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HADRONIC INTERFEROMETRY IN HEAVY ION COLLISIONS

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1 INTRODUCTION AND SUMMARY

1.1 Interferometry

In 1801 Young provided a proof for the wave nature of light by using amplitude interference in his famous double-slit experiment. In this experiment coherent light from a distant source was passed through two slits separated by a distance d, and interference maxima and minima were observed on a distant screen. From elementary geometrical considerations, one can show that constructive interference occurs (at normal incidence) when

$$d \sin\theta_n = n\,\lambda \quad \text{for } n = 1, 2, \ldots \tag{1}$$

where θ_n is the angle of the n^{th} order maximum. Originally this equation was used to measure the wavelength λ of monochromatic light. In principle, however, it can also be used to determine an unknown slit separation ('source size'), d, by measuring θ_n and using light of a known wavelength.

For applications in astronomy, amplitude interference measurements are complicated by the fact that the relative phase of the two paths is not just given by the length difference, $d \sin\theta$, but contains also uncontrolable contributions from atmospheric distortions. Nevertheless, in 1920 Michelson was able to determine the angular diameter of a seven stars by using a stellar interferometer to determine the degree of coherence of light from distant stars illuminating two slits (1).

Hanbury Brown and Twiss overcame the limitations of stellar amplitude interferometry and developed two-photon intensity interferometry as a technique for astronomical distance measurements (2, 3, 4). In this technique, which is now commonly referred to as the Hanbury-Brown/Twiss (HBT) effect, one records the two-photon correlation function for incoming coincident photons as a function of their relative momentum. This correlation function can be written as

$$R(\vec{k}_1, \vec{k}_2) = \frac{\langle n_{12}\rangle}{\langle n_1\rangle\langle n_2\rangle} - 1\ . \tag{2}$$

Here $\langle n_{12}\rangle$ is the probability of detecting two coincident photons of wavenumber $\vec{k}_1$ and $\vec{k}_2$ in detectors 1 and 2, and $\langle n_i\rangle$ is the probability of detecting a photon of momentum $\vec{k}_i$ in detector i ($i = 1, 2$). Equation 2 contains only count rates, which are proportional to the absolute squares of the amplitudes. As a consequence, HBT interferometry is insensitive to phase shifts introduced by atmospheric disturbances. It can be used with very large base lines and delivers superior resolution. This was first shown in (4) by measuring the angular diameter of Sirius.

The physical basis of the HBT effect is that two photons have a non-zero correlation function due to the symmetrization of their wave functions, a consequence of the quantum statistics for identical particles. This can be understood by considering the simplified case of simultaneous photon emission from two distant point sources located at $\vec{r}_a$ and $\vec{r}_b$. Assuming propagation in vacuum, the coincidence rate will then be proportional to the symmetrized two-photon wavefunction, normalized by the incident fluxes:

$$\begin{aligned} n(\vec{k}_1,\vec{k}_2) &= \tfrac{1}{2}|\exp(i\vec{k}_1\vec{r}_a + i\vec{k}_2\vec{r}_b) + \exp(i\vec{k}_1\vec{r}_b + i\vec{k}_2\vec{r}_a)|^2 \\ &= \tfrac{1}{2}|\exp[i\tfrac{1}{2}(\vec{k}_1-\vec{k}_2)(\vec{r}_a-\vec{r}_b)] + \exp[i\tfrac{1}{2}(\vec{k}_1-\vec{k}_2)(\vec{r}_b-\vec{r}_a)]|^2 \\ &= 2\cos^2[\tfrac{1}{2}(\vec{k}_1-\vec{k}_2)(\vec{r}_a-\vec{r}_b)] \qquad (3) \end{aligned}$$

The correlation function depends on the relative momentum of the two photons, $\vec{q} = \frac{1}{2}(\vec{k}_1 - \vec{k}_2)$, and on the spatial separation of the two sources, $\Delta r = (\vec{r}_a - \vec{r}_b)$. The correlation function corresponding to simultaneous emission from an extended and incoherently emitting source is obtained by integrating this equation over the spatial extent of the source. In this simplified case, the correlation function is given by the Fourier transform of the source function.

1.2 Interferometry in Subatomic Physics

The ideas of Hanbury Brown and Twiss can also be applied to other particles. The generalization to other pairs of identical particles is straight forward, both for bosons (such as photons, pions and other mesons) and fermions (such as nucleons).

It was soon realized that intensity interferometry could also be used for size determinations in subatomic physics. Goldhaber et al. (5, 6) studied angular distributions of π mesons in proton-antiproton annihilation processes and found that the emission probability of coincident identical pions was strongly affected by their Bose-Einstein statistics, which (in analogy to Equation 3) causes an enhancement of the correlation function at zero relative momentum, $q = 0$. The width of the maximum at $q = 0$ depends on the radius of the interaction volume (6). Later, Shuryak (7) pointed out that pion correlations are not only sensitive to the spatial dimensions of the source, but also to the time dependence of the emission process. In that sence, intensity interferometry is sensitive to the space-time characteristics of the emitting system. Other work (8, 9, 10, 11, 12, 13, 14, 15) has developed pion intensity interferometry into a quantitative tool for the investigation of subatomic pion sources.

Pion interferometry has received renewed interest in the field of relativistic heavy ion collisions of beam energies per nucleon around 1 GeV. Experiments have collected information on the apparent pion source size (16, 17, 18, 19, 20, 21, 22), and their results have been compared to the results of intranuclear cascade calculations (23).

Intensity interferometry is now strongly pursued at ultra-relativistic energies as well. Topics of particular interest are the interplay between source dynamics and final state interaction (24, 25, 26, 27), pion correlations from an exploding source (28, 29, 30), pion correlations calculated via microscopic space-time models (31, 32), and pion interferometry as a probe for the possible formation of a quark-gluon plasma (33, 34, 35, 36). Several sets of data have been taken (37, 38, 39, 40, 41), and their analysis should greatly clarify the situation.

Other authors have worked on the relationship between short-range correlations and intermittency (42, 43), three and more pion correlations (44, 45), pion interferometry and anti-deuteron production (46) and coherent production of pions (47, 48, 49, 50). Intensity interferometry using kaons has also been investigated (51, 52, 53), because kaons probe different parts of the space-time geometry of heavy-ion reactions due to their longer mean free path in hadronic matter and their lower contamination from long-lived resonances. First results on $2K^+$ correlations from central ^{28}Si + Au collisions at 14.6 A GeV have now been reported by the E802 collaboration (54, 55).

The future for this subfield at the Relativistic Heavy Ion Collider (RHIC) looks very bright. Several thousand pions are expected to be produced in a single central collision. This large number will decrease the statistical uncertainty associated with the measurement, allowing all six dimensions of information in the two-pion correlation function to be probed. In fact, a reasonable correlation functions could be constructed from a single event.

Even though the emission probability of high energy photons in heavy ion collisions is small, the two-photon HBT effect was also studied theoretically (56) for this case. The interpretation of two-photon correlation functions in nuclear collisions is complicated by the fact that neutral pions decay into two energetic photons and thus provide a very strong background signal. Nevertheless, first experimental results have already been obtained by using the Two-Arm-Photon-Spectrometer (TAPS) (57).

1.3 Interferometry for Fermions

One can also use identical particles obeying the Fermi-Dirac statistics for intensity interferometry studies. Koonin (58) proposed to use two-proton intensity interferometry to obtain pictures of high energy nuclear collisions.

There are several advantages of using protons for intensity interferometry in heavy ion collisions. First, they are already present in nuclei and can therefore be readily emitted without requiring large energy expenditures for the creation of their rest mass as is the case for pions and other mesons. Hence, two-proton emission in intermediate energy collisions imposes less severe phase space constraints than two-meson emission. Therefore, protons can be used as a probe at much lower energies. Second, the two-proton relative wave function contains the prominent ^{2}He-'resonance', which leads to enhanced sensitivity of the correlation function to the source size. Lastly, protons are easy to detect with the required resolution.

For heavy ion collisions, two-proton interferometry has been studied intensively during the last few years. Investigations have focussed on the spatio-temporal extension of the source (59, 60, 61, 62, 63, 64, 65, 66), on the dependence of the extracted source radii on the 'violence' of the collision (67, 68, 69), on lifetime effects in evaporation from compound nuclei (70, 71, 72, 66, 73, 74, 75), and on longitudinal and transverse correlation functions (76, 65, 66). Correlation functions have also been studied as a function of projectile and target mass (62, 77, 78) and kinetic energy of the proton pairs (60, 79, 80, 61, 69, 62, 76, 64, 65, 66, 78).

Recent progress has been centered around the theoretical computation of two-proton correlation functions from nuclear transport theory (64, 65, 66, 78). In this framework, it is now possible to understand the dependence of the correlation functions on the parameters discussed above. Comparisons of this theory to experimental data (64, 66, 78) have now established two-proton intensity interferometry as a quantitative tool to study heavy ion reaction dynamics. We will discuss these new results in the main part of this review.

There have also been theoretical (81, 82, 83, 84) and experimental (85,

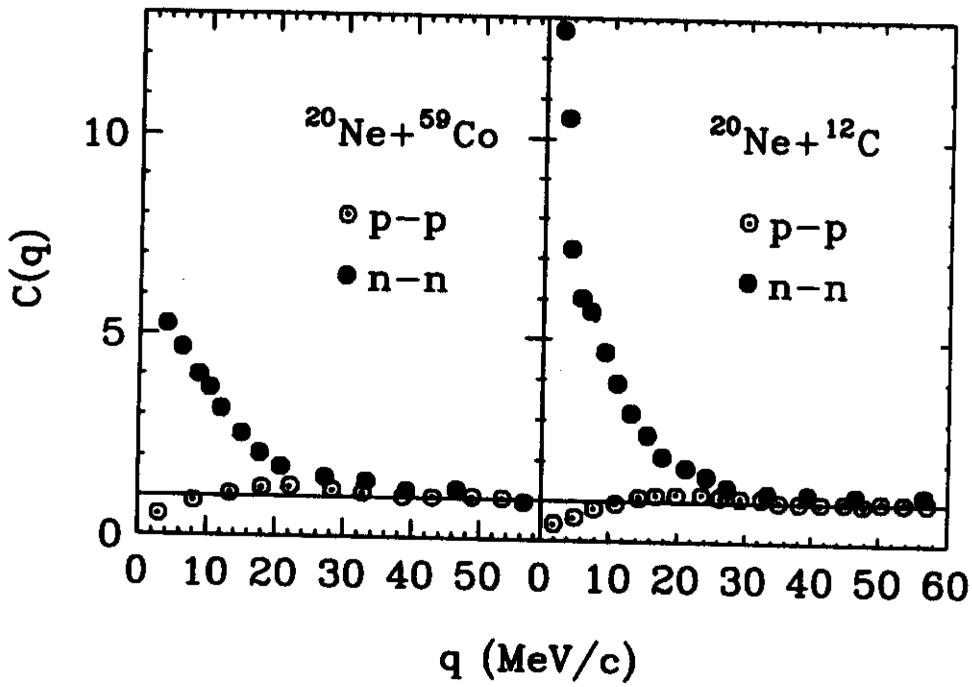

Figure 1: Correlation functions for pp and nn correlations in ^{20}Ne + C and ^{20}Ne + Co reactions at E/A = 30 MeV. The data are taken from (98).

86, 87, 88, 89, 90, 91, 92, 93, 94, 95) investigations of large-angle proton-correlations in heavy-ion collisions. At these large relative angles, one finds that the two-proton correlation function is dominated by direct proton knock-out (85, 86) and by the effects of the conservation laws of total energy (82), particle number (96), momentum (81, 87, 93, 83, 84), and angular momentum (97).

Recently, several groups have begun to study two-neutron correlations (99, 97, 100, 98). This probe has the advantage that there is no Coulomb interaction between the two neutrons or between the neutrons and the emitting source. However, there are bigger experimental obstacles due to the relatively small neutron detection efficiencies of good-resolution detectors and due to problems associated with 'cross-talk' between neighboring detectors. The difference between the two-proton and two-neutron correlation functions is illustrated (98) in Figure 1. For small source radii, the two-neutron correlation function exhibits a strong maximum at zero relative momentum, caused by the same resonance that brings about the maximum in the p-p correlation function. However, in the n-n case, there is no Coulomb repulsion which causes the p-p correlation function to go to zero at $q = 0$.

And, finally, there is also a history of studies of fragment-fragment correlation functions (101, 102, 103, 80, 68, 93, 69, 61, 62, 104, 105, 106, 107, 108) Here, however, the HBT effect due to quantum statistics is irrelevant, and only Coulomb repulsion (102, 106, 107, 108) between the fragments and decays of excited prefragments or particle-unstable states (109, 110, 111, 112, 113, 72) are important.

In the remainder of this review we will present the current status of two-proton intensity interferometry in heavy ion collisions, where much progress was made during the last two years. For a current review of the use of other probes the reader is referred to (114). In general, we did not quote unrefereed work such as conference proceedings or preprints. However, there are some recent conference proceedings published in book form which contain additional papers and references on the present subjects of interferometry and transport theory (115, 116, 117).

2 THEORETICAL BASIS

2.1 Interferometry with Strongly Interacting Probes

The original HBT effect is solely caused by the quantum statistics of the two identical particles. Final state *interactions* of the two outgoing particles were considered negligible. For photons, this assumption is exact. Already for charged pions, one must also consider their mutual Coulomb interaction, while it is still a reasonable approximation to neglect their strong interaction, because low energy $\pi\pi$ scattering is dominated by a slowly varying $I = 2$ phase shift (114). There are, however, some recent investigations of the role of the strong interaction in pion interferometry (118, 119). An unwanted contribution of the strong interaction to charged-pion interferometry is given by resonance decays such as $\eta' \rightarrow \eta\pi^+\pi^-$, $\eta \rightarrow \pi^0\pi^+\pi^-$, in which two like-charged pions are dynamically correlated (120).

For proton emission, the effects of the strong interaction are very important and have been studied in early works (58, 121, 122). The shape of the correlation function at small relative angle reflects then the interplay between the short-range nuclear interaction (including the ^{2}He-'resonance' (123)), the long-range Coulomb interaction, and the Pauli exclusion principle (58, 124).

2.2 Formalism

The theoretical expression for the two-proton correlation function, $C(\vec{P},\vec{q})$, can be written as (58, 28, 65)

$$C(\vec{P},\vec{q}) = R(\vec{P},\vec{q}) + 1 = \frac{\Pi_{12}(\vec{p}_1,\vec{p}_2)}{\Pi_1(\vec{p}_1)\Pi_1(\vec{p}_2)}$$
$$= \frac{\int d^4x_1 d^4x_2\, g(\frac{1}{2}\vec{P},x_1) g(\frac{1}{2}\vec{P},x_2) \left|\phi\left(\vec{q},\vec{r}_1-\vec{r}_2+\frac{\vec{P}(t_2-t_1)}{2m}\right)\right|^2}{\int d^4x_1\, g(\frac{1}{2}\vec{P},x_1)\int d^4x_2\, g(\frac{1}{2}\vec{P},x_2)}\,, \qquad (4)$$

where $\vec{P}$ and $\vec{q}$ are the total and relative momenta, $\vec{P} = \vec{p}_1 + \vec{p}_2$ and $\vec{q} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$, respectively, and ϕ is the relative wave function. x_1 and x_2 are the space-time points of the emission of protons 1 and 2. Π_1 is the single- and Π_{12} is the two-particle emission probability. $g(\vec{p},x) \equiv g(\vec{p},\vec{x},t)$ is the single-proton phase space emission function.

A derivation of Equation 4 can be found in (65). It is based upon the assumption that the final-state interaction between the two detected protons dominates, that final-state interactions with all remaining particles can be neglected, that the correlation functions are determined by the two-body density of states as corrected by the interactions between the two particles, and that the emission function $g(\vec{p},x)$ varies slowly as a function of momentum $\vec{p}$ (i.e. $g(\vec{p},x) \approx g(\vec{p}\pm\vec{q},x)$).

Within this approximation, the correlation function depends only on the final relative positions of all particles with momentum $\frac{1}{2}\vec{P}$. This can be illustrated more clearly by rewriting Equation 4 as:

$$C(\vec{P},\vec{q}) = \int d^3r\, F_{\vec{P}}(\vec{r})\, |\phi(\vec{q},\vec{r})|^2 \,. \qquad (5)$$

Here $\vec{r} = \vec{r}_1 - \vec{r}_2$ is the relative coordinate of the two emitted particles and the function $F_{\vec{P}}(\vec{r})$ is defined as

$$F_{\vec{P}}(\vec{r}) = \frac{\int d^3R\, f(\frac{1}{2}\vec{P},\vec{R}+\frac{1}{2}\vec{r},t_>) f(\frac{1}{2}\vec{P},\vec{R}-\frac{1}{2}\vec{r},t_>)}{\left(\int d^3r\, f(\frac{1}{2}\vec{P},\vec{r},t_>)\right)^2}\,, \qquad (6)$$

where $\vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$ is the center-of-mass coordinate of the two particles, and the Wigner function $f(\vec{p},\vec{r},t_>)$ is the phase space distribution of particles with momentum $\vec{p}$ and position $\vec{r}$ at some time $t_>$ after both particles have been emitted:

$$f(\vec{p},\vec{r},t_>) = \int_{-\infty}^{t_>} dt\, g(\vec{p},\vec{r}-\vec{p}(t_> - t)/m, t)\ . \tag{7}$$

For a given momentum $\vec{P}$, the correlation function has three degrees of freedom, $\vec{q}$, which are a function of $F_{\vec{P}}(\vec{r})$. Therefore correlation function measurements should allow the extraction of $F_{\vec{P}}(\vec{r})$, the normalized probability of two protons with the same momentum $\vec{P}/2$ being separated by $\vec{r}$.

Alternatively, one may use correlation function measurements to test various theoretical models capable of predicting $g(\vec{p},\vec{r},t)$ and thus making specific predictions about the correlation functions. This approach is more realistic in its goals as a full six-dimensional determination of $C(\vec{P},\vec{q})$ is very difficult in practice.

2.3 Sensitivity of the Two-Proton Correlation Function

In this section, we follow the discussion of (65).

The two-proton relative wave function appearing in Equation 5, $\phi(\vec{q},\vec{r})$, is influenced by identical particle interference, short-range hadronic interaction, and Coulomb repulsion.

For noninteracting identical particles, the squared wave function has the form

$$|\phi(\vec{q},\vec{r})|^2 \propto 1 \pm \cos(2\,\vec{q}\cdot\vec{r}) \tag{8}$$

where the upper sign stands for bosons, and the lower sign is for fermions of identical spin projection. (For spin-half particles with random spin orientations, the correlation function is reduced to one-half at $|\vec{q}| = 0$ and returns to unity with a width $\Gamma_q \approx 1/R$.)

The Coulomb interaction reduces the correlation function to 0 as $|\vec{q}| \rightarrow 0$. Since typical nuclear sources are small compared to the two-proton Bohr radius, $R_b = 2\,\hbar^2/(e^2\,M_p) = 57.6$ fm, the shape of this Coulomb dip depends only on the Gamov penetration factor (15)

$$G(|\vec{q}|) = \frac{2\pi\,\alpha M_p}{|\vec{q}|\ [\exp(2\pi\alpha M_p/|\vec{q}|) - 1]}\ . \tag{9}$$

The strong interaction is an excellent size gauge for nuclear systems in the case of two-proton interferometry. This is because of the appearance of

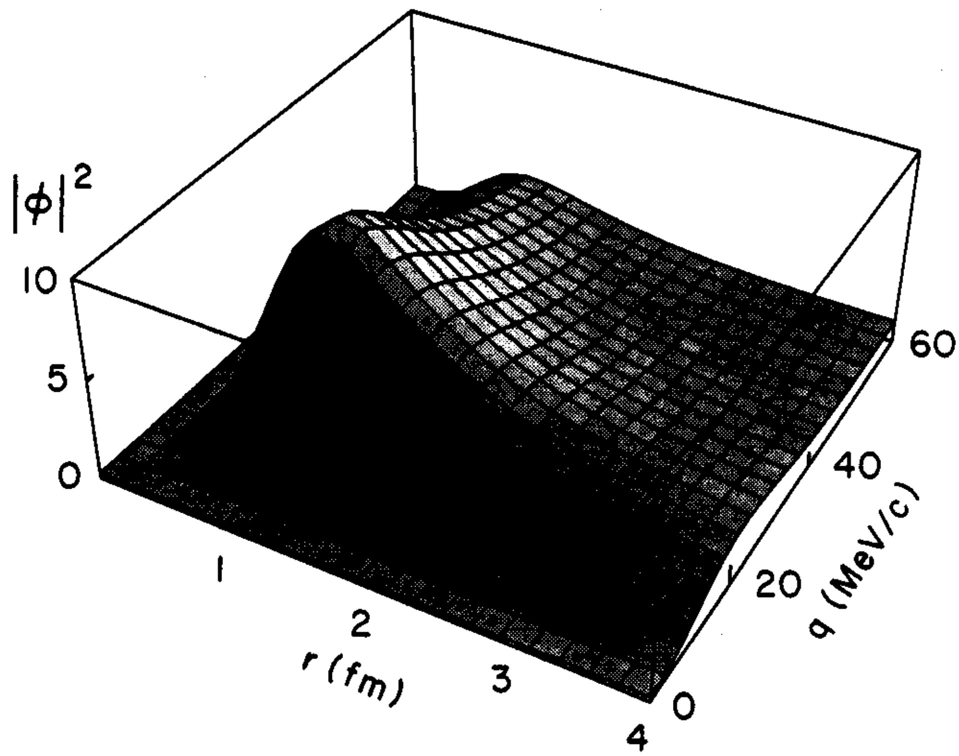

Figure 2: Two-proton relative wave function, $|\phi^2|$, as a function of the two-proton distance r and relative momentum q for $\theta = 45°$.

the ^{2}He-'resonance', a bump in the two-proton spectrum at $|\vec{q}| \approx 20$ MeV/c, the height of which is roughly proportional to R^{-3} (110, 114, 65). Due to the absence of Coulomb suppression at $q = 0$, two-neutron correlation functions exhibit even greater sensitivities.

In (65) the relative wave function was calculated by numerically solving the Schrödinger equation with the Reid soft core potential (125) for the $\ell = 0$ and $\ell = 1$ partial waves. Gong et al. used the full solution Coulomb waves, $\phi_c(\vec{q}, \vec{r})$, and added the modification $\delta\phi(\vec{q}, \vec{r})$ due to the strong interaction in the two lowest ℓ channels. In principle, $|\phi|^2$ is dependent on the six variables $\vec{q}$ and $\vec{r}$. Due to rotational symmetries, however, $|\phi|^2$ can be completely determined by three independent variables, for which we choose $r = |\vec{r}|$, $q = |\vec{q}|$, and $\theta = \cos^{-1}(\vec{r} \cdot \vec{q}/rq)$. In Figure 2, we display $|\phi|^2$ as a function of q and r for $\theta = 45°$. Due to the dominance of the s-wave interaction,

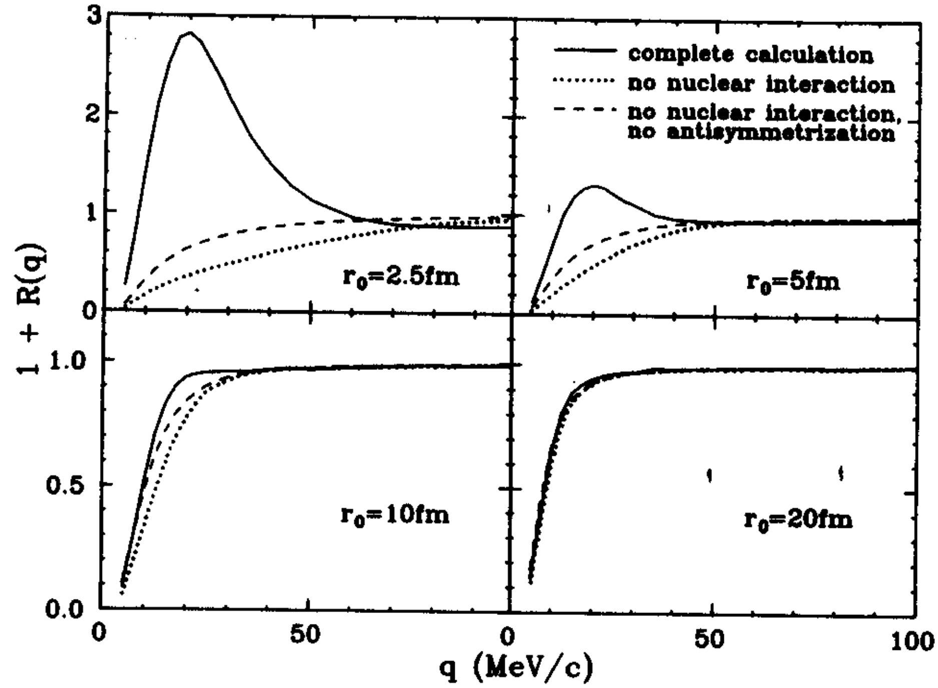

Figure 3: Two-proton correlation functions calculated with a source parameterization according to Equation 10. The solid lines represent the complete calculations including the effects of quantum statistics and of the Coulomb and strong interaction. The dashed lines represent the case of only Coulomb interaction, and the dotted line is for Coulomb interaction plus the effect of the Fermi-Dirac statistics for the two protons. [After (65)]

pictures for other values of θ look similar to Figure 2 in the coordinate and momentum space intervals considered here. At larger relative momenta and coordinates, one can see differences at different angles θ due to the p-wave interaction.

It is instructive to compute the two-proton correlation function in a simple source geometry to illustrate its sensitivity to the three effects discussed above. In (65) a zero-lifetime Gaussian source parameterization

$$g_0(\vec{p}, \vec{r}, t) = \rho_0 \exp(-r^2/r_0^2)\, \delta(t - t_0) \tag{10}$$

was chosen, where r_0 is the radius of the source. Figure 3 illustrates the effects of the different contributions to the two-proton final state interaction for sources of different radii. The Coulomb interaction completely dominates the shape of the correlation function for very large source radii. For $r_0 < 20$

fm, the correlation function becomes increasingly sensitive to the effects of antisymmetrization and the strong interaction, which clearly dominates at $r_0 = 2.5$ fm due to the prominent ^{2}He resonance.

One may illustrate the sensitivity of the shape of the correlation function on the lifetime and/or the temperature of the emitting source by choosing a simple parameterization,

$$g_t(\vec{p}, \vec{r}, t) = g_0\, \theta(R_s - r)\, \theta(t - t_0)\, p \exp\left(-\frac{p^2}{2M_pT} - \frac{t - t_0}{\tau}\right) . \tag{11}$$

Here, T and τ denote the (constant) source temperature and lifetime, respectively, R_s is the fixed source radius, and t_0 is the (fixed, but arbitrary) time at which the source starts to radiate. Calculations based on this parameterization are carried out in (65), and it is shown that the two-proton correlation functions are very sensitive to the lifetime τ in the interval [0,100] fm/c, but do not change noticeably for lifetimes greater than 300 fm/c. For this study, the fixed parameters were chosen to have values $T = 6$ MeV and $R_s = 5$ fm.

One may also investigate the sensitivity of the correlation functions to the relative angle between the sum momentum $\vec{P}$ and the relative momentum $\vec{q}$ of the pair,

$$\psi = \cos^{-1}\left(\frac{\vec{P}\cdot\vec{q}}{Pq}\right) . \tag{12}$$

It was predicted by Pratt (24, 28) that the transverse or sideward, $C(\psi = 90°)$, and longitudinal or outward, $C(\psi = 0°)$, correlation functions should exhibit a characteristic difference for long-lived sources. (Such a long-lived source could possibly signal the hadronization of a quark-gluon plasma created in ultra-relativistic heavy ion collisions (28, 34)) Bertsch (33, 34) showed in ultra-relativistic cascade calculations that this effect yields measurably different longitudinal and transverse correlation functions. The JANUS group (22) has experimetally observed a directional dependence of the two-pion correlation function for light projectile and target systems at the Bevalac. But the beam energy in this case ($E/A < 2$ GeV) was definitely too low to expect this observation to have any connection with the physics of the quark-gluon plasma. For intermediate energy heavy ion collisions and realistic cuts in ψ, however, Gong et al. (65) find that the differences between transverse and longitudinal two-proton correlation functions is typically only of the order of about 10% for lifetimes between 30 fm/c and 100 fm/c, and even lower outside this interval.

3 EXPERIMENTAL DETERMINATION OF CORRELATION FUNCTIONS

3.1 Normalization

For collisions at fixed impact parameter, the correlation function $1+R(\vec{P},\vec{q})$ is related to the single- and two-particle yields, $Y(\vec{p})$ and $Y(\vec{P},\vec{q})$:

$$Y(\vec{P},\vec{q}) \equiv Y(p_1,p_2) = \lambda[1 + R(\vec{P},\vec{q})]Y(\vec{p}_1)Y(\vec{p}_2) \ , \tag{13}$$

where $\vec{p}_1$ and $\vec{p}_2$ denote the momenta of the two detected particles, $\vec{P} = \vec{p}_1+\vec{p}_2$ is the total momentum of the particle pair, and $\vec{q}$ is the momentum of relative motion (defined in the center-of-momentum frame of the particle pair, where $\vec{P} = 0$; non-relativistically, $\vec{q} = \mu\vec{v}_{\text{rel}}$). The constant λ can be determined from the condition that $R(\vec{P},\vec{q}) = 0$ for sufficiently large relative momenta for which modifications of the two-particle phase-space density due to quantum statistics or final-state interactions become negligible.

In general, experimental determinations of two-particle correlation functions involve averages over impact parameter as well as implicit integrations over some of the six variables $\vec{P}$, $\vec{q}$ (15, 126). Most experimental correlation functions are determined according to the relation (114, 126):

$$\sum Y(\vec{p}_1,\vec{p}_2) = \lambda'[1 + R(\zeta)]\sum Y'(\vec{p}_1,\vec{p}_2) \ . \tag{14}$$

In Equation 14, $Y'(\vec{p}_1,\vec{p}_2)$ is the "background" yield, λ' is a normalization constant which ensures proper normalization at large relative momenta, and ζ denotes the variables for which the explicit dependence of the correlation function is evaluated (the most common choice is $\zeta = |\vec{q}|$). For each experimental gating condition (representing implicit integrations over a number of variables), the sums on both sides of Equation 14 are extended over all energy and detector combinations corresponding to the given bins of ζ. The experimental correlation function is defined in terms of the ratio of these two sums. Comparisons with theoretical results must take this definition into account (66, 126).

Two different approaches are commonly used for the construction of the background yield. In the "singles technique", the background yield is taken as proportional to the product of the single particle yields, measured with the same external trigger conditions as the true two-particle coincidence yield (59, 60, 127, 103, 69, 61, 76, 71, 64, 66, 128):

$$Y'(\vec{p}_1, \vec{p}_2) \propto Y(\vec{p}_1)Y(\vec{p}_2)\ . \tag{15}$$

In the "event-mixing technique", the background yield is generated by mixing particle yields from different coincidence events (12, 20, 67, 129, 95, 130, 77, 131, 132):

$$Y'(\vec{p}_1, \vec{p}_2) = \sum_{n \neq m} \delta^3(\vec{p}_1 - \vec{p}_{1,n})\, \delta^3(\vec{p}_2 - \vec{p}_{2,m})\ . \tag{16}$$

Here, the indices n and m label the n^{th} and m^{th} recorded two-particle coincidence events, and $\vec{p}_{1,n}$ and $\vec{p}_{2,m}$ denote the momenta of particles 1 and 2 recorded in events n and m, respectively. In most analyses, the index n runs over all recorded coincidence events and the index m is varied according to $m = n + k$, with typically $0 < k < 1000$. For large data sets, there is no need to eliminate the diagonal elements in the summation in Equation 16, and one may sum over all combinations of n and m (126).

If single- and two-particle data represent very different averages of impact parameter, the use of the singles technique may lead to serious distortions of the correlation function (133). Furthermore, less interesting correlations, resulting for example from phase space constraints due to conservation laws (96, 87, 60, 89, 93, 83, 84), may be suppressed by using the event-mixing technique. These advantages of the event-mixing technique have to be weighed against the disadvantage that the event-mixing technique attenuates the very correlations one wishes to measure (20, 126). The degree of this attenuation depends on the experimental apparatus and the magnitude of the correlations. Hence, quantitative analyses require careful Monte-Carlo simulations. For the extraction of undistorted correlation functions iterative procedures have been developed (20).

For statistical emission processes in which the emission of a single particle has negligible effect on further emissions, single and two-particle yields should originate from similar regions of impact parameters. In such instances, the singles technique appears to be the preferential choice (126).

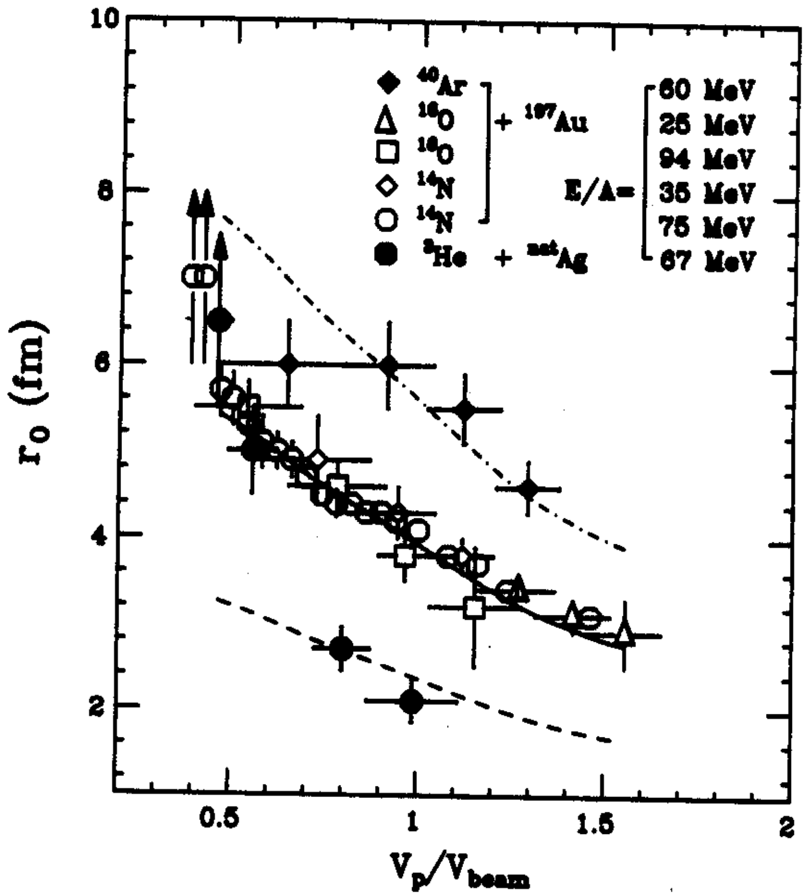

Figure 4: Systematics of Gaussian source radii extracted for a variety of reactions. [After (78)]

3.2 Source Size

It is customary to compare different correlation functions obtained from experiments or theories by fitting them to correlation functions generated with the aid of Equation 4 from simple zero-lifetime Gaussian sources parameterized according to Equation 10. The 'source size' r_0 is a free parameter in these fits. One should, however, keep in mind that the assumption of zero lifetime is unrealistic. Therefore one extracts generally different source sizes for the same emitting system, when fitting to correlation functions generated for different bins in $\vec{P}$, the sum momentum of the two emitted protons.

Nevertheless, it is instructive to perform such fits, because they simplify the comparison of correlation functions observed for different systems. A first systematic compilation of extracted source radii from two-pion correlation measurements was performed by Bartke (21), who showed that the extracted radius parameter scales with the radius of the projectile.

Another systematic study of this kind is shown in Figure 4, which is taken from Reference (78). The data points are the values of r_0 extracted from parameterizations with Equation 10. There are displayed as a function of $v_p/v_{\rm beam}$, where $v_{\rm beam}$ is the beam veolcity, and $v_p = \frac{1}{2}(p_1 + p_2)/M_p$ is the average velocity of the coincident proton pair. In this comparison the 'source sizes' for ^{14}N and ^{16}O projectiles track each other almost completely. They are interpolated by the solid line. The dashed and dot-dashed lines are obtained by scaling the solid line by factors of $(3/14)^{1/3}$ and $(40/14)^{1/3}$, respectively. For the emission of very energetic protons ($v_p/v_{\rm beam} > 0.5$) the radius parameter scales approximately with projectile radius. For the emission of lower energy protons, however, lifetime effects dominate the correlation functions, and a simple geometrical interpretation in terms of the radii of target and projectile becomes meaningless.

One effect that can be responsible for the observed pair-momentum dependence of the extracted source radius is cooling of the source. As a consequence of the falling temperature, the characteristic lifetime for high-energy particles is much shorter than the one for low-energy ones (65). This effect was first studied in a numerical simulation of two-proton correlation functions by Boal and DeGuise (79). Meaningful interpretations of the two-proton correlation function require, in general, calculations capable of predicting the full space-time dependence of the emission function. This type of calculations will be introduced in the following.

4 CALCULATION OF CORRELATION FUNCTIONS VIA NUCLEAR TRANSPORT THEORY

4.1 *Heavy Ion Transport Theory*

Equation 4 only requires knowledge of the proton one-body distribution function. Therefore it is in principle possible to generate small-angle correlation functions with any theory that calculates the time evolution of the one-body distribution function. Such calculations have recently been performed by Gong et al. (64, 65, 66, 78). The microscopic theory used there is based on the Boltzmann-Uehling-Uhlenbeck (BUU) equation. This equation was first postulated by Nordheim (134, 135) and later worked on by Uehling and Uhlenbeck (136). It was first applied to the heavy ion transport problem by Bertsch et al. (137, 138, 139), by Stöcker et al. (140, 141, 142, 143), and by Grégoire et al. (144, 145, 146, 147). Recent review articles on nuclear transport theories can be found in (148, 149, 150, 151, 152, 153).

The BUU equation describes the time evolution of the nuclear one-body density Wigner distribution $f(\vec{r},\vec{p},t)$ under the influence of the nuclear mean field (left side) and individual nucleon-nucleon collisions (right side):

$$\begin{aligned}\frac{\partial}{\partial t}f(\vec{r},\vec{p},t) \quad &+ \quad \frac{\vec{p}}{m}\vec{\nabla}_r\, f(\vec{r},\vec{p},t) - \vec{\nabla}_r U\,\vec{\nabla}_p f(\vec{r},\vec{p},t) \\ &= \quad \frac{g}{2\pi^3 m^2}\int d^3q_{1'}\, d^3q_2\, d^3q_{2'} \\ &\quad \delta\left(\frac{1}{2m}(p^2+q_2^2-q_{1'}^2-q_{2'}^2)\right)\cdot\delta^3(\vec{p}+\vec{q}_2-\vec{q}_{1'}-\vec{q}_{2'})\cdot\frac{d\sigma}{d\Omega} \\ &\quad \cdot\Big\{f(\vec{r},\vec{q}_{1'},t)\,f(\vec{r},\vec{q}_{2'},t)\Big(1-f(\vec{r},\vec{p},t)\Big)\Big(1-f(\vec{r},\vec{q}_2,t)\Big) \\ &\quad -f(\vec{r},\vec{p},t)\,f(\vec{r},\vec{q}_2,t)\Big(1-f(\vec{r},\vec{q}_{1'},t)\Big)\Big(1-f(\vec{r},\vec{q}_{2'},t)\Big)\Big\},\end{aligned} \tag{17}$$

In all present numerical implementations of the solution of Equation 17, the test particle method (154) is used to describe to propagation of the one-body distribution under the influence of the mean field potential U. It is supplemented by an intranuclear cascade (155, 156, 157, 158, 159, 160) to solve the collisions integral. The test particle collisions respect the Pauli exclusion principle due to the presence of the factors $1-f$, which are numerically implemented via a Monte Carlo rejection method.

4.2 Large-Angle Correlations

The solution of Equation 17 represents the time evolution of the single particle distribution function f. In the limit of $t \to \infty$ it is therefore possible to predict all single particle observables such as proton spectra (138) in this theory. It is also possible to predict the production cross sections of secondary particles (137, 140, 139).

Predictions for two-particle correlations can only be made, if these correlations are simply consequences of the conservation laws for momentum,

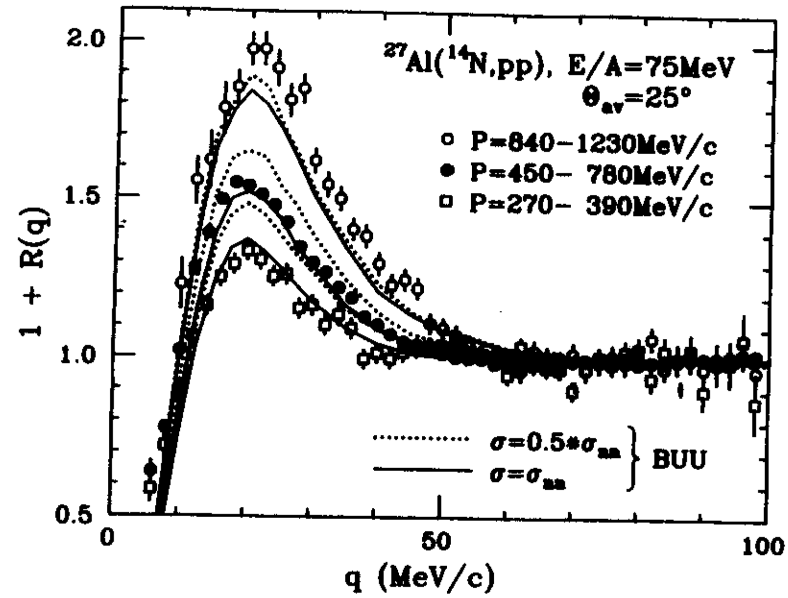

Figure 5: Two-proton correlation functions as a function of relative momentum q, measured for the reaction ^{14}Ne+^{27}Al at E/A=75 MeV for three different gates on total momentum of the proton pair, P, (plot symbols), are compared to the calculation based on the BUU theory (lines) for two different values of the in-medium nucleon-nucleon cross section. [After (66)]

energy, angular momentum, and particle number. Such a calculation was first performed by Bauer (83). It was shown that two-proton correlation functions measured at large angles (89, 90, 91, 93) can be successfully reproduced by the BUU theory, provided that total momentum conservation is correctly taken into account. In a second study, Ardouin et al. (161, 162) found that the variation of the large angle correlation function with polar angle θ is largely due to angular momentum effects.

4.3 Small-Angle Correlations

Using the convolution techniques of Equation 4, one can use the output of Equation 17 and calculate two-particle correlation functions for small relative momenta. This was first done by Gong et al. (64, 65, 66, 78).

As an example, Figure 5 (taken from (66)) shows correlation functions generated from ^{14}N + ^{27}Al collisions at E/A = 75 MeV. The measured two-proton correlation function is indicated by the plot symbols and is displayed for three different gates on the total momentum, P, of the proton pair. One can see the expected features: A maximum of $C(q)$ at $q \approx 20$ MeV/c, reduction of $C(q)$ as $q \to 0$, and $C(q) \to 1$ as $q \to \infty$.

The solid and dotted lines in Figure 5 represent calculations based on the BUU theory. For the dotted lines, it was assumed that the in-medium nucleon-nucleon cross section is one-half of the free value; and for the solid lines the full free cross section was used. (In both cases, the corrections to due the Pauli exclusion principle were of course also taken into account). Medium modifications of the nucleon-nucleon cross sections were recently suggested by Cugnon et al. (163), based on formalism developed in (164, 165). However, from Figure 5 one can see that, in particular for the two lower momentum bins, the free space value is preferred. Gong et al. (66) find that at E/A = 75 MeV the two-proton correlation function in the intermediate momentum bin (pair velocity $\approx \frac{1}{2}$ beam velocity) are very sensitive to the value of the in-medium nucleon-nucleon cross section, and their extracted value is:

$$\sigma^{NN}_{\rm medium} \approx \sigma^{NN}_{\rm free} \tag{18}$$

This finding is in agreement with theoretical and experimental studies on the disappearance of nuclear collective flow, where Ogilvie et al. (166, 167) report similar numbers.

The calculations shown in Figure 5 are able to reproduce the experimental fact that the correlation function has a different height for different P-bins, which gives rise to the apparent pair-velocity dependence of the extracted Gaussian source size discussed in section 3.2. From this agreement we have to conclude that the effect is due to the reaction dynamics, and that a simple zero-lifetime Gaussian fit to extract the apparent source size must not be associated with the spatial extent of a stationary source of negligible lifetime.

4.4 Initial State Correlations

Up to now, our discussion always assumed that the calculated correlation functions are completely determined by the final state interactions between the two outgoing protons, and that initial state correlations are completely negligible. Thus, a chaotic source was assumed.

However, correlations in the initial state may not always be negligible. The two most important initial state correlations discussed in the literature are coherent particle production and initial state spin correlations.

The idea of coherent production of particles in nuclear collisions via a Bremsstrahlung-type process was introduced by Heisenberg (168). The consequences of coherent particle production on two-pion interferometry have been studied in several theoretical approaches (47, 48, 15, 50, 44). This leads to the introduction of the degree of coherence

$$D(\vec{P}) = \frac{n_0(\vec{P})}{n_0(\vec{P}) + n_{\mathrm{ch}}(\vec{P})} , \tag{19}$$

where n_0 = number of coherently produced pions, and n_{ch} = number of chaotically produced pions (15). The two-pion correlation function at $q = 0$ is then given by $C(P, q{=}0) = 2 - (D(\frac{1}{2}P))^2$. (Another commonly used variable is the chaoticity parameter λ, where $\lambda = 1 - D^2$.) Today, however, we can say that there is no experimental evidence for a collective production of pions or high-energy photons in intermediate energy (E/A $\approx$ 100 MeV - 1 GeV) heavy ion collisions. Instead, all production cross sections of secondary particles seem to be explained in terms of incoherent nucleon-nucleon collisions (169, 151, 170). In other energy regimes the picture is not quite as clear, and the interpretation of the chaoticity parameter in ultra-relativistic heavy ion collisions is still a topic of current research.

A second source of initial state correlations is due to the spin, which has been studied for two-proton correlation functions from the reaction ^{3}He + Ag (78). The two-proton wave function can in general be written as

$$|\phi(\vec{q}, \vec{r})|^2 = \alpha |\phi_s(\vec{q}, \vec{r})|^2 + (1 - \alpha) |\phi_t(\vec{q}, \vec{r})|^2 , \tag{20}$$

where ϕ_s and ϕ_t are the singlet and triplet spatial wave functions. In large systems with many protons, one has to good approximation $\alpha = 1/4$, the statistical weight. Zhu et al. (78) find, however, that they have to set $\alpha = 0.45$ to reproduce their data in the ^{3}He + Ag reaction by calculations based on the BUU theory. This was interpreted as due to contributions from breakup in which some fraction of the two-proton coincidences in this system arise from proton pairs in ^{3}He with their initial (singlet) spin correlation undisturbed.

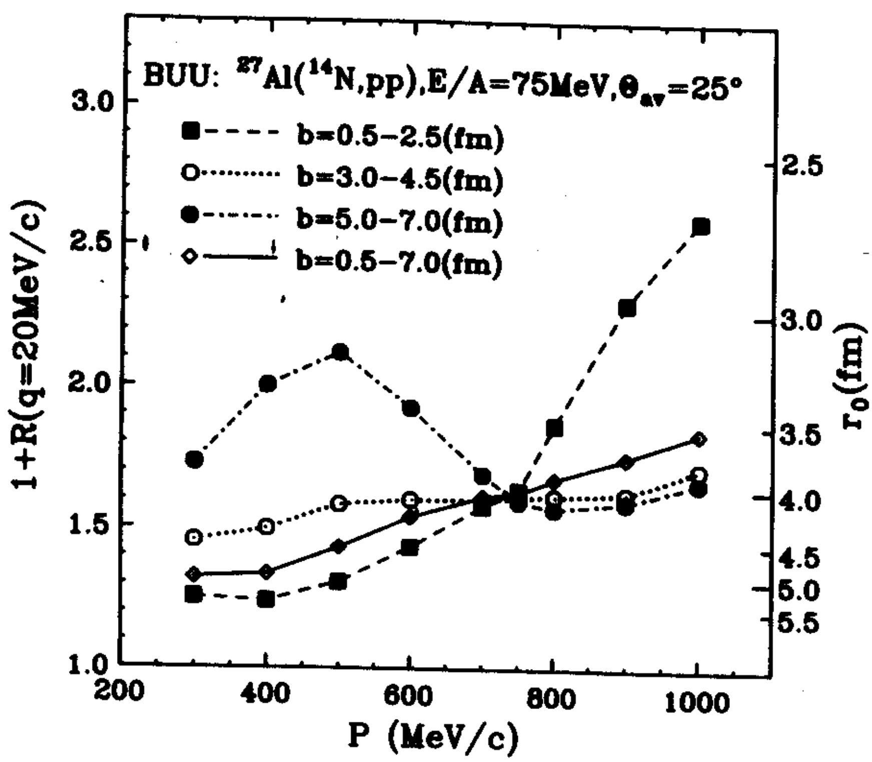

Figure 6: Momentum and impact parameter dependence of the height of the maximum of the two-proton correlation function for the reaction $^{14}N + ^{27}Al$ at E/A = 75 MeV, as predicted by calculations based on the BUU theory. [After (65)]

4.5 *Impact Parameter Selection*

Up to now, only very few experimental attempts (67, 68, 69) were made to study the impact parameter dependence of two-particle correlation functions, and no quantitative comparisons with dynamical calculations have been performed.

Details of theoretical Wigner functions strongly depend upon the impact parameter of the collision. As a consequence, much information is lost in inclusive experiments. Gong et al. (65) have calculated this loss of information for the $^{14}N + ^{27}Al$ reaction at E/A = 75 MeV. Their results are summarized in Figure 6. The circular and square shaped points in the figure show the heights of the maxima of two-proton correlation functions as predicted by the BUU theory for various cuts on the total momentum, P, of the proton pairs and on impact parameter, b. The diamond shaped points show predictions for impact parameter averaged measurements. (For orientation, the right hand scale give the corresponding Gaussian source parameters.) For different ranges of impact parameter, the predicted correlation functions exhibit qualitatively different dependences on the total momentum of the emitted proton pairs. Particularly striking is the strong momentum dependence predicted for central collisions. The calculations indicate that one should be able to extract a wealth of information about the space-time evolution of the reaction zone by detailed investigations of the momentum and impact parameter dependence of two-proton correlation functions. Such experiments would require high statistics and efficient impact parameter selections.

5 SUMMARY AND OUTLOOK

The field of intensity interferometry for heavy ion collisions has moved from a level of qualitative descriptions to a level, on which quantitative results can be obtained from experiment and theory.

At the ultra-relativistic energies ($E/A \geq 10$ GeV), present interest is focussed on two-pion correlations. Here the primary goal of intensity interferometry is to learn about the lifetime of the fireball created in the reaction. From this information, it should be possible to make further conclusions on the possible formation of quark-gluon-plasma. Future experiments at RHIC energies will be able to utilize the very large number of pions created, and one may be able to extract correlation functions on an event-by-event basis.

At intermediate energies ($E/A \approx 100$ MeV), the two-proton intensity interferometry technique is preferred. This probe was the main topic of the present review. Recent experiments have been able to yield quantitative information on the space-time extension of nuclear reactions, i.e. source radius, lifetime, and energy distribution. This is accomplished by comparing experimental data to the results of nuclear transport calculations.

We have only begun to scratch the surface of the overall potential of intensity-interferometry techniques in heavy ion collisions. Limited information on, for example, the in-medium nucleon-nucleon cross section was obtained. In the future, it should be possible to perform calculations and measurements at beam energies of $E/A \approx 1$ GeV. Here, there is potential to obtain further information on the response of nuclear matter to compression, which in turn should lead to additional information on the high-density, high-temperature part of the nuclear equation of state.

To reach this goal there are two challenges that have to be met. One is of experimental nature: The next generation of experiments should provide information on impact parameter dependence of the correlation functions. The other is theoretical: Additional work needs to be done to arrive at reliable calculations simulating the reaction dynamics in a consistent fashion for both two-particle interferometry and the emission of clusters of nucleons.

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