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ABSTRACT

Damping of nuclear collective vibration8 is studied on the basis of a Boltzmann-type transport equation with memory effects. It is shown that the memory effects play an essential role for a proper treatment of the damping properties of collective vibrations. A simple analytical formula is derived for the spreading widths at finite temperatures which provides a reasonable description for the gross properties of the damping of giant dipole and giant quadrupole resonances over a broad range of nuclei.

In order to describe damping of collective nuclear vibration8 (giant resonances) several approximate procedures have been developed in recent year8 [1–2]. In particular, the linearized limit of the extended time-dependent Hartree-Fock (TDHF) theory provide8 a convenient framework for describing both the structure and the damping of collective vibiations. However, it was pointed out several years ago that the memory effects in the collision term of the extended TDHF equation must be taken into account for a proper treatment of the damping widths [3–4]. In medium mass and heavy nuclei, the overwhelming contribution to the damping widths arises from the collisional spreading of the collective state due to decay into two particletwo hole states. Without taking the memory effects into account, the collisional damping is severely understimated and

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in fact it vanishes at zero temperature in the linearized limit of the collision term [5]. In the present work we calculate the spreading width of collective vibrations due to collisional damping in the basis of a semi-classical transport equation with a memory dependent collision term. We obtain a simple analytical expression which provides a reasonable description for the observed damping widths of isoscalar and isovector giant resonances in medium mass and heavy nuclei at zero temperature, as well as at finite temperatures.

The structure of giant resonance excitations can be very well described in phase-space as scaling vibrations with associated momentum space distortions [5–8]. Here we wish to study the damping properties of giant resonances rather than their full description. For this purpose it is sufficient to consider the distribution function f(p,t) in momentum space and calculate the relaxation rates for distortions of various multipolarity. First, we discuss the isoscalar vibrations and in the next step, we extend the treatment to the isovector modes. For simplicity, we consider a homogeneous system with a sharp surface, and assume that the spin-isospin averaged momentum distribution is determined by a Boltzmann-type transport equation with a memory dependent collision term as [3],

$$\frac{\partial}{\partial t} f(\mathbf{p},t) \equiv K(f) = \int_0^{\infty} d\tau \int d^3p_2\, d^3p_3\, d^3p_4\, \frac{W}{2\pi} \left(e^{i\tau\Delta\epsilon} + e^{-i\tau\Delta\epsilon}\right)$$

$$\cdot \left[(1-f_1)\,(1-f_2)\,f_3\,f_4 - f_1\,f_2\,(1-f_3)\,(1-f_4)\right]_{t-\tau} \tag{1}$$

where all the occupation factors $f_j = f(p_j,t)$ are evaluated at time $t-\tau$, $\Delta\epsilon = \epsilon_3+\epsilon_4-\epsilon_1-\epsilon_2$ and W denotes the spin-isospin averaged transition rates

$$W = \frac{g}{(2\pi\hbar)^3} \cdot \frac{4}{m^2} \cdot \frac{d\sigma}{d\Omega}\, \delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_3-\mathbf{p}_4) \tag{2}$$

with $g = 4$ as the degeneracy factor. Usually a Markovian approximation is introduced into the collision term by neglecting the τ-dependence of the occupation factors in the collision term (memory effects), then the time integration over history gives rise to two-body energy conserving transition rates with $\delta(\Delta\epsilon)$ in the collision term. However, the Markovian approximation breaks down in situations when the characteristic time of the occupation factors f(p,t) is in the same order as the memory time of the collision kernel in eq. (1) [3]. Giant resonance excitations constitute such a situation in which the nuclear density in both r-space and p-space execute small amplitude, high frequency vibrations of a certain multipole shape.

For describing damping properties of small amplitude vibrations, we linearize the memory dependent transport eq. (1) around a finite temperature equilibrium $f_0(p)$,

$$f(p,t) = f_0(p) + \delta f(p,t) = f_0(p) + \chi(p,t)\,\frac{\partial}{\partial\epsilon} f_0 \tag{3}$$

where

$$\chi(p,t) = \alpha_L(t)\, p^L\, P_L(\theta)$$

$$= (\alpha_L\, e^{i\Omega t} + \text{c.c.})\, p^L\, P_L(\theta) \tag{4}$$

characterizes the multipolarity of the distortion of the momentum distribution from spherical equilibrium with $P_L(\theta)$ as a Legendre function, and Ω is, in general, a complex frequency of the harmonic vibrations. In the linearized limit, since the time dependence of the occupation factors are known explicitly, the time integration over history in eq. (1) can be done, and the memory incorporated linearized transport equation becomes

$$\frac{\partial}{\partial t}\,\delta f(p,t) \equiv \delta K = I \cdot \delta f$$

$$= \int d^3p_2\, d^3p_3\, d^3p_4\, \frac{W}{2}\,[\delta(\Delta\epsilon-\Omega) + \delta(\Delta\epsilon+\Omega)]\,(\delta\tilde{F} - \delta F) \tag{5}$$

where

$$\delta F = \delta \overset{o}{f}_1 \overset{o}{f}_2 (1-\overset{o}{f}_3)(1-\overset{o}{f}_4) + \overset{o}{f}_1 \delta f_2 (1-\overset{o}{f}_3)(1-\overset{o}{f}_4)$$

$$- \overset{o}{f}_1 \overset{o}{f}_2 \delta \overset{o}{f}_3 (1-\overset{o}{f}_4) - \overset{o}{f}_1 \overset{o}{f}_2 (1-\overset{o}{f}_3) \delta f_4 \tag{6}$$

and $\delta\tilde{F}$ is obtained from eq. (6) by interchanging the indices as $1 \leftrightarrow 3$, $2 \leftrightarrow 4$. An expression for the collision rate similar to the linearized collision term in eq. (5) was proposed by Landau under the assumption that a finite energy is lost during a binary collision [9–10].

We define the relaxation rate of the distorted momentum distribution as [11],

$$\frac{1}{\tau_s} = -\frac{\int d^3p \;\; \chi(p) \;\; \delta K_s}{\int d^3p \;\; \chi(p) \;\; \delta f_s}$$

$$= \frac{\int d^3p_1 \; d^3p_2 \; d^3p_3 \; d^3p_4 \; \frac{W}{4} \; \delta(\Delta\epsilon - \Omega) \;\; \Delta\chi \;\; (\delta F \; - \; \delta\tilde{F})}{\int d^3p \;\; \chi^2 \; \frac{\partial}{\partial\epsilon} f_0} \tag{7}$$

where $\Delta\chi = \chi_1 + \chi_2 - \chi_3 - \chi_4$ with $\chi_j = \chi(p_j)$. In obtaining this result several terms have been combined using the symmetry properties of the collision term, and we insert a subscript "s" which refers to the situation that τ_s is the relaxation rate for the isoscalar modes. Incorporating the memory effects in the transport eq. (1) enforces the energy conservation between the collective state, with energy $\hbar\Omega$, and the more complicated two particle-two hole states in the relaxation rate, and properly accounts for the available phase-space for damping. On the other hand, the Markovian approximation consists of replacing the proper energy conserving factor $\delta(\Delta\epsilon - \Omega)$ with $\delta(\Delta\epsilon)$ in eq. (7); consequently, the resultant expression severely restricts the available phase-space for damping. In particular, at zero temperature all two-body collisions are Pauli blocked and hence the collisional width vanishes in the Markovian approximation. Using the fact that transitions are concentrated around the vicinity of the Fermi surface, and employing the standard coordinate transformation [12], the energy and angular parts of the momentum integrals in eq. (7) can approximately be factored as

$$\frac{1}{\tau_s} = \left[\frac{m}{2}\right]^3 \int d\epsilon_1\, d\epsilon_2\, d\epsilon_3\, d\epsilon_4\, \delta(\Delta\epsilon - \Omega)\, Z \cdot I_s\,. \tag{8}$$

Here

$$Z = \frac{\partial}{\partial\epsilon_1} \overset{o}{f}_1 \overset{o}{f}_2 (1-\overset{o}{f}_3)(1-\overset{o}{f}_4) + \frac{\partial}{\partial\epsilon_1} \overset{o}{f}_1 (1-\overset{o}{f}_2) \overset{o}{f}_3 \overset{o}{f}_4 \tag{9}$$

and the angular part of the integral with the notation of ref. [12] is given by

$$I_s = \frac{\int d\Omega_1\; d\Omega\; d\phi \left(\cos\frac{\theta}{2}\right)^{-1} W\; (\Delta P_L)^2}{\int d\Omega\; P_L^2} \tag{10}$$

with $\Delta P_L = P_L(\theta_1) + P_L(\theta_2) - P_L(\theta_3) - P_L(\theta_4)$. The factorization of the energy and angular integrals introduces about 10% error in the calculations of the relaxation rates. For temperatures which are small compared to the Fermi energy, the energy integrals in eq. (8) can be done exactly [13], and we find

$$\frac{1}{\tau_s} = \left[\frac{m}{2}\right]^3 \left[\frac{1}{2}\hbar^2\,\Omega^2 + \frac{2}{3}\pi^2\,T^2\right] I_s\,. \tag{11}$$

In order to describe the damping of isovector modes we need to generalize the previous treatment by distinguishing proton and neutron degrees of freedom in the relaxation process. The relaxation of the momentum distributions of protons and neutrons are determined by coupled transport equations,

$$\frac{\partial}{\partial t} f_p(\mathbf{p},t) = K_{pp}(f) + K_{pn}(f)$$
$$\frac{\partial}{\partial t} f_n(\mathbf{p},t) = K_{nn}(f) + K_{np}(f) \tag{12}$$

where K_{pp}, K_{pn}, . . . are the memory dependent collision terms between protons and neutrons, which are given by expressions similar to the collision term in eq. (1). In isoscalar modes, proton and neutron distributions vibrate in phase with the same amplitude,

$$\delta f_p(\mathbf{p},t) = +\delta f_n(\mathbf{p},t) = \alpha_L(t)\, p^L P_L(\theta) \frac{\partial}{\partial \epsilon} f_o \tag{13}$$

whereas, in the isovector modes they vibrate out of phase,

$$\delta f_p(\mathbf{p},t) = -\delta f_n(\mathbf{p},t) = \alpha_L(t)\, p^L P_L(\theta) \frac{\partial}{\partial \epsilon} f_o\,. \tag{14}$$

The equation of motion describing isoscalar vibrations is obtained by linearizing eqs. (12) and adding them side by side, $\delta f_s = \delta f_p + \delta f_n$,

$$\frac{\partial}{\partial t} \delta f_s \equiv \delta K_s = I_{pp} \cdot \delta f + I_{nn} \cdot \delta f + I_{pn} \cdot \delta f + I_{np} \cdot \delta f \tag{15}$$

where I_{pp}, I_{pn}, ... are the linearized collision kernels corresponding to the collision terms K_{pp}, K_{pn}, ... in eqs. (12), and the distortion δf is given by eq. (13) for protons and neutrons. This is the same equation, eq. (5), that we use to describe the damping of the isoscalar vibrations in the previous section. The equation of motion describing the isovector vibrations is obtained by subtracting the linearized version of the coupled transport eqs. (12) side by side, $\delta f_v = \delta f_p - \delta f_n$,

$$\frac{\partial}{\partial t}\,\delta f_V \equiv \delta K_V = I_{pp}\cdot\delta f - I_{nn}\cdot\delta f + I_{pn}\cdot\delta f - I_{np}\cdot\delta f \tag{16}$$

where the distortion δf is given by eq. (14) for both protons and neutrons. The relaxation rate τ_V for an isovector distortion is defined by replacing δK_s and δf_s in eq. (7) with the corresponding quantities δK_V and δf_V. Performing energy integrals, we obtain for the relaxation rate,

$$\frac{1}{\tau_V} = \left(\frac{m}{2}\right)^3\left[\frac{1}{2}\hbar^2\,\Omega^2 + \frac{2}{3}\pi^2\,T^2\right]I_V \tag{17}$$

where the angular part of the integral, with the notation of ref. [12], is given by

$$I_V = \frac{\int d\Omega_1\ d\Omega\ d\phi\ \left(\cos\frac{\theta}{2}\right)^{-1}\left[\tilde{W}_{pn}\ (\Delta P_L)^2 + W_{pn}\ (\Delta\tilde{P}_L)^2\right]}{\int d\Omega\ P_L^2} \tag{18}$$

with $\Delta\tilde{P}_L = P_L(\theta_1) - P_L(\theta_2) - P_L(\theta_3) + P_L(\theta_4)$, $\tilde{W}_{pn} = (W_{pp} + W_{nn})/2$. Here W_{pp}, W_{nn}, and W_{pn} are the transition rates for proton-proton, neutron-neutron and proton-neutron collisions, respectively, which are determined by the corresponding scattering cross-sections as in eq. (2) with $g = 2$.

Eqs. (11) and (17) allow us to calculate the relaxation rates for distortions of any multipolarity, and hence determine the spreading width of collective vibrations at finite temperatures using the relations

$$\Gamma_s = \frac{\hbar}{\tau_s} \quad \text{and} \quad \Gamma_V = \frac{\hbar}{\tau_V}\,. \tag{19}$$

Here we consider a few lowest order isoscalar ($L = 2,3$) and isovector ($L = 1,2$) vibrations. Assuming isotropic scattering cross-sections, the relevant angular integrals I_s and I_V in eqs. (10) and (18) can be evaluated analytically, and we find simple analytical formulas for the spreading widths. The formula for the width of isoscalar quadrupole and octupole vibrations (L = 2,3) is given by

$$\Gamma_{\rm S} = \hbar \frac{4\pi^2}{2L+1} \sigma_{\rm S}\, \rho\, v_{_F} \left[\left[\frac{\hbar\Omega}{\pi \epsilon_{_F}}\right]^2 \frac{3}{4} + \left[\frac{T}{\epsilon_{_F}}\right]^2 \right] \tag{20}$$

where $\sigma_{\rm S} = (\sigma_{\rm pp} + \sigma_{\rm nn} + 2\sigma_{\rm pn})/4$ is the spin-isospin averaged cross-section and ρ, $v_{_F}$ denote the nuclear matter density and Fermi velocity, respectively. Similarly, the formula for the spreading widths of isovector dipole and quadrupole vibrations (L = 1,2) is given by

$$\Gamma_{\rm V} = \hbar \frac{4\pi^2}{2L+1} \sigma_{\rm V}\, \rho\, v_{_F} \left[\left[\frac{\hbar\Omega}{\pi \epsilon_{_F}}\right]^2 \frac{3}{4} + \left[\frac{T}{\epsilon_{_F}}\right]^2 \right] \tag{21}$$

where $\sigma_{\rm V} = \sigma_{\rm pn}/2$ for L = 1 and $\sigma_{\rm V} = (\sigma_{\rm pp} + \sigma_{\rm nn} + 2\sigma_{\rm pn})/4$ for L = 2. The effect of partial cancellation between particle and hole amplitudes of the collisional damping is automatically incorporated into eqs. (20) and (21) for the widths [1–2]. In particular this cancellation becomes complete for the isoscalar monopole mode as seen from the fact the angular integral $I_{\rm S}$ in eq. (10) vanishes for L = 0. As a result, the spreading width of the monopole vibration vanishes in the linearized limit of the collision term in eq. (1).

We apply the formulas (20) and (21) to describe the damping of isovector giant dipole and isoscalar giant quadrupole resonances. In numerical calculations we use a Fermi momentum of $k_{_F} = 1.34/\text{fm}$ to obtain the Fermi energy, the Fermi velocity and the nuclear matter density as $\epsilon_{_F} = 37$ MeV, $v_{_F} = 0.28c$ and $\rho = 0.16/\text{fm}^3$. For numerical calculations we also need the cross-sections $\sigma_{\rm pp}$, $\sigma_{\rm nn}$, and $\sigma_{\rm pn}$. The largest contribution to the damping comes from the collisions of nucleon pairs with relative momentum close to the Fermi momentum. In the calculations we use free space cross-sections at this momentum, $\sigma_{\rm pp} = \sigma_{\rm nn} \simeq 25$mb and $\sigma_{\rm pn} \simeq 50$mb, which give a spin-isospin averaged cross-section of $\sigma = 37.5$mb. Figure 1. shows the results of calculations and a comparison with the measured damping widths of dipole and quadrupole resonances at zero temperature as a function of the atomic mass number of nuclei. For the mass dependence of the resonance energies for spherical medium mass and heavy nuclei we use the formulas

$$\hbar\Omega_D = 80A^{-1/3}\text{MeV}, \quad \hbar\Omega_Q = 64A^{-1/3}\text{MeV}. \tag{22}$$

The mass dependence of the damping widths in the calculation, $A^{-2/3}$, arises entirely from the mass dependence of resonance energies [14]. As seen from the figure, except for light nuclei, calculations describe rather well the average trend of damping widths of giant dipole and giant quadrupole resonances over a broad mass range. The over prediction of widths in the lower mass region is mainly due to the effects of direct nucleon emission which is neglected in the calculations. In principle, the energy integrals in the decay rates, eqs. (8) and (17), must be restricted to the bound single particle states which substantially reduces the collisional damping in light nuclei. In addition to the collisional damping, the escape with $\Gamma\uparrow$ due to direct nucleon emission must be incorporated into/ the calculations. The escape width is only a small portion of the total width in medium-mass and heavy-nuclei, but increases with decreasing mass of the system [1–2]. Figure 2. shows the calculated spreading width of the giant dipole resonance in ^{108}Sn, in comparison with the experimental data as a function of excitation energy E* which is related to temperature through the relation $E^* = a\,T^2$ with $a = \pi^2 A/4\epsilon_F$. Both the experimental and the calculated widths increase linearly with excitation energy, but the slope of the calculated curve is somewhat smaller than the experimental one. This behavior is again due to the escape width which is not included into the calculations. The escape width is small at zero temperature but increases with excitation energy, and it must be incorporated into the calculations for a proper description of the damping widths at finite temperatures. The calculations of damping widths including the effects of direct nucleon emission and the escape width will be presented in a subsequent publication.

In summary, we have presented a transparent study of the collisional relaxation of nuclear collective vibrations on the basis of a Boltzmann-type transport equation. It is shown that for a proper treatment of the damping properties of small amplitude high frequency collective vibrations, it is essential to incorporate the memory effects into the collision term of the transport equation. We have derived a simple analytical formula for the spreading width of isoscalar and isovector resonances in nuclei at finite temperature. Our calculations provide a reasonable description for the gross properties of the damping widths of giant dipole and giant quadrupole resonances at zero and at finite temperatures. Some shortcomings of the calculations can be attributed to the escape width due to direct nucleon emission which has not been incorporated into the calculations yet.

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FIGURE CAPTIONS

1. The atomic mass number dependence of the damping width for the giant dipole resonance (upper part) and the giant quadrupole resonance (lower part).

2. The excitation energy dependence of the damping width for the giant dipole resonance in ^{108}Sn nucleus.

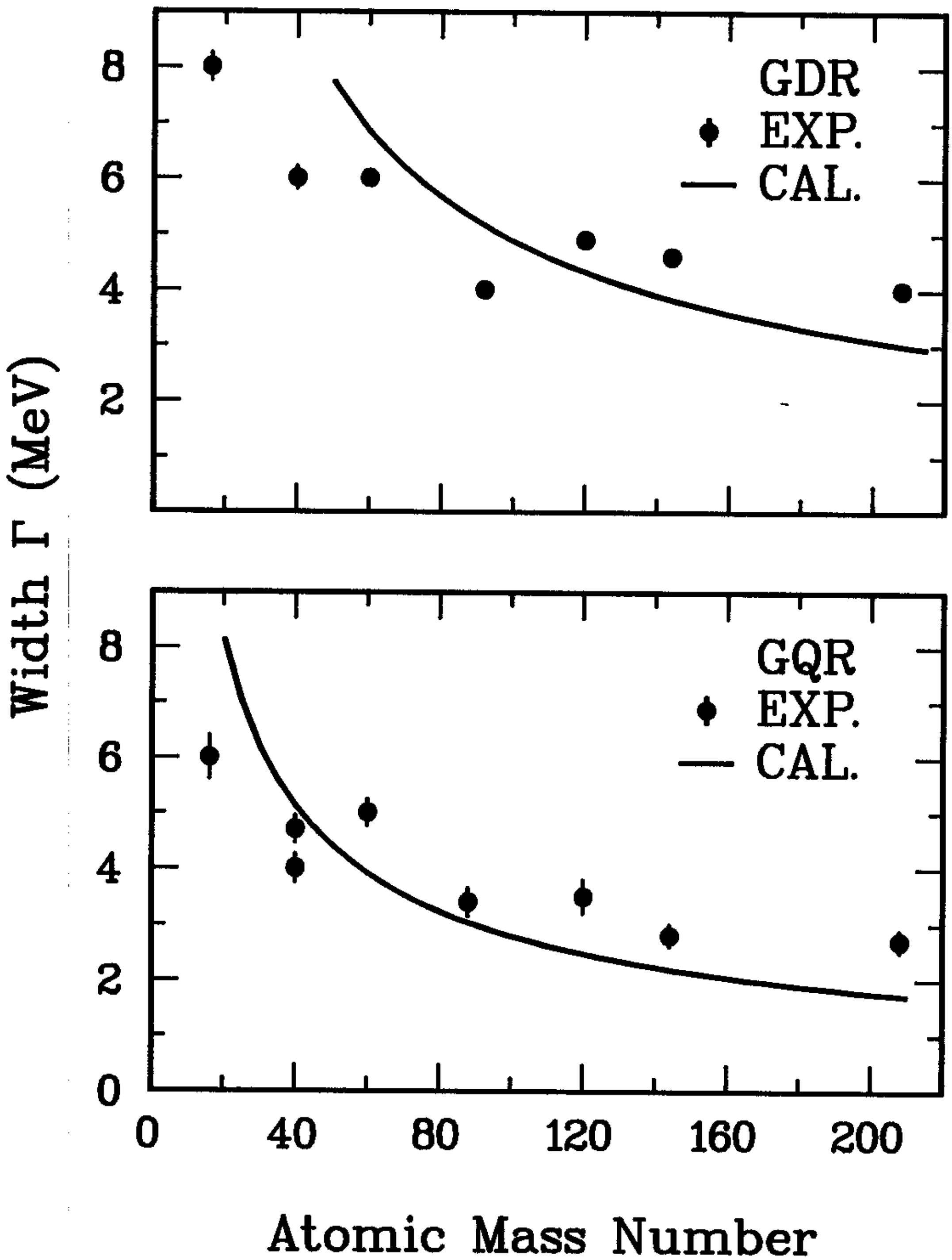
GDR
EXP.
CAL.
GQR
EXP.
CAL.
8
6
4
2
0
0
40
80
120
160
200
Width Γ (MeV)
Atomic Mass Number

FIGURE 1

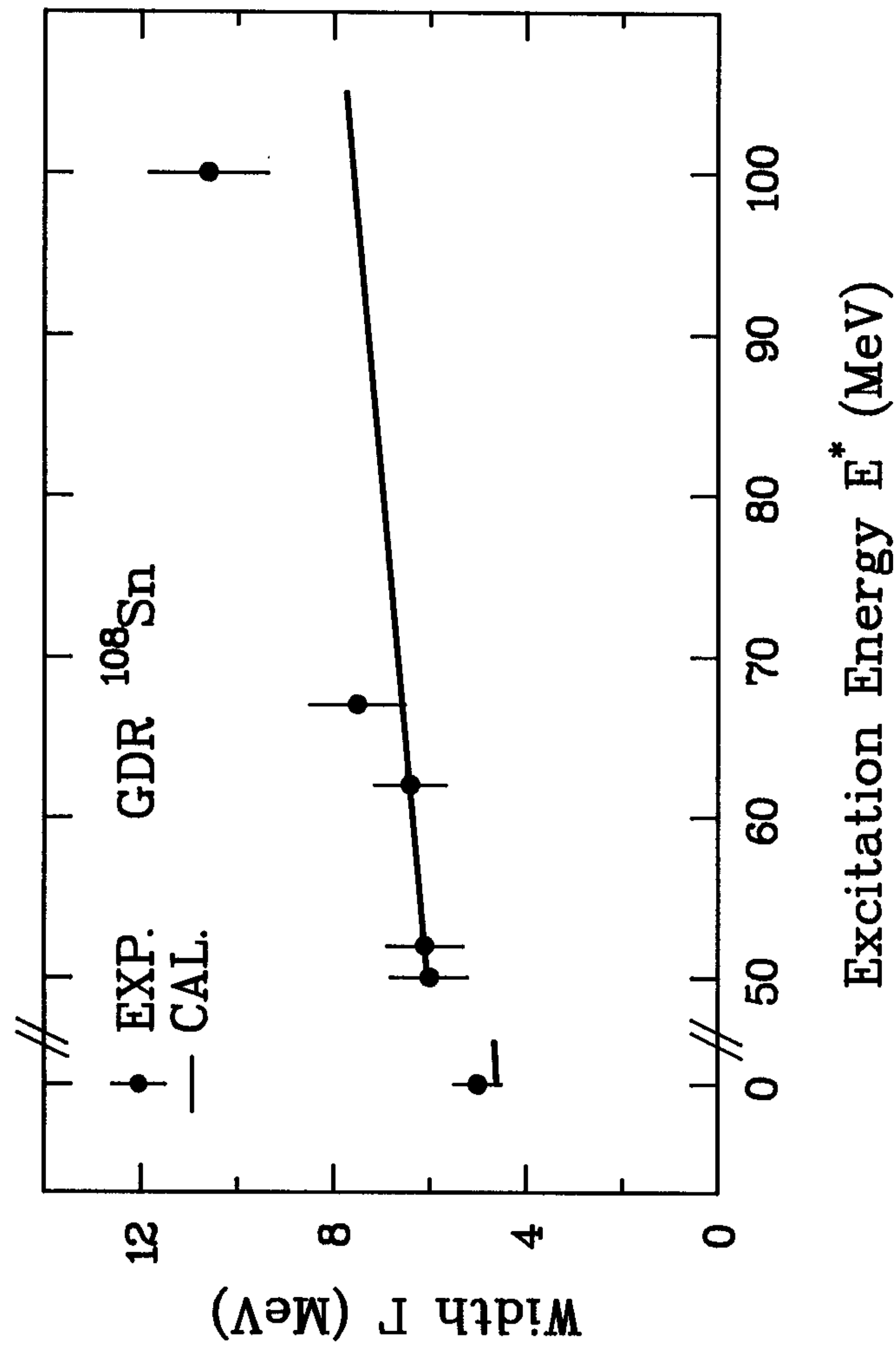
Width Γ (MeV)
Excitation Energy E* (MeV)
EXP.
CAL.
GDR 108Sn
0
4
8
12
0
50
60
70
80
90
100

FIGURE 2