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Abstract

Formulas for particle correlation functions and cluster emission probabilities are derived in terms of the single-particle phase-space density and production rate that are available from the popular dynamic models relying on nucleon degrees of freedom and nucleon-nucleon collisions. The expressions can be used to calculate the production of $A > 2$ clusters from the dynamic models. Koonin's formula is obtained as a special case.

Correlations between particles emitted with low relative momentum from heavy-ion reactions have been studied extensively in recent years [1]. Detailed data are available on proton-proton pairs, other light-fragment, as well as pion pairs. From the data one expects to learn about the development of the reactions in space and time [1] and on temperatures reached in the collisions [2,3]. When determining the space-time structure, the data have been typically parametrized in terms of Gaussian sources, with coincidence cross sections given by a convolution of the two-body wavefunction with the sources [4,5]. The process of correlated emission is related to that when particles are emitted as one cluster. Formulas involving convolutions of cluster wave functions with the single-particle density matrices, have been used to describe the light-fragment production [6].

The single-particle features such as proton spectra are quite well reproduced by the dynamic models relying on nucleon degrees of freedom and independent nucleon-nucleon collisions [7-9]. In the past, Remler *et al.* [10,11] have devised a procedure for calculating the deuteron production from the models. Pratt *et al.* [12,13] applied a formula proposed by Koonin [4] to obtain the nucleon-nucleon correlation functions from the dynamics. In this letter we derive relations of the coincidence cross sections, and cross sections for fragment emission, to the quantities that are calculated in a quasiparticle limit in the dynamic models [7-9]. We generalize a result of Remler *et al.* and we derive the original Koonin formula.

We begin with the following representation of the probability density for detecting A free nucleons or a cluster from an event:

$$\frac{d\mathcal{P}}{d\nu} = \lim_{t\to\infty} \frac{1}{A!} \int d\mathbf{x}_1 \ldots d\mathbf{x}_A \int d\mathbf{x}_1' \ldots d\mathbf{x}_A'$$
$$\times\, \Phi_\nu^{(-)*}(\mathbf{x}_1,\ldots,\mathbf{x}_A)\,\Phi_\nu^{(-)}(\mathbf{x}_1',\ldots,\mathbf{x}_A')$$
$$\times\, \langle\psi^\dagger(\mathbf{x}_A',t)\ldots\psi^\dagger(\mathbf{x}_1',t)\psi(\mathbf{x}_1,t)\ldots\psi(\mathbf{x}_A,t)\rangle\ . \qquad (1)$$

Here ν stands either for the momenta $\mathbf{p}_1,\ldots,\mathbf{p}_A$ of individual nucleons, or the center of mass momentum $\mathbf{P}$ of a cluster, in conjunction with any discrete quantum numbers; the $\Phi_\nu^{(-)}$ is an antisymmetrized wavefunction with outgoing boundary conditions for the case under consideration. An operator $\psi^\dagger(\mathbf{x},t)$ in (1) creates a particle at time t and position $\mathbf{x}$. Spin-isospin indices are suppressed. The expectation value in (1), divided by the factorial, is a time-dependent A-body density matrix. This is a reduced density matrix of the full evolving system. The relation between the probability density, averaged over impact parameter, and the cross section σ_ν is

$$\frac{d\mathcal{P}}{d\nu} = \frac{1}{\sigma}\frac{d\sigma_\nu}{d\nu}\ , \qquad (2)$$

where σ is the reaction cross section. For a fixed impact parameter, a correlation function [1] is obtained directly from (1).

We shall transform the expectation value in (1) using the equation of motion

$$\left(i\frac{\partial}{\partial t} + \frac{\nabla_1^2}{2m} + \ldots + \frac{\nabla_A^2}{2m}\right)<\psi^\dagger(\mathbf{x}_A',t')\ldots\psi^\dagger(\mathbf{x}_1',t')\psi(\mathbf{x}_1,t)\ldots\psi(\mathbf{x}_A,t)>$$

$$= <\psi^\dagger(\mathbf{x}_A',t')\ldots\psi^\dagger(\mathbf{x}_1',t')[\psi(\mathbf{x}_1,t)\ldots\psi(\mathbf{x}_A,t),V]> , \quad (3)$$

where the Laplacians act on unprimed coordinates, and V represents static two-body interaction. It can be shown by induction that

$$[\psi(\mathbf{x}_1)\ldots\psi(\mathbf{x}_A),V]$$

$$= \sum_{k<\ell}\int d\mathbf{x}_k' d\mathbf{x}_\ell' V(\mathbf{x}_k,\mathbf{x}_\ell;\mathbf{x}_k',\mathbf{x}_\ell')\psi(\mathbf{x}_1)\ldots\psi(\mathbf{x}_k')\ldots\psi(\mathbf{x}_\ell')\ldots\psi(\mathbf{x}_A)$$

$$+ \sum_k (-1)^{k-1} j(\mathbf{x}_k)\psi(\mathbf{x}_1)\ldots\psi(\mathbf{x}_{k-1})\psi(\mathbf{x}_{k+1})\ldots\psi(\mathbf{x}_A) , \quad (4)$$

where

$$j(\mathbf{x}) = [\psi(\mathbf{x}),V] . \quad (5)$$

When inserted into (3), the first term from the r.h.s. of (4) yields interactions only between the particles from the reduced density, while the second term with the operator j yields, in general, coupling to an additional particle in the system. We use J_A to denote the second sum on the r.h.s. of (4), and we rewrite eq. (3) as

$$\left(i\frac{\partial}{\partial t} + \frac{\nabla_1^2}{2m} + \ldots + \frac{\nabla_A^2}{2m}\right)<\psi^\dagger(\mathbf{x}_A',t')\ldots\psi(\mathbf{x}_1,t)\ldots\psi(\mathbf{x}_A,t)> - \sum_{k<\ell}\int d\mathbf{x}_k'' d\mathbf{x}_\ell''$$

$$\times V(\mathbf{x}_k,\mathbf{x}_\ell;\mathbf{x}_k'',\mathbf{x}_\ell'')<\psi^\dagger(\mathbf{x}_A',t')..\psi(\mathbf{x}_1,t)..\psi(\mathbf{x}_k'',t)..\psi(\mathbf{x}_\ell'',t)..\psi(\mathbf{x}_A,t)>$$

$$= <\psi^\dagger(\mathbf{x}_A',t')\ldots\psi^\dagger(\mathbf{x}_1',t')J(\mathbf{x}_1,\ldots,\mathbf{x}_A,t)> . \quad (6)$$

On considering the arguments of operators $\psi^\dagger$ in (6), and on introducing the retarded and advanced A-body Green's functions $G_A^{0\pm}$ that satisfy

$$\left(i\frac{\partial}{\partial t} + \sum_k \frac{\nabla_k^2}{2m}\right)G_A^{0\pm}(\mathbf{x}_1,\ldots,\mathbf{x}_A,t;\mathbf{x}_1',\ldots,\mathbf{x}_A',t') - \sum_{k<\ell}\int d\mathbf{x}_k'' d\mathbf{x}_\ell''$$

$$\times V(\mathbf{x}_k,\mathbf{x}_\ell;\mathbf{x}_k'',\mathbf{x}_\ell'')G_A^{0\pm}(\mathbf{x}_1,\ldots,\mathbf{x}_k'',\ldots,\mathbf{x}_\ell'',\ldots,\mathbf{x}_A,t;\mathbf{x}_1',\ldots,\mathbf{x}_A',t')$$

$$= \delta(t - t')\frac{1}{A!}\sum_\sigma \operatorname{sgn}\sigma \prod_k \delta(\mathbf{x}_k - \mathbf{x}_{\sigma(k)}') \ , \tag{7}$$

where σ are permutations of A indices, we obtain

$$<\psi^\dagger(\mathbf{x}_A',t)\ldots\psi^\dagger(\mathbf{x}_1',t)\psi(\mathbf{x}_1,t)\ldots\psi(\mathbf{x}_A,t)>$$

$$= \int dt''\int d\mathbf{x}_1''\ldots d\mathbf{x}_A''\int dt'''\int d\mathbf{x}_1'''\ldots d\mathbf{x}_A''' G_A^{0+}(\mathbf{x}_1,\ldots,\mathbf{x}_A,t;\mathbf{x}_1'',\ldots,\mathbf{x}_A'',t'')$$

$$\times <J_A^\dagger(\mathbf{x}_1''',\ldots,\mathbf{x}_A''',t''')J_A(\mathbf{x}_1'',\ldots,\mathbf{x}_A'',t'')>$$

$$\times G_A^{0-}(\mathbf{x}_1''',\ldots,\mathbf{x}_A''',t''';\mathbf{x}_1',\ldots,\mathbf{x}_A',t) \ . \tag{8}$$

The functions $G^{0\pm}$ may be decomposed in terms of a complete set of normalized A-body states ν with energies E_ν:

$$G_A^{0\pm}(\mathbf{x}_1,\ldots,\mathbf{x}_A,t;\mathbf{x}_1',\ldots,\mathbf{x}_A',t')$$

$$= \mp i\theta(\pm(t - t'))\sum\!\!\!\!\!\!\int d\nu \ e^{-iE_\nu(t-t')}$$

$$\times \Phi_\nu^{(-)}(\mathbf{x}_1,\ldots,\mathbf{x}_A)\Phi_\nu^{(-)*}(\mathbf{x}_1',\ldots,\mathbf{x}_A') \ . \tag{9}$$

On combining (1), (8), and (9) we get

$$\frac{d\mathcal{P}}{d\alpha} = \frac{1}{A!}\int dt\int d\mathbf{x}_1\ldots d\mathbf{x}_A\int dt'\int d\mathbf{x}_1'\ldots d\mathbf{x}_A' \ e^{-iE_\nu(t-t')}$$

$$\times \Phi_\nu^{(-)*}(\mathbf{x}_1,\ldots,\mathbf{x}_A)\Phi_\nu^{(-)}(\mathbf{x}_1',\ldots,\mathbf{x}_A')$$

$$\times <J_A^\dagger(\mathbf{x}_1',\ldots,\mathbf{x}_A',t')J_A(\mathbf{x}_1,\ldots,\mathbf{x}_A,t)> \ . \tag{10}$$

Equation (10) is a direct generalization of a formula for the one-body case that can be found e.g. in ref. [14]. A similar structure as in (10) has also been given for the elastic scattering of composites off nuclei [15].

In order to obtain a calculable expression from (10), we introduce the following approximation:

$$<\psi^\dagger(\mathbf{x}_A',t')..\psi^\dagger(\mathbf{x}_2',t')j^\dagger(\mathbf{x}_1',t')j(\mathbf{x}_1,t)\psi(\mathbf{x}_2,t)..\psi(\mathbf{x}_A,t)>$$

$$\simeq <j^\dagger(\mathbf{x}_1',t')j(\mathbf{x}_1,t)> \sum_\sigma \mathrm{sgn}\sigma \prod_{2\le k} <\psi^\dagger(\mathbf{x}_{\sigma(k)}',t')\psi(\mathbf{x}_k,t)> . \tag{11}$$

With (11), the expectation value of the product of A-body sources becomes

$$<J_A^\dagger J_A> = \sum_\sigma \mathrm{sgn}\sigma \sum_k <j^\dagger(\mathbf{x}_{\sigma(k)}',t')j(\mathbf{x}_k,t)> \prod_{k\ne\ell} <\psi^\dagger(\mathbf{x}_{\sigma(\ell)}',t')\psi(\mathbf{x}_\ell,t)>. \tag{12}$$

In the quasiparticle limit that underlies the dynamic models of collisions [7-9], the factors are

$$<\psi^\dagger(\mathbf{x}',t')\psi(\mathbf{x},t)> \simeq \int \frac{d\mathbf{p}}{(2\pi)^3} f(\mathbf{p};\mathbf{R},T)\; e^{i\mathbf{p}(\mathbf{x}-\mathbf{x}')}\; e^{-i\varepsilon_{\mathbf{p}}(t-t')} . \tag{13}$$

Here f is a Wigner function, $\varepsilon_{\mathbf{p}} = \varepsilon_{\mathbf{p}}(\mathbf{R},T)$ is single-particle energy, and $\mathbf{R} = (\mathbf{x} + \mathbf{x}')/2$, $T = (t + t')/2$. The expectation value of the product of single-particle sources $<j^\dagger j>$ may be expanded diagrammatically in terms of $<\psi^\dagger\psi>$, $<\psi\psi^\dagger>$, and the interaction [16,17]. The irreducible part in the expansion, containing no single particle lines,

$$<j^\dagger(\mathbf{x}',t')j(\mathbf{x},t)> = <j^\dagger(\mathbf{x}',t')j(\mathbf{x},t)>_{\mathrm{irred}} + \dots , \tag{14}$$

is identified after Wigner transformation with the single-particle production rate $-i\Sigma^<$ in standard notation [16],

$$\langle j^\dagger(\mathbf{x}',t')j(\mathbf{x},t)\rangle_{irred}$$

$$= \int\frac{d\omega}{2\pi}\int\frac{d\mathbf{p}}{(2\pi)^3}\,(-i)\Sigma^<(\mathbf{p},\omega;\mathbf{R},T)\; e^{i\mathbf{p}(\mathbf{x}-\mathbf{x}')}\; e^{-i\omega(t-t')}\quad . \tag{15}$$

For $\omega = \varepsilon_\mathbf{p}$ the production rate multiplied by a blocking factor is identical with the gain term in kinetic equations [9,16]. The other terms in the expansion (14), cf. ref. [16], are associated with the propagation following production. On using $\langle j^\dagger j\rangle_{irred}$ for $\langle j^\dagger j\rangle$, we get from (10), (12), (13), and (15),

$$\frac{d\mathcal{P}}{d\nu} = A\int dT\int d\mathbf{R}_1\ldots d\mathbf{R}_A\int\frac{d\omega}{2\pi}\int\frac{d\mathbf{p}_1}{(2\pi)^3}\cdot\cdot\frac{d\mathbf{p}_A}{(2\pi)^3}2\pi\delta(E_\nu - \omega - \varepsilon_{\mathbf{p}_2} - \cdot\cdot - \varepsilon_{\mathbf{p}_A})$$

$$\times\,(2\pi)^3\delta(\mathbf{P}_\nu - \mathbf{p}_1 - \mathbf{p}_2 - \cdot\cdot - \mathbf{p}_A)(-i)\Sigma^<(\mathbf{p}_1,\omega;\mathbf{R}_1,T)$$

$$\times\, f(\mathbf{p}_2;\mathbf{R}_2,T)\ldots f(\mathbf{p}_A;\mathbf{R}_A,T)\,g_\nu(\tilde{\mathbf{p}}_1,\ldots,\tilde{\mathbf{p}}_{A-1};\tilde{\mathbf{R}}_1,\ldots,\tilde{\mathbf{R}}_{A-1})\,. \tag{16}$$

Here $\mathbf{P}_\nu$ is the momentum of state ν,

$$\Phi_\nu^{(-)} = \exp(i\mathbf{P}_\nu(\mathbf{x}_1+\ldots+\mathbf{x}_A)/A)\;\varphi_\nu^{(-)}, \tag{17}$$

g_ν is the Wigner transform

$$g_\nu(\tilde{\mathbf{p}}_1,\ldots,\tilde{\mathbf{p}}_{A-1};\tilde{\mathbf{R}}_1,\ldots,\tilde{\mathbf{R}}_{A-1})$$

$$= \int d\tilde{\mathbf{r}}_1\ldots d\tilde{\mathbf{r}}_{A-1}\,e^{-i\sum\tilde{\mathbf{p}}_k\tilde{\mathbf{r}}_k}\;\varphi_\nu^{(-)}(\tilde{\mathbf{R}}_1+\tilde{\mathbf{r}}_1/2,\ldots,\tilde{\mathbf{R}}_{A-1}+\tilde{\mathbf{r}}_{A-1}/2)$$

$$\times\,\varphi_\nu^{(-)*}(\tilde{\mathbf{R}}_1-\tilde{\mathbf{r}}_1/2,\ldots,\tilde{\mathbf{R}}_{A-1}-\tilde{\mathbf{r}}_{A-1}/2)\;, \tag{18}$$

and we use tilde to indicate the Jacobi coordinates and conjugate momenta,

$$\tilde{\mathbf{x}}_k = \mathbf{x}_{k+1} - (\mathbf{x}_1 + \ldots + \mathbf{x}_k)/k\;,$$

$$\tilde{\mathbf{p}}_k = (k\mathbf{p}_{k+1} - (\mathbf{p}_1 + \ldots + \mathbf{p}_k))/(k+1) \ .$$

Finally, the production rate in the kinetic models [7-9] is [16]

$$-i\Sigma^{<}(\mathbf{p},\omega;\mathbf{R},T) = \int \frac{d\mathbf{p}_1}{(2\pi)^3} \frac{d\mathbf{p}'}{(2\pi)^3} \frac{d\mathbf{p}_1'}{(2\pi)^3} (1 - f(\mathbf{p}_1;\mathbf{R},T)) f(\mathbf{p}';\mathbf{R},T)$$

$$\times f(\mathbf{p}_1';\mathbf{R},T) |\mathcal{M}_{NN\to NN}|^2 \, 2\pi\delta(\omega + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}'} - \varepsilon_{\mathbf{p}_1'})$$

$$\times (2\pi)^3 \delta(\mathbf{p} + \mathbf{p}_1 - \mathbf{p}' - \mathbf{p}_1') \ . \quad (19)$$

As it stands, eq. (16) does not include the possibility of absorption due to further interactions following the production. This can be accounted for by summing up the other terms at the r.h.s. of eq. (14). We postpone the full derivation to a future publication, and quote only a final expression that is an obvious generalization of eq. (16),

$$\frac{d\mathcal{P}}{d\nu} = A\int dt \int d\mathbf{x}_1 \ldots d\mathbf{x}_A \int dt' \int d\mathbf{x}_1' \ldots d\mathbf{x}_A' \, \chi_\nu^{(-)*}(\mathbf{x}_1,\ldots,\mathbf{x}_A,t)$$

$$\times \langle j^\dagger(\mathbf{x}_1',t') j(\mathbf{x}_1,t)\rangle_{irred} \langle \psi^\dagger(\mathbf{x}_2',t')\psi(\mathbf{x}_2,t)\rangle$$

$$\times \ldots \langle \psi^\dagger(\mathbf{x}_A',t')\psi(\mathbf{x}_A,t)\rangle \chi_\nu^{(-)}(\mathbf{x}_1',\ldots,\mathbf{x}_A',t') \ . \quad (20)$$

Here the wavefunction is

$$\chi_\nu^{(-)}(\mathbf{x}_1,\ldots,\mathbf{x}_A,t) = \lim_{t'\to\infty} \int d\mathbf{x}_1' .. d\mathbf{x}_A' \, G_A^-(\mathbf{x}_1,\ldots,\mathbf{x}_A,t;\mathbf{x}_1',\ldots,\mathbf{x}_A',t')$$

$$\times e^{-iE_\nu t'} \Phi_\nu^{(-)}(\mathbf{x}_1',\ldots,\mathbf{x}_A') \ . \quad (21)$$

The function G_A^- satisfies a more general equation than (7) and $\chi^{(-)}$ satisfies a more general equation than $\Phi^{(-)}$, by inclusion of a time-dependent, generally nonlocal, complex optical potential for individual nucleons:

$$\Sigma^{-} = \mathrm{Re}\,\Sigma + i\Gamma/2 \ . \qquad (22)$$

The sign of the imaginary part is such that the norm of the wavefunction decreases in the propagation backward in time, $\|\chi\|^2 \le \|\Phi\|^2$.

Equations (16)-(20) constitute the main result of this letter. In contrast to some expressions in the past, ours have been derived without reference to ad hoc quantities. The expressions involve no adjustable parameters. The production rate for the A-body state is given by a product of the single-particle rate and the single-particle phase-space densities for particles that are already present, which we consider a natural result. The substitution of

$$\varepsilon_{\mathbf{p}_1} + \varepsilon_{\mathbf{p}_2} + \ldots + \varepsilon_{\mathbf{p}_A} \qquad (23)$$

for E_ν in (16), exact when φ is a plane wave, yields direct extension of the gain term in the approach by Remler *et al.* [10,11] to correlations (see also ref. [18]), and to other fragments than deuteron. Remler *et al.* have obtained their result from the consideration of discontinuous changes in nucleon velocity, in an analogy to that how one obtains a photon bremsstrahlung cross-section.

In general, the single-particle rate in (16) is evaluated off the energy shell. Following a collision the nucleon is off-shell and may form a cluster with other nucleons. The other nucleon in the nucleon-nucleon collision, c.f. eq. (19), acts as a catalyzer permitting the energy and momentum to be conserved. Further discussion of interactions with several particles in the initial state is given in ref. [19]. In equilibrium, a thermal occupation factor can be associated [16] with the single-particle rate, sharing with the rate the energy-argument. Using the cluster wavefunctions [20,21], we can determine temperatures $T \approx \sum\varepsilon - E_\nu$, above which energy conservation in the coalescence of nucleons into clusters ceases to matter. We get 20 MeV for d, 70 MeV for t and ^{3}He, and 140 MeV for α. Appearance of a thermal occupation factor can be shown in equilibrium directly for $\langle J_A^\dagger J_A \rangle$, and is important for justifying the temperature determination from resonance yields. Equation (20) shows that the interaction product(s) can be deflected by the mean field or absorbed on the way to detectors. For the correlations, the medium acts as a dirty lens.

An appeal of eqs. (16-21) is that the equations follow from a direct microscopic derivation and are amenable to an implementation into the existing dynamic codes. For the first time it is possible to calculate the production of $A > 2$ clusters. The codes provide the nucleon Wigner-functions at all times, the off-shell (if needed) rate can be calculated with minor modifications to existing procedures, and optical potential for χ can be obtained as well. Optimally, the effects of optical potential can be estimated along a classical c.m. trajectory of the nucleon set ν. A simpler procedure would just consist in the use eq. (16) with those emission processes where the particles from the set do not collide any more in the reaction.

The formula proposed in ref. [4] may be derived from eq. (20), following a set of approximations, giving additional insight into the equation. The necessary approximations in (20) include the quasiparticle approximation for the c.m. motion of ν with the internal state unaffected by the exterior, the approximation (13), and, after the integration over microscopic time and space variables, the substitution of (23) for E_ν, and $\mathbf{P}_\nu/A \equiv \bar{\mathbf{p}}$ for $\mathbf{p}_k$ in f and $\Sigma^<$, under the assumption of low relative momenta. Further, the values of f are obtained from the integration of production and absorption in the past along an assumed straight-line trajectory,

$$f(\bar{\mathbf{p}};\mathbf{R},T) = \int_{-\infty}^{T} dT' \ (-i)\Sigma^<(\bar{\mathbf{p}},\varepsilon_{\bar{\mathbf{p}}};\mathbf{R}+\frac{\bar{\mathbf{p}}}{m}(T'-T),T')$$

$$\times \exp\left(-\int_{T'}^{T} dT'' \ \Gamma(\bar{\mathbf{p}},\varepsilon_{\bar{\mathbf{p}}};\mathbf{R}+\frac{\bar{\mathbf{p}}}{m}(T''-T),T'')\right) , \qquad (24)$$

The norm of the wavefunction for ν can be found from integration of absorption from the future towards the past, giving, after symmetrization of the times,

$$\frac{d\mathcal{P}}{d\nu} = \int dT_1 \int d\mathbf{R}_1 \ldots \int dT_A \int d\mathbf{R}_A \ D(\bar{\mathbf{p}};\mathbf{R}_1,T_1)\ldots D(\bar{\mathbf{p}};\mathbf{R}_A,T_A)$$

$$\times \ |\varphi_\nu(\tilde{\mathbf{z}}_1,\ldots,\tilde{\mathbf{z}}_{A-1})|^2 \ . \qquad (25)$$

The variables for φ are $\mathbf{z}_k = \mathbf{R}_k - \frac{\bar{\mathbf{p}}}{m} T_k$, and D is the distribution of the last collisions of particles, in time, space, and momentum, or otherwise the representation of the sources as seen from the outside through the medium

$$D(\bar{\mathbf{p}};\mathbf{R},T) = -i\Sigma^{<}(\bar{\mathbf{p}},\epsilon_{\bar{\mathbf{p}}};\mathbf{R},T)$$

$$\times \exp\left(-\int_T^{\infty} dT' \; \Gamma(\bar{\mathbf{p}},\epsilon_{\bar{\mathbf{p}}};\mathbf{R}+\frac{\bar{\mathbf{p}}}{m}(T'-T),T')\right) \; , \qquad (26)$$

here in the absence of rarefaction. Note that an expression such as (26) cannot be obtained by simply Wigner transforming the expectation value exhibited at the l.h.s. of (14). The structure of (25) is easiest to comprehend when ignoring absorption. Starting from (16) one then directly arrives at (25). Explicitly, the steps in the derivation of (25) with (26) will be given in a forthcoming paper. While our derivation relates the function D to the single-particle rates $-i\Sigma^{<}$ and Γ, the distribution is actually readily available from the existing dynamic codes [7-9]. In the case of two-particle coincidence cross section, eq. (25) reduces to the equation proposed in ref. [4]. We think that our derivation of Koonin's formula is more fundamental than the one in previous work [13]. Other than for the calculation of correlation functions, eq. (25) can serve to give a rough estimate of the A = 2,3 cluster production in high-energy reactions.

In order to obtain expressions useful in the description of cluster-cluster correlations, or e.g. the production of a cluster by a lighter cluster picking up more nucleons, one needs to go beyond the approximation (11) and isolate the few-body densities [19]. Also one needs to go beyond (11) to take into account the dynamic effect of a catalyzing nucleon onto the A-body rate. For clusters, the effect can be estimated from the cluster breakup cross-section, on substituting $|\mathcal{M}_{N...N\to\nu N}|^2$ for $\int|\mathcal{M}_{NN\to NN}|^2 g_\nu$ in (16), see ref. [19]. The effect can be quite sizeable for low relative energies.

Concerning eqs. (1) and (10), one should mention that one might consider expectation values with more time-arguments than in (3), with a reduction resulting in more single-particle sources, and the wavefunctions of Bethe-Salpeter. For A = 2 there is no advantage, as an isolated particle can only survive when on the energy shell for macroscopic times. At least in principle, the procedure might have some advantage for more particles, because one could describe the buildup of correlation between the particles before the last one is emitted.

To conclude, we have given here new formulas for calculating the probabilities for emission of a set of nucleons with definite momenta, and for cluster emission. The formulas reveal microscopic aspects of the emission, and they stem from a more fundamental derivation than the expressions used in the past. It should be mentioned that the case of boson emission goes very much along the same lines as the case of fermion emission, although care must be taken of the generally relativistic aspects. A more detailed study of the subject and numerical applications are in progress.

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