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HEAVY ION EXCITATION OF GIANT RESONANCES: A BRIDGE FROM THE ELASTIC SCATTERING TO THE INELASTIC DATA

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HEAVY ION EXCITATION OF GIANT RESONANCES: A BRIDGE FROM THE ELASTIC SCATTERING TO THE INELASTIC DATA *

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Abstract

We develop data-to-data relations involving the giant-resonance cross sections and the elastic cross section for heavy ion collisions at intermediate energies. The usefulness of this novel method is shown by applications to the $^{17}O +^{208} F'b$ at $E_{Lab} = 84$ $MeV/nucleon$.

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1. Introduction

The Coulomb excitation of giant multipole resonances in heavy ion collisions at intermediate energies allows a detailed study of multipolarity content, relative strength, decay branching ratios and other aspects of these collective states[1)]. When compared to purely electromagnetic probes, heavy ions supply strong nuclear fields, which may complicate the analysis. Usually, one uses DWBA codes[2)], which, when compared to the experimental data, allow the extraction of the multipolarity content and relative strength of the excitation. Recently[3)], a simpler approach based on the use of the eikonal aproximation to the distorted waves was shown to be quite adequate in describing the excitation of giant resonances in heavy ion induced reactions.

In this article we derive data-to-data (DTD) relations involving the inelastic cross section for the excitation of giant resonances on the one hand and the elastic cross section on the other. The competition between the Coulomb and the nuclear interaction implies that the deflection function reaches a maximum at the so-called rainbow angle, θ_r. The region $\theta > \theta_r$ is classically forbidden and is called the dark side of the Coulomb rainbow. For $\theta \gg \theta_r$ absorption sets in and the cross sections for elastic scattering becomes very small. On the dark side of the Coulomb rainbow, where the DTD relations strictly hold, the inelastic cross section for a given multipolarity is found to go as $C_\lambda\, q^2\, \sigma_{el}(q)$, where q is the momentum transfer. Thus we predict that in the relevant angular region the shape of the inelastic angular distribution does not depend on the multipolarity λ. These relations allow a reliable and easier analysis of the experimental data, as shown by an application to the $^{17}O + ^{208}Pb$ system. Details of the calculations will be presented elsewhere[4)]. Data-to-data relations have been derived earlier for intermediate energy proton-nucleus scattering by Amado *et al.*[5)]. The fundamental difference between our work and ref. 5 is the very important Coulomb effects in the heavy-ion system.

The amplitude for the transition of the nucleus from the ground state $|0>$ to the excited state $|\lambda\mu>$ is given, within the eikonal approximation[3)], by $[q = 2k\sin(\theta/2)]$

$$f^{\lambda\mu}_{N,C}(\theta) = \frac{ik}{2\pi\hbar v} \int e^{i\mathbf{b}.\mathbf{q}+i\chi(b)} < \mathbf{r},\, \lambda\mu|U_{N,C}|\mathbf{r},\, 0 > d^2b\, dz\,, \tag{1}$$

where $N(C)$ stands for the nuclear (Coulomb) contribution, $U_{N,C}$ is the nucleus-nucleus interaction, and $\chi(b)$ is the total eikonal phase. For comparison, the elastic scattering amplitude is given by

$$f_{el}(\theta) = \frac{ik}{2\pi} \int e^{i\mathbf{b}.\mathbf{q}} \left[1 - e^{i\chi(b)}\right] d^2b = ik \int J_0(qb) \left[1 - e^{i\chi(b)}\right] d^2b. \tag{2}$$

The phase $\chi(b)$ is related to the potential by the usual eikonal formula

$$\chi(b) = -\frac{k}{E} \int_{-\infty}^{\infty} dz \left[U_N\left(\sqrt{b^2+z^2}\right) + U_C\left(\sqrt{b^2+z^2}\right)\right]. \tag{3}$$

The nuclear potential, $U_N(r)$, is evaluated using the "$t\rho\rho$" aproximation[6] with due care to medium effects in the calculation of the nucleon-nucleon t-matrix. Since the inelastic transitions considered are peripheral, the amplitudes are sensitive to the surface region only. Therefore, as in ref. 3, we employ Gaussian forms for the densities $\rho = \rho_G\, e^{-r^2/\alpha^2}$ that are adjusted to reproduce the tail region of the realistic densities. This fit imply the relations $\alpha_i = \sqrt{2\,a\,R_i}$ and $\rho_{G,i} = (\rho_0/2)\, e^{R_i/2a}$ with $i = 1,2$; $\rho_0 = 0.17\ fm^{-3}$; $a = 0.65\ fm$ and $R_i = 1.2\ A_i^{1/3}\ fm$. The resulting U_N becomes Gaussian[3] and the corresponding nuclear phase comes out to be

$$\chi_N(b) = -\pi^2\, \frac{k}{E} < t_{NN} > \frac{\alpha_1^3 \alpha_2^3}{\alpha^2}\, \rho_{G,1}\, \rho_{G,2}\, e^{-b^2/\alpha^2}\,. \tag{4}$$

where $\alpha = \sqrt{\alpha_1^2 + \alpha_2^2}$.

In figure 1 we show the elastic scattering angular distribution for the system $^{17}O + ^{208}Pb$ at $E_{Lab} = 84\ MeV/nucleon$ obtained with $< \sigma_{NN} >= 60\ mb$ and $t_{Real}/t_{Imag} = 1$, as in ref. 3. The data[7] are very well reproduced with our calculation. Further we have assessed that within the angular range of interest $0 \le \theta < 6°$ the cross section is completely near-side[8] dominated and represents a nice case of Coulomb rainbow scattering with the rainbow angle being $\theta_r = 3.3°$, as can be seen clearly in figure 2 which shows the analogy of the deflection function, which we apropriately call here, following ref. 9, the momentum transfer function. The rainbow momentum transfer, $q_r(b)$, which corresponds to θ_r is $1.86\ fm^{-1}$.

When analysing the inelastic amplitude it is clear that the Coulomb-rainbow scattering effect is also present through the phase, which is the same as the one that appears in the elastic one. For $q < 1.86\ fm^{-1}$ two stationary phase contributions dominate f_{el} and f_{inel}. These are the nuclear (inner branch) and the Coulomb (outer branch) and the cross sections clearly exhibits the known Coulomb-nuclear interference. On the dark-side of the rainbow there is only one complex stationary phase contribution. This is a mixture of Coulomb+nuclear scattering. Therefore, no interference arises neither in f_{el} nor in f_{inel}.

It is in this region that one would expect a simple linear relationship between the amplitudes to hold. Since the inelastic amplitude involves an integrand containing the derivative of the potential [Tassie Model[10)]], which can be related to the derivative of the eikonal phase with respect to b [see eq. (3)], a simple integration by parts of eq. (1) indicate that $f_{inel}(q)$ should be proportional to $q\ f_{el}(q)$ in this angular region. Further, since the inelastic amplitude can be written in the form[3)] (after integration over the azimuthal angle),

$$f_{N,C}^{\lambda\mu} = \frac{i^{1+\mu} C_\lambda\, k}{\hbar v} \int_0^\infty db\, b\, J_\mu(qb)\, e^{i\chi(b)} \int_{-\infty}^{\infty} dz\, r^\lambda U_{N,C}\Big(\sqrt{b^2+z^2}\Big)\ P_{\lambda\mu}(\theta)\,, \qquad (5)$$

the q dependence arises entirely from the Bessel function. Moreover, the integral over z can be related to derivatives, with respect to b, of $\chi(b)$, eq. (2), if one uses the Gaussian approximation for the tails of the densities. Since the b's that contribute are large (roughly, the sum of the two radii of the two nuclei), and taking q to be near the rainbow value, one may use the asymptotic form of $J_\mu(qb)$, namely $J_\mu(qb) \approx \sqrt{2/\pi qb}\,\cos\big(qb - \mu\pi/2 - \pi/4\big)$. Therefore, one would expect that the λ and the μ dependence to be almost irrelevant to the q-dependence. Accordingly, the inelastic amplitudes in the dark side should have similar q-dependence, irrespective to the multipolarity.

Explicit derivation of our DTD relations can be done following the arguments presented above, with the use of the detailed form of $P_{\lambda\mu}(\theta)$ in terms of b and z. With the help of the Tassie model [10)] we have obtained the DTD relations for the excitation of the isovector giant dipole (IVGDR) and isoscalar giant quadrupole (ISGQR) resonances. We find for the IVGDR, $\sigma_{1^-}(q)$, the following

$$\sigma_{1^-}(q) = \frac{3}{4\pi}\, |\frac{C_1}{\eta}|^2\, q^2\, \sigma_{el}(q)\,, \qquad (6)$$

where

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v} \qquad \text{and} \qquad C_1 = \frac{\pi^{3/2}}{\sqrt{3}}\, \frac{Z_P e^2}{\hbar v}\, \beta_1\, R_T\, \rho_{G,T}\, \alpha_T^3\,, \qquad (7)$$

where P (T) refers to the projectile (target) and β_1 is the deformation parameter[2].

For the excitation of ISGQR, an analogous data-to-data relation may be obtained. Using a uniform stationary phase approximation[11] it can be shown that the inelastic amplitude in the shadow region is directly proportional to $b_r\, f_{el}(\theta)$, where b_r is the Coulomb rainbow impact parameter (see fig. 2). For the $^{17}O + ^{208}Pb$ system at 84 $MeV/nucleon$, we find $b_r = 11.5\ fm$. In the dark region we find the data-to-data relation

$$\sigma_{2^+}(q) = \frac{5}{64\pi}\, |C_2|^2\, q^2\, b_r^2\, \sigma_{el}(q)\,, \tag{8}$$

where

$$C_2 = \frac{2\pi^{3/2}}{\sqrt{5}}\, \beta_2\, \frac{\alpha^{12}}{\alpha_P^7 \alpha_T^5}\,. \tag{9}$$

In figures 3 and 4 we show our data-to-data relation calculation for $\sigma_{1^-}(q)$ (fig. 3) and $\sigma_{2^+}(q)$ (fig. 4) using the elastic scattering cross section shown in fig. 1 and $\beta_1 R_T = 0.42\ fm$ and $\beta_2 R_T = 0.55\ fm$. The values for the deformation lenghts cited above are the same as those used in ref. 3 in which eikonal-DWBA calculation was employed. The agreement of the DTD relations with the data is quite good in the angular region where they are applicable.

In conclusion, we have shown that it is possible to derive data-to-data relations involving the inelastic cross sections for the excitation of giant resonances and the elastic cross section in heavy ion scattering at intermediate energies. The successful application of our DTD relations to the $1^-$and 2^+ giant resonance excitation in the system $^{17}O + ^{208}Pb$ at $E_{lab} = 84\ MeV/nucleon$ clearly showed the usefulness of our theory. Further application to other systems is in progress.

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Figure Captions

Fig. 1 - Elastic cross section data for the system $^{17}O +^{208} Pb$ at 84 $MeV/nucleon$. Data are from ref. 7. The solid curve is our theoretical prediction.

Fig. 2 - The momentum transfer function $q(b)$ versus b. See text for details.

Fig. 3 - Cross section for the excitation of isovector giant dipole resonance. Data are from ref. 7. The solid curve was obtained from the data-to-data relation given by eqs. (6) and (8), with deformation parameter $\beta_1 R = 0.42\ fm$.

Fig. 4 - The same as in fig. 3, but for the isoscalar giant quadrupole resonance. We used here $\beta_1 R = 0.55\ fm$.

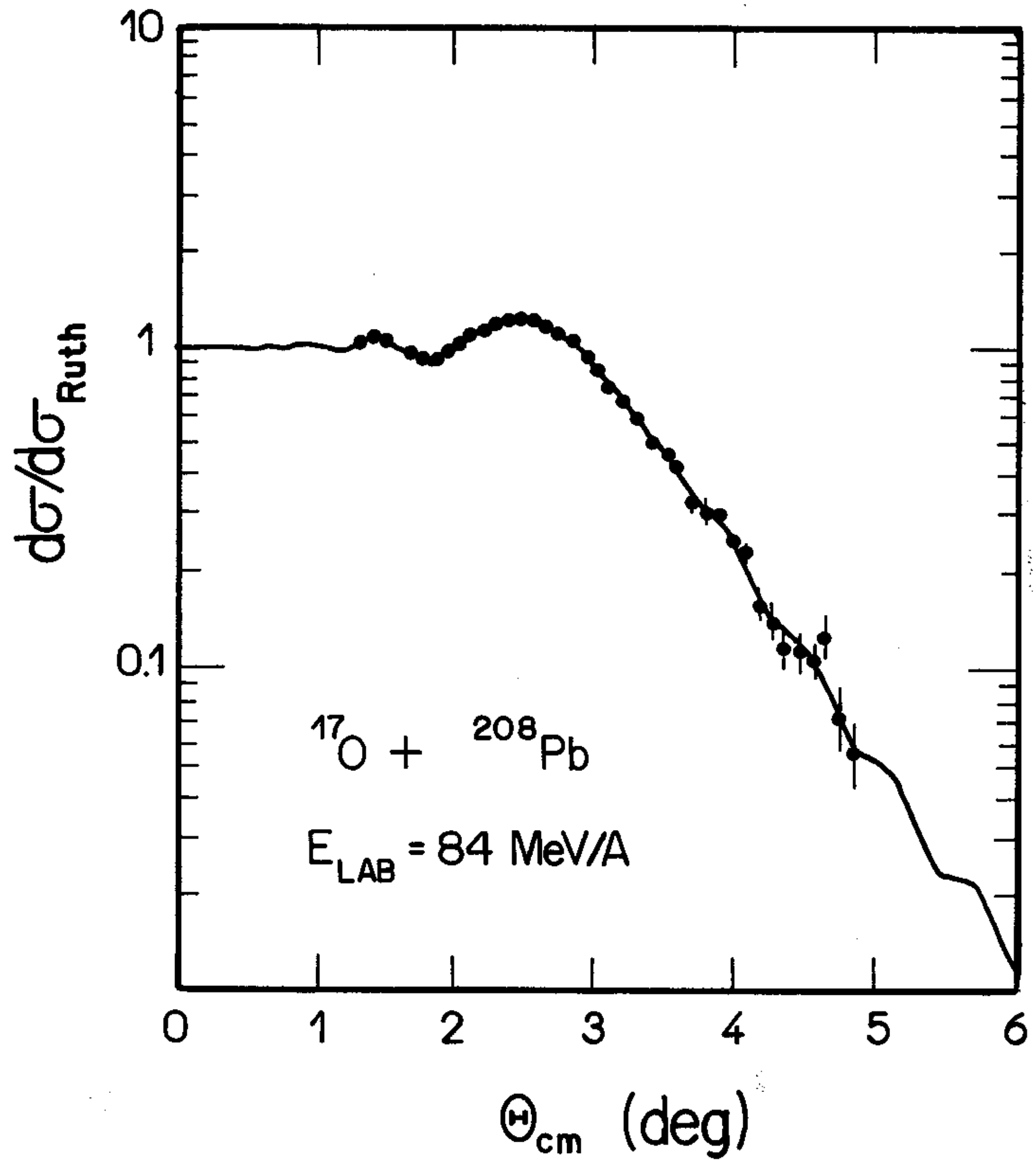
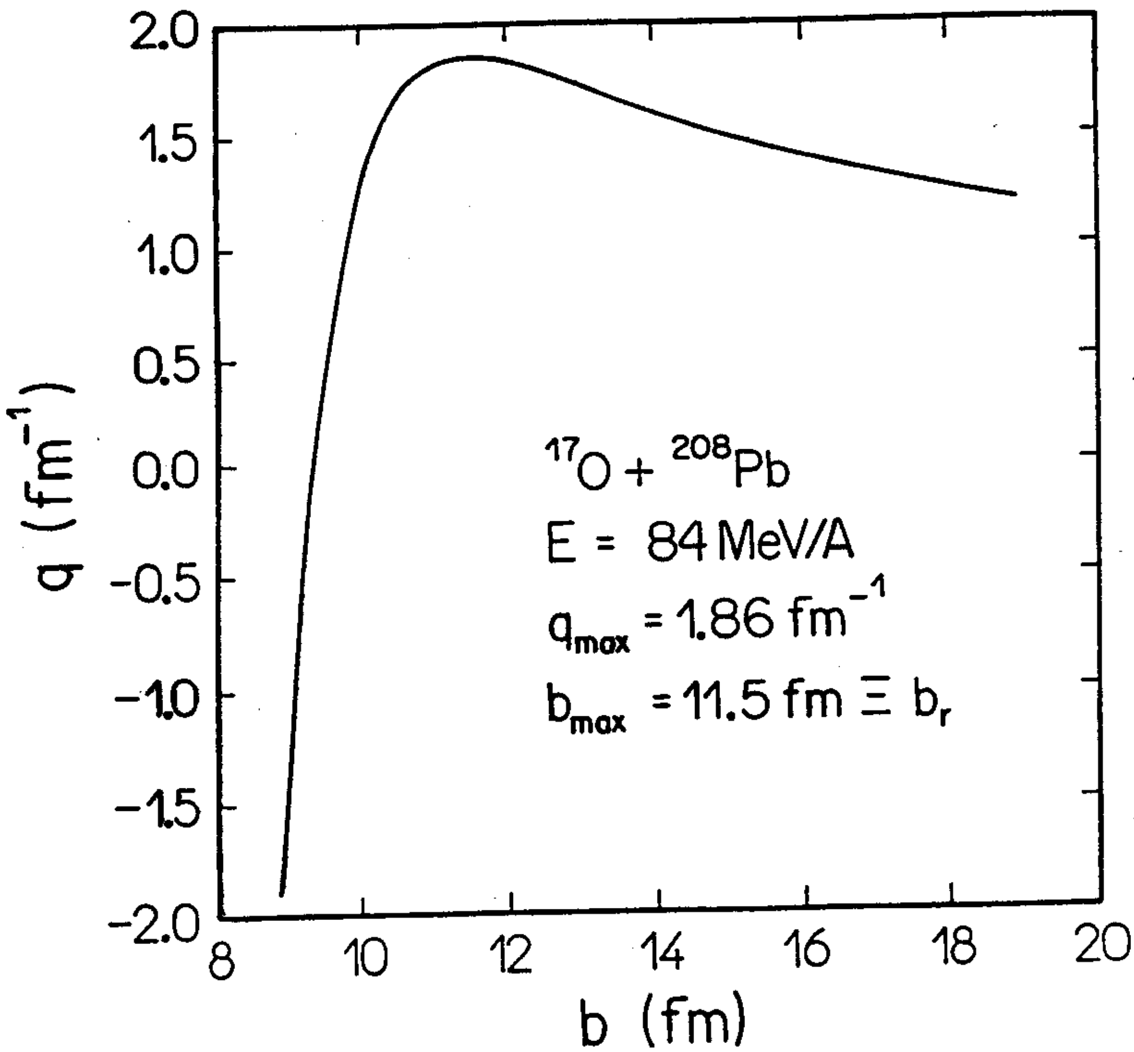

10
1
0.1
$d\sigma/d\sigma_{Ruth}$
$^{17}O + {}^{208}Pb$
E_{LAB} = 84 MeV/A
0
1
2
3
4
5
6
Θ_{cm} (deg)

Figure 1



2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
q (fm⁻¹)
$^{17}O + ^{208}Pb$
E = 84 MeV/A
$q_{max} = 1.86\ fm^{-1}$
$b_{max} = 11.5\ fm \equiv b_r$
8
10
12
14
16
18
20
b (fm)

Figure 2

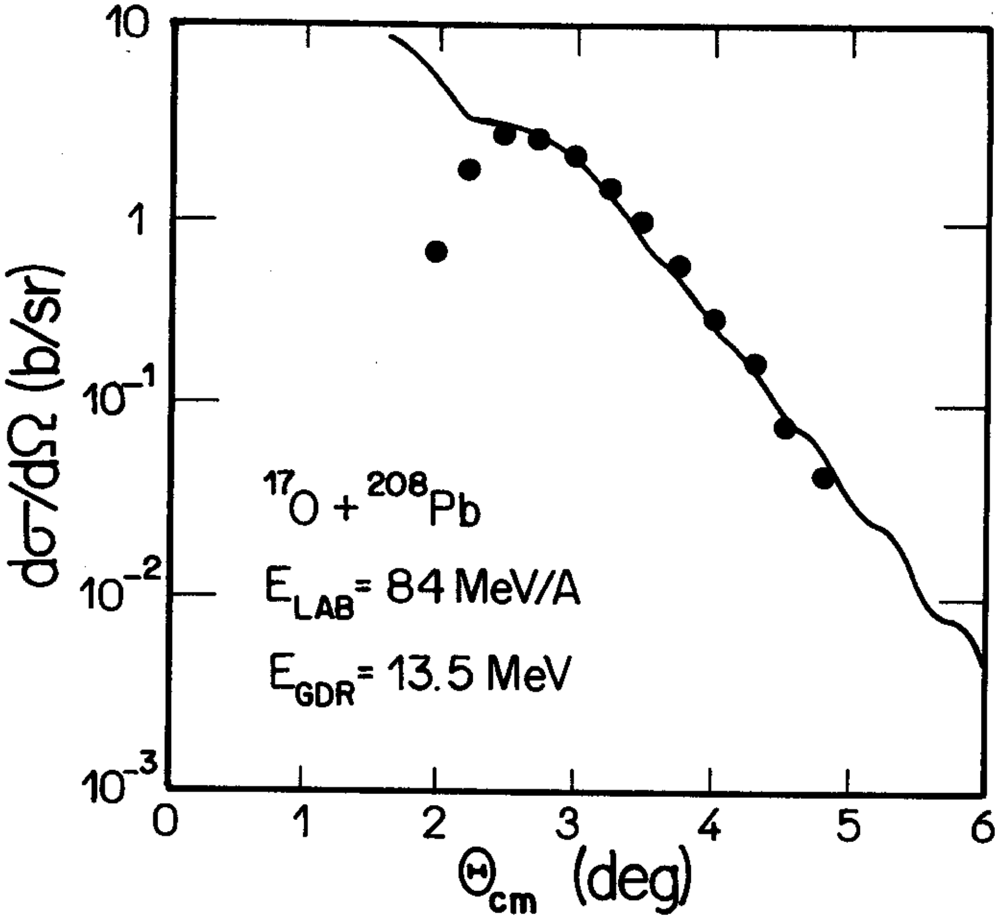


Figure 3

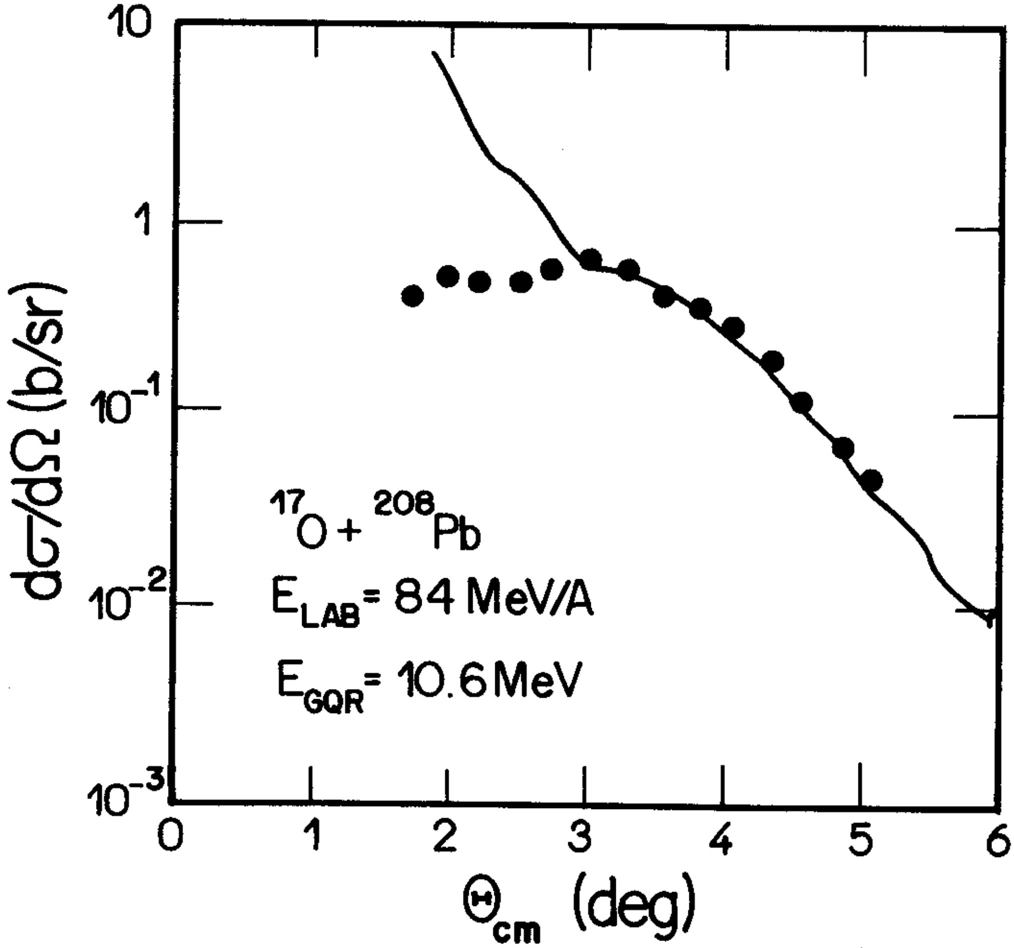


Figure 4