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NUCLEAR FISSION WITH DIFFUSIVE DYNAMICS

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We investigate the dynamics of nuclear fission, assuming purely diffusive motion up to the saddle point. The resulting Smoluchowski equation is solved for conditions appropriate to the ^{16}O + $^{142}Nd \rightarrow {}^{158}Er$ reaction at 207 MeV. The solution is characterized by an equilibration time τ_0 for the system to reach steady state, and the fission decay rate in steady state, A. We find that the equilibration time τ_0 plays a very small role in determining the number of prescission neutrons. The diffusion coefficient extracted from the experimental data is larger than the theoretical in the work of Bush *et* al. by a factor of 5 $\sim$ 11.

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It is necessary to understand the dynamics of large amplitude shape change in order to describe a number of processes for which the statistical model fails. This includes several effects associated with fission, including the emission of prescission neutrons[1–3] and the emission of giant dipole photons[4]. On the theoretical side, large amplitude shape dynamics is most commonly treated assuming the average motion can be described by Newtonian mechanics with inertial and linear friction parameters[5]. Recently, Bush *et al.* proposed a model of the shape dynamics of highly excited nuclei from quite a different point of view[6]. They start with the basic assumption that the highly excited nucleus can be described as an incoherent mixture of Hartree-Fock configurations at a given energy. They concentrate exclusively on the diffusion in the nuclear shape degrees of freedom avoiding a discussion of inertia, friction or indeed any collectivity in the motion. Based on this picture, they have calculated the diffusion coefficient for quadrupolar shape fluctuations using the residual interaction to mix Hartree-Fock configurations.

In frictional dynamics, the Kramers theory[7] may be used to find the fission decay rates. A corresponding theory can be derived from the diffusive dynamics using the Smoluchowski equation. We show how this is done below. The end result is equivalent to the Kramers theory, in the strongly overdamped limit of that theory. We then discuss the numerical application to prescission neutron multiplicities. In Ref. [6], the theoretical diffusion rate was an order of magnitude slower than found in the analysis of Ref. [2]. Our analysis here of the measurement of Ref. [3] is a factor of 5 different from the theoretical prediction, due mostly to the larger number of prescission neutrons extracted from the later experiment.

We start from a generic rate equation

$$\frac{dP_i}{dt} = -(\Gamma_{i,i-1} + \Gamma_{i,i+1})P_i + \Gamma_{i-1,i}P_{i-1} + \Gamma_{i+1,i}P_{i+1}$$

where P_i is the probability of finding the system in the state i and $\Gamma_{i,j}$ is the rate at which the system moves from one state to its immediate neighbor. This is a generalization of the treatment of Ref. [6], which assumed all $\Gamma_{i,j}$ equal. To be more realistic, we have to consider the effect of the level density on the rates. Detailed balance requires that $\Gamma_{i,j}$ and $\Gamma_{j,i}$ be in the ratio of the final state level densities ρ_j and ρ_i. In terms of these level densities, the rate equation may be expressed

$$\frac{dP_i}{dt} = -v(\rho_{i-1}P_i + \rho_{i+1}P_i - \rho_i P_{i-1} - \rho_i P_{i+1})$$

where v is a constant. To reduce the above to the Smoluchowski equation, we assume that the states i are uniformly separated in deformation by an amount $\Delta\beta$ and treat P as a continuous function of β. The finite differences are expanded $P_{i+1} = P_i + \Delta\beta\frac{\partial P}{\partial\beta}$, *etc.* The result to order $(\Delta\beta)^2$ is the Smoluchowski equation,

$$\frac{\partial P}{\partial t} = -\frac{D_\beta}{T}\frac{\partial}{\partial\beta}(-\frac{\partial V}{\partial\beta}P) + D_\beta\frac{\partial^2 P}{\partial\beta^2} \quad (1)$$

where the diffusion coefficient D_β may be expressed in terms of microscopic quantities (such as the residual interaction $V_{i,j}$) as[6]

$$D_\beta = \frac{2\pi}{\hbar}\sum_j(\beta_i - \beta_j)^2 \mid V_{i,j} \mid^2 \delta(E_j - E).$$

In deriving Eq. (1), we have also defined the temperature as the inverse logarithmic derivative of the level density and used the chain rule, $\frac{\partial\rho}{\partial\beta} = \frac{\partial\rho}{\partial E^*}\frac{\partial E^*}{\partial\beta}$. Several authors have applied the Smoluchowski equation to nuclear fission processes with the fission variable as the generalized coordinate[8, 9]. It is interesting to note that the fluctuation-dissipation theorem, relating the coefficient of the drift term to that of the fluctuation term, follows directly from the above derivation.

A convenient feature of Eq. (1) is that the diffusion coefficient can be scaled out of the equation by defining a new time variable $\tau = D_\beta t$. Thus we need only solve the equation with $D_\beta = 1$ and all solutions corresponding to different D_β's can be obtained simply by rescaling the time. This scaling property is well-known in previous studies using the Smoluchowski equation[8, 9]. We will calculate the decay rate by considering the the probability current $I(\beta,t)$ which crosses β at time t. This may be expressed as

$$I(\beta,t) = D_\beta[\frac{1}{T}(\frac{\partial V}{\partial \beta})P + \frac{\partial P}{\partial \beta}]. \tag{2}$$

The decay rate λ is defined

$$\lambda(\beta,t) = \frac{I(\beta,t)}{\int_{-\infty}^{\beta} d\beta' P(\beta')}.$$

It is seen to obey the same scaling, $\lambda(D_\beta) = \Lambda D_\beta$, where Λ is the decay rate for $D_\beta = 1$.

We now derive the stationary decay rate for Eq. (1) for a particle trapped inside a parabolic barrier. The assumption of the steady state means that we will solve the time-independent diffusion equation

$$-\frac{1}{T}\frac{\partial}{\partial \beta}(-\frac{\partial V}{\partial \beta}P) + \frac{\partial^2 P}{\partial \beta^2} = 0.$$

We first have to specify the form of the potential $V(\beta)$. We assume the potential has a parabolic shape, upward-curving near the equilibrium deformation β_1 and downward-curving near the barrier top at β_2,

$$\begin{aligned} V(\beta) &= \tfrac{1}{2}k_1(\beta-\beta_1)^2, \quad \text{when } \beta \le a \\ &= U_B - \tfrac{1}{2}k_2(\beta-\beta_2)^2, \text{ when } \beta > a. \end{aligned} \tag{3}$$

Here U_B is the barrier height, and the parameter a is determined to make the parabolas join smoothly. Following Kramers' derivation[7, 10], we write the probability distribution in terms of a new variable ξ as

$$P(\beta) = \xi(\beta)\exp(-\frac{V(\beta)}{T}).$$

The ξ satisfies the equation

$$\frac{\partial^2 \xi}{\partial \beta^2} - \frac{1}{T}\frac{\partial V}{\partial \beta}\frac{\partial \xi}{\partial \beta} = 0.$$

This equation has the solution

$$\xi(\beta) = \frac{1}{Z_o}\int_\beta^\infty \exp(\frac{V(\eta)}{T})d\eta \tag{4}$$

where Z_o is a constant to be determined. The other constant in the solution of the second-order differential equation is fixed by demanding that ξ vanish at large β. To fix Z_o, we note that in the vicinity of β_1 the distribution will be thermal, i.e. described by the function

$$P(\beta) = \frac{1}{Z}\exp(-\frac{V(\beta)}{T}) \approx \frac{1}{Z}\exp(-\frac{k_1}{2T}(\beta-\beta_1)^2)$$

where $\frac{1}{Z}$ is a normalization constant which will be set equal $\xi(\beta_1)$. Carrying out the integration in Eq. (4), only the integrand near β_2 is important, and we find

$$\xi(\beta_1) \approx \frac{1}{Z_0}\int \exp(-\frac{k_2\eta^2}{2T})d\eta = \frac{1}{Z_o}\sqrt{\frac{2\pi T}{k_2}} = \frac{1}{Z}.$$

We now evaluate the probability current at the barrrier, Eq. (2). This is

$$I(\beta_2) = [\frac{1}{T}\frac{\partial V}{\partial \beta}P + \frac{\partial P}{\partial \beta}]_{\beta=\beta_2} = -\frac{1}{Z}\exp(-\frac{U_B}{T})\sqrt{\frac{k_2}{2\pi T}}.$$

The total probability inside the barrier is approximately

$$\begin{aligned} S(\beta_2) &= \frac{1}{Z}\int_0^{\beta_2} \exp(-\frac{1}{2T}k_1(\beta-\beta_1)^2)d\beta \\ &\approx \frac{1}{Z}\int_{-\infty}^{\infty} \exp(-\frac{1}{2T}k_1(\beta-\beta_1)^2)d\beta = \frac{1}{Z}\sqrt{\frac{2\pi T}{k_1}}. \end{aligned}$$

Finally, the stationary value of the decay rate is the ratio of the current to the total probability,

$$\Lambda_o = -\frac{I(\beta_2)}{S(\beta_2)} = \frac{\sqrt{k_1 k_2}}{2\pi T}\exp(-\frac{U_B}{T}). \tag{5}$$

This result can be compared with the Kramers' stationary decay rate[7]

$$\lambda_k = \frac{\omega_1}{2\pi}\,[\{1+(\frac{\gamma}{2\omega_2})^2\}^{\frac{1}{2}} - \frac{\gamma}{2\omega_2}]\exp(-\frac{U_B}{T})$$

where γ is the reduced friction coefficient. To connect this with Eq. (5), we explicitly introduce the inertial parameter M in the Kramers formula. The oscillator frequencies ω_i are expressed as $\omega_i = \sqrt{\frac{k_i}{M}}$. The reduced friction coefficient γ is related to the diffusion coefficient by the Einstein relation[11], $\gamma M = \frac{T}{D_\beta}$. Then in the limit of large γ the Kramers' formula reduces to

$$\lambda_k \approx \frac{\omega_1\omega_2}{2\pi\gamma}\exp(-\frac{U_B}{T}) = \frac{\sqrt{k_1k_2}}{2\pi\gamma M_2}\exp(-\frac{U_B}{T}) = \frac{\sqrt{k_1k_2}}{2\pi T}\,D_\beta\exp(-\frac{U_B}{T})$$

identical to our result, as should be expected.

We now display some of the characteristics of the solution of the Smoluchowski equation. We choose parameters corresponding to the nucleus ^{158}Er following Ref. [12]. The decay rate is most sensitive to the height of the fission barrier U_B and the difference $\beta_s = \beta_2 - \beta_1$ between the yrast state and the saddle point deformations. The fission barrier height is quoted in Ref. [12] for a range of angular momenta and we use those, interpolating or extrapolating to angular momenta not included in their table. The deformation at the saddle point is determined using their Fig. 2, which shows the barrier as a function of the separation of the two mass centers. To convert to an equivalent deformation β, we demand that quadrupole moments be the same as for a spheroid with the deformation β_2. This procedure is rough, but high accuracy is not possible without a more detailed consideration of the fission path when calculating the diffusion coefficient. The saddle point deformation does not depend strongly on angular momentum; we take a fixed value $\beta_2 = 1.20$. The yrast deformation β_1 is determined from the rotating liquid drop model. This also does not depend strongly on angular momentum for the range considered; our value here is $\beta_1 = 0.42$. The potential energy function is shown in Fig. 1 and the values of the parameters are given in Table I.

We now describe the results of numerical solution of the Smoluchowski equation. We assume an initial probability density

$$P(\beta, 0) \propto \exp(-\frac{V(\beta)}{T_o})$$

with $T_o = 0.3$ MeV as suggested in Ref. [13]. In Fig. 2, we plot time evolution of $P(\beta, \tau)$ for ^{158}Er at $J = 65\hbar$ and $T = 2.5$ MeV as a function of τ in two cases, (a) with and (b) without the fission barrier. The point of this comparison is to show that the system reaches a stationary decaying state in one case but not in the other. The location of the saddle point is indicated by an arrow in the figures. The two cases (a) and (b) are clearly distinguished in Fig. 2 where the role of the fission barrier becomes especially transparent.

We mention briefly the sensitivity of the decay rate to the parametrization of Eq. (3). According to Eq. (5), the stationary value Λ_o of the decay rate depends on the barrier height U_B and the nuclear temperature as well as the oscillator parameters k_i. In our calculations, we have used two parabolas with the same oscillator parameters $k_1 = k_2$. Once U_B and β_s are fixed, there is not much room left to vary the oscillator parameters. Moreover, $\sqrt{k_1 k_2}$ and hence Λ_o is not affected by more than just a few percent even though we change the ratio $\frac{k_1}{k_2}$ as much as possible. We therefore judged it sufficient to take the parabolas with the same oscillator parameter in our main calculations.

In Fig. 3, we show the calculated $\Lambda(\tau)$ across the saddle point as a function of τ for two cases, $J = 65\hbar$ and $72\hbar$ of ^{158}Er. In this figure, we can clearly see the transient behaviour and the saturation of decay rate which occur under the influence of fission barrier. On the other hand, the decay rate which is calculated from the pure diffusion equation without any fission barrier does not show any evidence for saturation as can be seen from the dashed line in Fig. 3. But note the over-shooting of the decay rate at higher partial waves, which has been also discussed by Lu *et al.*[14]. In the last two columns of Table I, we list the stationary values of decay rate Λ_o^{Th} which is evaluated using Eq. (5) and Λ_o^{Num} which is obtained from our numerical calculations. The stationary value Λ_o of the fission decay rate shows a strong dependence on A for fixed J and also a strong dependence on J for fixed A. However, the origin for such dependence in two cases is quite different. In Table I, we also list the nuclear temperature T together with the potential parameters. When J is fixed, the main reason Λ_o decreases as A becomes smaller is the decrease in T due to the cooling by neutron evaporation. Here the shape of the potential barrier remains more or less the same for all the nuclei participating the chain. On the contrary, when A is fixed, the temperature does not depend on J very much, but the barrier height changes with J, giving a larger rate Λ_o for higher J.

The equilibration time τ_o may be defined by the time interval until Λ reaches its stationary value. The τ_o is found to exhibit more or less the same value $\tau_o = 0.068$ for all J's and for all A's. This also corresponds to an extremum point in the dashed line (without the fission barrier) of Fig. 3 where τ_o is indicated by an arrow. Our result suggests that τ_o does not depend on T nor the shape of the potential. The equilibration time t_o which corresponds to a specific D_β can be found from the scaling $t_o = \frac{\tau_o}{D_\beta}$. Therefore, t_o is inversely proportional to D_β and hence proportional to the reduced friction coefficient γ as a result of the fluctuation-dissipation theorem. This phenomenon has been also studied by Weidenmüller and Zhang[8] and they found such dependence of t_o on γ too in the overdamped situation, but could not decide clearly the dependence of t_o on U_B and T. Our result shows that τ_o is the same for any combination of U_B and T.

We finally turn to the calculation of neutron multiplicity prior to fission. The decay rates will change as neutrons are emitted due to the cooling of the system, reflected in the temperature dependence of the statistical neutron decay rate and of the fission decay rate $\lambda = \frac{\Lambda}{D_\beta}$ where D_β is proportional to T^3[6]. We perform this part of the calculation with a numerical cascade. Let $Y_s(t)$ be the occupation probability of the s-th nucleus in the neutron decay chain which terminates at the s_o-th step either by fission or by neutron emission made impossible due to short of available excitation energy $E_s^* = E_{s-1}^* - 2T_{s-1} - B_{n,s-1}$. Here T_s and $B_{n,s}$ are the nuclear temperature and the neutron separation energy of the s-th nucleus, respectively. Then $Y_s(t)$ satisfies a set of coupled equations

$$\frac{d}{dt}Y_s(t) = \lambda_{n,s-1}Y_{s-1}(t) - [\lambda_{n,s} + \lambda_{f,s}(t)]Y_s(t) \qquad s = 1, 2, \cdots s_o \qquad (6)$$

where $\lambda_{n,s}$ and $\lambda_{f,s}$ are the neutron emission rate and the fission decay rate at the s-th step, respectively, and $\lambda_{n,0} = \lambda_{n,s_o} = 0$ at beginning and end of the neutron decay chain. The neutron emission rate is calculated by the Weisskopf formula[15]

$$\hbar\lambda_n = \frac{2mR^2}{\pi\hbar^2}(\frac{E^* - B_n}{\alpha}) \exp(2\sqrt{\alpha(E^* - B_n)} - 2\sqrt{\alpha E^*})$$

where m is the mass of the emitted neutron and R is the radius of the nucleus. In order to calibrate this formula, α is adjusted to give the neutron emission rate of 50 keV/$\hbar$ which is calculated in a more elaborated statistical model for $^{16}O +^{142} Nd \rightarrow^{158} Er$ at 207 MeV and $J = 65\hbar$[16]. Also let $Y_{s,f}(t)$ be the probability of ending the process with fission at time t at the s-th step. It then satisfies

$$\frac{dY_{s,f}}{dt} = \lambda_f(t)Y_s(t)$$

and hence when $t = +\infty$, it becomes

$$Y_{s,f}(\infty) = \int_0^\infty \lambda_f(t)Y_s(t)dt.$$

The neutron multiplicity prior to fission is defined by adding all the neutrons released from the entire decay chain,

$$\nu(J) = \sum_{s=1}^{s_o}(s-1)Y_{s,f}(\infty) / \sum_{s=1}^{s_o} Y_{s,f}(\infty)$$

where J is the angular momentum for which the calculation is originally performed and the observed neutron multiplicity ν is obtained by taking average among the participating partial waves.

In Fig. 4, we present our results on the neutron multiplicity ν prior to fission. First, we have solved Eq. (6) numerically assuming D_β does not depend on T along the decay chain. The thin dashed line is obtained by taking $\lambda_f(t) = \Lambda(t)D_\beta$ while the thin solid line is the result of neglecting the transient behaviour of the fission decay rate and taking $\lambda_f(t) = \Lambda_o^{Num} D_\beta$ where Λ_o^{Num} is the constant stationary value of $\Lambda(t)$ (see the last column of Table I). These two thin lines clearly demonstrate that the transient behaviour in the fission decay rate does not affect the prescission neutron number very much. Bhatt *et al.* have also reached the same conclusion[17]. The reason is as follows. For small values of D_β, the neutron decay rate λ_n is much larger than the fission decay initially (because of the diffusion hindrance). As neutrons are emitted, the system cools, eventually favouring fission because of its lower barrier. On average, the fission occurs when the two decay rates are comparable. Thus, in this limit the initial transit time t_o plays no role. In the unphysical opposite limit of large values of D_β, the t_o becomes too short compared to the neutron decay lifetime to affect ν either. Therefore, we conclude that most of the enhancement in ν comes from the reduction of the fission decay rate in the overdamped process[18]. In view of this, we have calculated ν neglecting the effect of the transient time completely. The thick solid line of Fig. 4 is the calculated result taking the temperature dependence of D_β into account. The fission decay rate of the s-th nucleus is then given by

$\lambda_{f,s} = \Lambda_{o,s}^{Num} D_\beta(\frac{T_s}{T_1})^3$ where D_β is the diffusion coefficient in the parent nucleus ^{158}Er. This temperature dependence causes emission of up to almost one more neutron than in the case of the temperature independent D_β.

Gavron *et al.* measured the prescission neutron multiplicity for the reaction $^{16}O + {}^{142}Nd \rightarrow {}^{158}Er$ at 207 MeV and reported the value $\nu = 2.7 \pm 0.4$[2]. However, there has been a conflicting report on the neutron multiplicity by Hinde *et al.* in their more recent measurement for the same reaction at the slightly lower energy of 178 MeV[3]. They found that $\nu = 4.2 \pm 0.3$. Since ν is expected to increase with increasing excitation energy[14], the conflict between the two measurements is quite severe. Assuming ν increases by 0.4 for each 10 MeV of excitation energy[14], the expected ν at 207 MeV would be 5.4. One might hope that a microscopic calculation of ν would shed some light on this problem. We examined the recent theoretical predictions for D_β by Bush *et al.*[6]. They obtained $D_\beta = 3.8 \pm 1.6$ keV/$\hbar$ for ^{158}Er at $T = 2.5$ MeV which is shown by a shaded area in Fig. 4. The uncertainties in the theoretical D_β are due to the Monte-Carlo averages and sums. In Fig. 4, the data by Gavron *et al.* (lower rectangle) and by Hinde *et al.* (upper rectangle) are also shown. From the thick solid line, we can extract the range of D_β corresponding to the two measured ν's; D_β(Gavron)= 42.0 ± 9.0 keV/$\hbar$ and D_β(Hinde)= 17.7 ± 3.0 keV/$\hbar$. They are larger than the theoretical value of Bush *et al.* by a factor of 11.1 and 4.7, respectively. Clearly, the theory of Bush *et al.* is incompatible with either measurement. Further progress might result from examining other effects of the fission hindrance, for example the giant-dipole photon spectrum[4]. It would be interesting to see what values of D_β are favoured by this data.

In summary, we have solved the Smoluchowski equation for quadrupolar deformations. The solutions exhibit two nice properties which help us to analyze the fission process systematically. One is the scaling relation and the other is an analytical expression for the stationary value of the decay rate. The latter is just the overdamped limit of the well-known Kramers' stationary decay rate. We find that the equilibration time is pretty much the same for any combination of the height of the fission barrier and the nuclear temperature. One of the important results in our calculations is that the transient behaviour in the fission decay rate plays a very small role in determining the prescission neutron multiplicity quite contrary to the underdamped process. It is due to the fact that the Smoluchowski equation scales the equilibration time and the fission decay rate in an opposite way. The diffusion coefficient extracted by our calculation from the existing two data is larger by a factor of $5 \sim 11$ than the theoretical estimation of Bush *et al.* based on their recent derivation of the diffusion coefficient from mixing of Hartree-Fock configurations[6]. On the other hand we predict that the neutron multiplicity of $^{16}O + ^{142}Nd \rightarrow ^{158}Er$ at 207 MeV which corresponds to the theoretical D_β would be $\nu^{Th} = 6.6 \pm 0.5$. which can be compared to $\nu \approx 5.4$ extrapolated from the Hinde's result according to a recent calculation on the neutron multiplicity in terms of the excitation energy[14].

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[18] This result is quite contrary to the underdamped case in the Kramers model where t_o increases as D_β becomes larger[12].

TABLES

TABLE I. Values of temperature (T), oscillator parameter (k) and barrier height (U_B) adopted for the potential energy function plotted in Fig. 1 and used in calculations of the stationary value of decay rate (Λ_o) for various spins and nuclei. The decay rate (Λ_o^{Th}) in the sixth column is evaluated by Eq. (5) and the decay rate (Λ_o^{Num}) in the last column is from the solution of the Smoluchowski equation.

J ($\hbar$)	Nucleus	T (MeV)	k (MeV)	U_B (MeV)	Λ_o^{Th}	Λ_o^{Num}
65	^{158}Er	2.59	40.1	6.10	0.234	0.232
	^{155}Er	2.19	39.4	6.00	0.186	0.184
	^{152}Er	1.75	38.5	5.86	0.123	0.122
	^{149}Er	1.07	37.4	5.69	0.027	0.027
69	^{158}Er	2.54	26.9	4.09	0.337	0.339
72		2.50	16.5	2.51	0.385	0.406
75		2.45	5.6	0.85	0.257	0.327

FIGURES

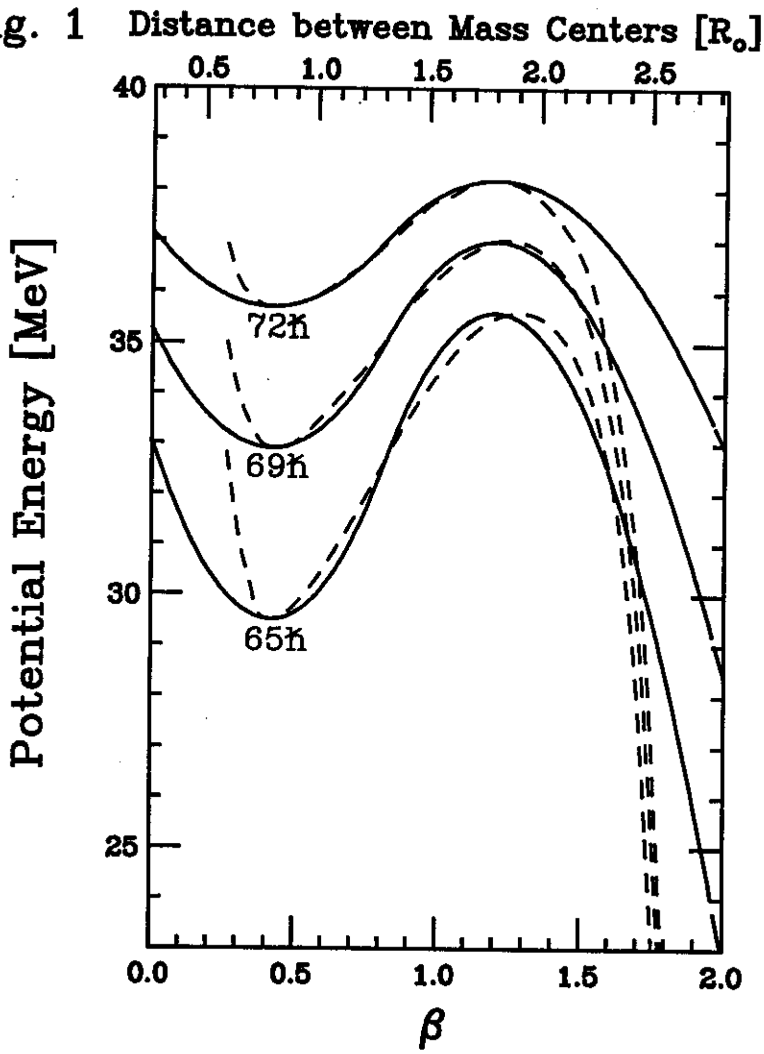
FIG. 1. Our potential energy functions (solid line) for ^{158}Er at $J = 65\hbar$, $69\hbar$ and $72\hbar$ are compared to those given in Ref. [12] (dashed line). The potential parameters adopted are given in Table I.

FIG. 2. Evolution of the probability distribution $P(\beta, \tau)$ along τ (a) with and (b) without fission barrier. Locations of the saddle point are indicated by an arrow.

FIG. 3. Decay rate $\Lambda(\tau)$ across the saddle point for ^{158}Er at $J = 65\hbar$ and $72\hbar$ is plotted as a function of τ. The dashed line is the decay rate calculated from the pure diffusion equation without the fission barrier. The arrow indicates the transient time τ_o. Both axes of the Λ and the τ are dimensionless. For a given diffusion coefficient D_β, they are simply translated by $\lambda = \Lambda D_\beta$ and $t = \tau / D_\beta$.

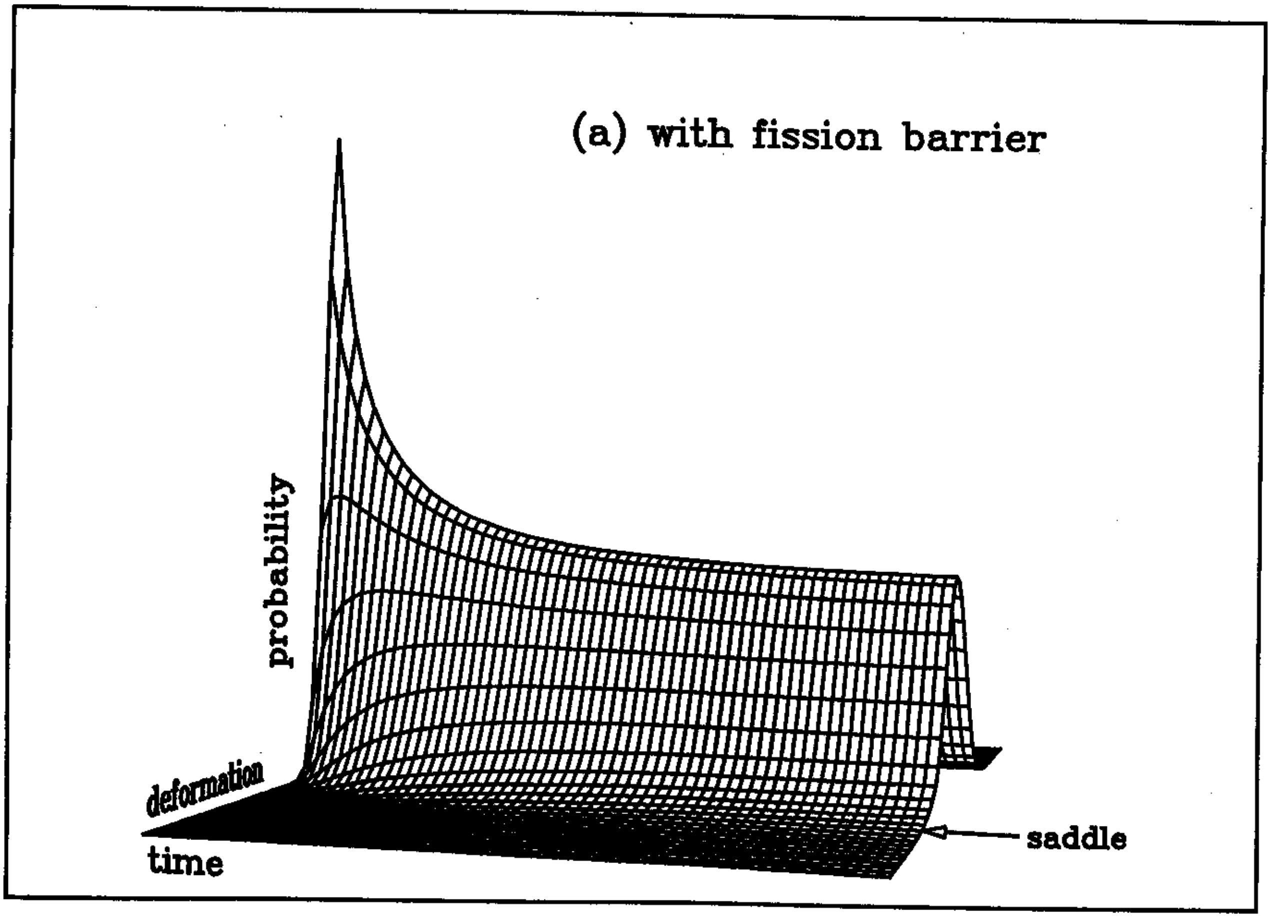
FIG. 4. Prescission neutron multiplicity is plotted in terms of the diffusion coefficient D_β in the parent nucleus ^{158}Er. The thin lines are obtained assuming D_β does not depend on T while the thick line takes the full account of T dependence along the decay chain. The time dependent fission decay rate $\lambda_f(t)$ has been used in the (thin) dashed line but only the stationary value of it has been considered for the (thin and thick) solid line thus excluding the transient time completely. The shaded area represents the range of D_β derived in Ref. [6]. Two rectangles show the recent data on ν by Gavron *et al.*[2] (lower one) and by Hinde *et al.*[3] (upper one).

Fig. 1



Distance between Mass Centers [R_o]
0.5
1.0
1.5
2.0
2.5
40
35
30
25
Potential Energy [MeV]
72ħ
69ħ
65ħ
0.0
0.5
1.0
1.5
2.0
β

Fig. 2



(a) with fission barrier
probability
deformation
time
saddle

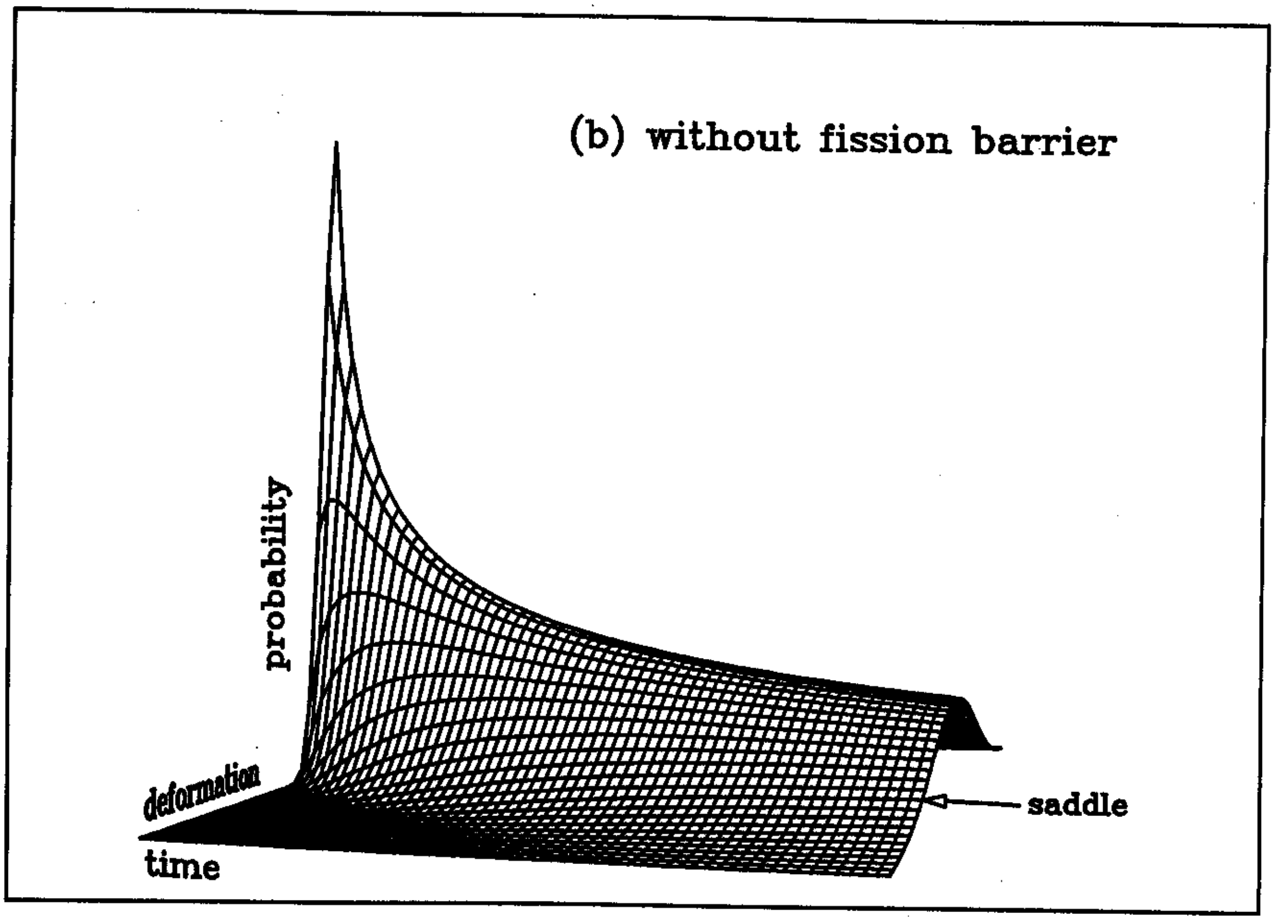
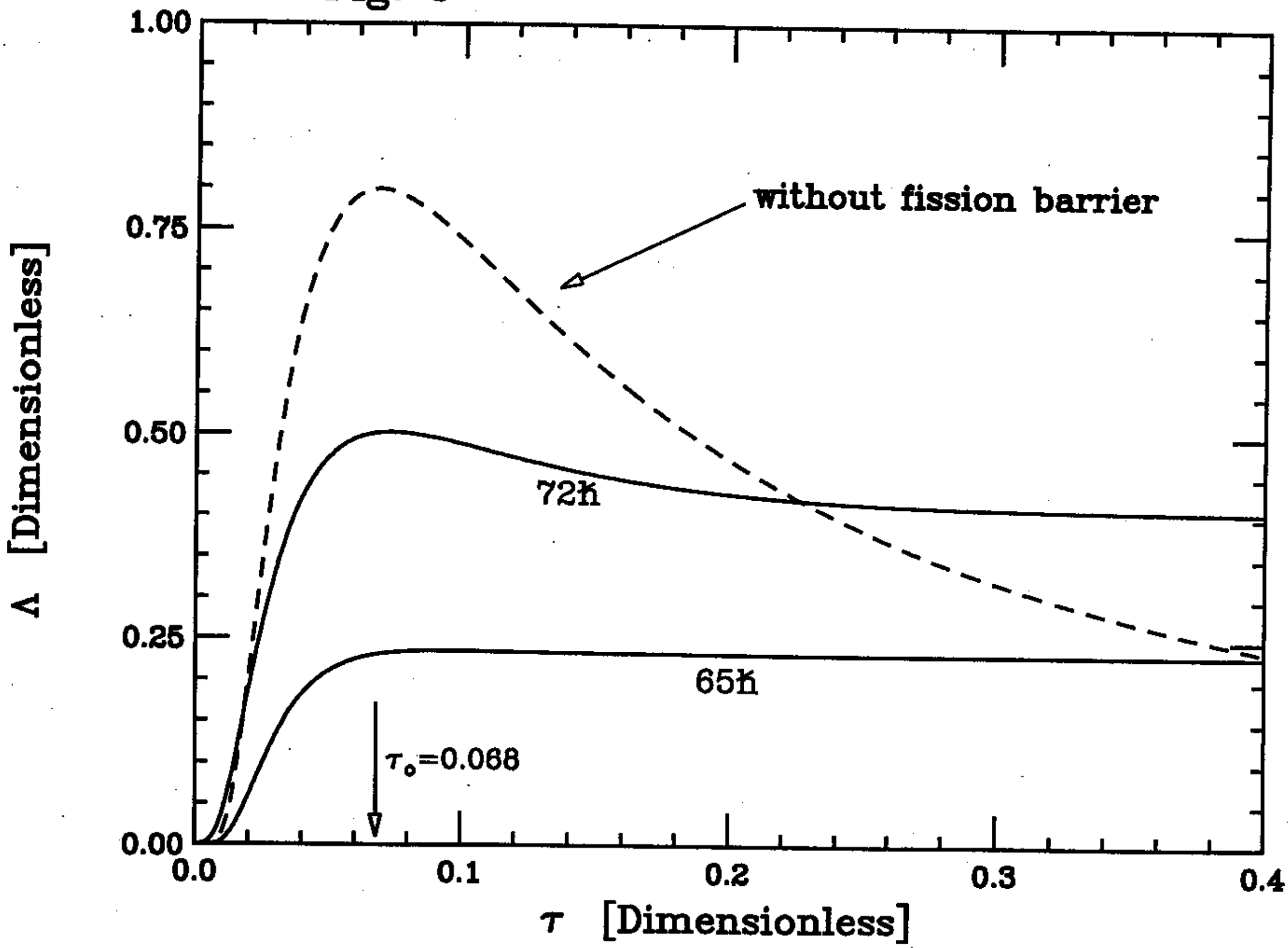
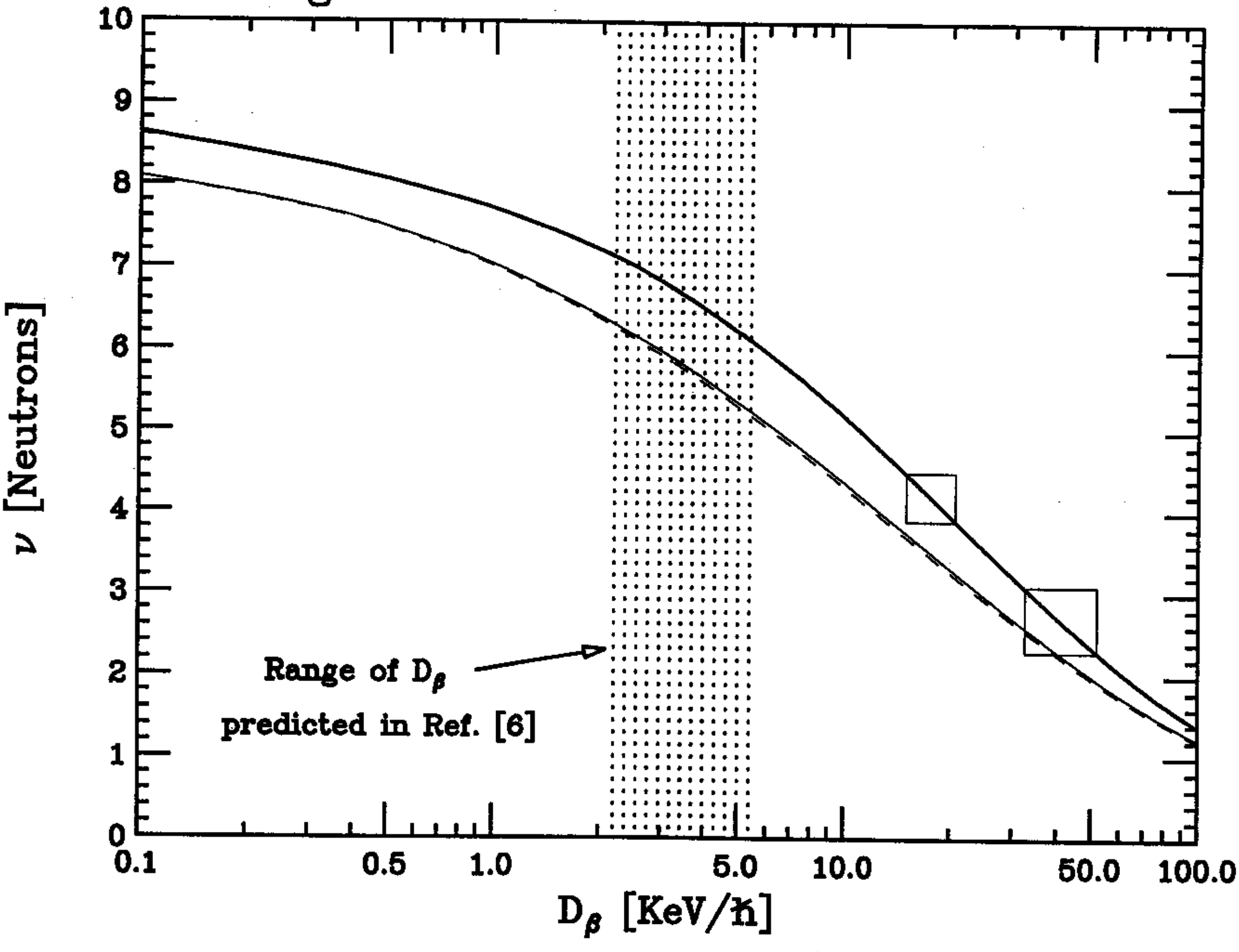

(b) without fission barrier
probability
deformation
time
saddle

Fig. 3



1.00
0.75
0.50
0.25
0.00
without fission barrier
72ħ
65ħ
τ_o=0.068
0.0
0.1
0.2
0.3
0.4
Λ [Dimensionless]
τ [Dimensionless]

Fig. 4


10
9
8
7
6
5
4
3
2
1
0
0.1
0.5
1.0
5.0
10.0
50.0
100.0
ν [Neutrons]
D_β [KeV/$\hbar$]
Range of D_β
predicted in Ref. [6]