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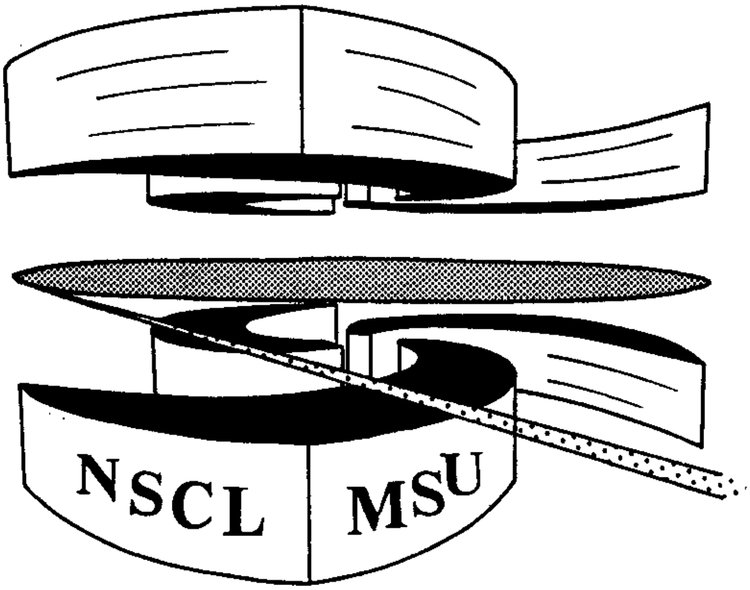

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# Bosonic kinetics and the pion transverse momentum in heavy ion collisions

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**Abstract** : Using a new technique to solve the kinetic equation including bose statistics, we show that the shape of the pion transverse momentum spectrum is sensitive to the hadronization time in ultrarelativistic heavy ion collisions. The magnitude of the soft pion component observed in central 200 GeV/n 0 + Au collisions can be reproduced with a hadronization time of about 7 fm/c. This explanation implies that the hadrons are produced out of chemical equilibrium.

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Negative particle and neutral pion spectra from ultra-relativistic heavy ion collisions at CERN show a markedly enhanced low transverse momentum component when compared to corresponding minimum bias proton–proton spectra.[1–3] This "soft pion" puzzle has been much discussed in the literature. One suggestion has been that the hadronic gas would cool by collective expansion,[4] but more detailed considerations of the hydrodynamics found too small an effect.[5] The decay of excited baryons gives low momentum pions,[6] and this mechanism may well account for the soft pions in situations where the baryon density is high, such as near target rapidity, or at the lower energies of AGS experiments.[7] For SPS energies, on the other hand, there are probably not enough baryons in the central region to account for the data.[8] We also mention the possibility that soft pion interactions within hot, dense matter affect the dispersion relation and the momentum distribution in the final state.[9] Finally, Kataja and Ruuskanen[10] have pointed out that the transverse momentum spectra are compatible with a thermal Bose–Einstein distribution if there is a strong excess of pions with respect to chemical equilibrium at freezeout. However, collective flow would wash out the enhancement thus obtained,[10] while the corresponding mean free paths ($\lambda \sim 1$ fm) are too short to be compatible with estimated freezeout radii ($R \sim 7$ fm),[11, 12] and the rapidity distributions are too narrow by a factor of two.

In this work, we shall seek a more detailed picture of the hadronic evolution of the collision, to see whether the assumptions of Kataja and Ruuskanen can plausibly be supported in models of kinetics. One such kinetic equation is the bosonic Boltzmann equation,[13] which describes the time evolution of the phase space distribution function $f(\vec{r},\vec{p},t)$ for bosons :

$$\frac{Df_1}{Dt} = \frac{1}{4E_1} \int d\omega_2 d\omega_3 d\omega_4 \, (2\pi)^4 \, \delta^4(k_1+k_2-k_3-k_4) \, |T|^2 \, \{f_3 f_4 \tilde{f}_1 \tilde{f}_2 \; - \; f_1 f_2 \tilde{f}_3 \tilde{f}_4\} \; , \quad (1)$$

where $d\omega_i = d^3k/[(2\pi)^3 2E_i]$, $E_i^2 = |\vec{p}|^2 + m^2$, $|T|^2$ is the square of the invariant scattering amplitude, and the factors $\tilde{f}_i \equiv 1 + f_i$ account for bose enhancement. These factors will drive particles to occupy preferentially regions of higher phase space density, i.e., toward small transverse momenta and rapidities. Gavin and Ruuskanen[14] have solved Eq. (1) using a relaxation time approximation for the right hand side. Here we shall use a more general test–particle method to simulate the evolution of the system. To our knowledge, the only previous treatment of the kinetics for bosons was by Zeldovich and Levich in the theory of plasmas.[15]

We shall consider specifically the central 200 GeV/n $^{16}$O + Au data of the NA35 collaboration.[1] This experiment measures negative particles, mostly pions, with a ~ 10% admixture of negative kaons, electrons and anti–protons.[16] For simplicity, we solve Eq. (1) for an isospin–symmetric pion gas only, and assume that all particles in the experimental data are negative pions. An integration of the rapidity distribution therefore shows that the system should be initialized with ~ 440 pions. Inclusion of other negatives would reduce this number, and hence the phase space density of pions, by 10%. The results obtained here are thus subject to a systematic error of this order of magnitude. Further, we have ignored the excited state mesons and baryons which will be present in the initial state and influence the evolution of the system. Exclusion of the baryons is probably justified for the mid–rapidity range considered here. The heavy mesons omitted are mainly $\rho$'s, which will decay rapidly. We therefore consider the evolution of the pions only with an effective average hadronization, or "pionization," time $\tau_0^{eff}$ for the system which is $\gtrsim 1-3$ fm/c larger than the actual average hadronization time.

Some further model assumptions are needed to specify the initial conditions for Eq. (1). We shall suppose Bjorken–like initial conditions are applicable.[17] Locally thermal distributions are boosted by an amount that depends on the longitudinal particle coordinate via

$$z = \tau_0^{eff} \sinh y_{boost} \quad , \tag{2}$$

where $\tau_0^{eff}$ is the effective pionization time and $y_{boost}$ is the boost rapidity variable. The initial transverse spatial distribution is assumed to follow the transverse density profile of the oxygen projectile. Further, we fit the initial local momentum distribution to the minimum bias pp transverse spectrum.[1] This is well described by a thermal source of temperature 135 MeV and zero chemical potential, boosted uniformly over a rapidity interval $\Delta y_{boost} = 3.6$.[18] In pp collisions the effect of bose statistics is not expected to be significant, because of the small size of the source. We shall ignore the high $p_\perp$ portion of the momentum spectrum, since this corresponds to hard processes for which our kinetic approach is ill–suited.

In Fig. 1(a) we show the initial ($t = \tau_0^{eff}$) rapidity distribution generated as described above (dashed line), together with the experimental AB data (circles).[1] The latter has been shifted by $\Delta y = -2.3$ to the c.m. frame. The calculated points include a factor of 1/3 to account for the pion degeneracy. Fig. 1(b) shows the corresponding initial transverse momentum distribution (dashed line), with the experimental data. The rapidity cuts for the input distribution are $-0.3 < y < 0.7$, from the shift chosen for the rapidity data. A problem encountered is that integration of the experimental $dN_-/dy$ from $y = 2$ to 3 gives $N_- = 38 \pm 2$, while the $dN_-/(p_\perp dp_\perp)$ curve has fewer pions, $N_- = 35 \pm 5$. Since we choose the pion number to fit the rapidity distribution, the $p_\perp$–normalization is therefore decreased by $\sim$ 15% to account for the difference in the data, and the fact that the experimental $dN_-/dy$ includes pions from hard processes as well.

Eq. (1) is solved using the "full–ensemble" test particle method.[19, 20] To do this, assume that the distribution function $f(\vec{r}, \vec{p}, t)$ is bounded by a constant $F_{max}$ at all times, and replace the transition matrix in (1) by $|T'|^2 \equiv (1 + F_{max})^2 |T|^2$. Collisions between test particles in phase space elements $d\omega_1, d\omega_2$ are then accepted with a probability of $(1 + f_3)(1 + f_4)/(1 + F_{max})^2$, where $f_3, f_4$ are the final phase space occupations. In order to sample phase space adequately, and to avoid surface effects,[20] the number of test particles per pion must be chosen to be rather large, $\gtrsim 100$ for $F_{max} \sim 10$. This method has been shown to yield the correct collision rates in thermal equilibrium, and the correct Bose–Einstein equilibrium states,[21] but ceases to be practical as $\mu_\pi \to m_\pi$, when $f$ becomes unbounded.

The pionization time controls the longitudinal dimension of the system, and therefore the density and collision rate, but also the time $t_i = \tau_0^{eff} \cosh y_{boost}$ at which a given particle $i$ begins to participate in the evolution of the system. We therefore expect the magnitude of the low–$p_\perp$ enhancement to depend sensitively on $\tau_0^{eff}$, and should be able to extract it from the AB data given the initial conditions discussed above. Of course, the collision rate, and hence the size of the enhancement, also depend also on the value of the cross section that appears in Eq. (1). We use $\sigma = 23$ mb, which is a thermal average[22] at $T = 135$ MeV of $\pi$–$\pi$ cross sections determined from scattering phase shift data.

We first show the results of the calculation for $\tau_0^{eff} = 7$ fm/c, which give our best fit. The solid line in Fig. 1(a) shows the rapidity distribution at time $t = 29$ fm/c from an evolution of a system with the initial conditions given by the dashed curve. The solid line has been subjected to the NA35 software cuts ($|\vec{p}_{lab}| > 0.1$ GeV/c and $0.5 < y_{lab} < 4.5$). We see that the rapidity distribution has narrowed somewhat relative to the input distribution, to a full width at half maximum of $\Delta y_{FWHM} = 3.4$. An effect in this direction is also seen in the heavy ion data ($\Delta y_{FWHM} = 3.1$) relative to the pp data ($\Delta y_{FWHM} = 3.5$).[1] The solid line in Fig. 1(b) shows the corresponding transverse momentum distribution. The low–$p_\perp$ part of the curve is well reproduced, suggesting a pionization time of $\sim 7$ fm/c.

To see the sensitivity to $\tau_0^{eff}$, we show in Fig. 2 the $p_\perp$–spectra obtained from a longer pionization time of 9 fm/c (long dashed curve). Since the accompanying narrowing of the rapidity distribution is less pronounced than before, we fit the AB data using a smaller initial $\Delta y_{boost}$ of 3.4. The bose enhancement effect on the $p_\perp$ curve is reduced in comparison to the calculation of Fig. 1, and no longer consistent with the data. Finally, for a smaller $\tau_0^{eff}$ of 5 fm/c, the soft pion enhancement is too pronounced, as is seen from the dot–dashed line in Fig. 2. Here we have taken $\Delta y_{boost} = 3.8$, since the rapidity distribution narrows considerably during the evolution of the system. The curves in Fig. 2 clearly indicate that the soft pion enhancement is rather sensitive to $\tau_0^{eff}$. These calculations therefore suggest limits of $6 \lesssim \tau_0^{eff} \lesssim 8$ fm/c, bearing in mind that the simplifying assumptions made regarding the data have introduced some systematic error. This range is consistent with source size and duration parameters extracted from $\pi$–$\pi$ correlations.[11]

Our findings have a number of implications for the main objective of ultrarelativistic collisions, to discover a dense phase of matter such as the quark–gluon plasma and measure its properties. The result that the hadronization time scale is rather long on the QCD time scale is encouraging, because the system then has adequate time to come to a local equilibrium. However, the finding that the pions are apparently produced out of chemical equilibrium suggests that any phase transition is a weak one. We would expect the conversion of matter via a strong first–order transition to produce particles near chemical equilibrium. Also, the hadronization time of 7 fm/c is somewhat short if a first order transition with a large latent heat is involved. Of course, this time represents some sort of average and small fractions of the matter could persist to much greater times.

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# FIGURE CAPTIONS

FIG. 1. (a) The input ($t = \tau_0^{eff}$, dashed line) and final ($t$ = 29 fm/c, solid curve) rapidity distributions for negative pions, and $\tau_0^{eff} = 7$ fm/c. The circles are the NA35 data for central 200 GeV/n $^{16}$O + Au → neg. collisions,[1] shifted by $\Delta y = -2.3$. (b) The initial (dashes) and final (solid line) transverse momentum distributions for $\tau_0^{eff} = 7$ fm/c in the rapidity interval $-0.3 < y < 0.7$. The circles are AB data $(2 < y_{lab} < 3)$.[1]

FIG. 2. Transverse momentum spectra at $t$ = 29 fm/c. Long–dashed curve : $\tau_0^{eff} =$ 9 fm/c and $\Delta y_{boost}^{initial} = 3.4$; Dot–dashed line : $\tau_0^{eff} = 5$ fm/c and $\Delta y_{boost}^{initial} = 3.8$. The normalization and rapidity cuts are as in Fig. 1(b). The circles are the AB data.[1]

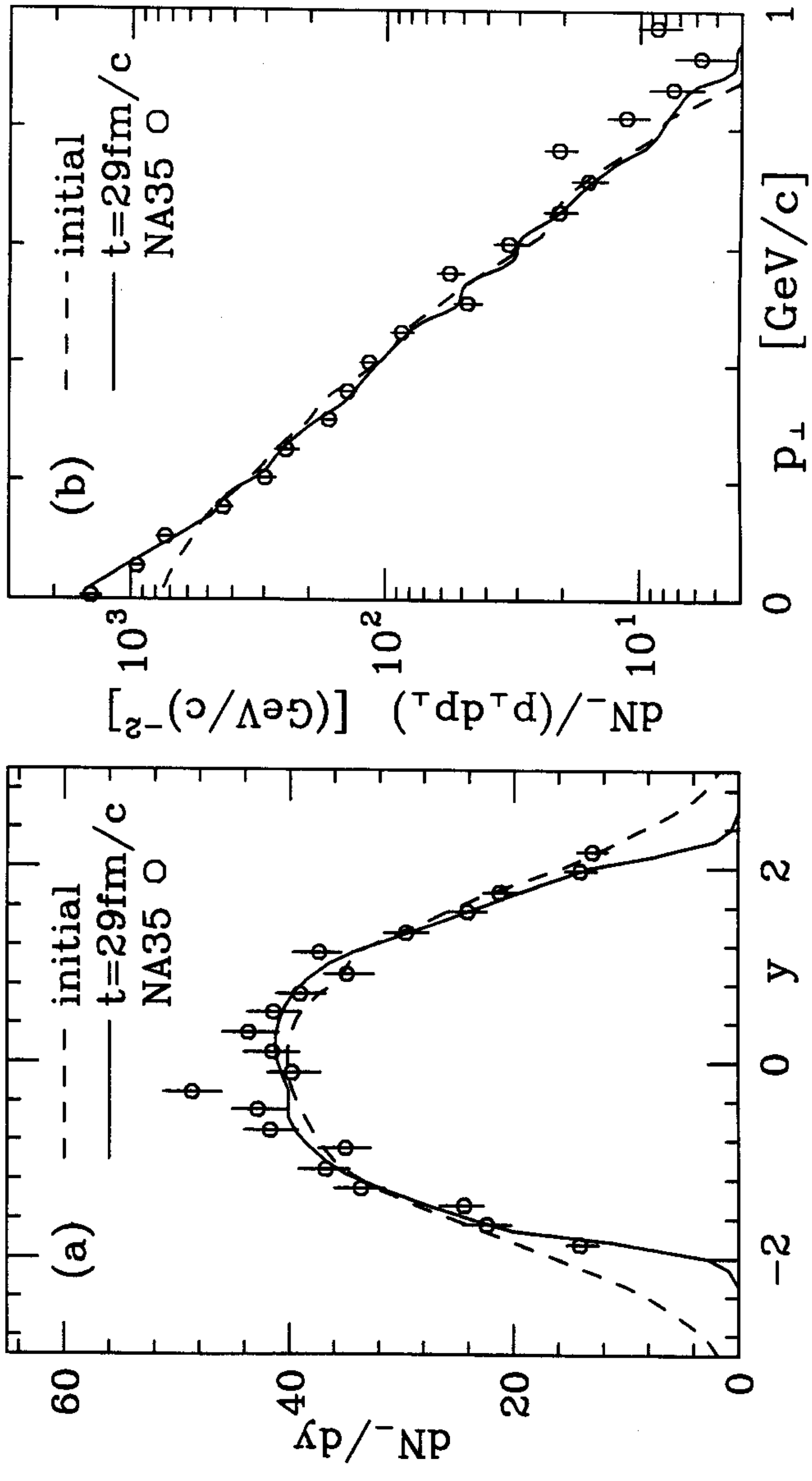
(a)
initial
t=29fm/c
NA35
dN_/dy
60
40
20
0
−2
0
2
y
(b)
initial
t=29fm/c
NA35
dN_/(p⊥dp⊥) [(GeV/c)^−2]
10^3
10^2
10^1
0
1
p⊥ [GeV/c]

FIG.1

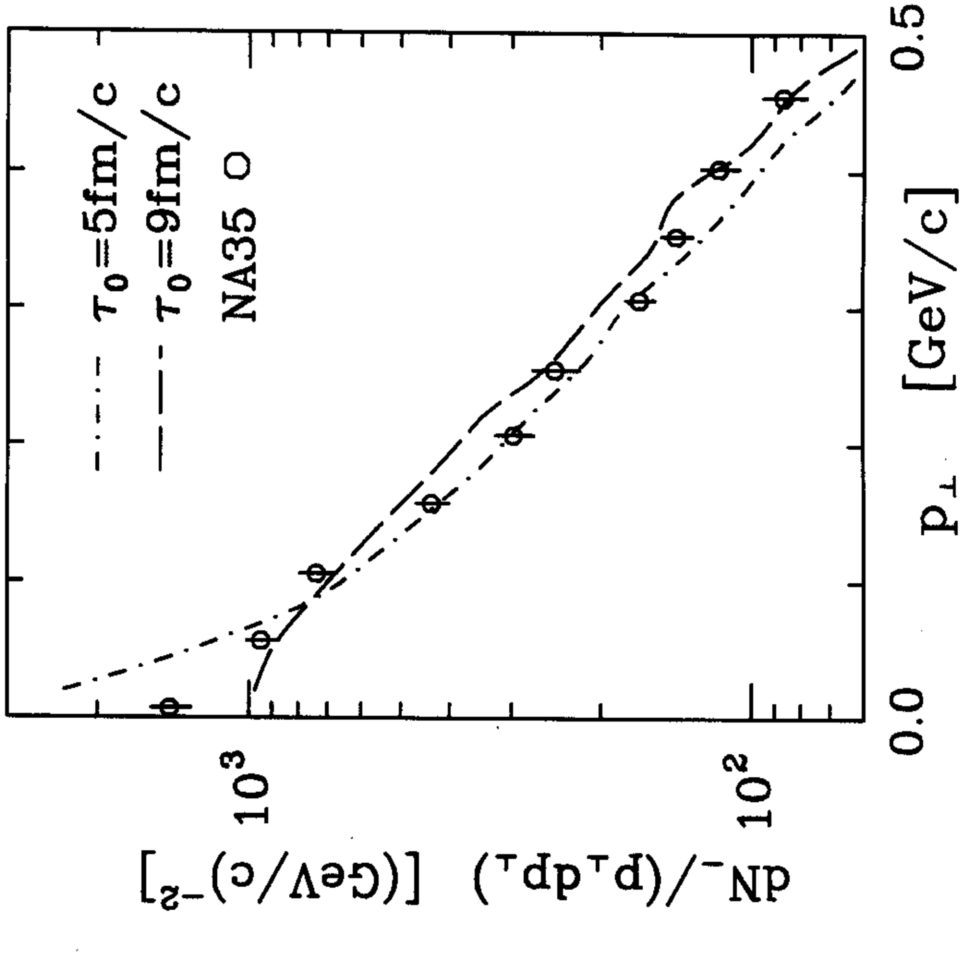
$\tau_0$=5fm/c
$\tau_0$=9fm/c
NA35 ○
$10^3$
$10^2$
0.0
0.5
$p_\perp$ [GeV/c]
$dN_-/(p_\perp dp_\perp)$ [(GeV/c)$^{-2}$]

FIG.2