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ELASTIC AND INELASTIC SCATTERING OF UNSTABLE NUCLEI

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ABSTRACT

Some problems related to the study of the halo size, excitation, and applications of reactions induced by unstable nuclei are discussed. Particularly, the elastic scattering of protons by neutron-rich nuclei, and the description and application of Coulomb excitation of such nuclei is presented.

1 - Introduction

The study of nuclear reactions induced by secondary radioactive beams has raised many interesting questions concerning the reaction mechanisms, nuclear structure, and applications to other fields[1]). In this work, some of these questions are reviewed, and special care is given to (a) the determination of the neutron-halo size by means of a study of proton elastic scattering, (b) the use of radioactive beams to access information on the radiative capture cross sections of astrophysical interest, (c) the Coulomb dissociation of ^{11}Li, with comparison among different models, and (d) non-perturbative character of Coulomb dissociation of weakly-bound nuclei.

2 - Elastic Scattering of Protons from ^{11}Li and the Neutron Halo

The most simple scattering process, i.e. elastic scattering, has not been measured experimentally with secondary radioactive beams. This basically stems from the low current of the secondary beam. However, it is of interest to do a preliminary qualitative calculation to assess the importance of the halo in the elastic distribution. Recent study of this question in the context of heavy ions has been made[2)] with the major conclusion being that systems such as $^{11}Li+^{12}C$ behave in as much the same way as, e.g., $^{12}C+^{12}$ C. Stronger refractive effects were found for $^{11}Li+^{12}$ C.

Here, we consider[3)] the much simpler problem of the elastic scattering angular distribution of $p+^{11}Li$ at E_{Lab} = 100 MeV. Glauber's theory is used, with the $p+^{11}Li$ optical potential obtained within the "$t\rho$" approximation[4)] with ρ_{Li}, the matter density of ^{11}Li, calculated within constrained Hartree-Fock theory[5)]. Medium effects on the nucleon-nucleon t matrix, are fully taken into account. The effect of the halo neutrons on $d\sigma/d\Omega$ is assessed through comparison with the $p+^{12}$ C and $p+^{9}Li$ systems at $E_{Lab}=$ 100 and 800 MeV. This is displayed in fig. 1.

We note from fig. 1 (a) that the position of the second minimum occurs at $\theta = 53°$ in ^{9}Li, $\theta = 47°$ in ^{11}Li, $\theta = 46°$ in ^{12}C, clearly showing the influence of the matter distribution in ^{11} *Li.* The above features should be easily tested experimentally.

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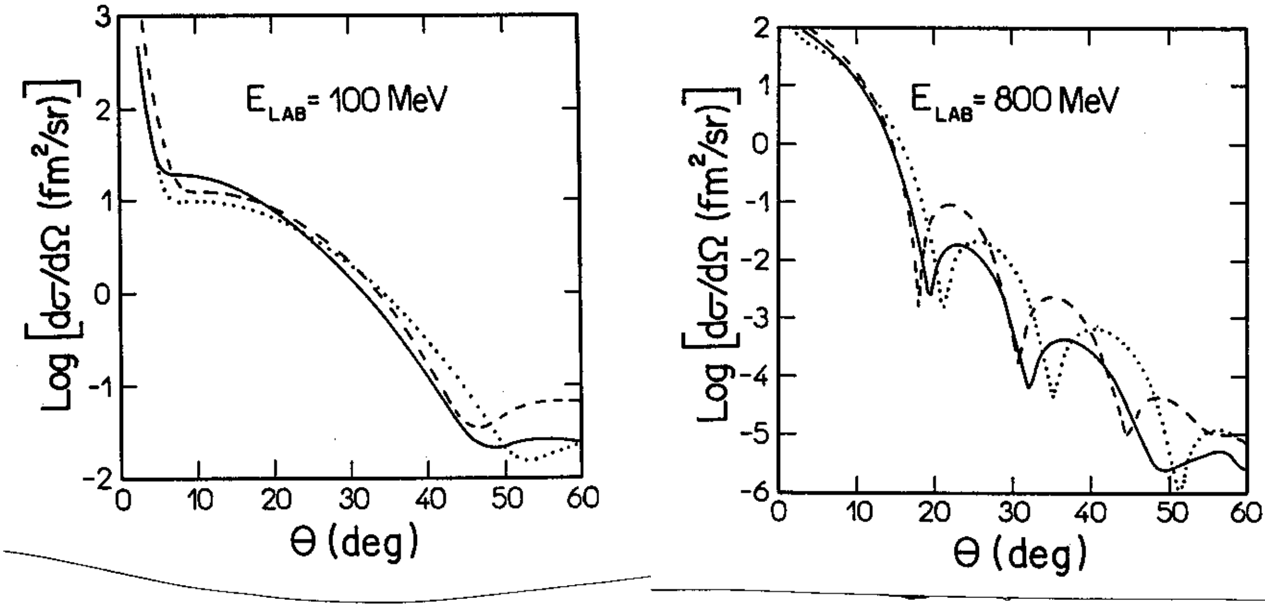

Fig. 1 - (a) The elastic scattering cross sections for the three systems: $p+^{9}Li$ (dotted), $p+^{12}C$ (dashed), and $p+^{11}Li$ (solid). (b) Same as in (a), but at $E_{Lab}=$ 800 MeV.

3 -Astrophysical S-Factors from Radioactive Beam Dissociation

The radiative capture reaction is one of the most important in the formation of various elements in the universe. One example of such reactions is $^{7}Be(p,\gamma)^{8}B$, the decay of which is believed to yield about 70-80% of the neutrinos of our Sun, that are detectable using the $^{37}Cl(\nu,e^{-})^{37}Ar$ reaction. Although there are some experiments on proton capture reactions on ^{7}Be, which has a half-live of 54 days, disagreement among the results are found[6)].

Besides the possibility to study entirely new phenomena, unobservable with current methods, radioactive beams can give direct information on the astrophysical S-factors for radiative capture reactions leading to the formation of unstable nuclei. This possibility arises because of the huge Coulomb dissociation cross sections of radioactive projectiles, due to their low binding energy. The Coulomb dissociation cross section for the reaction $a+A\rightarrow b+c+A$ is given by[7)]

$$\sigma_{CD}=\sum_{\pi\lambda}\int n_{\pi\lambda}(\varepsilon)\,\sigma_{\gamma}^{\pi\lambda}(\varepsilon)\,\frac{d\varepsilon}{\varepsilon} \tag{1}$$

where $\sigma_{\gamma}^{\pi\lambda}(\varepsilon)$ is the photo-disintegration cross section $\gamma+a\rightarrow b+c$, with the photon energy ε, and multipolarity $\pi=E$ (electric) or M (magnetic), and $\lambda=1,2,...$ (order). The photo-disintegration cross section is related to the radiative capture cross section through the detailed balance theorem. The radiative capture cross section is usually written in terms of the astrophysical S-factor as

$$\sigma(b+c\rightarrow a+\gamma)=\frac{S(E)}{E}\ \exp\Big[-2\pi\eta(E)\Big] \tag{2}$$

where E is the centre-of-mass energy of relative motion of the $b+c$-system, $\eta = Z_b Z_c e^2/\hbar v$, with $v=\sqrt{2\mu E}/\hbar$, μ is the reduced mass of $b+c$. This definition factors out the steep increase of the radiative capture cross sections at low relative energies.

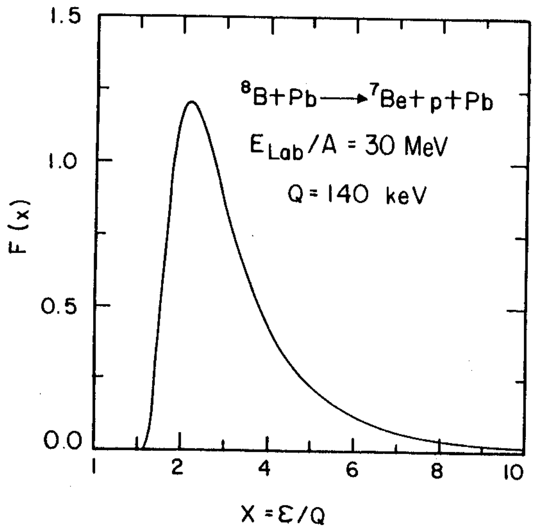

Fig. 2 - Spectral distribution function $F(x)$ for the dissociation of ^8B projectiles, incident on ^{208}Pb targets at 30 $MeV/nucleon$, as a function of the ratio between the total energy transfered by the Coulomb field to ^8B and its proton separation energy.

The radiative capture cross section is a sum over all multipolarities. In most cases, one (or two) multipolarity dominates the process. This also occurs with the Coulomb-induced break-up cross section given by eq. (1), depending on the function $n_{\pi\lambda}(\varepsilon)$. These functions are interpreted as the *number of equivalent (virtual) photons*, incident on nucleus a, provided by the Coulomb field of nucleus A. They can be calculated for all bombarding energies with good accuracy, as was demonstrated in ref. 8. In very high energy collisions, simple analytical formulas were obtained[7)]. Also, for all multipolarities, $n_{\pi\lambda}(\varepsilon)$ decreases rapidly with ε, thus enhancing the lower energy part of the photo-disintegration cross section, which enters in eq. (1).

Let us assume that a single multipolarity prevails in the radiative capture cross section, as well as in the sum of eq. (1). Then, using eq. (2), the detailed balance theorem, and defining $x=\varepsilon/Q$, where Q is the energy release in the radiative capture, we find

$$\sigma_{CD}=\frac{(2J_b+1)(2J_c+1)}{(2J_a+1)}\ \frac{\mu c^2}{Q^2}\ \int_1^\infty F(x)\ S(x)\ dx \tag{3}$$

where

$$F(x) = n_{\pi\lambda}(x)\ \exp\Big[-2\pi\eta(x)\Big]\ \frac{1}{x^3} \tag{4}$$

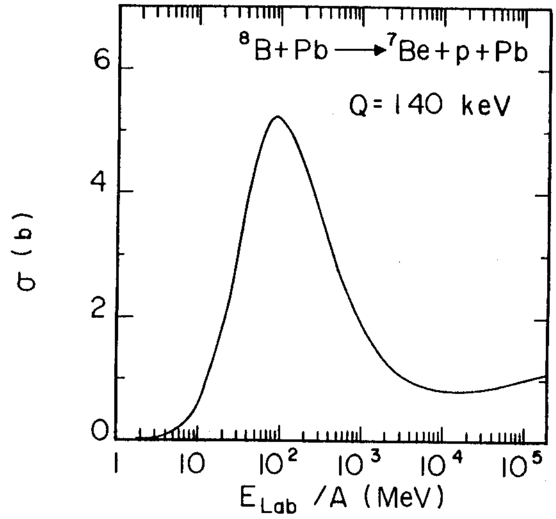

Fig. 3 - Total Coulomb dissociation cross section of ^8B projectiles, incident on ^{208}Pb targets, as a function of the laboratory energy per nucleon.

The function $F(x)$ peaks strongly at $x = x_0 \approx 2$. That is, in the absence of a resonance in the radiative capture cross section at low energies, the Coulomb dissociation selects preferentially the kinetic energy of the fragments equal to their binding energy Q. Indeed, this is shown in figure 2, where we plot $F(x)$ for the reaction $^8B + Pb \rightarrow {}^7Be + p + Pb$ at 30 MeV/nucleon, assuming an $E1$-transition. The reaction $^7Be(p,\gamma)^8B$ has a resonance peak at $E \approx 730\ keV$, which is far from this peak ($E_0 \approx 140\ keV$). Based on this result, and assuming that $S(E)$ is a slowly-varying function of E, we may write the eq. (4) as

$$\sigma_{CD} = \frac{(2J_b+1)(2J_c+1)}{(2J_a+1)}\ \frac{\mu c^2}{Q^2}\ S(E \approx Q) \int_1^\infty F(x)\, dx\,. \tag{5}$$

This allows the direct determination of astrophysical S-factor for low relative energies E, of order of the Q-value. For weakly bound nuclei, as ^8B, this implies energies around 140 keV. This is about the lowest energy so far measured[9)] for the reaction $^7Be(p,\ \gamma)^8B$. The value of $S(E \approx 0)$ which is needed for this reaction, was obtained from the measurements of Filippone et al.[9)] by assuming that the S-factor is constant for $E_p < 500\ keV$. But extrapolations based on other methods and calculations[6)] indicates that the S-factor for this reaction at low energies is not known as definetely as the assumed uncertainty of 9% would imply. Therefore, this reaction seems to require further investigation. Although restricted to the values of $S(E)$ around $E = Q$, the measurement of the total cross section for Coulomb dissociation can shed more light on these questions.

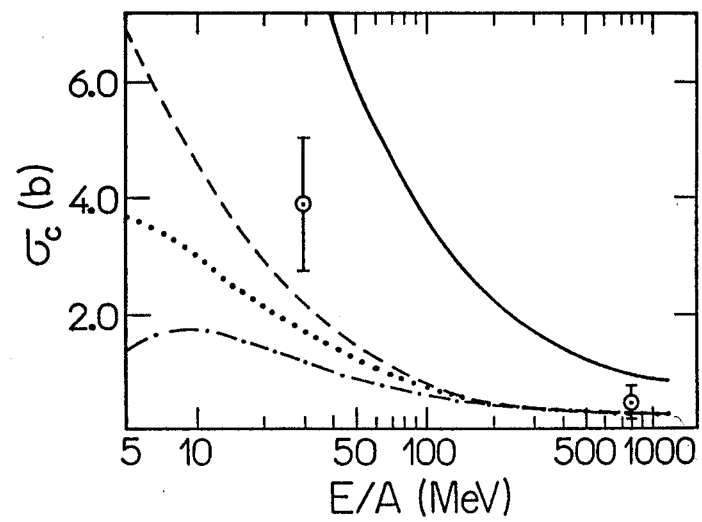

Fig 4 - Cross sections for the Coulomb dissociation of ^{11}Li projectiles incident on ^{197}Au targets, as a function of the laboratory energy per nucleon. The curves are obtained with different theoretical models (see text). Data are from refs. 17 and 18.

In figure 3 we show the Coulomb dissociation cross section for the reaction $^8B + Pb \rightarrow {}^7Be+p+Pb$, as a function of the laboratory energy per nucleon of the ^8B projectiles. We use eq. (5), with the commonly acccepted value[9] of $S(E=0) = 0.024\ keV\ b$. We see that the largest value of σ_{CD} is attained at $E_{Lab} \approx 100\ MeV/nucleon$, and is about 5.3 b. Therefore, reactions around this energy allows the extraction of $S(E \approx Q)$ with largest accuracy. The approximation given by eq. (6) is better, the narrower the peak, as displayed in figure 2, is. In fact, $S(E \approx Q)$ must be viewed as an average of $S(E)$ over energies around the peak. Experimental strategies based on this method should also account for the variation of the width of the peak over the bombarding energy. It is found that $S(E)$ in eq. (5) is in fact averaged out over the range $\Delta E = Q{-}3Q$, depending on the bombarding energy.

4 - Coulomb Dissociation of ^{11}Li: Comparison Among Different Models

Generally, the $E1$ multipolarity dominates the Coulomb excitation cross section given by eq. (1). Therefore, the Coulomb dissociation of ^{11}Li allows the study of the $E1$ strength function of this exotic nucleus. This has been indeed studied by several authors recently [See, e.g., refs. 10-16]. Unfortunately, up to now, only a few experimental data are available[17-18], for the Coulomb dissociation of ^{11}Li at 800 $MeV/nucleon$, and at 30 $MeV/nucleon$, respectively.

Let us compare the Coulomb dissociation cross-sections of ^{11}Li projectiles incident on ^{197}Au, using for the dipole strength function $dB(E1)/dE$ different models discussed in the literature. In figure 4 we show a comparison among the cross sections obtained with the modified independent particle model[10] (dashed), the correlated model [11] (dotted), the hybrid RPA-cluster model[13] (dashed-dotted), and the cluster model[12] (full).

One sees that none of the models accounts for the low energy data point (E_{lab} = 30 MeV/nucleon). Whereas the cluster model overestimates the cross section, the other models fall short in value. The recent calculation of Lenske and Wambach[14], using the quasiparticle RPA method, also fall short in value (the cross section for this case is not shown in the figure, as it almost coincides with the independent particle result). Further theoretical studies and experiments are clearly needed to settle the matter.

5 - Non-Perturbative Character of Coulomb Excitation of Weakly-Bound Nuclei

The Coulomb excitation amplitude of weakly-bound projetiles (A_P, Z_P) incident on a target (A_T, Z_T) is given by[7,8]

$$a_{0f} = -i\frac{Z_T e}{\hbar v}\sqrt{\frac{8\pi}{3}}\,\frac{\xi}{a_0}\,e^{-\pi\xi/2}\left[D_{0f}^{(1)}\left(K'_{i\xi}(\frac{\epsilon\xi}{\gamma})+\frac{\sqrt{\epsilon^2-1}}{\gamma\epsilon}\,K_{i\xi}\,(\frac{\epsilon\xi}{\gamma})\right)\right.$$
$$\left.-\,D_{0f}^{(-1)}\left(K'_{i\xi}(\frac{\epsilon\xi}{\gamma})-\frac{\sqrt{\epsilon^2-1}}{\gamma\epsilon}\,K_{i\xi}\,(\frac{\epsilon\xi}{\gamma})\right)\right], \qquad (6)$$

where $K_{i\xi}$ are the modified Bessel functions of imaginary indices, $K'_{i\xi} = dK_{i\xi}(x)/dx$, $\xi = \omega a_0/\gamma v$, $a_0 = Z_P Z_P e^2/m_0 v^2$, $\epsilon = \sqrt{1+\gamma^2 b^2/a_0^2}$, and $m_0 = m_N \,.\, A_P A_T/(A_P+A_T)$. The formula (6) is valid for all bombarding energies.

$D_{0f}^{(m)}$ are the electric dipole matrix elements with angular momentum projection m. These matrix elements are easily calculated within the cluster model[12]. This allows us to estimate the dissociation probability as a function of the impact parameter b. Using this model it is easy to show that the dissociation probability $P_{0f}(b)$ exceeds unity for the break-up of ^{11}Li projectiles incident on uraniun at intermediate energies ($\sim$ 100 $MeV/nucleon$), and b = 15 fm. This points to the failure of perturbation theory for Coulomb excitation of such weakly-bound nucleus. As we learned from the last section, this result maybe fortuitous, since the cluster model seems to overestimate the experimental data. However, a closer study of this question maybe fruitful[19], since such behavior could also be manifest with other excitation models for weakly-bound nucleus.

For impact parameters b such that $\omega b/\gamma v \ll 1$, we are allowed to use the sudden approximation. In this case, a simple expression can be obtained for the transition probability to the continuum of the $b + c$ system due to the Coulomb excitation. It reads

$$P_{0f}(b) = 1 - \frac{4\eta^2}{\mathcal{C}^2}\left[atan\left(\frac{\mathcal{C}}{2\eta}\right)\right]^2 \qquad \text{where} \qquad \mathcal{C} = \frac{2Z_T e^2}{\hbar v b}\left(Z_b\,\frac{m_c}{m_a} - Z_c\,\frac{m_b}{m_a}\right). \quad (7)$$

Application of this equation to the reaction mentioned above gives a total cross section (obtained by integrating eq. 7 over impact parameters, from a minimum value $R = R_a + R_T$) which is smaller than that obtained with use of eq. (6) by 10%. This apparently indicates that only for a small range of impact parameters the first-order perturbation theory needs modification. In order to have insight on the accuracy of this statement it maybe useful to perform a coupled-channels calculation. To this aim we assume that the initial state is the only one which is bound, and that the continuum is represented by orthonormal wave packets of the form

$$\phi_{E_j}(\mathbf{r}) = \frac{1}{\Delta E_j}\int_{E_j-\Delta E_j/2}^{E_j+\Delta E_j/2} dE\,\phi_E(\mathbf{r})\,, \qquad (8)$$

where $\phi_E(\mathbf{r})$ are wave functions for the continuum, with relative kinetic energy of the $b+c$ system equal to E. A limited set of grid points with spacing ΔE_j are used. The interaction potential is given by the coupling of the Coulomb field of a straight-line moving charge with the transition charge of the nucleus. Such procedure leads to the determination of the break-up probability $P_j(t)$ as a function of the time. A preliminary calculation[19] has shown that $P_{0f}(t) = \sum_{j\neq 0} P_j(t)$ grows up, reaches a maximum at $t = 0$ when the nuclei are at the distance of closest approach, and, interestingly, falls down until it reaches a constant value for $t > b\hbar v/E_j$. This indicates that, although $P_{0f}(t \to \infty)$ maybe small, the continuum-continuum couplings at $t = 0$ are important, and that higher order effects are needed. That is, perturbation calculation is invalid for the break-up of loosely-bound nuclei colliding with a large Z nucleus, since unitarity is violated, even if the final probabilities are small. Such effect has indeed been reported previously by Momberger et al., in a study of electron-positron production in relativistic heavy ion collisions[20]. Since the production of the pair maybe viewed as the ionization of an electron from the negative energy see, the similarity of these results are expected.

5 - Conclusions

From the discussion presented in this article one concludes that: (a) proton elastic scattering by neutron-rich nuclei is a direct and useful method to deduce the halo size of these nuclei, (b) the Coulomb excitation mechanism, which has applications in, e.g., astrophysics, is not completely understood, and (c) intriguing models to describe the features of the reaction and excitation mechanisms are needed. This explains the rapidly growing interest of the field in recent years.

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