Michigan State University

National Superconducting Cyclotron Laboratory

INDUCED EFFECTIVE GAUGE FIELDS AND CHAOS IN LARGE AMPLITUDE COLLECTIVE MOTION

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AUREL BULGAC and DIMITRI KUSNEZOV

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INDUCED EFFECTIVE GAUGE FIELDS AND CHAOS IN LARGE AMPLITUDE COLLECTIVE MOTION

Aurel BULGAC[a] and Dimitri KUSNEZOV[b]

[a] Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing MI 48824-1321, USA

[b] Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, CT 06511-8167, USA

Abstract

Both experlment and theory strongly support the idea that in any treatment of many-body systems, one can safely perform a separation of variables into fast and slow degrees of freedom. Standard wisdom then suggests that at a phenomenological level one can limit the analysis of the low energy properties of a many-body system to an effective description in terms of a Hamiltonian with only kinetic and potential energy terms. In a rlgorous treatment of large amplitude motion however, effective gauge fields arise naturally and their introduction is unavoidable. In the simplest picture, one can show that effective gauge fields are manifestations of level or band crossings. Their influence on the dynamics is extremely important: quantum numbers can become fractional and the collective trajectories become chaotic. An important consequence of this is a new picture of dissipation in large amplitude collective motion.

1. INTRODUCTION

One can probably safely say that the problem of Large Amplitude Collective Motion (LACM) is likely the most important contribution to the baggage of theoretical tools generated in theoretical nuclear physics. Nuclei have proved to be very singular quantum objects so far, as being the only many-body systems where LACM has been observed in its full glory and studied in a rather detailed manner. LACM has a long, rich and distinguished history, with an impressive breadth of results, concepts and ideas and encompassing an extensive range of phenomena, unequaled by any other nuclear problem so far. One can predict that the methods and results of LACM will very soon find their way into other physics subfields, such as atomic clusters, chemical physics and field theoretical models.

In spite of its long history, more than forty years, and the impressive amount of theoretical effort we are still quite a long way from delivering a final product. In their landmark paper, Hill and Wheeler [1] outlined a whole series of problems, many of them still waiting for a satisfactory solution. The idea of introducing collective (slow, extrinsic, relevant) and intrinsic (noncollective, fast, irrelevant) degrees of freedom hardly asks for a missionary. There is no doubt in anyone's mind that this is the correct way to proceed in order to understand and describe a staggering amount of experimental data. However, the implementation of this obvious idea in the nuclear case in particular, it is far from being a simple matter. For quite a while, we have seemed to accept that the Adiabatic Time–Dependent Hartree–Fock (ATDHF) theory [2] was finally the long sought after answer to this problem. The only hurdle to be passed seemed to be a well defined procedure to extract, within ATDHF ideology, the collective variables [3], without resorting to "physical intuition", requantize them and in this way finish once and for all with the phenomenology and be able to do real "science". Upon a more critical analysis however, one cannot fail to establish that we are still far from a reasonable solution. Surprisingly enough, the basis for the criticism comes from the same old paper by Hill and Wheeler [1]. What have we missed in this long process? Hill and Wheeler [1] make a point that a theory of LACM cannot be formulated in a simple way, by introducing or deriving a collective Hamiltonian/Lagrangian in terms of a kinetic and potential energy terms. To illustrate their image of LACM, Hill and Wheeler describe in qualitative terms the way a nucleus undergoes deformation. There is a marked difference between small and large amplitude deformations of a quantum many–body system. If one tries to slightly deform a nucleus, one quickly finds that their stiffness is rather high, while the stiffness for large deformations is surprisingly low. We all believe that the mean–field picture for the motion of nucleons inside the nucleus is essentially proven, at least for not too high excitation energies, and that the nuclear matter is incompressible. These two ingredients are essentially enough to give a rather simple and convincing qualitative description of the nuclear deformation [1,4]. For small deformations one has to deform the Fermi sphere and this is rather costly, since it is a volume effect. However, it suffices to deform a nucleus a little and one can easily rearrange the nucleons among the single–particle levels and find a new, almost spherical Fermi surface. In proceeding this way, one comes across a level crossing. Level crossing can be simply looked upon as a rather annoying small detail (which in particular leads to large variations of the inertial parameters computed within ATDHF theory [5]), but otherwise irrelevant to the physics. Hill and Wheeler [1] saw a problem right away: at the level crossings the adiabatic hypothesis is violated and one has to take into account the Landau-Zener phenomenon. In simple terms that means that one cannot limit the description of the collective motion to one single potential surface, but has to start with a multisurface potential energy. A multisurface potential energy is not something we are accustomed with, and at this point another school of thought emerged. LACM cannot be adiabatic, since the Landau-Zener transition probabilities are very close to one and instead one has to consider just the opposite limit, the diabatic picture or something closer to it. In terms of macroscopic (collective) variables, this amounts also to the introduction of a new physical phenomenon, friction or dissipation in LACM and, at the same time, in order to retain the simple picture we are all used to, of a single potential energy surface. During its evolution the collective energy of a nuclear system is degraded into "heat", i.e. internal excitation of the intrinsic degrees of freedom [6]. One can easily make the parallel between this type of description and the popular BUU approach to heavy–ion reactions, where one introduces a single–particle distribution functions and allows for essentially no fluctuations [7]. It became clear that fluctuations have to be put back in [8] in order to be able to describe multi–fragmentation at least. A diabatic picture of LACM will obviously suffer from the same illness as BUU, no fluctuations (or too little of them) and instead one will have to resort to either an ensemble description or go back to Hill and Wheeler suggestion [1] and use a multisurface potential energy surface. In condensed matter physics such an approach has been advocated for quite a long time by Tully [9], under the name of surface-hopping. It is not a theory in the strict sense, but a rather sensible solution to this problem.

To make the matters even more complicated, recently a new phenomenon has emerged, generically known as Berry's phase [10], and which it will likely force us to change gears again. In constructing an effective collective Hamiltonian/Lagrangian, one has now to make room for dynamically generated gauge-fields, which, depending on the degree of the sophistication of the theory, could be either abelian or non–abelian [11]. The presence of a vector potential will obviously lead to some new effects. The quantum numbers can become fractional and this has been already put in evidence in simple molecules [12]. The rather unexpected shape of the fission path presented by Negele [13] as a result of an Imaginary TDHF calculation of the fission of S_{32} into two O_{16} nuclei can likely be understood only in terms a "Dirac magnetic monopole" at the level crossing.

The presence of level crossings, or in other words, of effective gauge fields in a collective Hamiltonian/Lagrangian will lead also, as we will demonstrate here, to chaotic behaviour. Level crossings are probably the strongest source of chaoticity in LACM and this is very likely to modify in an essential manner the way we understand friction and dissipation in collective motion.

2. ONE LEVEL CROSSING

Before embarking into the analysis of a real many–body system, where many level crossings are present and the dynamics seems to be rather complicated, we would like to spend some time on the one level crossing problem. A suitable model Hamiltonian will be the following [14]

$$H = \frac{\mathbf{P}^2}{2M} + V(Q) + \frac{\kappa}{2} f(Q)\mathbf{Q}\cdot\boldsymbol{\sigma}. \tag{1}$$

Here $\mathbf{P}$ and $\mathbf{Q}$ will designate the slow variables, M a large collective mass and $V(Q)$ a shallow collective potential. The last term in Eq. (1) describes the coupling between the collective variables $\mathbf{Q}$ and the fast ones, the Pauli matrices $\boldsymbol{\sigma}$, and where κ is a coupling constant and $f(Q)$ is some formfactor. In particular, one can associate one collective variable with the quadrupole deformation and the other two with the real and imaginary parts of the pairing field. Most of the time we shall treat the slow variables as classical, however this approximation can be easily improved through either a semiclassical or even quantum treatment if needed. The Hamiltonian (1) can describe a two–surface potential system, which can correspond e.g. to the ground and an excited state bands.

We shall represent the fast degrees of freedom through the density matrix ρ

$$\rho = \frac{1}{2}\begin{pmatrix} 1+z & x-iy \\ x+iy & 1-z \end{pmatrix}, \tag{2}$$

where $\mathbf{r} = (x, y, z) = Tr(\rho\boldsymbol{\sigma})$ are real, with $r^2 = x^2 + y^2 + z^2 \leq 1$ (equality for the case of a pure state only, $\rho^2 = \rho$).

If one treats the slow variables as classical, the equations of motion for this system can be derived from the following Lagrangian [14,15]

$$\mathcal{L} = \mathbf{P}\cdot\dot{\mathbf{Q}} + \frac{z(x\dot{y} - y\dot{x})}{2(x^2+y^2)\sqrt{x^2+y^2+z^2}} - \left[\frac{\mathbf{P}^2}{2M} + V(Q) + \frac{1}{2}\kappa f(Q)\mathbf{Q}\cdot\mathbf{r}\right]. \tag{3}$$

The second term is the gauge field of a Dirac monopole [10], which, when integrated over a closed loop, is the exact quantum nonintegrable phase, which modifies the Bohr-Sommerfeld quantization rule. The equations of motion are

$$\dot{\mathbf{Q}} = \frac{\mathbf{P}}{M}, \quad \dot{\mathbf{P}} = -\frac{\partial V(Q)}{\partial \mathbf{Q}} - \frac{1}{2}\kappa\mathbf{r}, \quad \dot{\mathbf{r}} = \kappa\mathbf{Q}\times\mathbf{r}. \tag{4a, b, c}$$

(Here, for simplicity, we have assumed that $f(Q) = 1$.) In this form they can be thought of as fully quantum, if Eqs.(4a,b) are interpreted as Heisenberg equations of motion for the corresponding operators (Eq. (4c) is already the Schrödinger equation $i\dot{\rho} = [H, \rho]$ in a disguised form). It is easy to establish the existence of the following integrals of motion

$$r = \sqrt{x^2 + y^2 + z^2}, \quad E = \frac{\mathbf{P}^2}{2M} + V(Q) + \frac{\kappa \mathbf{Q} \cdot \mathbf{r}}{2}, \quad \mathbf{J} = \mathbf{Q} \times \mathbf{P} + \frac{1}{2}\mathbf{r}. \qquad (5-7)$$

In the case of a pure state for the fast variables $r = 1$. The second integral of motion is the total energy of the system. The last integral of motion has a most unusual structure. It looks like the angular momentum for the slow degrees of freedom, except for the last term. If the slow motion is quantized J is half-integer. (If the fast modes are in a mixed state, then $r < 1$ and consequently J becomes fractional/real.)

A standard Born–Oppenheimer approach to this model corresponds to assuming $\mathbf{Q} \cdot \mathbf{r} = \pm Q, \quad (r = 1)$. The equations of motion for the slow variables then read

$$\dot{\mathbf{Q}} = \frac{\mathbf{P}}{M}, \quad \dot{\mathbf{P}} = -\frac{\partial V_\pm(Q)}{\partial \mathbf{Q}} = -\left(\frac{\partial V(Q)}{\partial \mathbf{Q}} \pm \frac{\kappa \mathbf{Q}}{2Q}\right), \qquad (8a, b)$$

where $V_\pm(Q) = V(Q) \pm \kappa Q/2$ represent the two adiabatic potential surfaces, the lower one being a "Mexican hat". At this level the adiabatic approximation has several deficiencies:

- Instead of $\mathbf{J}$ only $\mathbf{L} = \mathbf{Q} \times \mathbf{P}$ is conserved. As a result the classical collective trajectory is confined to a plane.
- Even though in the exact treatment the force is everywhere defined and nonsingular, in the Born-Oppenheimer approximation the force is singular at the origin.
- The Bohr-Sommerfeld quantization rule ($\hbar = 1$)

$$\oint \mathbf{P} \cdot d\mathbf{Q} = 2(n + \alpha_M)\pi \qquad (9)$$

  does not include the correction coming from the Berry's phase. α_M is the correction coming from turning points or the Maslov index.

One can try to improve upon this simple Born–Oppenheimer approximation by taking into account the induced Berry's gauge potential into the Lagrangian. This amounts to replacing the Lagrangian in Eq. (3) by

$$\mathcal{L}_{BO} = \mathbf{P} \cdot \dot{\mathbf{Q}} \pm \frac{Q_3(Q_1\dot{Q}_2 - Q_2\dot{Q}_1)}{2(Q_1^2 + Q_2^2)\sqrt{Q_1^2 + Q_2^2 + Q_3^2}} - \left[\frac{\mathbf{P}^2}{2M} + V(Q) \pm \frac{\kappa Q}{2}\right], \qquad (10)$$

which follows from assuming $\mathbf{Q} \cdot \mathbf{r} = \pm Q, \quad (r = 1)$. The ensuing equations of motion are then

$$\dot{\mathbf{Q}} = \frac{\mathbf{P}}{M}, \quad \dot{\mathbf{P}} = -\left(\frac{\partial V(Q)}{\partial \mathbf{Q}} \pm \frac{\kappa \mathbf{Q}}{2Q} \pm \frac{\mathbf{Q} \times \mathbf{P}}{2MQ^3}\right). \qquad (11a, b)$$

The main difference with Eqs. (8a,b) is in the presence of the induced "magnetic force", arising from the "Dirac monopole" at the origin. Now $\mathbf{P}$ and $\mathbf{Q}$ are not canonical variables anymore. One can easily check that in this improved adiabatic approximation:

- The conserved total angular momentum is

$$\mathbf{J}_{BO} = \mathbf{Q} \times \mathbf{P} \pm \frac{\mathbf{Q}}{2Q} = \mathbf{Q} \times \mathbf{P} + \frac{1}{2}\mathbf{r} \qquad (12)$$

  and the classical collective trajectory is not confined to a plane anymore.

- The scalar potential force is identical to the one in standard Born-Oppenheimer approximation, but the "induced magnetic force" is even more singular at the origin. However, since in the adiabatic approximation $\kappa \to \infty$ and $\dot{\mathbf{Q}} \to 0$, the role of the "magnetic force" is rather small, except in the immediate vicinity of the origin. The strength of this induced "magnetic force" is independent of κ and its role is merely to ensure the conservation of the "correct total angular momentum". Its strength should somehow vanish when $\kappa \to 0$ and increase in the opposite limit. This fact alone should shed strong doubts on the limits of validity of this improved adiabatic approximation, which has been discussed in the literature lately, in connection with the Berry's phase phenomenon or the Molecular Aharonov–Bohm effect. This form of the effective magnetic field follows simply from the kinematical constraint imposed on the "spin" variables.
- The Bohr–Oppenheimer quantization rule now reads

$$\oint \left[\mathbf{P} \cdot d\mathbf{Q} \pm \frac{Q_3(Q_1 dQ_2 - Q_2 dQ_1)}{2(Q_1^2 + Q_2^2)\sqrt{Q_1^2 + Q_2^2 + Q_3^2}} \right] = 2(n + \alpha_M)\pi, \tag{13}$$

  and it includes the contribution from the Berry's phase as it should. However, the range of validity of this formula is very likely limited (see the above comment).

One should expect this improved Born–Oppenheimer approximation to be valid only for extremely low potential energies, where the "spin" is almost perfectly antialigned with the "magnetic field" and the total energy is almost identical to $V_-(Q)$. This is a rather strong limitation for the purposes of LACM. Even though the potential $V(Q)$ might be rather shallow, the amplitude of the motion is large and the potential energy $V(Q)$ can undergo rather large variations. There is no reason to expect that along the collective trajectory the kinetic energy is always small. This amounts to the fact that the "spin" cannot remain antialigned with the "magnetic field" and strong deviations from the (improved) adiabatic approximation will occur.

As we shall illustrate in the following, level crossings are a very strong source of chaoticity. We shall first consider the case of bound trajectories and harmonic oscillator potential $V(Q) = M\omega^2 Q^2/2$ ($f(Q) = 1$). Through an appropriate canonical transformation one can easily find such a representation where $M = \omega = 1$ and as a result the mixed classical–quantum equations of motion (4) become

$$\dot{\mathbf{Q}} = \mathbf{P}, \quad \dot{\mathbf{P}} = -\mathbf{Q} - \frac{1}{2}\kappa\mathbf{r}, \quad \dot{\mathbf{r}} = \kappa\mathbf{Q} \times \mathbf{r}. \tag{14a, b, c}$$

These equations have been solved for a variety of initial conditions and values of the coupling constant κ. For $\kappa \ll 1$ the trajectories are those of a slightly perturbed harmonic oscillator and the motion is essentially regular. For $\kappa \geq 1$ the onset of chaos is evident for almost any intial conditions. Only if the total energy is very close to the bottom of the "Mexican hat" the motion is regular. For higher total energies however, the trajectories can be characterized as chaotic most of time. One can characterize the degree of "chaoticity" by computing a number of different quantities: Lyapunov exponents, correlation integral [16], etc. For different initial values we have found Lyapunov exponents of the order unity. The correlation integral is the fractal dimension of the trajectory. The dimension of a periodic (regular) trajectory is one, while for a irregular one the value of the correlation integral/dimension is greater than one and less or equal (for an ergodic trajectory) to the dimension of the space. For the system (14), with 9 degrees of freedom and 5 integrals of motion, the maximum value for the correlation integral is therefore 9-5=4. For a reasonable set of initial conditions, with total energy above the minimum of the lower adiabatic potential surface, we have obtained for the dimension of the trajectories values around 3. Consequently, the motion is not ergodic, however quite irregular. A couple

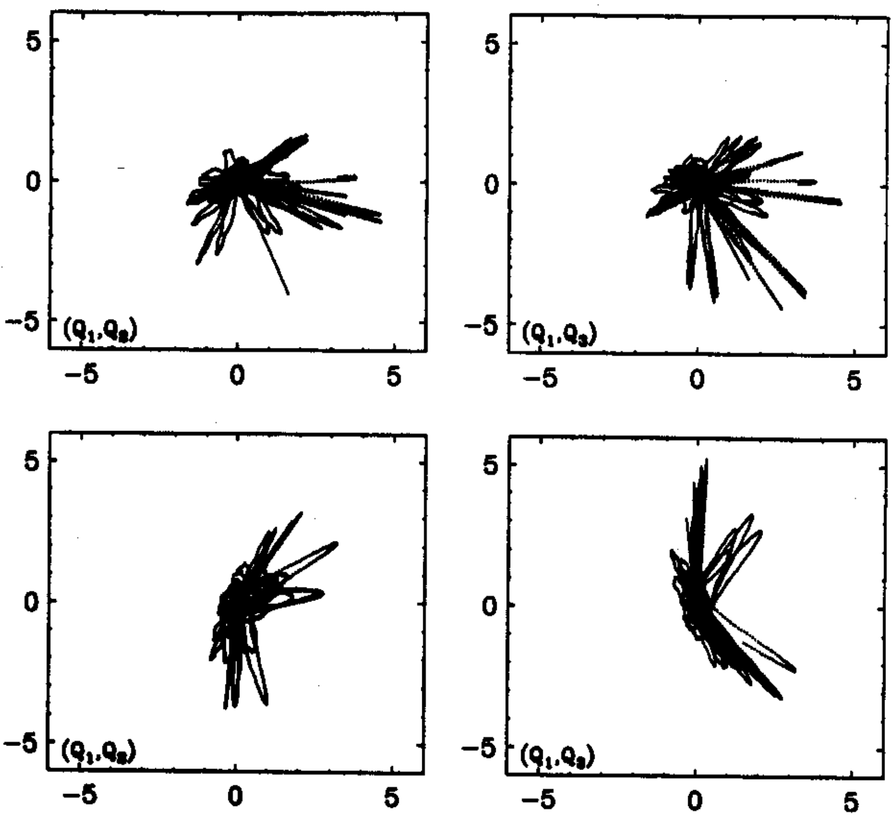

Figure 1: Chaotic trajectories for Eqs. (4) with $\kappa = 5$ for two sets of initial conditions (upper and lower parts respectively).

of representative trajectories for the system (14) are shown in Fig. 1. The rather complicated pattern of these trajectories cannot be reproduced with a scalar potential force only (due to nonplanarity of the orbit, presence of multiple turning points) and the effect of the "effective gauge potential" is obvious. While we have examined a particularily simple model (1), it can be shown that these features persist on the large part for general Hamiltonians as well as with N levels[17].

A more interesting case is the "scattering" of a particle off a level crossing. In a realistic many–body system one has more or less isolated level crossings, and the collective trajectory can be approximately viewed as a sequence of "scatterings" off a large number of funnels, with a variety of impact parameters, initial momenta and orientations of the "spin". We have chosen the following model situation

$$V(Q) = V_0 \exp\left(-\frac{Q^2}{2Q_0^2}\right), \quad f(Q) = \frac{\tanh(Q)}{Q}, \tag{15}$$

which describes two levels with constant separation far from the level crossing. In Figs. 2 and 3 we display the scattering angles θ, ϕ, final absolute momentum and the time delay, for a range of initial conditions. The emerging scattering pattern is unexpectedly complex and can be characterized as chaotic scattering [18]. The "transition probability" from one level to another (not shown here) has a similar aspect, it is not a monotonous smooth function but a fractal, in

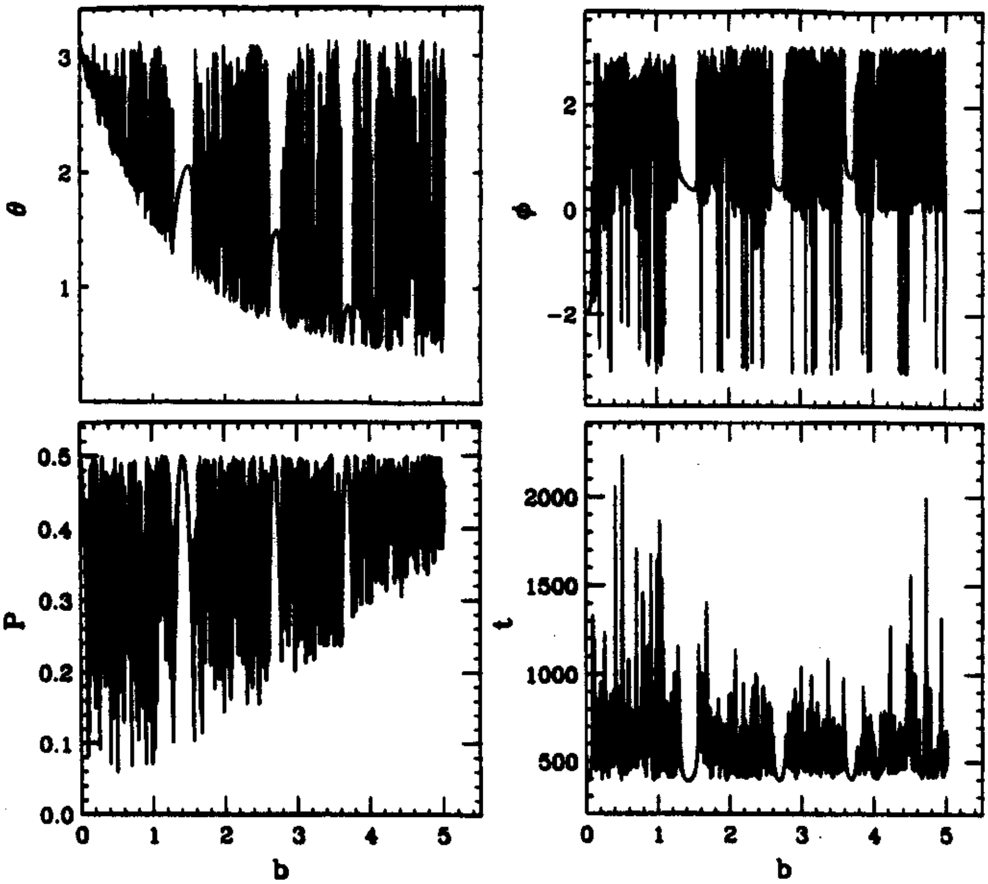

Figure 2: The final direction of the momemtum specified by the spherical coordinates θ and ϕ, final absolute value and the time delay for initial momentum and spin along the Oz axis as a function of the initial impact parameter.

contrast with what one would have expected from a Landau–Zener picture.

An equally complex picture emerges in the 1–dimensional case as well, when the particle is allowed to move along the Oz axis only. In Fig. 4 we display the final momentum of a particle as a function of the initial orientation of the "spin". The most surprising thing now is that we cannot "predict" whether the particle, which impinges on the level crossing from the left in this case, will go to the right or to the left, but instead can bounce either to the left or to the right in an essentially random manner.

The occurrence of fractals in this relatively simple problem can be linked with the existence of a Cantor set of unstable periodic orbits embedded in the continuum [18]. The "effective magnetic field of the monopole" acts like a trap and trajectories originating from far away come very close in the phase space to these unstable periodic orbits and are almost caught by them. One might argue that quantum effects will wash out all these irregularities and everything will become smooth. Blümel and Smilansky [19] demonstrated that classical chaotic scattering manifests itself at the quantum level as Ericson fluctuations in particular. Jung and Pott [20] showed also that the fluctuations in the quantum cross sections have a fractal nature as well. The classical treatment we have adopted here can easily be turned into a semiclassical approach.

The systems we have analysed here have rather high symmetry (Jahn–Teller type), the functional form of the potential $V(Q)$ and the coupling between the fast and slow degrees of

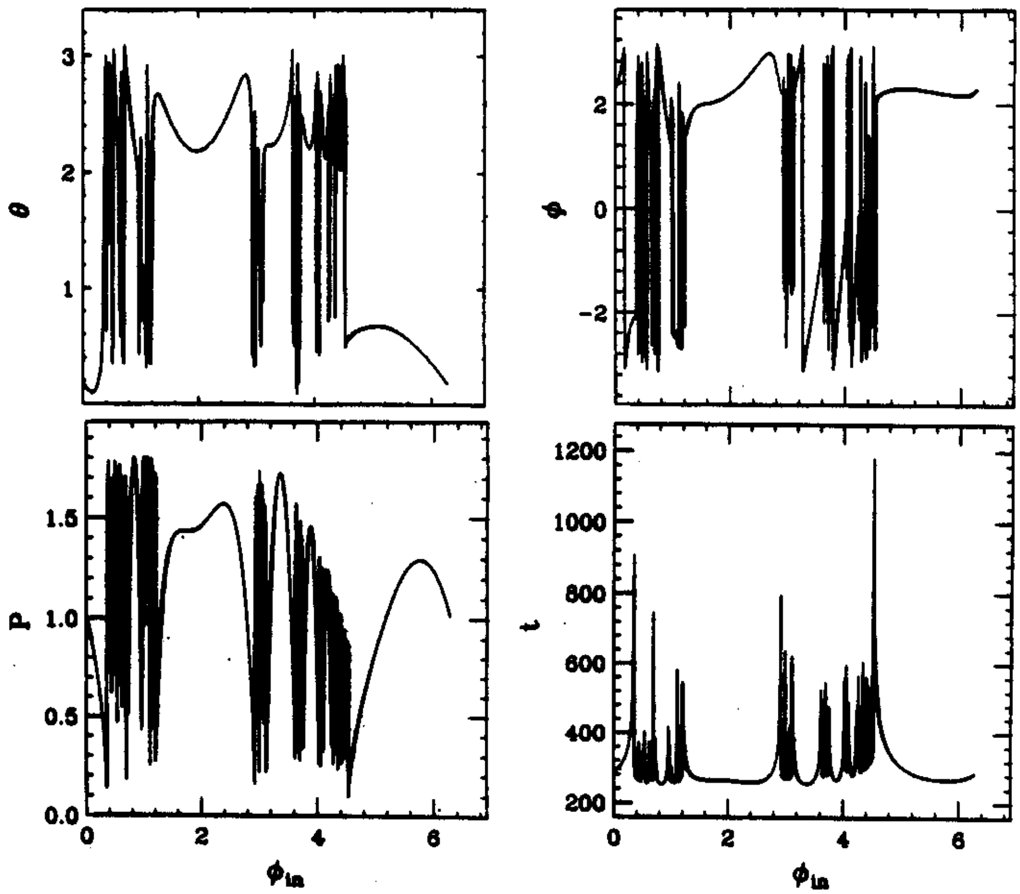

Figure 3: The same as in Fig. 2 but for initial momentum along the Oz axis and fixed impact parameter as a function of the initial orientation of the spin $\mathbf{r}(0) = (\cos\phi_{in}, \sin\phi_{in}, 0)$.

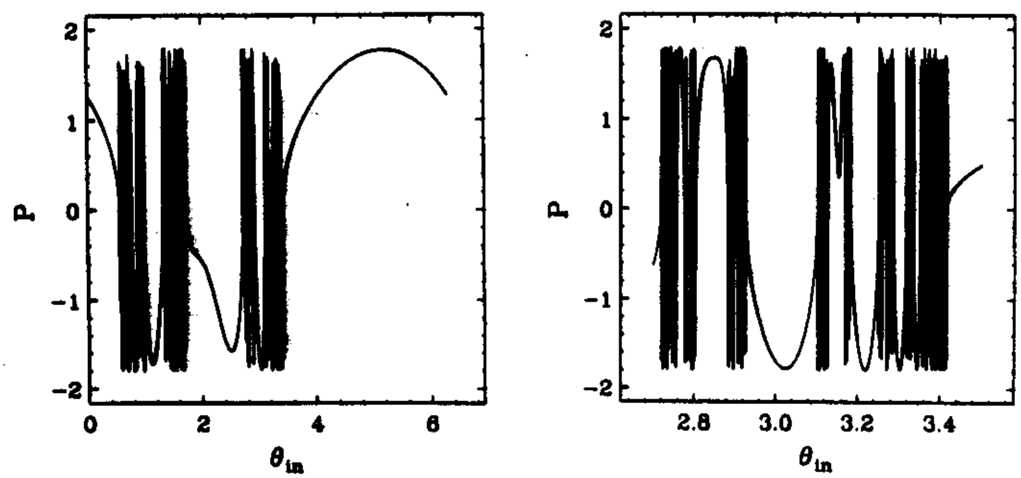

Figure 4: The final absolute value of the momemtum as a function of the initial orientation of the spin $\mathbf{r}(0) = (\cos\phi_{in}, \sin\phi_{in}, 0)$ for 1–dimensional motion

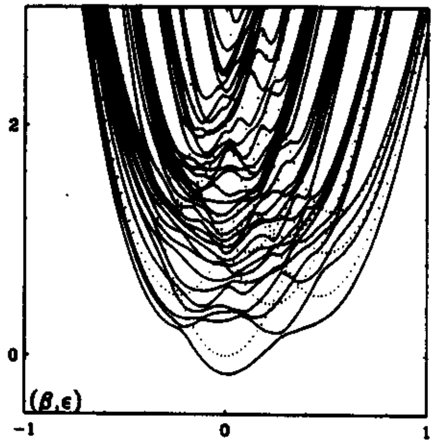

Figure 5: An example of a multisurface potential energy. The dotted lines are the diabatic surfaces and the solid lines are the adiabatic surfaces for the Hamiltonian (16).

freedom allow for the existence of the integral of motion (7). As a result, both the level crossing and the "monopole" occur at the same point in space in the adiabatic approximation. The fact that a system with so high symmetry displays such a complex dynamics is at least unexpected.

3. MANY LEVEL CROSSINGS

A many–body system has a far more complex behaviour due to the large number of level crossings. A caricature of a multisurface potential energy of a "realistic" nucleus, which undergoes large amplitude collective motion, not merely small amplitude vibrations around a local minimum, is presented in Fig. 5. These particular energy surfaces correspond to four independent particles in a harmonic oscillator mean–field potential with axial symmetry ($\omega_x = \omega_y = \exp(\beta)$, $\omega_z = \exp(-2\beta)$). Distinct configurations were generated by using different occupation number probabilities as Hill and Wheeler have done [1]. In a simplified picture, one can imagine that at each band crossing there is a "Dirac magnetic monopole". The "effective magnetic field" generated by each "monopole" is felt in the immediate neighborhood of each funnel or diabolic point mainly. The residual interaction was "modeled" with either uniform or Gaussian random numbers. At least for the naked eye the two types of "residual interaction" produce similar adiabatic energy surfaces. The corresponding Hamiltonian now reads

$$H = \frac{P^2}{2M} + h, \quad h_{ii} = V_i(\beta), \quad h_{ij} = h_{ji} = (1-\delta_{ij})\Delta_{ij}, \tag{16}$$

where $V_i(\beta) = \sum_k[(n_x(k) + n_y(k) + 1)\exp(\beta) + (n_z(k) + 1/2)\exp(-2\beta)]$ is the mean–field energy of a configuration, M is a collective mass and Δ_{ij} are either uniform or Gaussian random numbers. An actual collective Hamiltonian should include other collective variables as well, e.g. nonaxial deformation, pairing (partially mocked here through Δ_{ij}'s), rotation degrees of freedom, etc.

Let us assume that the motion starts with vanishing collective velocity somewhere far from the absolute energy minimum of this fictitious nucleus. Once it reaches the first band crossing, depending on the particular values of the collective coordinates and momenta and the population of the two adjacent bands, after passing past the funnel or diabolical point, the system can end up in any state. In the absence of a "magnetic force" the system will most likely follow the valley of the lower potential energy surface. The "magnetic force" will however curve the trajectory (or even capture the system for a while) and at the same time the "spin" (which is related with the relative population of the two adjacent bands) can turn around. Near the funnel or diabolical point, the collective and intrinsic degrees of freedom are strongly coupled and the motion is essentially chaotic. One cannot expect either the adiabatic or the diabatic picture to be a truthful representation of the dynamics. An essential element is of course the fact that the collective motion is really multidimensional, one cannot limit the analysis to only one degree of freedom, since the "magnetic field" will propel the system into additional directions. The final picture will be even more complicated by the fact that one has to consider the propagation of a packet of initial conditions. In such a case quantum interference effects in the slow variables will also become important. The system can start near one band crossing and reach another one, after passing near several intermediate ones through a variety of paths. Then the interference among different trajectories should be included and (due, in particular, to the essentially random distribution of funnels or diabolical points) very likely is destructive in character [21]. The trajectory will also have a quite complicated pattern, since near every funnel or diabolical point, it will be bend or even forced to make a few turns. As a result the actual length of the trajectory can become very large and the time required to get away from the initial coordinates rather long. On a coarse scale it can appear as a random walk, in spite of the fact that very little dissipation is actually present. This is due to the fact that a system is very "unwilling" to get very far from an initial configuration, and not because it gets excited and there is a strong dissipation of the collective energy, but rather due to something which looks like Anderson localization, a characteristic behaviour of quantum particles in a random potential. The motion seems to be slower than one would expect, even though the nucleus might not get very excited by the time it reaches the potential minimum.

In Fig. 6 we display two "collective" trajectories for the Hamiltonian (16). The initial intrinsic state was always taken as a "pure" adiabatic wave function at the corresponding deformation and the initial collective momentum was zero. The collective motion looks rather complicated and hardly resembles what one might have expected. On average the collective velocity is extremely small, due to the repeated "turns". Sometimes the collective energy is completely turned into internal excitation of the internal degrees of freedom and the nucleus gets stuck. For different initial conditions some of the collective energy is degraded into "heat", but a significant amount of the collective energy is preserved and the "nucleus" undergoes rather large amplitude undamped oscillations. One cannot describe the emerging picture as neither adiabatic or diabatic and a diffusive picture is also not the most appropriate description one can think about. Obviously, the fluctuations are rather large and a new type of approach is needed.

The description of the collective degrees of freedom we have adopted here still needs serious improvements. If one intends to retain the (semi)classical description of LACM, the most important element seems to be a generalization of classical mechanics, which will allow for multiple valued coordinates and momenta and at the same time permit coupling with non–abelian effective gauge fields. Qualitatively, this is similar to the description of superfluidity, where at the same point in space one has a normal and a superfluid component, which move with different velocities and inertia. Whether one should relinquish the "particle" approach to collective motion in favour of a continuum approach, it is not clear at the moment, but it might prove to be the only viable alternative. The nucleus can be on different potential surfaces and therefore have different values of the collective momentum. Even though a nucleus starts moving on one definite potential surface, as soon as it encounters a level crossing the trajectory can bifurcate. At this point Tully [9] advocates the introduction of a stochastic element, the probability to jump from one adiabatic surface to another. The presence of a stochastic element in an otherwise totally deterministic theory, where no thermostat is present, it is certainly questionable from the theoretical point of view. For one reason, it can lead to too much chaos.

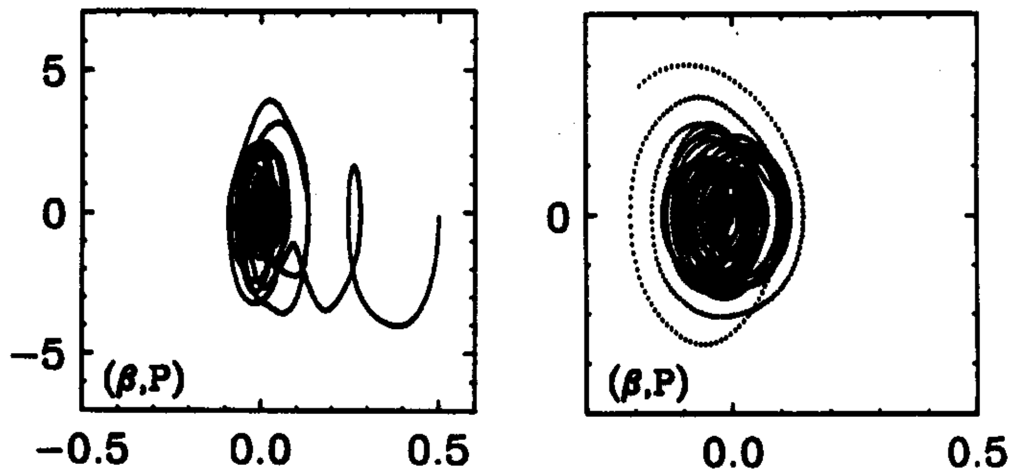

Figure 6: Two collective trajectories for the Hamiltonian (16).

It is also unclear how a requantization procedure can be performed, once a stochastic element is introduced *ad hoc* at the classical level. On the other hand, a fully quantum description of the collective motion is in principle possible, and many of these rather technical issues (new type of classical mechanics) will not arise. The obvious disadvantage is of course the fact that we will have to be able to solve a multidimensional time–dependent Schrödinger equation. A (semi)classical approach, which even though does not exists yet, should be both simpler and more intuitive.

4. CONCLUSIONS

We did not try to present here a coherent theory of LACM, since in our opinion at the present time such a theory does not exists yet. Whether such a theory is possible and moreover, whether it will have any practical value it is unclear. It might easily prove to be too complicated for any practical purposes. It should allow us however to reach a deeper understanding of the dynamics of complex many–body systems.

We have pointed to a series of phenomena, which seemed to have eluded our attention till now. One can expect that a proper treatment of the type of chaotic behaviour, induced by the level crossings, should allow us to understand not only the collective dynamics, but moreover, the role and the character of dissipation in nuclear collective motion [22]. Our rather liberal use of the term dissipation should not misslead the reader, there is no nondeterministic or irreversible element in the theory (unless introduced *ad hoc*). Level crossings will simply make the dynamics look irreversible on relatively short time scales, the Poincaré return times being very large. The peculiar role played by Landau–Zener transitions in mesoscopic systems and the occurrence of a new type of dissipative behaviour has been advocated by Wilkinson [23] (see also [21]). The main difference between nuclei and systems considered in Refs. [21,23] is that the Landau–Zener transitions in mesoscopic systems are externally driven, while in nuclei (and very likely in atomic clusters as well) the slow and fast modes are strongly coupled and as a result the slow motion becomes chaotic.

The number of elements entering the emerging picture of LACM and of related phenomena is truly astounding: i) one has to introduce several collective degrees of freedom; ii) one has to include a multisurface potential energy; iii) using a loose language, one has to introduce "effective magnetic fields"; iv) the motion becomes chaotic; v) interference phenomena should be properly taken into account for long trajectories; vi) the quantum numbers become fractional; vii) very likely, in a correct description one has to resort to a non–abelian gauge description; viii) dissipation or friction are likely much more intricate phenomena than we have suspected until now; ix) quasilocalization in collective coordinates; and the list seems simply to keep growing.

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