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EVENT-MIXING ANALYSIS OF TWO-PROTON CORRELATION FUNCTIONS

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Abstract:
Two-proton correlation functions constructed via an event-mixing analysis are compared to recently published correlation functions for the $^{14}N+^{27}Al$ reaction at E/A=75 MeV which were constructed by generating the denominator from single particle spectra. For the present case, the two techniques yield virtually identical results.

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The technique of intensity interferometry allows the extraction of valuable information about the space-time characteristics governing the decay of hot nuclear systems [1-34]. Two-particle correlation functions offer a convenient way of presenting such information. Most theoretical treatments are based upon the assumption that the final-state interaction between the two detected particles dominates, that final-state interactions with all remaining particles can be neglected, and that the correlation functions are determined by the two-body density of states as corrected by the interactions between the two particles. Hence, the theoretical expression for the correlation function is the ratio of available states with and without interaction between the particle pair. This ratio depends on the spatial dimensions of the region of phase space occupied by the emitted particles [1,3,4,9,11,18,31].

For collisions at fixed impact parameter, the correlation function $1+R(\vec{P},\vec{q})$ is related to the single- and two-particle yields, $Y(\vec{p})$ and $Y(\vec{p}_1,\vec{p}_2)=Y(\vec{P},\vec{q})$:

$$Y(\vec{P},\vec{q}) = Y(\vec{p}_1,\vec{p}_2) = C\,(1+R(\vec{P},\vec{q}))\,Y(\vec{p}_1)Y(\vec{p}_2)\ , \qquad (1)$$

where $\vec{p}_1$ and $\vec{p}_2$ denote the momenta of the two detected particles, $\vec{P}=\vec{p}_1+\vec{p}_2$ is the total momentum of the particle pair, and $\vec{q}$ is the momentum of relative motion (defined in the center-of-momentum frame of the particle pair, where $\vec{P}=0$; non-relativistically, $\vec{q}=\mu\vec{v}_{rel}$). The constant C can be determined from the condition that $R(\vec{P},\vec{q})=0$ for sufficiently large relative momenta for which modifications of the two-particle phase-space density due to quantum statistics or final-state interactions become negligible.

Since the impact parameter of a subatomic collision cannot be measured with precision, experimental determinations of two-particle correlation functions involve averages over impact parameter. Furthermore, it is virtually impossible to collect sufficient statistics to allow the determination the full six-dimensional dependence of the correlation function upon $\vec{P}$ and $\vec{q}$. Hence, implicit integrations are carried out in the construction of experimental correlation functions. For example, experimental correlation functions are mostly defined according to the relation [1]:

$$\Sigma\, Y(\vec{p}_1,\vec{p}_2) = C^{\dagger}\,(1+R(\zeta))\,\Sigma\, Y^{\dagger}(\vec{p}_1,\vec{p}_2)\ . \qquad (2)$$

In Eq. 2, $Y^{\dagger}(p_1,p_2)$ is the "background" yield, $C^{\dagger}$ is a normalization constant which ensures proper normalization at large relative momenta, and ζ is the variable for which the explicit dependence of the correlation function is evaluated (the most common choice is ζ=q). For each experimental gating condition (representing implicit integrations over a number of variables), the sums on both sides of Eq. 2 are extended over all energy and detector combinations corresponding to the given bins of ζ. The experimental correlation function is defined in terms of the ratio of these two sums. Comparisons with theoretical results must take this definition into account, see also the discussion given in the appendix of ref. [32].

Two different approaches are commonly used for the construction of the background yield. In one approach (referred to as the "singles technique"), the background yield is taken as proportional to the product of the single particle yields, measured with the same external trigger conditions as the true two-particle coincidence yield [5,6,10,13-17,21,25,26,29,30,32,33]:

$$Y^{\dagger}(\vec{p}_1,\vec{p}_2) \propto Y(\vec{p}_1)Y(\vec{p}_2) \ . \tag{3}$$

In the other approach (referred to as the "event-mixing technique"), the background yield is generated by mixing particle yields from different coincidence events [3,7,8,19,20,22-24,27,28,34]:

$$Y^{\dagger}(\vec{p}_1,\vec{p}_2) = \sum_{n \neq m} \delta^3(\vec{p}_1-p_{1,n})\delta^3(\vec{p}_2-p_{2,m}) \ . \tag{4}$$

Here, the indices n and m label the n-th and m-th recorded two-particle coincidence events, and $p_{1,n}$ and $p_{2,m}$ denote the momenta of particles 1 and 2 recorded in events n and m, respectively. In most analyses, the index n runs over all recorded coincidence events and the index m is varied according to m=n+k, with typically 0<k<1000.

The main advantage of the event-mixing technique is its simplicity, as no singles measurements are necessary. In some situations, single- and two-particle data could represent different averages of impact parameter. In such cases, the use of the singles technique could lead to serious distortions of the correlation function [27] which could complicate comparisons with theoretical predictions. Furthermore, less interesting correlations, resulting for example from phase space constraints due to conservation laws [6,35-40], may be suppressed by using the event-mixing technique. However, the event-mixing technique also attenuates the very correlations one wishes to measure [7]. The degree of attenuation depends on the phase space acceptance of the experimental apparatus and on the magnitude of the correlations. Quantitative analyses require careful Monte-Carlo simulations. For the extraction of undistorted correlation functions iterative procedures have been developed [7].

For statistical emission processes in which the emission of a single particle has negligible effect on further emissions, single and two-particle yields should originate from similar regions of impact parameters. In such instances, the singles technique appears to be the preferential choice since it avoids the attenuation of the very correlations one wishes to measure. There are, however, scenarios where the singles technique may become inappropriate. For example, emission to extreme forward angles may have large contributions from breakup reactions in which only one particle is emitted [27]. In such instances, single and two-particle detection will select different classes of collisions.

Two-proton correlations generated with the singles technique display a strong dependence on the total energy (or momentum) of the emitted particle pairs [6,15-17,21,29,30,32]. In a recent paper [27], the issue was raised that this dependence might be an artifact of the singles technique employed for the construction of the experimental correlation function. In this paper we address this question and give a quantitative comparison of two-proton correlation functions constructed by the two techniques. For this purpose, we have re-analyzed the high-statistics data of refs. [30,32] taken for the $^{14}N+^{27}Al$ reaction at E/A=75 MeV.

In our event-mixing analysis, we have first projected out "pseudo-singles" spectra from the two-proton yields:

$$Y^{*}(\vec{p}) = \sum_{i,\,n} \delta^{3}(\vec{p}-\vec{p}_{i,n}) \ , \qquad (5)$$

where $\vec{p}_{i,n}$ is the momentum of the i-th proton (i=1,2) detected in the n-th two-proton coincidence event. These pseudo-singles yields were then inserted into

Eq. 3 to generate the background yield. This procedure is equivalent to summing Eq. 4 over all event indices n and (n+m), thus also allowing a contamination of contributions from true coincidences (m=0). In our analysis, this contamination is entirely negligible ($<10^{-6}$). The background constructed via Eqs. 3 and 5 provides the maximum statistical accuracy which can be obtained with the event-mixing technique.

Useful insight can be gleaned by comparing true singles and pseudo-singles yields. In Fig. 1, energy spectra are compared for three representative regions of angle covered by the experimental apparatus of refs. [30,32]. In Fig. 2, angular distributions are compared for three different energy intervals. Clearly singles and pseudo-singles yields are very similar.

Figure 3 gives a comparison of two-proton correlation functions constructed with the two techniques, using the same momentum gates as displayed in Fig. 14 of ref. [32]. Very similar results are obtained by the two techniques. In particular, both techniques give very similar momentum dependences of the two-proton correlation functions. As expected, the event-mixing technique gives correlation functions which are slightly attenuated in comparison with the singles technique. The differences are small (typically smaller than 5%) and only of marginal statistical significance.

We also explored whether the results were stable with respect to the number of detectors employed in the experiment. In Fig. 4, we show energy-integrated two-proton correlation functions from data in which the entire hodoscope (top panel), only 23 detectors (center panel) and only 7 detectors were incorporated into the analysis. (Integration over all outgoing particle energies was performed to ensure good statistical accuracy for the analysis employing only 7 detectors.) Again, the two techniques give rather similar results. Here again, however, correlation functions obtained with the event-mixing technique are slightly attenuated as compared to those constructed with the singles technique reflecting the presence of correlations in the background yields constructed from event-mixing analyses [7].

In conclusion, the momentum (or energy) dependence of two-proton correlation functions reported in refs. [30,32] can be considered as firmly established. A re-analysis of the data of refs. [30,32] in terms of a background constructed by event-mixing gives correlation functions which are very similar (though slightly attenuated) to those published previously.

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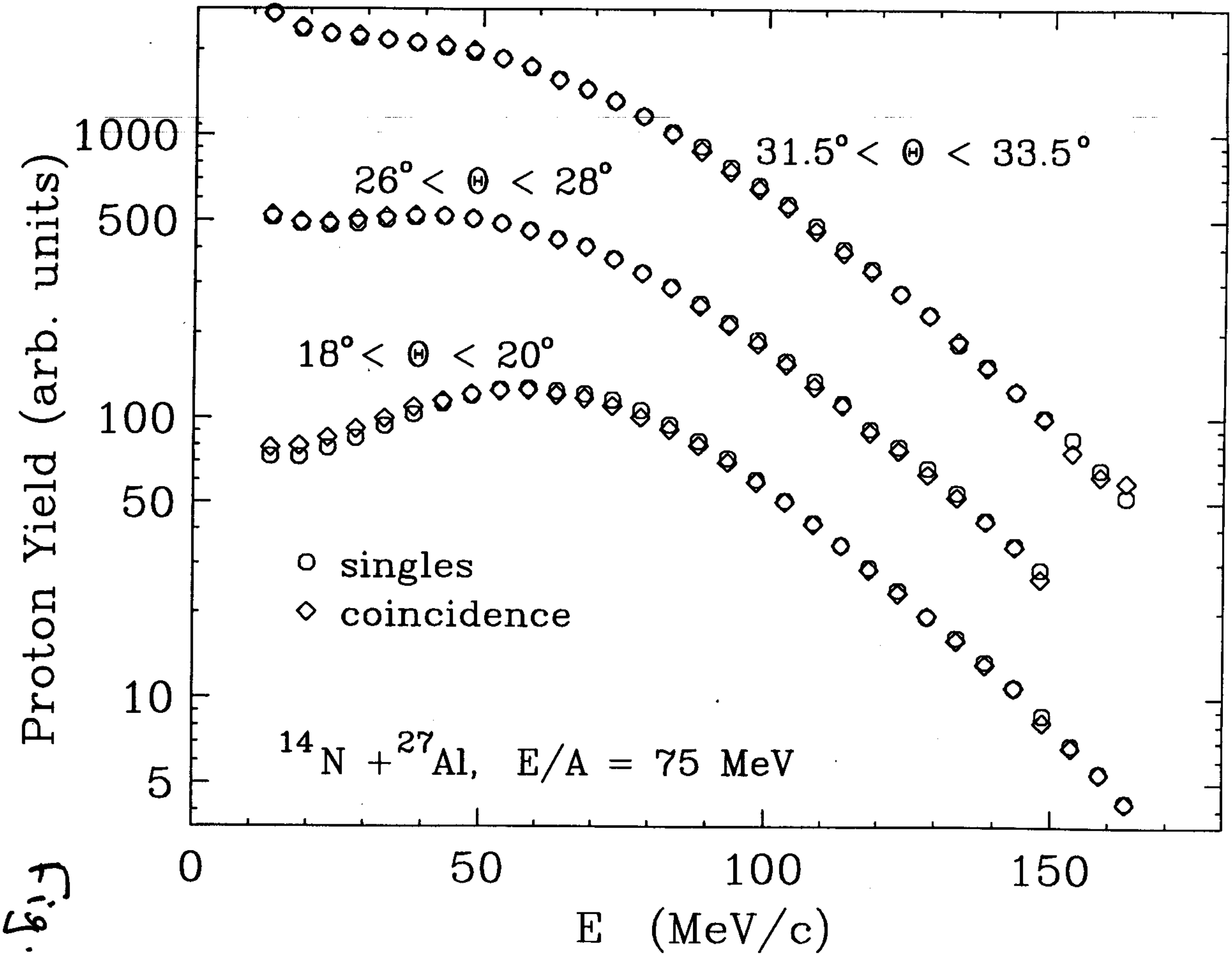
Figure Captions:

Fig. 1: Comparison of singles (circles) and pseudo-singles (diamonds) energy spectra measured [30,32] for the $^{14}N+^{27}Al$ reaction at E/A=75 MeV. (Statistical errors are smaller than the size of the symbols.)

Fig. 2: Comparisons of singles (circles) and pseudo-singles (diamonds) angular distributions measured [30,32] for the $^{14}N+^{27}Al$ reaction at E/A=75 MeV. (Statistical errors are smaller than the size of the symbols.)

Fig. 3: Two-proton correlation functions measured [30,32] for the $^{14}N+^{27}Al$ reaction at E/A=75 MeV. Open and solid points represent correlation functions constructed by the singles and event-mixing techniques, respectively. Momentum cuts are indicated in the figure. Statistical errors (larger than the size of the data points) are only shown for open points, solid points have errors of same magnitude.

Fig. 4: Energy integrated two-proton correlation functions measured [30,32] for the $^{14}N+^{27}Al$ reaction at E/A=75 MeV. Open and solid points represent correlation functions constructed by the singles and event-mixing techniques, respectively. Top, center, and bottom panels represent correlation functions extracted from arrays consisting of 47, 23, and 7 detectors, respectively. Statistical errors (larger than the size of the data points) are only shown for open points, solid points have errors of same magnitude.



Proton Yield (arb. units)
1000
500
100
50
10
5
0
50
100
150
E (MeV/c)
31.5°< Θ < 33.5°
26°< Θ < 28°
18°< Θ < 20°
singles
coincidence
14N + 27Al, E/A = 75 MeV

Fig. 1

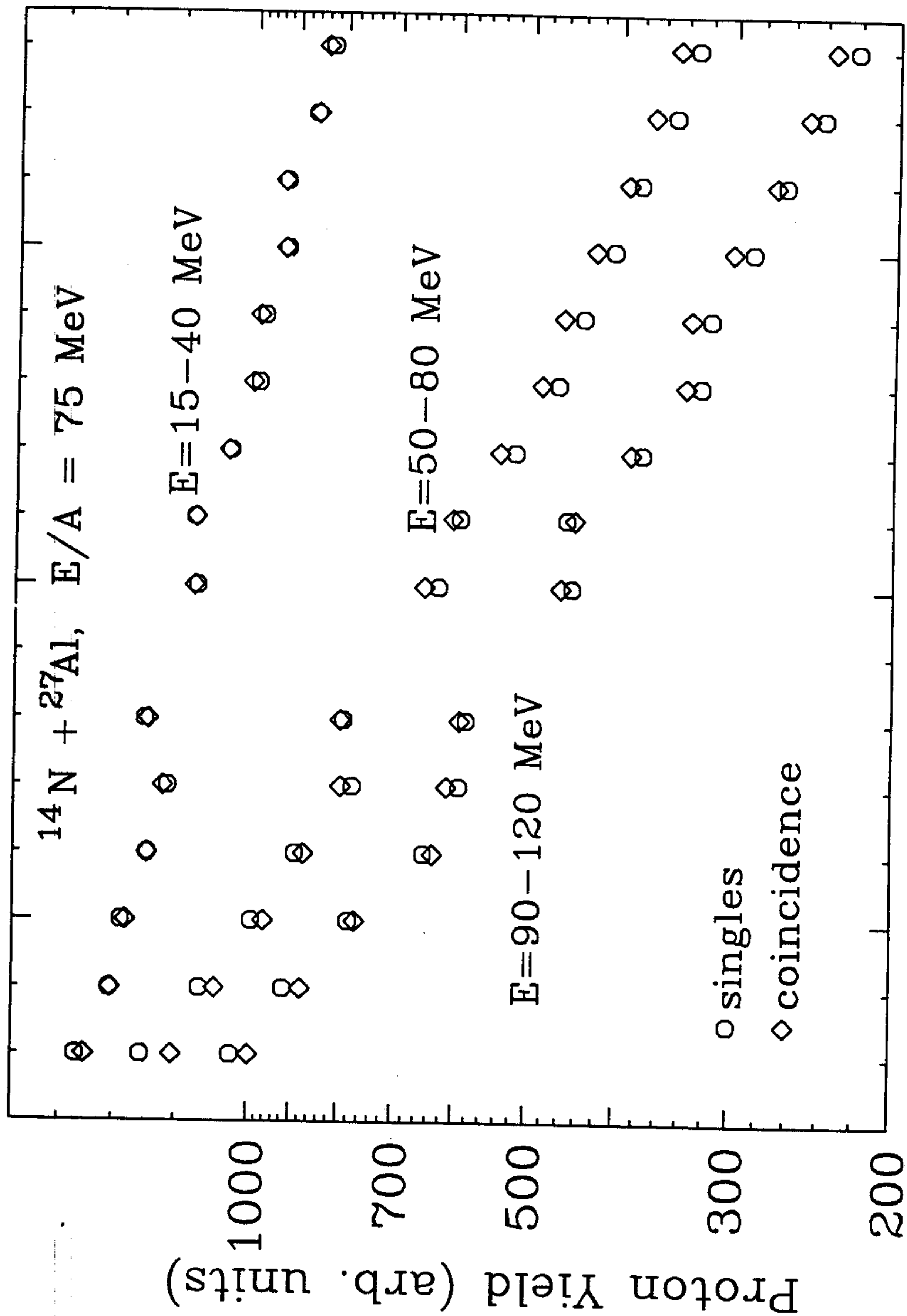
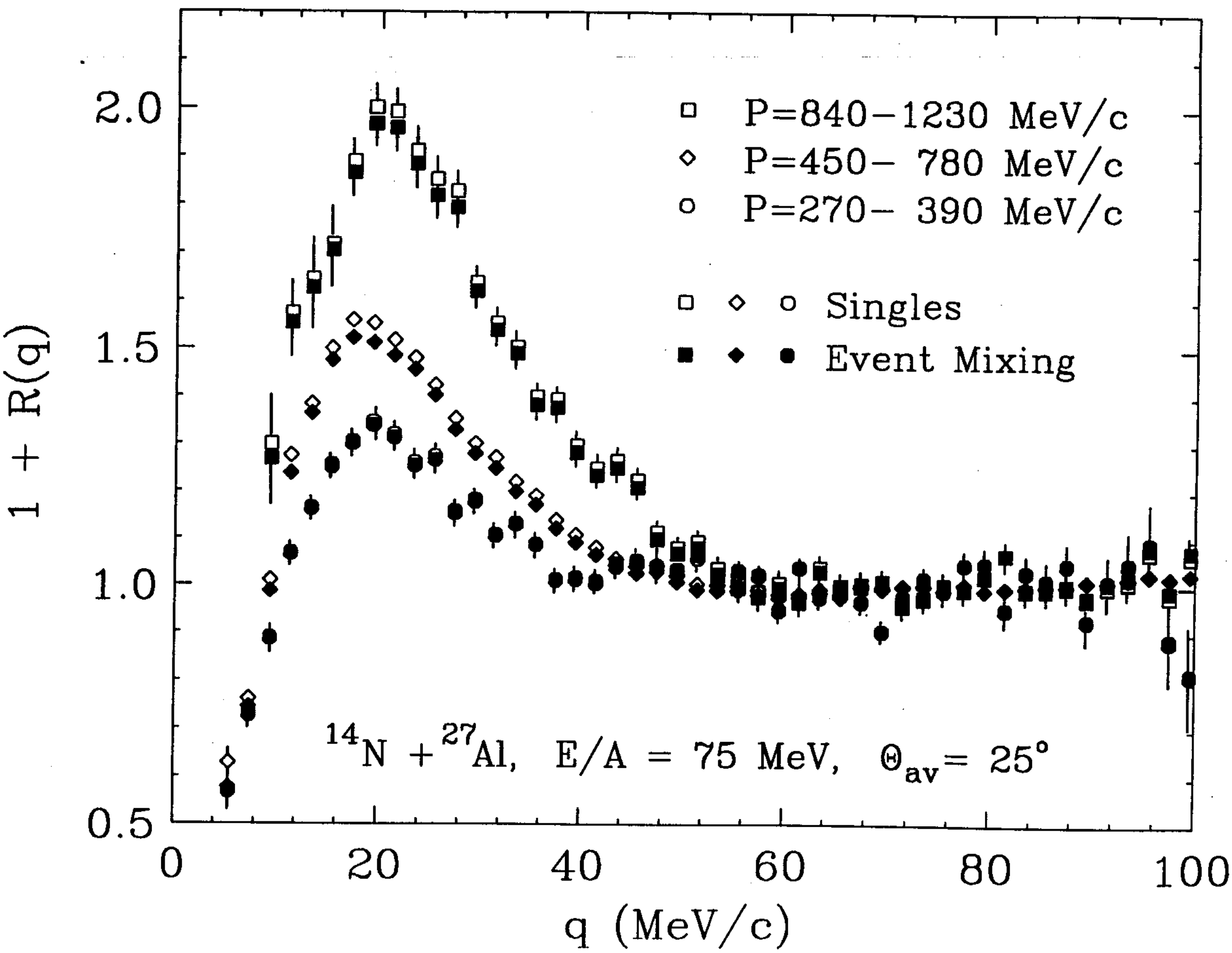

^{14}N + ^{27}Al, E/A = 75 MeV
E=15−40 MeV
E=50−80 MeV
E=90−120 MeV
singles
coincidence
Proton Yield (arb. units)
Θ (deg)
200
300
500
700
1000
20
25
30

Fig. 2



P=840−1230 MeV/c
P=450− 780 MeV/c
P=270− 390 MeV/c
Singles
Event Mixing
$^{14}N + ^{27}Al$, E/A = 75 MeV, Θ_{av} = 25°
1 + R(q)
q (MeV/c)
2.0
1.5
1.0
0.5
0
20
40
60
80
100

Fig. 3

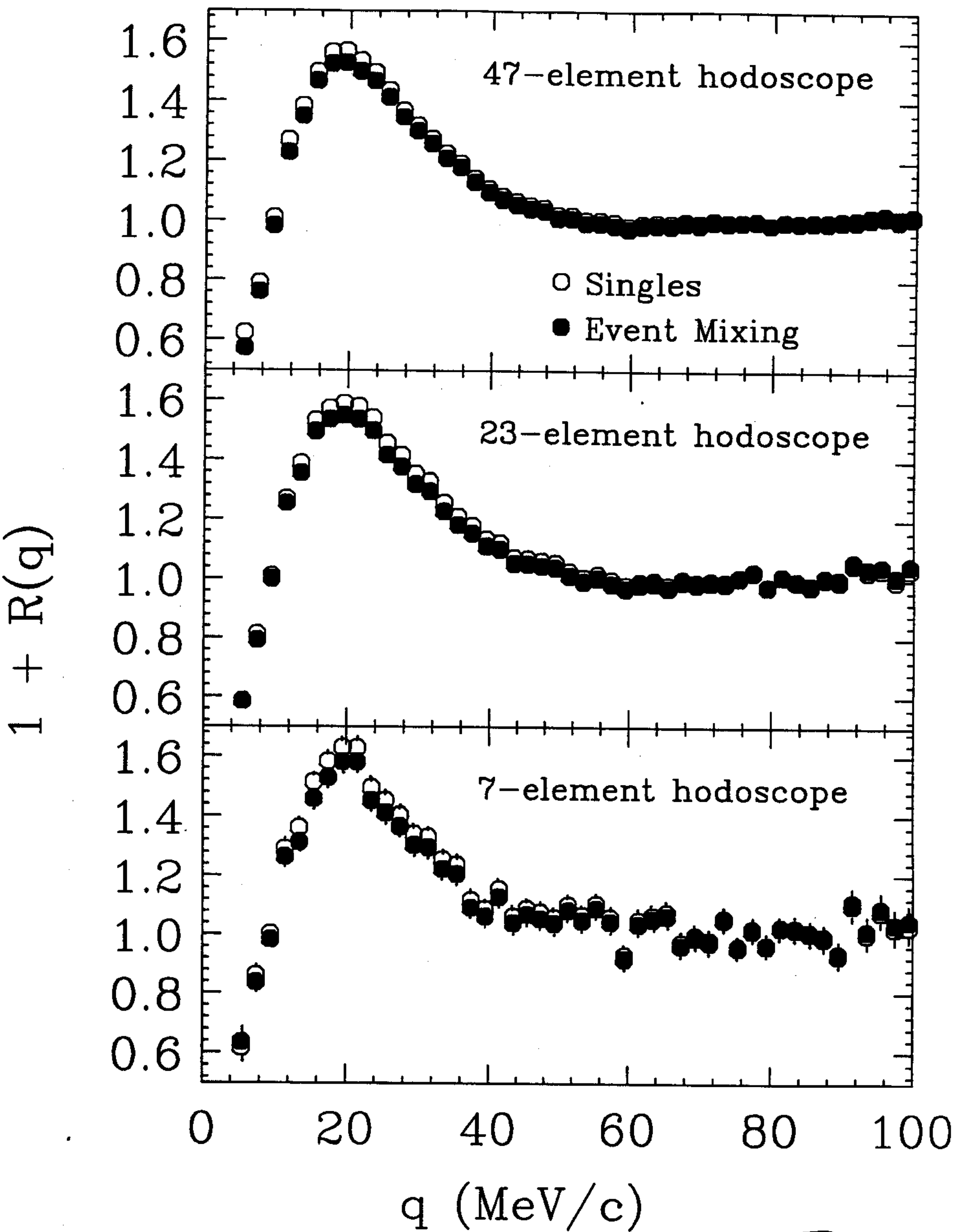

47−element hodoscope
Singles
Event Mixing
23−element hodoscope
7−element hodoscope
1 + R(q)
q (MeV/c)
0.6
0.8
1.0
1.2
1.4
1.6
0
20
40
60
80
100

Fig. 4