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Abstract: The Boltzmann equation with bosonic factors in the collision integral is solved for an expanding gas of mesons modelling the central rapidity region of central ^{16}O on Au collisions at 200 GeV/nucleon. The $\pi - \pi$ scattering-amplitude used in the collision integral is derived from a Bethe-Goldstone equation incorporating in-medium effects, and it is strongly reduced compared to the free space amplitude. Mean field effects are not included. In the purely pionic scenario, it is found that thermalization sufficient to explain the experimentally observed low-momentum peak in the pion spectra can be achieved if a hadronization time of $\sim$ 4.5 fm/c is assumed. Inclusion of the mesonic resonances reduces the effects of statistics as the pion phase space density is lower. Consequently a much shorter hadronization time of $\sim$ 1 fm/c is needed to describe the low $p_\perp$ data.

1. Introduction

In ultrarelativistic collisions of heavy ions a hot transient central region is formed that subsequently decays predominantly into π–mesons. One of the main goals of the theory of these collisions is to trace the evolution of the hot initial zone until the stage when the freezeout occurs. For this purpose one has to study the dynamics of the mesonic resonance gas. One question to be resolved is to what extent the observed pion spectra and, in particular, the enhancement over pp data at low momenta, should be associated with the hadronization process itself (see also Refs. [1],[2]). In other words, are π–π collisions in the rapidly expanding pion cloud effective and frequent enough to modify the initial distribution of the pions? If not, the experimentally observed distribution of the pions could well be a remainder of the hadronization process itself and may, for example, give us information on whether the pions originate from a hadronizing plasma blob or not.

To answer these questions, the evolution of the system was recently studied [3],[4] with a Boltzmann equation including Bose statistics.* The collision rate is considerable for large Bose final–state occupation factors and may cause a rapid condensation in momentum space. However, it has been shown [6],[7] that a more consistent treatment of π–π scattering in the gas results in an in–medium cross section which is considerably reduced in comparison to the free one. In equilibrium, the collision rate turns out to be not much larger than in a Boltzmann equation without final state Bose factors and the free π–π cross section [7].

In the present work we therefore calculate the evolution of the hot and dense meson gas using the Boltzmann equation with the Bose statistics and an in–medium cross section† for pions, as determined from the solution of a Bethe-Goldstone type equation. The in–medium scattering is discussed in Section 2. Specifically, we shall apply this formalism to consider pions in the rapidity range $2 < y_{lab} < 3$ from central ^{16}O on Au collisions at 200 GeV/nucleon. Section 3 describes the initial conditions for the hadronized

*Solutions of a pionic Boltzmann equation approximating the collision integral using a momentum independent relaxation time ansatz were considered in Ref. [5].

†Another aspect is the mean field [8], [9], which we do not consider here.

gas, assuming that the low momentum peaking is absent and that boost invariance holds for central rapidities. The important parameter for the hadronic evolution turns out to be the proper hadronization time that controls the initial density and expansion time of the meson gas [3]. We consider two scenarios: in the first, it is assumed that only pions are created during hadronization; in the second, mesonic resonances are included and the additional effect of their decay on the final pion spectra is studied. In the last Section we summarize and discuss these results.

2. Boltzmann equation with in-medium π–π scattering

The relativistic Boltzmann equation for the time evolution of pion phase space density $f_1(\vec{x},\vec{p})$ reads ($\hbar = c = 1$)

$$
\begin{aligned}
(p^0 \frac{\partial}{\partial t} + \vec{p}\frac{\partial}{\partial \vec{x}}) f_1(\vec{x},\vec{p}) \;&=\; -\frac{1}{4}\sum \int d\Gamma_2 d\Gamma_3 d\Gamma_4 \times \\
(2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \mid t \mid^2 &\{ f_1 f_2 \bar{f}_3 \bar{f}_4 - f_3 f_4 \bar{f}_1 \bar{f}_2 \} \quad , \qquad (1)
\end{aligned}
$$

where $d\Gamma_i = d^3p_i/[(2\pi)^3 2\omega(p_i)]$, $\omega(p_i) = \sqrt{m_\pi^2 + p_i^2}$, $\bar{f}_i \equiv 1 + f(\vec{x}_i,\vec{p}_i)$, m_π is the mass of the pion, with spatial coordinates $\vec{x}_i$ and momentum $\vec{p}_i$. Inclusion of resonances as dynamic degrees of freedom gives rise to additional source terms at the r.h.s. of Eq. (1).

The scattering of pions is described by a t–matrix that may be derived microscopically using the nonequilibrium Green's function technique (see Refs. [10]–[12], and references therein). The equation for the t–matrix is of the Bethe–Goldstone form (see also Ref. [13]); it describes the interaction of a pair of pions embedded in a pure pionic medium. The influence of the medium [6],[7] is accounted for through the phase space occupancy factors f, and by the single-particle energies in the propagator which, however, we shall take to be the free energies, as in Eq. (1). In the subsequent calculations we shall consider the π–π interaction in the isospin $I = 0$ s–wave and $I = 1$ p–wave channels. Corresponding to the resonances formed, the former is referred to as the σ–channel, and the latter as the ρ–channel. The transition matrix squared $|t|^2$ in (1) is decomposed into channel contributions α:

$$
\mid t \mid^2 = \sum_\alpha (2\ell_\alpha + 1)^2 \;\mid t_\alpha \mid^2 P^2_{\ell_\alpha}(\cos\theta) \quad , \qquad (2)
$$

where θ is the scattering angle. The solution of the t–matrix equation is greatly simplified if the π–π interaction amplitude is parametrized in a separable form [14]

$$V^{\alpha}_{\pi\pi} = \langle \pi\pi \mid V \mid \alpha \rangle \frac{1}{s - M^2_{\alpha} + i\epsilon} \langle \alpha \mid V \mid \pi\pi \rangle \ , \tag{3}$$

where $s = E^2 - \vec{P}^2$ is the center of mass energy squared of the pion pair having total momentum $\vec{P}$. We take the form factor to be

$$V_{\alpha}(k) = 4\pi \ \omega(k) \, g_{\ell_{\alpha}} \sqrt{2M_{\ell_{\alpha}}} \left(\frac{k}{k_{\ell_{\alpha}}} \right)^{\ell_{\alpha}} \left(1 + \frac{k^2}{k^2_{\ell_{\alpha}}} \right)^{-\ell_{\alpha}-1} \ , \tag{4}$$

with parameters $M_0 = 940$ MeV, $g_0 = 0.60 \, m_{\pi}^{-1/2}$, $k_0 = 2.71 \, m_{\pi}$ in the σ–wave channel, and $M_1 = 826.7$ MeV, $g_1 = 0.6684 \, m_{\pi}^{-1/2}$ and $k_1 = 3.34 \, m_{\pi}$ in the ρ–channel. The t–matrix in channel α then is

$$t_{\alpha}(k, k', P, s) = \frac{V_{\alpha}(k) \, V_{\alpha}(k')}{s - M^2_{\alpha} - \frac{1}{4\pi^2} \int_0^{\infty} \ dk \ k^2 \ \frac{V^2_{\alpha}(k) \ \langle \ 1 + f_1(k) + f_1(-k) \ \rangle}{\omega(k)(s - 4\omega^2(k) + i\epsilon)}} \ , \tag{5}$$

and the total cross section for channel α has the structure

$$\sigma_{\alpha} = \frac{2\ell_{\alpha} + 1}{128\pi\omega^2(k)} \mid t_{\alpha} \mid^2 \propto \frac{1}{(s - M^2_{\alpha} - a)^2 + b^2 \langle \ 1 + f_{k_s^+} + f_{k_s^-} \ \rangle^2} \ , \tag{6}$$

indicating that large occupation factors might cause, on average, a decrease in σ compared to its free space value [6],[7]. Very high occupation factors can actually lead to a Lee–Wick type bound state in Eq. (5), first appearing in the $\ell = 0, I = 0$ channel. In this paper we do not consider such effects. In solving the Boltzmann equation (1) with in-medium cross section we use the test particle method [15] extended to include Bose statistics [3],[16].

3. Results and Discussion

Let us first consider a simplified picture of the system after hadronization in which the pion scattering rate is assumed to be very high but pion annihilation processes such as $4\pi \rightarrow 2\pi$ are suppressed. The evolution of the gas would then be adiabatic with a distribution for the pions of the form

$$f(p) = [\exp\{(\sqrt{p^2 + m^2_{\pi}} - \mu_{\pi})/T\} - 1]^{-1} \ , \tag{7}$$

where μ_π and T are the chemical potential and temperature, respectively. In the adiabatic evolution the temperature would drop during the expansion and, with conserved pion number, the chemical potential could approach the pion mass. The pion spectrum would then be strongly peaked at low $p_\perp$ [1].

In the present kinetic framework, the Bose enhancement factors $(1+f)$ in Eq.(1) are responsible for driving the particles to regions of high phase space density via two–body collisions. The relevant question is therefore whether the collision rate is high enough, or the expansion slow enough, for the gas to come sufficiently close to thermal equilibrium. Previous calculations with free cross sections [3] showed that there is indeed enough thermalization in a pion gas to produce peaking at low $p_\perp$ consistent with the data. However, as discussed in the previous section, the pion scattering rate is decreased by medium effects, altering this conclusion quantitatively, and possibly qualitatively. Further, it is well known that high energy hadronic collisions produce a large abundance of heavy meson resonances, which reduce the pion phase space density and hence the pion scattering rate even further.

Therefore, to study the effectiveness of the collision processes in thermalizing the initially adopted distribution, we consider two different scenarios. In the first, we assume that only pions emerge from the hadronization of the transient central region. In the second scenario we shall assume that both pions and heavier resonances originate from the hadronization process. We consider specifically central ^{16}O on Au collisions at 200 GeV/nucleon and central rapidities $2 < y_{lab} < 3$, as measured by the NA35 collaboration [17].

Let us begin with the first scenario. Assuming boost invariance in the formation of pions at central rapidities, we take the initial pion distribution on the hyperbola $t^2 - z^2 = \tau_{eff}^2$ to be of the form [3]

$$\frac{d^6N}{dy_b\, dr_\perp^2\, dy\, dp_\perp^2} = \mathcal{N} \left[\exp\left[\beta\, m_\perp \cosh(y - y_b)\right] - 1\right]^{-1} \times\ \theta(\Delta y_b/2 + y_b)\, \theta(\Delta y_b/2 - y_b) \int_{-\infty}^{\infty} dz'\, \rho_0(r_\perp^2 + z'^2)\ , \quad (8)$$

where y is the particle rapidity, $p_\perp^2 = m_\perp^2 - m_\pi^2$ the transverse momentum, y_b is the boost rapidity that fixes the longitudinal position $z = \tau_{eff} \sinh y_b$, $r_\perp$ is the transverse position, and the normalization constant $\mathcal{N}$ is fixed by the total particle number. Further, ρ_O is the empirical projectile baryon density. The parameter τ_{eff} is the proper time needed for the formation of pions, and determines the spatial density of the initial distribution via its relation to the longitudinal position of the particles.

Integration over experimental rapidity distribution gives about 400 pions that we assume to emerge directly from the hadronization process. This fixes the normalization in Eq. (8). The width Δy_b of the boost rapidity is fixed to a value of 3.6 from the experimental rapidity distribution. We take the initial momentum distribution in the form of thermal distribution with zero chemical potential, i.e. exhibiting no low–$p_\perp$ peak.[‡] The initial distribution is indicated in Fig. 1 by the dashed line. It agrees with the NA35 data in the region $0.3 \lesssim p_\perp \lesssim 1$ GeV/c, while low $p_\perp$ the discrepancy is by about a factor of 2.

In the inset to Fig. 1 we show the low–$p_\perp$ behavior of the distribution evolved according to Eq. (1) for different values of the initial proper time τ_{eff}. One clearly sees that the lowering of τ_{eff} leads to an increased peaking of the distribution at low $p_\perp$. In other words, the increased initial pion density and collision rate drive the system to a state with high phase space density at low momenta. In the calculations with free π–π scattering cross sections [3], a value of $\tau_{eff} \sim 7$ fm/c was sufficient to generate the enhancement observed experimentally. In order to obtain the same result with the in–medium cross sections it is necessary to reduce τ_{eff} to a value of ~ 4.5 fm/c, i.e., the initial pion density has to be increased by about 60%. The corresponding spectrum is shown as the solid line in Fig. 1. The duration of the possible plasma stage therefore appears limited to $4 - 5$ fm/c, unless some of the peaking is generated in the hadronization process itself. However, we also note that this proper hadronization time must be interpreted as an average time for a mixed phase, i.e., globs of plasma might well survive for a much longer time.

We next examine how this conclusion is altered by inclusion of resonances. The resonances deplete the free pion density lowering the occupation factors,

[‡]The parameter β^{-1} in Eq. (8) is chosen to be 160 MeV, which roughly corresponds to the slope of the transverse momentum spectrum in proton-proton collisions. Further, the resulting particle rapidity distribution is bell–shaped, and describes the data well [3].

but may compensate for the decreasing effect of Bose enhancement by populating low momenta from decay "cooling" [18]-[20]. For exploratory reasons, we assume an extreme scenario in which mesons are initially in the ratio $\pi : \rho : \omega : \eta \sim 3 : 9 : 3 : 1$. These statistical weights are obtained when assuming that mesons result from (u,d)-symmetric quark-antiquark pair production. Phenomenological models used for quark jets, such as the Lund model [21], predict abundance ratios close to these values. So we start with 38 π, 114 ρ, 38 ω and 13 η mesons, and distribute them in phase space in analogy to Eq. (8). We describe the in-medium π–π scattering and the ρ-meson formation within the t-matrix approach. The initially formed ρ-mesons have a finite life-time [22] that is decreased over the free space value by the $(1+f)$ phase space occupancy factors of the pions. Further formation of ρ-mesons, however, is assumed to occur only in the point-like elastic π–π collisions, as described by the in-medium t-matrix formulation. In this way we avoid double counting, at the cost of increasing the pion phase space density somewhat. Elastic scattering of the pions with the ω and η mesons is treated by a Hauser-Feshbach type method outlined in Ref. [23].

We begin in Fig. 2 with results for the final negative pion distribution resulting from decays of the resonances alone (dotted line), with no particle propagation and no Bose $(1+f)$ factors. The low $p_\perp$-peaking of the NA35 data is not well described, in agreement with the results of Ref. [18]. We note here that the ω decay is treated using the momentum dependent decay matrix element [24]

$$\Gamma_{\omega\to 3\pi} \sim |\, \epsilon_{ijk} E_i \vec{k}_j \times \vec{k}_k \,|^2 \quad , \tag{9}$$

where ϵ_{ijk} is the usual completely anti-symmetric tensor. Using this form the pion decay spectrum is somewhat reduced at low $p_\perp$ when compared to the case of a constant probability across the Dalitz plot.

Next, the solid line in Fig. 2 shows the results for $\tau_{eff} = 4.5$ fm/c, the value obtained in the "pion-only" scenario. This time, however, the experimentally observed enhancement is not satisfactorily reproduced. This means that low momentum pions originating from resonance decays do not appear early enough in the expansion of the system to compensate for the lower pion phase space density, and there is less final state pion thermalization. To reproduce the data one has to consider much smaller hadronization times of the order $\tau_{eff} \approx 1$ fm/c (dashed curve in Fig.2), i.e., an initial meson density more than twice that in the "pions-only" scenario,[§] for the pions to thermalize sufficiently. In other words, our present calculations lead us to the conclusion that a pronounced peak as seen in the collision of ^{16}O on Au at 200 GeV/nucleon can only be explained by a rather early onset of the pion formation. This would give rise to strong Bose correlations and manifest itself in the low $p_\perp$ enhancement. If Bose correlations are not the (sole) origin of the peak structure at low $p_\perp$, it might likely be associated with the hadronization process itself. However, it remains to be seen whether these conclusions will be sustained in a more elaborated approach including further essential effects such as the mean field dynamics and baryonic degrees of freedom.

4. Conclusion

We have studied the collisions in an expanding pion gas by taking the in-medium π–π scattering into account. We find that the scattering rate in the gas for large final state Bose occupancy factors is largely compensated by an overall drop of the in-medium cross section, reducing the thermalization of the momentum distribution of the pions. In particular, investigating the low $p_\perp$ behaviour of the negative pion spectra from central collisions of ^{16}O on Au at 200 GeV per nucleon at central rapidities, it is found that a purely pionic scenario that starts with a high phase-space density as described by an average hadronization time of $\sim$ 4–5 fm/c, will thermalize sufficiently to explain the observed low $p_\perp$-peak in terms of Bose correlations alone. For a scenario based on a string fragmentation picture that initially includes higher mass meson resonances, the soft pion component of the spectra is only reproduced for an extremely high particle phase space density. The meson-resonance decays do not populate enough the low momentum region. In other words, Bose correlations can only describe the negative pion spectrum emerging from the collision of ^{16}O on Au at 200 GeV/nucleon if a comparatively short time interval for the onset of the pion formation is assumed.

[§] For the pure pion gas with $\tau_{eff} = 4.5$ fm/c, $f \overset{\propto}{\sim} 400/4.5$, while here, with $\tau_{eff} = 1$ fm/c, $f \overset{\propto}{\sim} 200/1$.

Despite the fact that our calculations consider the essential effect of the in–medium scattering of the pions, there are still some important effects to be included to make a final judgement of the soft pion puzzle. Essential aspects that need to be considered in a more elaborated approach are the inclusion of mean field effects, and the baryonic degrees of freedom, in particular states that produce several sequential decay pions.

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Figure Captions

Fig. 1. Transverse momentum spectrum of pions for $-0.5 < y < 0.5$. The circles are the NA35 data for central ^{16}O on Au collisions at at 200 GeV/nucleon [17], at central rapidities $2 < y_{lab} < 3$. The dashed line is the input spectrum at $t = \tau_{eff} = 4.5$ fm/c, while the solid curve is the final spectrum. The inset shows the low $p_\perp$ behaviour of the final spectrum for $\tau_{eff} = 5$ fm/c (dotted line), $\tau_{eff} = 4.5$ fm/c (solid), $\tau_{eff} = 4$ fm/c (long dashes), and $\tau_{eff} = 3$ fm/c (dash–dotted line). The short dashed line is the input distribution.

Fig. 2. As in Fig. 1, but for an expanding gas of π, ρ, ω and η mesons, and $\tau_{eff} = 1$ fm/c (dashes), 4.5 fm/c (solid line), and pure resonance decays (dots).

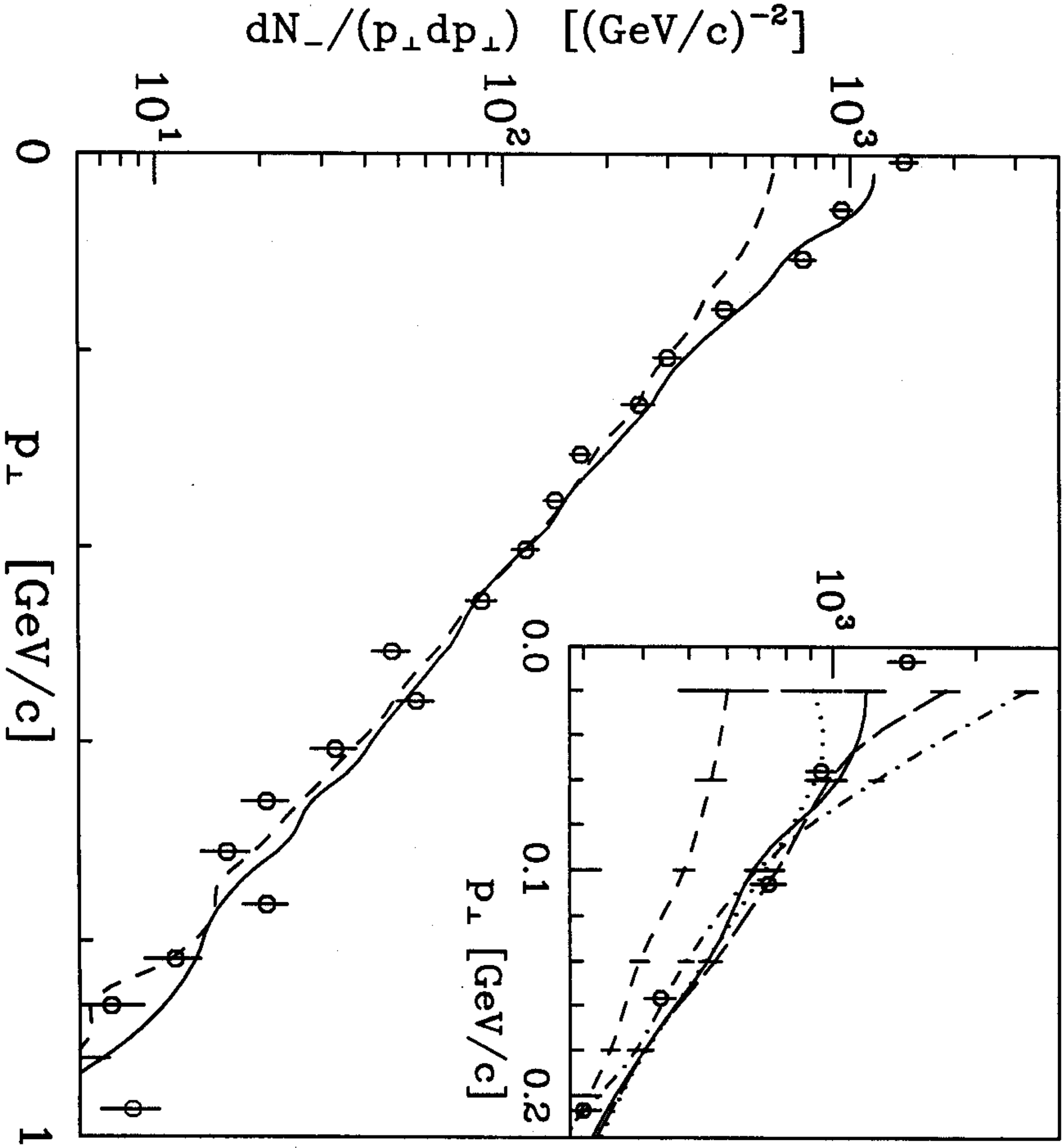
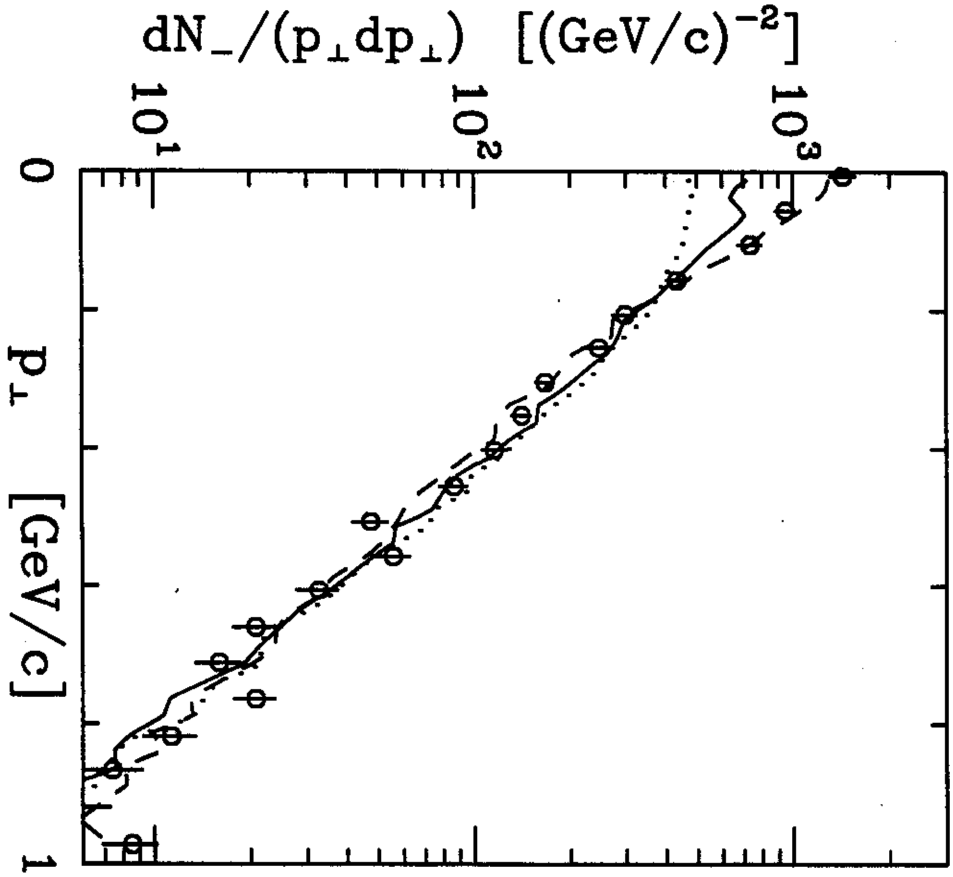


Fig.1 H.W.Barz et al



H.W. Barz et al Fig.2