Michigan State University

National Superconducting Cyclotron Laboratory

FORMATION OF COMPOSITES EMITTED AT LARGE ANGLES IN INTERMEDIATE AND HIGH ENERGY REACTIONS

Talk given at the International Workshop on Dynamical Fluctuations and Correlations in Nuclear Collisions, Aussois, France, March 16-20, 1992

P. DANIELEWICZ

MSUCL-832 **APRIL 1992**

MSUCL-832
April, 1992

Talk given at the International Workshop
on Dynamical Fluctuations and Correlations in Nuclear Collisions
Aussois, France, March 16-20, 1992.

FORMATION OF COMPOSITES EMITTED AT LARGE ANGLES IN INTERMEDIATE AND HIGH ENERGY REACTIONS

P. Danielewicz

National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA

Abstract

The importance of the nuclear mean-field, the various collision processes, and the Coulomb potential, for the formation of composites emitted at large angles in nuclear reactions, is discussed. It is demonstrated that the nucleon-nucleon collisions are by far most essential for the formation of composites at high energy. A model with composite formation during the collisions is applied to the intermediate-energy heavy-ion reactions.

1. INTRODUCTION

The production of light composites at large angles is of interest in the heavy-ion induced reactions where the composites are produced rather abundantly [1,2], and in the reactions induced by light projectiles [3,4]. The most often cited dynamic model of composite particle formation is possibly the one of Butler and Pearson [3]. The formation of a deuteron in their model is caused by the optical potential. The deuteron momentum distribution turns out to be proportional to the product of the momentum distributions of constituent nucleons. This model was successful in explaining a large body of data [4-6]. In the dynamic models proposed for heavy-ion induced reactions [7-9], deuterons are assumed to be formed in the aftermath of the nucleon-nucleon collisions. These models, are also successful in explaining data, yielding, in particular, the law of proportionality for the momentum distributions. Even in the forward-peaked direct pickup reactions, the momentum distributions of produced deuterons reflect the distributions of picked-up nucleons, and, in fact, can even be used for the determination of the momentum distribution in the target.

In this talk we shall examine what mechanism should, actually, dominate the production of composites at large angles. We shall relate the production to the breakup, and show, in particular, that the production in the model of Butler and Pearson is, physically, a process inverse to the so-called elastic breakup. Finally, we will represent results from a transport model of heavy-ion reactions with deuteron, triton, and helion (^{3}He) formation in the collisions, basing on these considerations.

We first discuss the mechanisms of the production and breakup using general terms (Sec. 2). We turn next to the breakup of energetic deuterons incident on a nucleus (Sec. 3). Subsequently, we present (Sec. 4) a model with deuteron production related to the deuteron breakup using microscopic balance. The model is similar to that used by Butler and Pearson [3] to explain data from proton-induced reactions. We then turn to the transport model for heavy-ion reactions (Sec. 5) and conclude (Sec. 6).

2. FORMATION AND BREAKUP OF COMPOSITE PARTICLES

A formation process running backward in time becomes a breakup process that may be easier to consider than the process of formation. Either the formation or breakup requires an external agent.

For the breakup or formation by the nuclear mean-field potential, the second derivative of the potential must be finite, or, correspondingly, the force must vary over the region occupied by a composite. A potential changing linearly with position and giving a constant force could only accelerate a composite as a whole. Further, it should be realized that in the case of a fast composite the potential must change perpendicular to the trajectory, in order to be effective. The potential changing along the trajectory of a composite or its constituents, would deliver the same force to all constituents only at some differing time instants.

The Coulomb potential is weak even for heavy nuclei, but it is of long range and the force acts only on protons making it easier to change the internal state of a composite. The interaction with individual nucleons is of short range implying that the constituents of a composite are separately affected. Respectively, the interaction should be effective in the formation and breakup, except at low energies when the collisions with individual nucleons are suppressed by the Pauli principle.

Let us turn to some available data for light composites incident on a nucleus. If the mean-field potential played an important role in the breakup, one might expect an associated structure in the surface region of the imaginary potential used to describe the elastic scattering of the composites. The optical potentials for deuterons were studied as a function of energy by Daehnick *et al.* [10], and for α particles by Budzanowski *et al.* [11]. There is a surface peaking of the potentials at low energy, but there is also such a peaking of the potentials for nucleons [12]. At 50 MeV/nucleon for deuterons and at 35 MeV/nucleon for α particles, the imaginary potentials become largely uniform throughout the volume of the considered target nuclei.

We now turn to some approximations and experimentations with the breakup cross-sections and then proceed to construct a model in which the formation is simply related to the breakup cross section. Similar analysis of the breakup to one in the next section has been done before using the coupled-channels method by Austern *et al.* [13] for 23 MeV deuterons incident on calcium.

3. APPROXIMATIONS AND EXPERIMENTATION WITH DEUTERON BREAKUP

We take the deuteron as the simplest case of a composite, and examine the matrix element for the deuteron breakup or formation in a potential V in the PWBA approximation. Although the approximation hardly ever works as such in nuclear reactions, it will be useful for the sake of comparison with the results of Butler and Pearson. With $\mathbf{Q} = (\mathbf{Q}^{\perp}, Q^{L})$ being the total momentum transfer, and $\mathbf{q}$ - the relative momentum of a nucleon pair the matrix element reads

$$\mathcal{M} = \int d^3R\, e^{iQR} \int d^3r\, \psi_{\mathbf{q}}^*(\mathbf{r})\, (V(\mathbf{R} + \mathbf{r}/2) + V(\mathbf{R} - \mathbf{r}/2))\, \psi_d(r) \quad (1)$$

The integration is over the c.m. position $\mathbf{R} = (\mathbf{b}, Z)$, and the relative position $\mathbf{r} = (\mathbf{r}^{\perp}, z)$. The function ψ_d is the deuteron wavefunction and $\psi_{\mathbf{q}}$ is the scattering wavefunction of a pair. The scattering amplitude f for the breakup is related to $\mathcal{M}$ via

$$f = m\mathcal{M}/\pi\ , \quad (2)$$

where m is nucleon mass. If the potential V is expanded in (1) around $\mathbf{r} = 0$, the lowest-order term that gives a finite contribution, involves a second derivative of V. Since a potential that has no dependence on $\mathbf{r}$, cannot cause a transition, one can rewrite (1) into

$$\mathcal{M} = \int d^3R\, e^{iQR} \int d^3r\, \psi_{\mathbf{q}}^*(\mathbf{r})\, (V(\mathbf{R} + \mathbf{r}/2) + V(\mathbf{R} - \mathbf{r}/2) - 2\langle V\rangle(R))\, \psi_d(r)\ , \quad (3)$$

where the average is with respect to deuteron wavefunction

$$\langle V\rangle(R) = \int d^3r\, |\psi_d(r)|^2\, V(\mathbf{R} + \mathbf{r}/2)\ . \quad (4)$$

The specific use of (4) in the matrix element, generally allows for the approximation of using plane rather than scattering waves in (3).

When calculating the total breakup cross section the squared matrix element is integrated over the scattering angle Ω and the relative momentum $\mathbf{q}$, as

$$d\sigma = |f|^2 d\Omega\, d^3q/(2\pi)^3\ . \quad (5)$$

This is also done when calculating the rate of the deuteron formation from protons and neutrons moving through a potential V [3]. At high energies the integration may be simplified, because the scattering is focused forward, and the longitudinal momentum transfer drops to zero (even though finite energy is needed for the internal excitation), yielding

$$\sigma^{Born} = \frac{m^2}{\pi^2}\int d\Omega \int \frac{d^3q}{(2\pi)^3}\, |\mathcal{M}|^2 \approx \frac{1}{4\pi^2 v^2} \int d^2Q^{\perp} \int \frac{d^3q}{(2\pi)^3}\, |\mathcal{M}|^2$$
$$= \frac{1}{v^2} \int d^2b\, \langle(\tilde{V} + \tilde{V} - 2\langle\tilde{V}\rangle)^2\rangle. \quad (6)$$

Here $\tilde{V}$ is the potential of a nucleon integrated over the longitudinal direction,

$$\tilde{V}(\mathbf{b}) = \int dZ\, V(\mathbf{b}, Z)\ , \quad (7)$$

and up to a factor the result (6) is nothing else but a term in the expansion of the breakup cross section in the Glauber approximation in series of the interaction potential.

The Glauber approximation is expected to work well at higher energies. In the adiabatic approximation the cross section for deuteron breakup is [14]

$$\sigma = \int d^2b\, T(b) = \int d^2b\, (1 - |\langle e^{i\chi}\rangle|^2)\ . \quad (8)$$

The phase function χ is given by

$$\chi(\mathbf{b}, \mathbf{r}^{\perp}) = -\frac{1}{v} \int dZ\, (U_p(\mathbf{b}+\mathbf{r}^{\perp}/2, Z) + U_n(\mathbf{b}-\mathbf{r}^{\perp}/2, Z))\ , \quad (9)$$

where v = P/2m is the velocity, and we now take the nucleon optical potential U consisting of real and imaginary parts, and parametrized for protons as

$$U_p(r_p) = V_p(r_p) + iW(r_p) = (V_0 + V_I(N - Z)/A)\ \rho(r_p)/\rho(0)$$
$$+ V_c(r_p) - i\sigma_{NN} v\rho(r_p)/2. \qquad (10)$$

In the above V_0 and V_I are the strengths of the isoscalar and isovector potentials, ρ is the nucleon distribution in target for which we use a parametrization taken from electron scattering, V_c is the Coulomb potential, and σ_{NN} is the nucleon-nucleon (NN) cross-section. For neutrons the isovector potential changes its sign and the Coulomb force is absent.

In Fig. 1 we show V and W for 100 MeV nucleons incident on a ^9Be nucleus. That was the target in the experiment that Butler and Pearson have analyzed the deuteron formation. The lowest energies of emitted particles were 100 MeV/nucleon. In Fig. 2 we show the breakup transmission coefficient [15] T, cf. Eq. (8), and contributions to the breakup cross section from different impact parameters $d\sigma/db$, for 100 MeV/nucleon deuterons incident on ^9Be. We also show in this figure the results obtained when artificially switching off the real and imaginary parts of the nucleon potential. Although the potentials are nearly the same, the effects of this operation are quite different in the two cases. When the real part is switched off, the results are almost unchanged. By contrast, when the NN collisions are switched off, the cross section drops to a small fraction of the original one. Note that ^9Be that is only little larger than a deuteron, particularly favors the breakup by the mean-field potential. This is because the forces can squeeze the deuteron acting from both sides. We further show with dotted lines the contributions to T and $d\sigma/db$ from the elastic breakup, i.e. processes in which the internal state of the target nucleus is not altered. The cross section for the elastic breakup is

$$\sigma^e = \int d^2b\ T^e(b) = \int d^2b\ \left(<|1 - e^{i\chi}|^2> - |<1 - e^{i\chi}>|^2 \right)$$

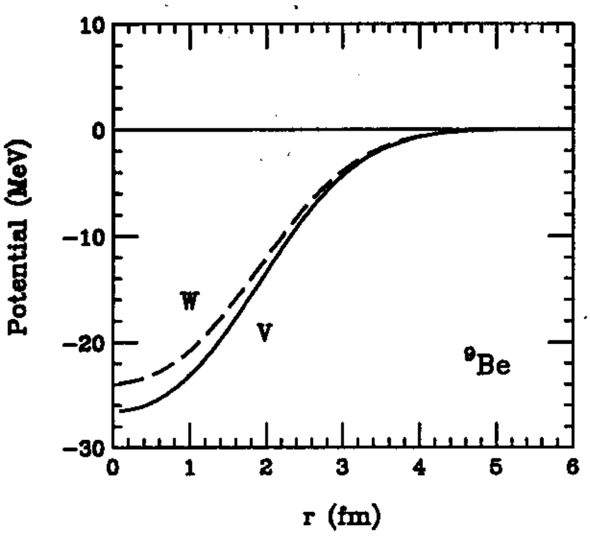


Figure 1. Real (V) and imaginary (W) parts of the optical potential of 100 MeV nucleons incident on ^9Be.

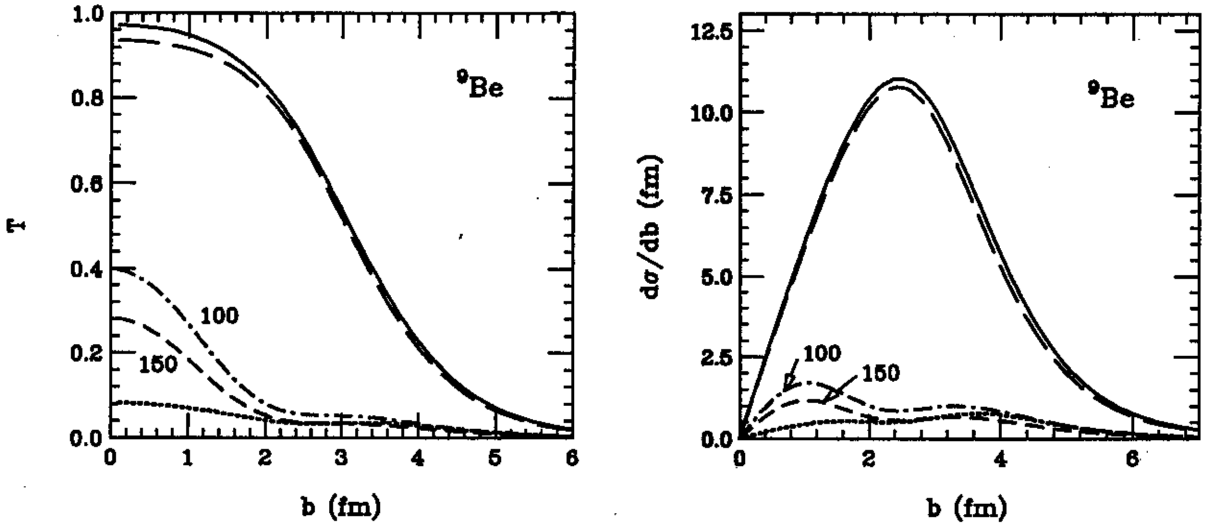


Figure 2. Breakup transmission coefficient T and contributions to the breakup cross sections from different impact parameters $d\sigma/db$, in the Glauber approximation, for deuterons incident on ^{9}Be. Except for the lines marked 150 that refer to 150 MeV/nucleon, all results are for the incident energy of 100 MeV/nucleon. The solid and long-dashed lines show results obtained using the full complex and purely imaginary potentials, respectively. The short-dashed and dash-dotted curves are obtained using only real potentials. The dotted lines represent T^e and $d\sigma^e/db$.

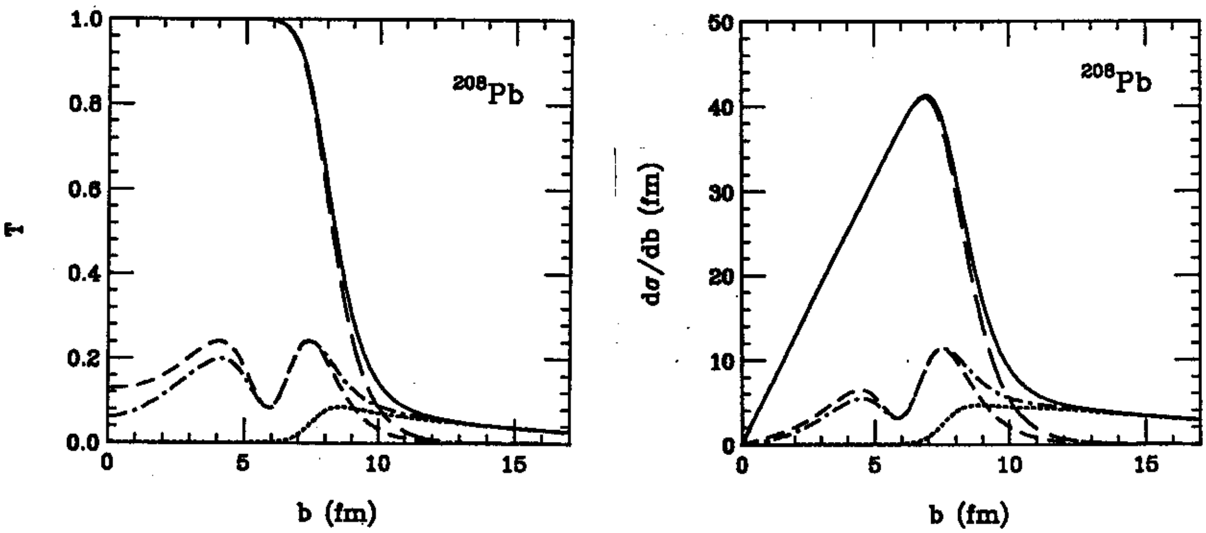


Figure 3. Breakup transmission coefficient T and contributions to breakup cross sections from different impact parameters $d\sigma/db$ in the Glauber approximation for the 100 MeV/nucleon deuterons incident on ^{208}Pb. The solid and long-dashed lines show results obtained using the full complex and purely imaginary potentials, respectively. The dash-dotted and short dashed curves represent results obtaining using a real potential only with and without the Coulomb and isovector potential combination, respectively. The dotted lines represent T^e and $d\sigma^e/db$.

$$= \int d^2b \left(1 - |\langle e^{i\chi}\rangle|^2 - (1 - \langle |e^{i\chi}|^2\rangle)\right) = \sigma - \sigma^{cl}. \qquad (11)$$

where the classical breakup cross section is

$$\sigma^{cl} = \int d^2b \left(1 - \langle \exp[-\sigma_{NN}\int dZ \ (\rho(r_p) + \rho(r_n))]\rangle\right) . \qquad (12)$$

The elastic breakup cross section drops, but does not completely vanish when the real potential is switched off and collisions are retained because of diffraction [16].

On the basis of the results shown in Figs. 2 and 3 we conclude that except for the Coulomb potential for heavy nuclei and large impact parameters, the real part of optical potential plays a quite insignificant role in the breakup at energies around 100 MeV/nucleon and above.

4. MODEL WITH DEUTERON FORMATION

In explaining the formation of deuterons emitted at large angles in proton-induced reactions, Butler and Pearson [3] have considered uniform fluxes, of protons and neutrons incident on the target nucleus that catalyzed the formation. In the following we consider such a situation and relate after certain simplifications the rate of deuteron formation under these circumstances to the breakup cross section on the same nucleus. This will allow us to carry over the conclusions reached for breakup to the formation.

The number of transitions per unit time leading to deuterons with energy E and momentum $\mathbf{P}$, for neutrons and protons incident on a nucleus, may be written as

$$\mathcal{N}(\mathbf{P}) = \frac{1}{(2\pi)^3} \int d^3p_p \int d^3p_n \ F_p(\mathbf{p}_p)F_n(\mathbf{p}_n) \int \frac{d^3Q}{(2\pi)^3} \sum_{\nu} |\mathcal{M}_{0\nu}^{p+n\to d}|^2$$
$$\times \ (2\pi)^3\delta(\mathbf{P} - \mathbf{p}_p - \mathbf{p}_n - \mathbf{Q}) \ 2\pi\delta(E - e_p - e_n) \qquad (13)$$

Here F is the number of nucleons per unit momentum space and volume, ν are final states of target nucleus, and in the δ-function for the energy conservation we ignore target excitations assuming high incident energies. The breakup cross section on the other hand may be written as

$$\sigma = \frac{1}{v} \int \frac{d^3p_p}{(2\pi)^3} \int \frac{d^3p_n}{(2\pi)^3} \int \frac{d^3Q}{(2\pi)^3} \sum_{\gamma} |\mathcal{M}_{0\gamma}^{d\to p+n}|^2$$
$$\times \ (2\pi)^3\delta(\mathbf{P} - \mathbf{p}_p - \mathbf{p}_n - \mathbf{Q}) \ 2\pi\delta(E - e_p - e_n). \qquad (14)$$

For the processes at high energy it should not be particularly matter, whether the target nucleus is initially in its ground or some excited state,

$$\sum_{\nu} |\mathcal{M}_{0\nu}^{p+n\to d}|^2 \approx \sum_{\nu} |\mathcal{M}_{\gamma\nu}^{p+n\to d}|^2 \ . \qquad (15)$$

Indeed, in (10) we just use density distribution. We average out the squared matrix element in (13) over a range of initial states that extends beyond the characteristic falloff range of the matrix element. We do the same in (14). We further use microscopic reversibility and also assume that the functions F vary weakly within the range characteristic for the momentum transfers and the momenta within deuteron wavefunction in order to get

$$\mathcal{N}(\mathbf{P}) \simeq (3/4)(2\pi)^3\, \sigma v F_p(\mathbf{P}/2) F_n(\mathbf{P}/2) \ . \qquad (16)$$

Thus the number of deuteron formation processes turns out to be simply proportional to the deuteron breakup cross section on the same target. The result (16) has some appealing features. For example, if one keeps only NN collisions and uses a classical cross section (12), then a result emerges that can be obtained following the approach of Ref. [17].

From (16) and the preceding discussion it is clear that the mean-field potential plays an insignificant role in the formation of deuterons at high energies. The Coulomb force might be significant for heavier systems, but one has to keep in mind that in the actual reaction processes the particles would rather move out radially from the reaction zone. Consequently, the Coulomb effects are not expected to play an essential role.

Let us now relate these results to those of Butler and Pearson, and see, in particular, how, in view of the above facts, they were able to describe quantitatively the data. With nucleons spread over a volume V and deuteron formation occurring within time interval τ, the relation between the distribution of deuterons and nucleons in momentum space becomes

$$\frac{d^3N_d}{dP^3} = \tau\mathcal{N} = \frac{3}{4}\,(2\pi)^3\,\frac{\sigma v\tau}{V^6}\,\frac{d^3N_p}{d(P/2)^3}\,\frac{d^3N_n}{d(P/2)^3} = B\,\frac{d^3N_p}{d(P/2)^3}\,\frac{d^3N_n}{d(P/2)^3} \qquad (17)$$

For a later use it will appear convenient to write B in the form

$$B = \frac{\tau}{V^6}\,R = \frac{\tau}{V^6}\,\frac{3}{4}\,(2\pi)^3\, v \int d^2b\; T(b) \ . \qquad (18)$$

For the reaction data [18] with ^{9}Be the coefficient of proportionality is $B = 0.016\ \mathrm{GeV}^3$, containing correction due to target absorption [3]. Butler and Pearson arrived at an expression corresponding to (6) in the formation rate, by making use of the perturbation theory both in the optical potential *and* in the proton-neutron interaction. The potential in their approach may be complex. They used an approximation corresponding to

$$\langle V\rangle(R) \simeq V(R) \qquad (19)$$

in (3) or (6), and were able to reproduce the data with an overall strength of the interaction potential of 25±5 MeV.

The results of Butler and Pearson may be interpreted in two different ways. One is that the real part of the potential alone could provide a justification for the observed number of deuterons. With a complex potential, and, clearly, a modulus square in (6), the formation becomes inverse to the elastic breakup, as the cross section in the breakup channel is nothing else but the expansion of (11) to lowest nonvanishing order in χ. Note that the full breakup cross section (8) has a term linear in the imaginary potential. Thus the second interpretation is that the process inverse to the elastic breakup could be taken as responsible for the deuteron formation. Of immediate physical relevance is the validity of the second statement, but we will show that neither of the statements is true.

As a whole the Born approximation works for real potentials and small systems, but not for a complex potential. Instead of rising, the elastic breakup cross section drops as can be seen in Figs. 2 and 3, when the imaginary potential is included. Let us then concentrate on the case of an exclusively real potential. In Fig. 4 we show the contributions to the rate R in (18) from different impact parameters. The constant B may be estimated using [3] $\tau \approx 2R_o/v$, $V \approx 4\pi R_o^3/3$, with $R_o \approx 2.5$ fm. Integrating over the impact parameters we get for B values of 0.013 GeV^3 and 0.09 GeV^3, when, respectively, using (6) with (19), and (8) (or equivalently (11)). If we estimate B in the same manner using the total breakup cross section calculated with a full complex potential, we get a value of 0.060 GeV^3, and with collisions only a value of 0.058 GeV^3, i.e. nearly four times larger than that describing the data! The values are inflated, because in relating the model to the reaction one chooses a density as for the nucleons all contained in nuclear volume characterized by R_o, but in the model the density is uniform over the whole space. Consequently, most of the produced deuterons contain nucleons from the exterior region. Using the fact that the expression (8) may be rewritten into a form involving deuteron density matrices rather than wavefunctions, we can recalculate the rate R, see Fig. 4, demanding that the nucleon impact parameters are less than R_o. Taking only the real potential we get $B = 7\cdot10^{-4}$ GeV^3, and with the full complex, or only imaginary potential, we get B = 0.009 GeV^3. Note that in actual reaction the formation should not always be expected to be a two-step process. The first collisions that direct nucleons towards large angles would also contribute to formation.

In Fig. 5 we show contributions to the rate when using a ^{27}Al target [3,18]. The data can be described taking B = 0.008 GeV^3. With [3] $R_o \approx 3.6$ fm, we get for the real potential employing (6) and (19), a value of B = 0.006 GeV^3, and using (8) a value of B = 0.003 GeV^3. When restricting the nucleon impact parameters to less than R_o, we get $B = 5\cdot10^{-4}$ GeV^3. For the complex and a pure imaginary potentials we get

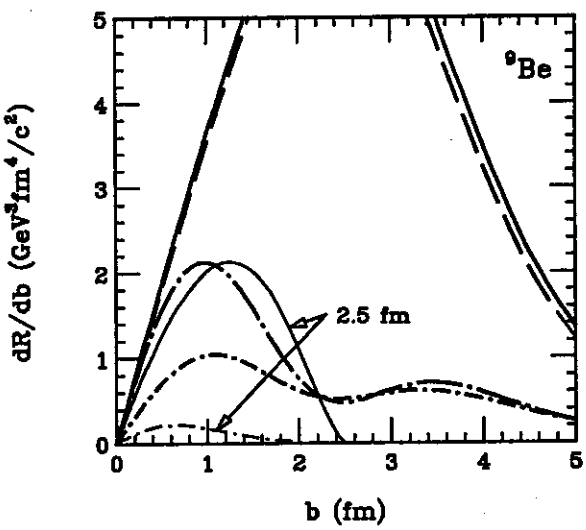


Figure 4. Contributions to the formation rate in the model of Section 4 from different impact parameters, dR/db, cf. (18), for a ^{9}Be target. The thick solid and long-dashed lines are obtained using the full complex and purely imaginary potentials, respectively. The thick short-dash-dotted line is obtained using only a real potential. The long-dash-dotted line is obtained using a real potential, the Born approximation, and (19). The thin solid and dash-dotted lines marked by 2.5 fm are obtained when restricting the nucleon's impact parameters to less than R_o for the complex and real potentials, respectively. The results obtained for a purely imaginary potential could be hardly distinguished in the latter case in the Figure from the results for the complex potential.

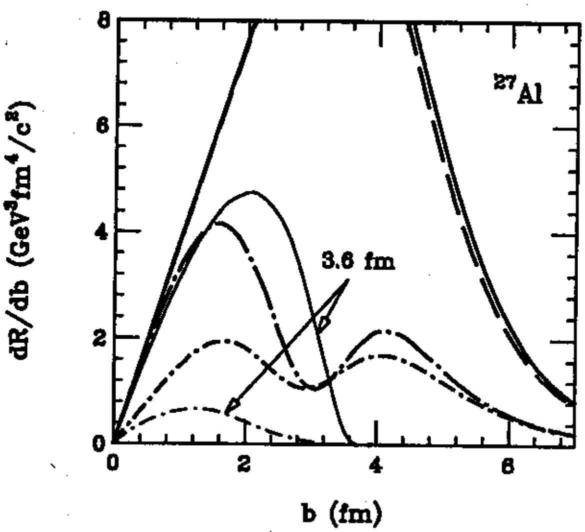


Figure 5. Same as Fig. 4 but for the ^{27}Al target. The results marked 3.6 fm are obtained when restricting the impact parameters.

a value of B =0.018 GeV^3, and when restricting the impact parameters we obtain B = 0.005 GeV^3.

Concerning the above results one might claim that a value of R_o should have be differently chosen. However, one should keep in mind that the relative importance of various types of interactions cannot be changed. Let us note, in passing, that the data may be easiest to parametrize in terms of the size of emission region just by using the assumption of local (collisional) balance for a region of momentum space

$$f_d = f_p f_n \ . \qquad (20)$$

Here f are Wigner functions, so that the functions F in (16) are F = $gf/(2\pi)^3$, where g is spin degeneracy factor. Nominally Eq. (20) corresponds to T = 1 for particles in (18), but it goes beyond the adiabatic approximation and nucleons are not primordial. With (20) we get for the proportionality factor a standard formula,

$$B = (3/4)\ (2\pi)^3/V \ , \qquad (21)$$

yielding R_o = 2.8 fm for ^{9}Be data, and R_o = 3.5 fm for ^{27}Al data.

Before moving on to the transport model let us discuss what we expect for lower energies. Lowering of the energy makes the collisions less essential, because of the Pauli principle. The importance of the mean-field potential should increase. Surprisingly, however it may be, in the calculations [19] of the interaction of 25.5 MeV deuterons with nuclei, the elastic breakup cross section is estimated to constitute only from 8% to 11% of total reaction cross section for nuclei ranging from ^{27}Al to ^{181}Ta. In a later calculation Baur *et al.* [15] find an elastic breakup cross section for the niobium target and the same energy, that is ~ 22% of the reaction cross section. The elastic breakup cross seems, though, not to depend strongly on the target [19]. So, extrapolated to lighter nuclei such as ^{12}C, it may be a significant part of the reaction cross section. A pickup process involves only a mean-field potential as no additional nucleon is knocked out. Examining the pickup cross sections on carbon nuclei in the literature, and comparing them with cross sections estimated using the transport model to be described in next Section, we convinced ourselves that the mean-field effects can indeed be important for deuterons emitted with energy up to 30 MeV, when the nuclei are small. In the process of fragmentation, when matter possibly reaches some instability, the mean-field effects may be of a major importance. In general, the potential at lower energies may be important just modifing the breakup or formation process.

5. TRANSPORT MODEL WITH COMPOSITE PARTICLE PRODUCTION

We now briefly discuss a model of heavy-collisions that incorporates deuteron production [9]. We will also show preliminary results with the production of tritons and helions.

In the model [9], the deuteron distribution function in space and momentum is a dynamic quantity, as are nucleon distribution functions. The deuterons are produced in the interaction of three nucleons, with the deuteron formation rate per unit time, momentum space, and volume, given in terms of Wigner functions by

$$\mathcal{K}^{>} = \sum_{N=n,p} \frac{4}{3}\int d^3p\ d^3p_1\ d^3p_2\ d^3p'\ |\mathcal{M}^{pnN\to d}|^2\ \delta(\mathbf{P} + \mathbf{p} - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}')$$
$$\times\ \delta(E + e - e_1 - e_2 - e')\ f_p f_n f_N\ . \qquad (22)$$

We suppress here factors of 2π, etc., that may be found in Ref. [9]. The breakup rate is given by an expression analogous to (22), and the matrix elements for the formation and breakup are related by

$$|\mathcal{M}^{ppn\to pd}|^2 = \frac{3}{4}\ |\mathcal{M}^{pd\to ppn}|^2\ . \qquad (23)$$

The matrix element for the breakup is parametrized in terms of NN cross section and deuteron wavefunction, here in momentum space, as

$$|\mathcal{M}^{pd\to ppn}|^2 = \mathcal{F}\ \frac{4\pi^2}{m^2}\ \sigma_{NN}\ |\psi_d|^2 + \dots\ . \qquad (24)$$

The dots indicate terms that make the expression symmetric in the variables. The cross section for deuteron breakup can be simply expressed in terms of the breakup element. The normalization factor $\mathcal{F}$ is adjusted to reproduce the total breakup cross section. At high energies $\mathcal{F}$ is close to 1 indicating that the nucleon breaking the deuteron or catalyzing the formation in the inverse channel, indeed interacts separately with constituent nucleons. However, at lower energies $\mathcal{F}$ is lower than 1. This can either indicate that the above assumption is not so well satisfied and/or the effects of momentum spread in deuteron on cross section are large. The production is suppressed in the transport model, in the regions of high phase-space density. The produced deuterons are assumed to move in mean-field potential with strength twice as large as that of nucleons. The A = 3 clusters are produced in the model through interactions that involve 4 nucleons.

The spectra calculated within the model are compared with the data in Figs. 6 and 7. The description is better at higher energies than at lower. In Fig. 7 we include the results obtained for ^{3}He and it is seen that we have problems describing the data in the fragmentation region. The model gives a fair description of the wide-angle proton and deuteron yields at high energies [9]. The yield ratios have been used in the past to extract the amount of entropy produced in collisions. Figure 8 compares the entropy actually produced in the reaction process to the entropy obtained from the d/p ratios. They agree to within 0.25 per nucleon at this bombarding energy. The Plastic Ball group has observed [21] relatively large triton yields in the reactions at intermediate and high energies. Our yields are compared with theirs in Fig. 9, and it seen that we can reproduce the d/p and ^{3}He/p ratios, but not t/p. Unlike

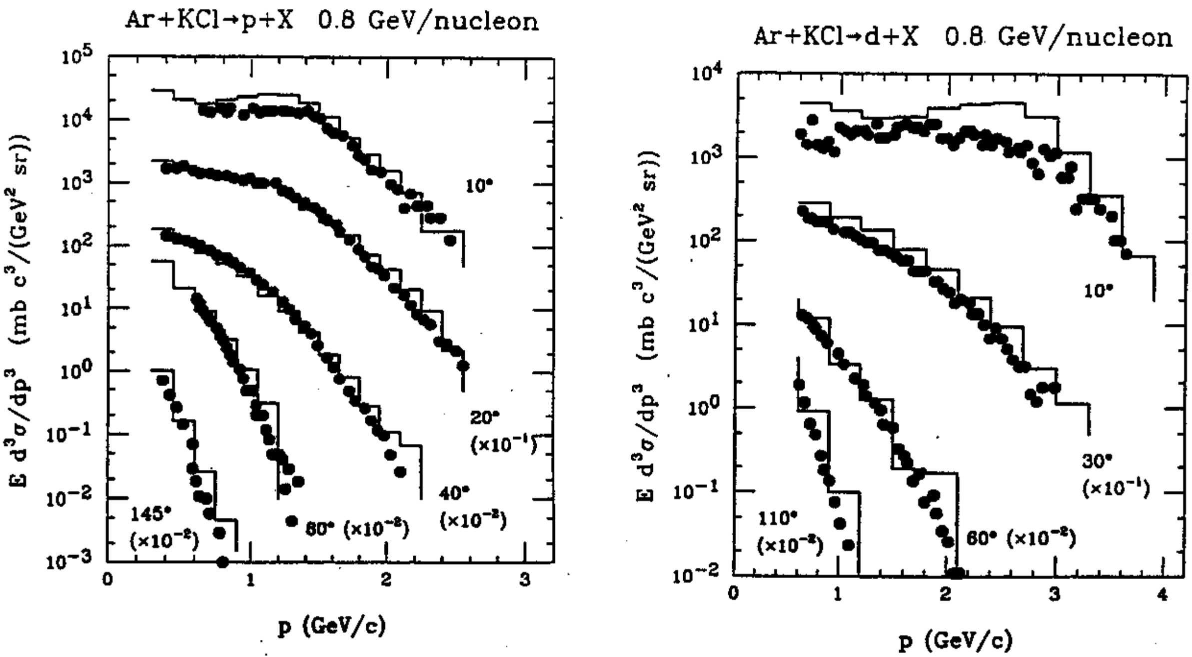


Figure 6. Proton and deuteron spectra from 800 MeV/nucleon Ar+KCl reaction. The data from Ref. [1] are indicated by dots and the results of the calculations by histograms.

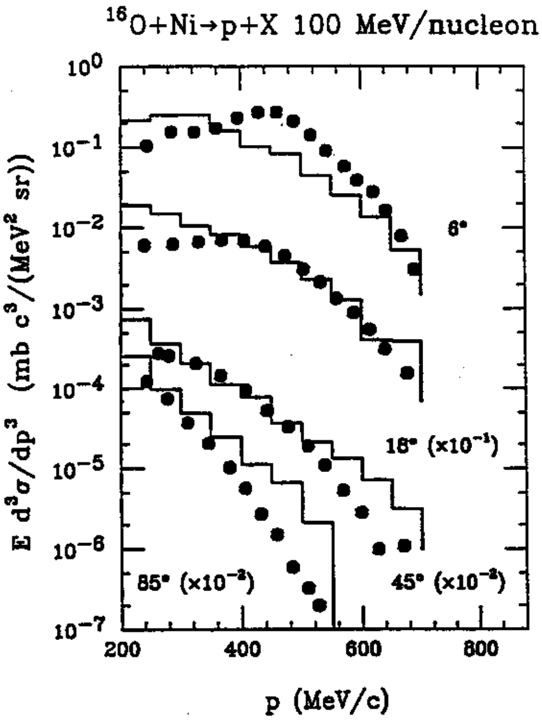
the measurements, our t/^{3}He ratios are roughly consistent with N/Z for the system. Comparison with other results of the Plastic Ball group for composite particles is forthcoming.

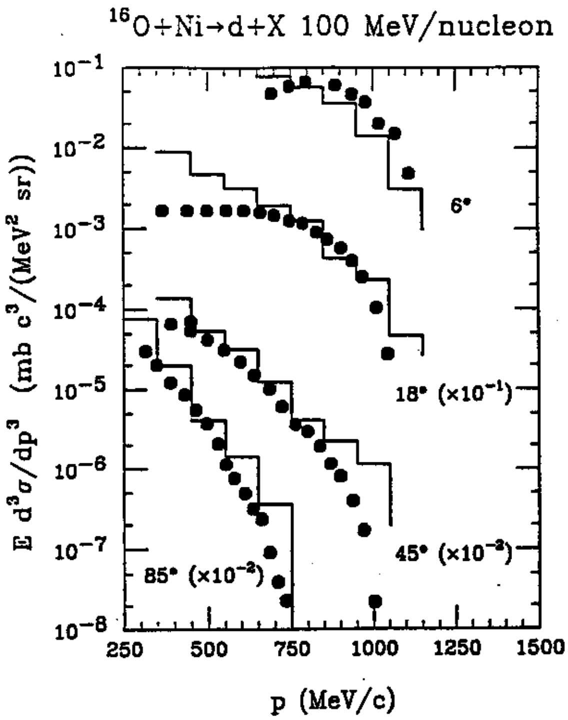
Composite particles are generally expected to be more sensitive to collective flow than are nucleons. Figure 10 displays the measured and calculated dispersions of the azimuthal angle with respect to the reaction plane, of particles emitted from ^{36}Ar + ^{197}Au reaction at 35 MeV/nucleon, as obtained by Tsang *et al.* Both calculations and data show, indeed, that deuterons distributions have stronger anisotropies associated with the reaction plane than proton distributions. With the changing centrality of reaction and polar angle the calculated and measured dispersions are observed to follow similar changes.

6. CONCLUSIONS

We have demonstrated that NN collisions are most essential in the formation of composites at high energy. The deuteron formation in the Butler Pearson model is physically an inverse process to elastic breakup. The process is *relatively* unimportant, and the agreement of the model with various data is purely coincidental.

We have formulated a transport model for heavy ion collisions with light composite production. The model gives fair description of yields and spectra at higher energies. For A = 3 fragments the isospin asymmetry is much weaker than exhibited by Plastic Ball data.





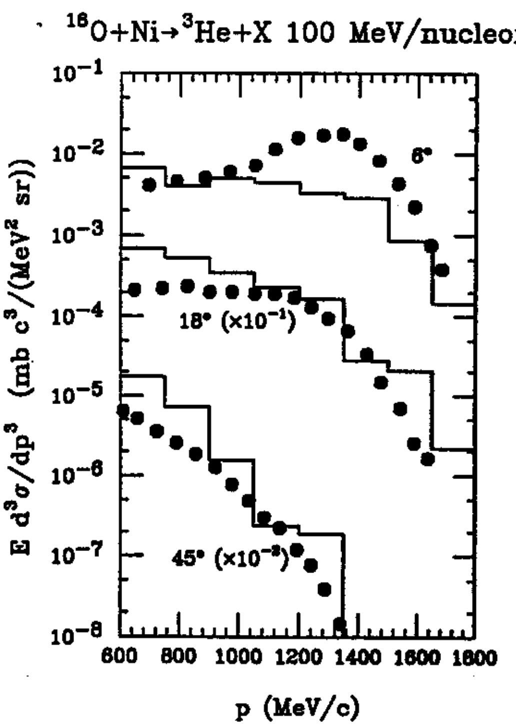


Figure 7. Proton, deuteron and helion spectra from ^{16}O+Ni reaction. Data from Ref. [20] are indicated by dots and the results of the calculations are given by histograms.

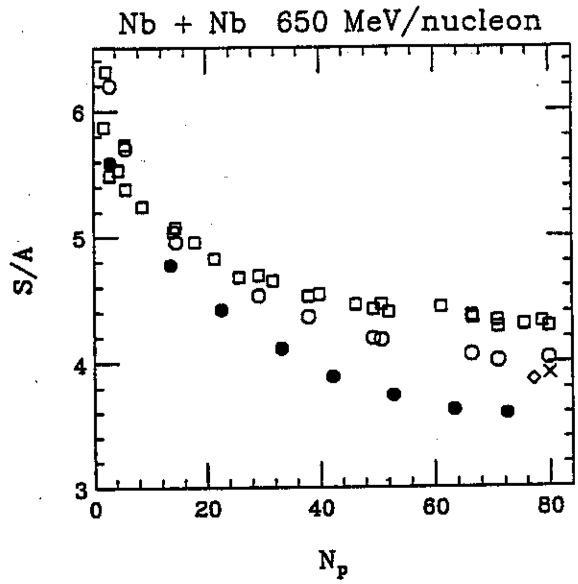


Figure 8. Entropy per nucleon produced in Nb+Nb collisions at 650 MeV/nucleon as a function of participant proton multiplicity. Open circles indicate the values calculated using actual Wigner functions. Open squares indicate estimates obtained from d/p ratios under assumption of equilibrium. Filled circles indicate the entropy extracted from the data using QSM model.

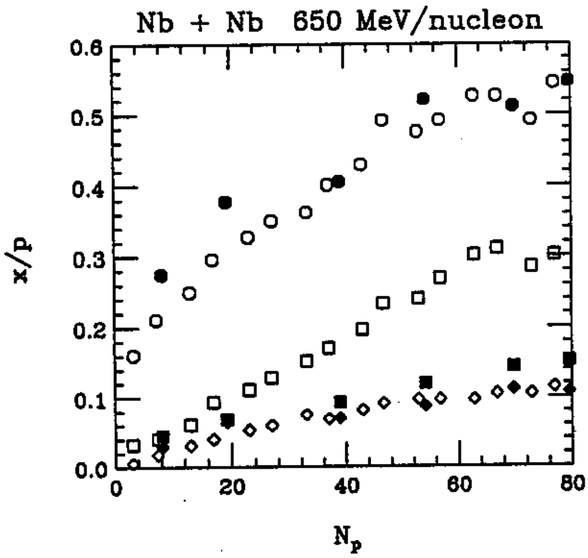


Figure 9. Ratios of composite yields to proton yields x/p as a function of the participant proton multiplicity N_p. The circles, squares and diamonds indicate, respectively, the d/p, t/p, and ^{3}He/p ratios. The measured values are shown with open symbols, and the calculated ones with the filled symbols.

The author has benefited from extensive discussions with Carlos Bertulani. The changes to the text suggested by Hartmut Schulz are greatly appreciated. This work was supported by the National Science Foundation under Grant No. PHY-9017077.

7. REFERENCES

1 S. Nagamiya *et al.*, Phys. Rev. C24 (1981) 971.
2 L. P. Csernai and J. I. Kapusta, Phys. Reports 131 (1986) 223.
3 S. T. Butler and C. A. Pearson, Phys. Rev. 129 (1963) 836.
4 A. J. Borisenko *et al.*, Sov. J. Nucl. Phys. 29 (1979) 800.
5 P. A. Piroue and A. J. Smith, Phys. Rev. 148 (1966) 1315.
6 P. Ciok, Nucl. Phys. A340 (1980) 445.
7 M. Gyulassy *et al.*, Nucl. Phys. A402 (1983) 596.
8 J. Aichelin and E. A. Remler, Phys. Rev. C35 (1987) 1291.
9 P. Danielewicz and G. F. Bertsch, Nucl. Phys. A533 (1991) 712.
10 W. W. Daehnick *et al.*, Phys. Rev. C21 (1980) 2253.

11 A. Budzanowski *et al.*, Phys. Rev. C17 (1978) 951.
12 F. D. Becchetti, Jr., and G. W. Greenlees, Phys. Rev. 182 (1969) 1190.
13 N. Austern *et al.*, Phys. Rev. C18 (1978) 1577.
14 R. J. Glauber, High Energy Collision Theory, in: Lectures in Theoretical Physics, v. 1, Interscience, New York, p. 315.
15 G. Baur *et al.*, Phys. Rev. C21 (1980) 2668.
16 A. I. Akhiezer and A. G. Sitenko, Phys. Rev. 106 (1957) 1236.
17 P. Danielewicz and P. Schuck, Phys. Lett. 274 (1992) 268.
18 V. L. Fitch *et al.*, Phys. Rev. 126 (1962) 1849.
19 J. Pampus *et al.*, Nucl. Phys. A311 (1978) 141.
20 R. L. Auble *et al.*, Phys. Rev. C28 (1983) 1552.
21 K. G. R. Doss *et al.*, Phys. Rev. C37 (1988) 163.
22 B. Tsang *et al.*, in preparation.

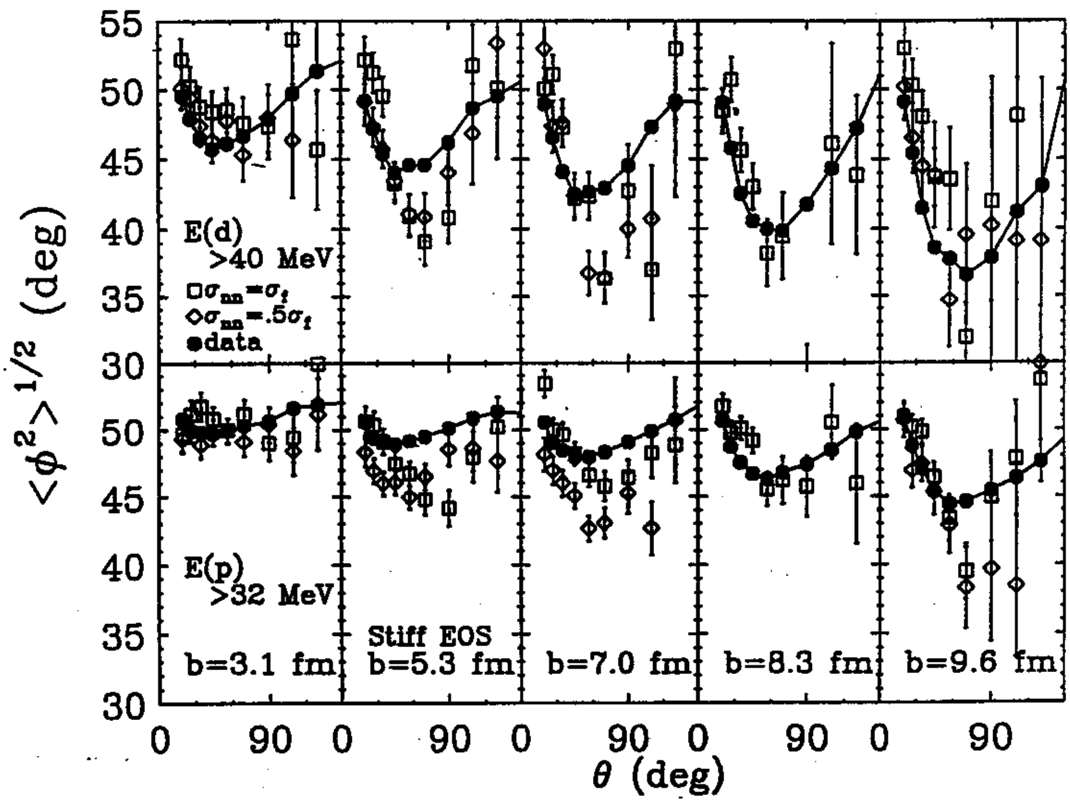


Figure 10. Angular dependence of azimuthal variances with respect to the true reaction plane for protons (lower part) and deuterons (upper part). Individual panels depict different cuts on estimated impact parameter. Filled circles: measured variances; open symbols: impact parameter dependence predicted by transport calculations using a stiff equation of state and $\sigma_{NN} = \sigma_{NN}^{free}$ (squares) and $\sigma_{NN} = 0.5\sigma_{NN}^{free}$.