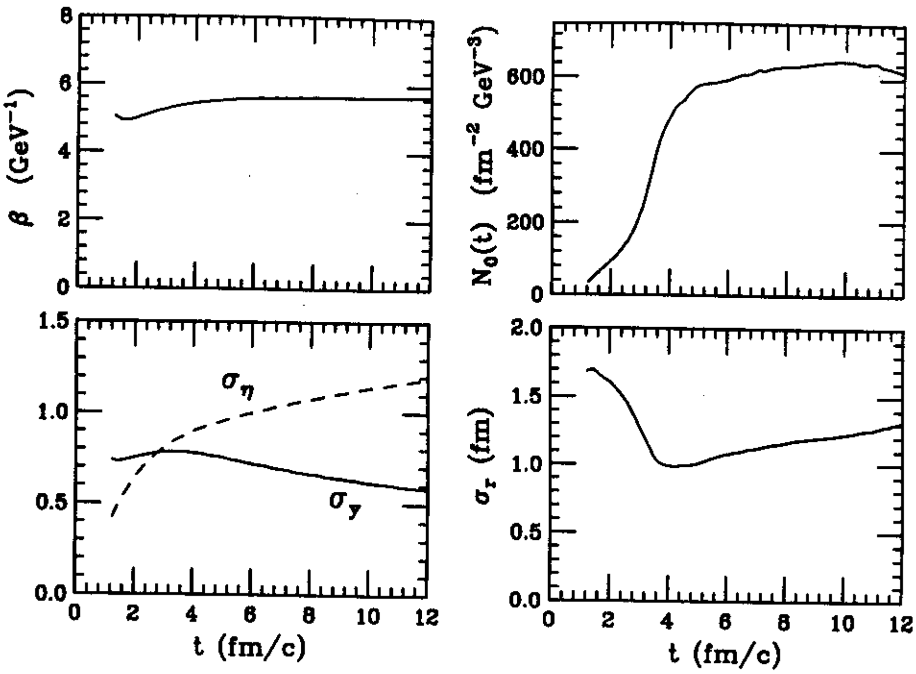

Fig. 1. Fitted parameters as a function of the fireball frame time variable t.

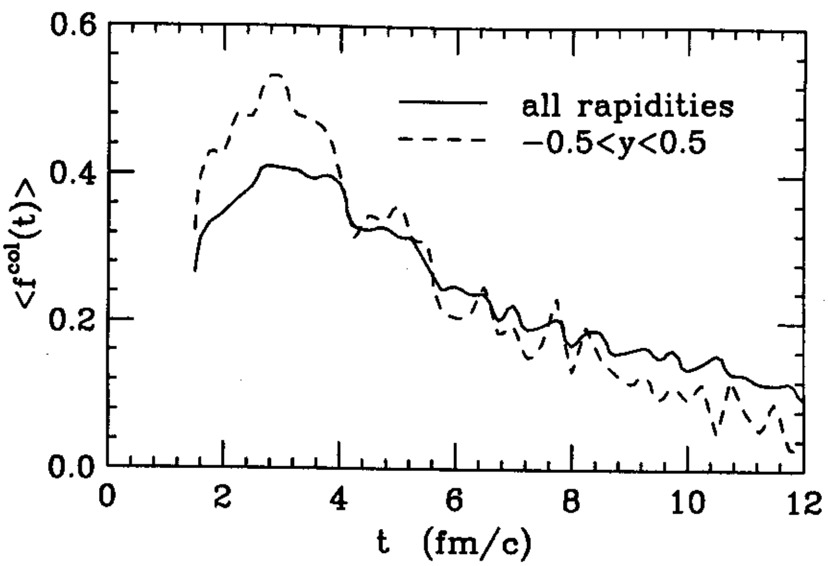

Fig. 2. The average value of the distribution function for pions produced in collisions or decays as a function of time.

$t \sim 1.5 - 6$ fm/c, $\langle f^{col} \rangle$ is an appreciable fraction of unity, indicating that Bose enhancement factors for the final state are not negligible.*

It is the low momentum region ($k_\perp \sim y \sim 0$) where the value of $f(\vec{r},\vec{p})$ is large (see parameterization Eq. 2),† and these states will be preferentially occupied in a Boson–cascade model. This further leads to a "positive feedback," i.e. preferential occupation of the region about $k=0$ early on leads to increased scattering into that region later on. The strength of this effect would depend on the initial conditions. Considerations here show that boson enhancement factors will play an important role in the evolution of the system. For this reason we describe in the next section a method for simulating the BBE, and test the technique by way of two examples in section 4.

3. Test particle method for a bosonic Boltzmann equation

The evolution of the phase space distribution function $f(\vec{r},\vec{k},t)$ for bosons may be described, for example, by the Boltzmann equation

$$\frac{Df_1}{Dt} = \frac{1}{4E_1}\int d\omega_2 d\omega_3 d\omega_4\,(2\pi)^4\,\delta^4(k_1+k_2-k_3-k_4)\,|T|^2\,\{f_1 f_2 \tilde{f}_3 \tilde{f}_4 - f_3 f_4 \tilde{f}_1 \tilde{f}_2\}\ . \quad (3)$$

The notation used here is described in these proceedings, Ref. [16]. We simulate Eq. 3 using a test particle method and accordingly represent f by $N=\widetilde{N}A$ point–like test particles, where A is the number of physical particles with degeneracy g

$$f(\vec{r},\vec{k}) \simeq \frac{(2\pi)^3}{g}\,\frac{1}{\widetilde{N}}\sum_{i=1}^{N}\delta^{(3)}(\vec{k}-\vec{k}_i(t))\,\delta^{(3)}(\vec{r}-\vec{r}_i(t)) \quad . \quad (4)$$

Further, we shall assume that the distribution function is bounded by F_{max}, and rewrite the collision term as

$$\frac{1}{4E_1}\int d\omega_2 d\omega_3 d\omega_4\,(2\pi)^4\,\delta^4(k_1+k_2-k_3-k_4)\,|T'|^2\,\{f_1 f_2 P_3 P_4 - f_3 f_4 P_1 P_2\}\ , \quad (5)$$

where $P_i \equiv (1+f_i)/(1+F_{max})$ is a final state blocking factor for the transition amplitude $|T'|^2 \equiv (1+F_{max})^2|T|^2$.

The simulation is now performed according to the method described, for example, in Refs. [14,17]. A collision is blocked if for the final state $(3,4)$

$$P_3\,P_4 = \prod_{i=3,4}\frac{F_{max}^{-1}+n_i/N_{max}}{F_{max}^{-1}+1} < x \in [0,1) \quad , \quad (6)$$

*For a system in global thermal equilibrium at temperatures given by the fits to the transverse spectra, the $\langle f^{col} \rangle$ at central rapidities correspond to a chemical potential of $70-110$ MeV in the time interval $t \sim 1.5-6$ fm, i.e. the equivalent thermal system is appreciably out of chemical equilibrium. Note that the large value of $\langle f^{col} \rangle$ is a consequence of the initial conditions.

†For example, at $t=2$ fm/c, the value of f is 7.5 at $y=k_\perp=r_\perp=\eta=0$, falling off to 2.3 at $y=k_\perp=0, r_\perp=\sigma_r(t=2\text{ fm/c})=1.6$ fm and $\eta=\sigma_\eta(t=2\text{ fm/c})=0.75$.

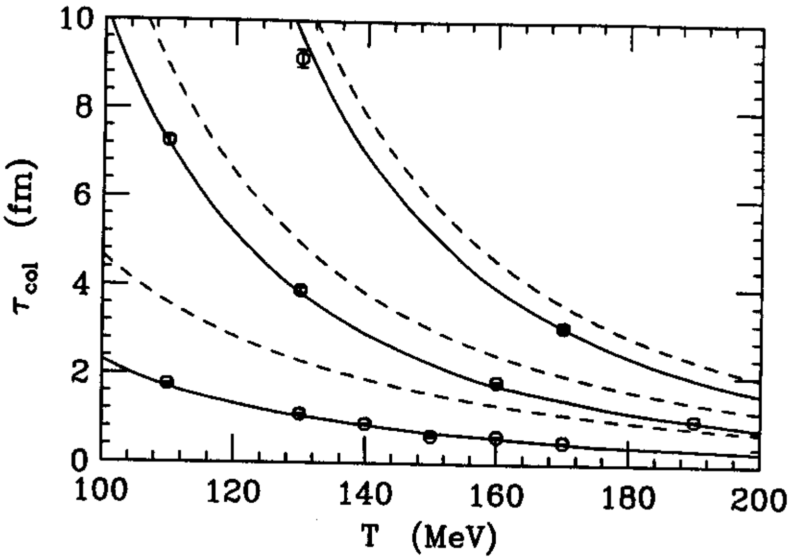

Fig. 3. Mean collision times in equilibrium as a function of temperature (see text).

where x is a uniform random number and n_i is the test particle occupation of a phase space sphere defined by radii r and k about the final states $(3,4)$:

$$g\,(4\pi/3)^2\,(rk)^3 \;=\; (2\pi)^3\,N_{max}/\widetilde{N}\,. \tag{7}$$

The value of N_{max} should be chosen large enough to avoid statistical fluctuations, yet small enough to sample phase space locally. The number of ensembles $\widetilde{N}$ is governed by practical considerations, but should satisfy $(1+F_{max})(\sigma/\widetilde{N})^{1/2} \ll L$, the linear dimension of the system,[17] and be such that the final state phase space is adequately sampled, *viz.* $\widetilde{N} \gtrsim O(1+F_{max})^2$.

4. Examples : mean collision times and equilibrium states

We shall illustrate the above simulation technique by way of a uniform spherical pion gas with periodic boundary conditions. For simplicity, consider the case of no mean field, and a constant elastic and isotropic cross section, so that $|T|^2 = 32\pi g^2 s\,\sigma$. We shall choose $\sigma = 30$ mb in the following.

In the first example, we consider test pions with a momentum distribution given by the Bose–Einstein function. The mean collision time is $\tau_{col} = \rho^0/2Z^0$, where Z^0 is the collision rate per unit volume in thermal equilibrium (see Ref. [16], these proceedings)

$$Z^0 \;=\; \frac{1}{4}\int \prod_{i=1,4} d\omega_i\,(2\pi)^4\,\delta^4(k_1+k_2-k_3-k_4)\,|T|^2\,f_1^0\,f_2^0\,\tilde{f}_3^0\,\tilde{f}_4^0\,. \tag{8}$$

The delta function is best removed by transforming the invariant integrals for particles 3 and 4 to relative and total momentum variables in the (1,2)–cm frame and evaluating the remaining 8 integrals by a Monte–Carlo technique. The resulting

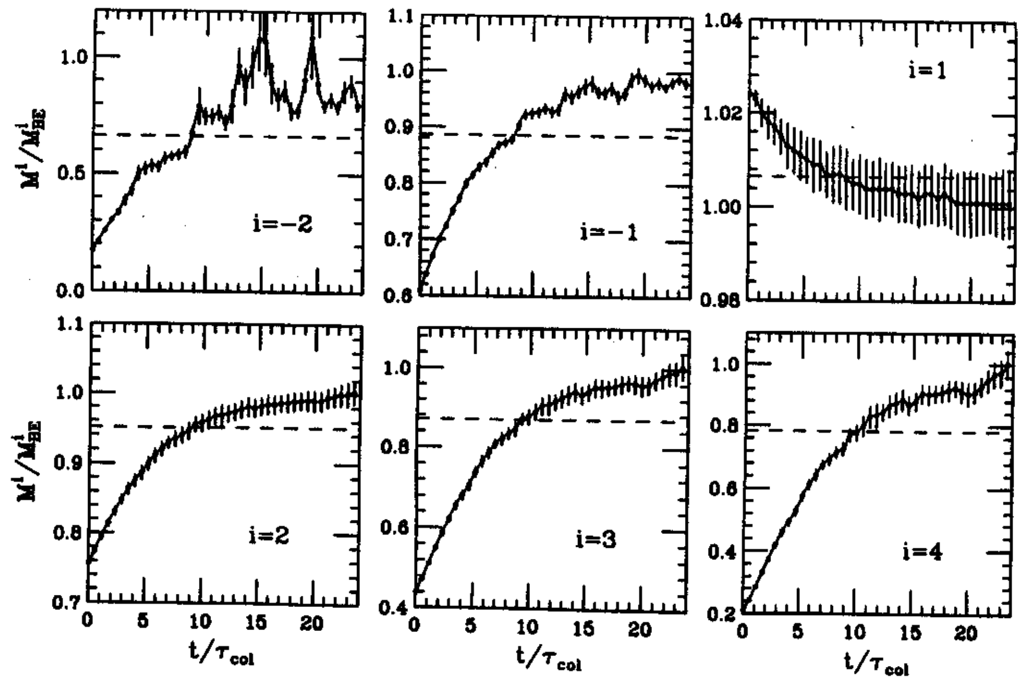

Fig. 4. Ratio of moments of the magnitude of the momentum vector to those of an equilibrated Bose–Einstein distribution (see text).

collision times are shown as function of temperature in Fig. 3 (solid lines; top to bottom: $\mu = -100$, 0 and 100 MeV). The points with error bars show the times obtained from a test particle simulation in a system with the indicated temperatures and chemical potentials. For comparison, we include in the graph corresponding Maxwell–Boltzmann collision times (dashed curves). The test particle simulation is seen to be in good agreement with the analytical result, though some of this is a simple density effect.

In the second example, we consider whether the simulation technique yields the correct equilibrium states. To this end, the initial momentum distribution is chosen to be isotropic with a fixed magnitude of $|\vec{p}| = p_0 = 427$ MeV/c, while the coordinate space distribution is chosen to be uniform with a particle density of 0.23 fm^{-3}. Since the collisions conserve 4–momentum and particle number, we expect the system to relax to a Bose–Einstein distribution with $T = 140$ MeV and $\mu = 100$ MeV. To determine if this is indeed the case, it is useful to define a ratio of the momentum moments as:

$$\frac{M^i}{M^i_{BE}} = \frac{\int d^3p \; |\vec{p}|^i \; f(\vec{p})}{\int d^3p \; |\vec{p}|^i \; f_{BE}(\vec{p}; T, \mu)} \; , \tag{9}$$

where f_{BE} is the distribution function for the corresponding equilibrated Bose–Einstein system. In Fig. 4 we show this ratio for $i = -2, -1, 1, 2, 3$ and 4 as a function of t/τ_{col}, where τ_{col} is the mean collision time in equilibrium. The error bars are standard errors in the mean from a calculation with 40 ensembles. Shown as dashed lines are the ratios of Eq. 9 for $f \equiv f_{MB}$, where f_{MB} is the Maxwell–Boltzmann distribution function for a system with $T = 124$ MeV and $\mu = 169$ MeV, i.e. with the same energy and particle density as the system under consideration. Clearly, the system relaxes to the correct Bose–Einstein equilibrium distribution, and not to the corresponding Maxwell–Boltzmann distribution.

5. Summary

We have considered a Boltzmann–cascade simulation of mesons produced in an O + Au collision at 200 GeV/n. Realistic initial conditions imply that the inclusion of final state enhancement factors is important for a correct description of the evolution of the system. To this end we have introduced a test particle method for solving the bosonic Boltzmann equation, which was shown to yield correct collision rates in equilibrium, and the correct Bose–Einstein equilibrium states. The next step is to consider a system with realistic initial conditions, in an attempt to reproduce both the transverse momentum spectra and the rapidity distributions observed in heavy ion collisions. Work towards this goal is in progress.

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