Michigan State University

National Superconducting Cyclotron Laboratory

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C.A. BERTULANI and H. SAGAWA

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C.A. Bertulani and H. Sagawa*

Cyclotron Laboratory, MSU, East Lansing, MI 48824-1321, USA
*) Department of Physics, Faculty of Science, University of Tokyo
Hongo 7-3-1, Bunkyo-ku, Tokyo 113, Japan

ABSTRACT

We study the differential cross sections of the soft multipole excitations of ^{11}Li with C-target by using a reaction model in which all input parameters are determined from other sets of experimental data. The results describe properly the differential cross sections of giant resonances in ^{208}Pb excited by a-projectile at a medium reaction energy E_{lab}=43 MeV/u without introducing any free parameters. It is predicted that the soft multipole states have the cross sections of the order of 100 mb/sr at forward angles using the medium energy radioactive beams at $E_{lab}=(30\simeq 70)$MeV/u , and could be feasible to distinguish each other experimentally by the angular dependence at very forward angle.

It is now one of the main issues of nuclear physics to study the excitation spectra of halo nuclei ,i.e., ^{11}Li, ^{11}Be and ^{14}Be[1, 2, 3]. The halo nuclei are characterized by extremely large radii compared with neighboring isotopes: the sizes of halo nuclei are substantially larger than that given by a standard formula $R=1.2A^{1/3}$ fm[4]. It is also pointed out that the loosely-bound nature of the neutrons is responsible to induce the large radii of these halo nuclei[5, 6].

A unique feature of the halo nuclei manifests itself in the excitation spectra at very low energy. Hansen and Jonson[5] predicted the existence of soft dipole mode in a halo nucleus ^{11}Li which is characterized by an extremely low excitation energy and substantially large dipole transition strength although it is described as a naive particle-hole excitation in terms of the shell model[7]. It was also pointed out[8, 9] that possible soft multipole excitations other than the dipole exist at the very low excitation energy $E_x=(1\sim2)$ MeV exhausting significant portion of the energy-weighted sum rule transition strength (EWSR). The mechanism which leads to the large transition strength is due to a strong threshold effect of the halo nuclei[10].

During last few years, several experimental efforts have been successfully made to obtain evidence of the soft dipole mode by using the pion double-charge exchange reactions $^{11}B(\pi^-,\pi^+)^{11}Li$[1], and also the triple coincidence experiments of the break-up reactions $^{11}Li^* \rightarrow {}^{9}Li + n + n$[2, 3]. The transition strength of the soft dipole state is found by these experiments at a very low excitation energy $E_x \simeq 1$ MeV with extremely large transition strength $B(E1) \simeq 1\ e^2fm^2$, despite the fact that the low-energy isoscalar dipole excitation is forbidden. Results of microscopic calculations by the random phase approximation (RPA)[6, 9] and also by the two-body Green's function method[11] are consistent with the experimental finding of the soft dipole state in ^{11}Li.

The RPA calculations show the existence of not only the soft dipole but also other excitation mode with the multipoles L=0 or 2 at the same energy region $E_x \simeq (1 \sim 2)$ MeV. It is obvious that the Coulomb excitation is dominated by the dipole transitions compared with any other multipoles when the transition strength is close to the single particle unit [12]. As is known, the monopole transition is forbidden in the Coulomb excitation. We need hadronic probes to obtain empirical information of the soft multipoles other than the dipole. An important difference between the Coulomb and the hadronic probes is that, being of long range, the Coulomb field measures the nuclear transition density as a whole, i.e., what enters the Coulomb excitation cross sections are the transition densities integrated over the coordinate space with a slowly varying weighting function, namely $r^\lambda Y_{\lambda\mu}(\theta, \phi)$. On the other hand, the hadronic probes are more sensitive to the spatial distribution of the transition densities, especially at the surface of the nucleus. Therefore, hadronic probes would be relevant to test more closely the nuclear structure model of halo nuclei. Smaller isoscalar targets like α-nucleus or ^{12}C have an advantage to excite the multipole excitations excluding the dipole since the hadronic interactions will overcome the Coulomb force. In this paper, thus, we study the reaction of ^{12}C + ^{11}Li at medium energy for the soft multipole excitations.

The DWBA is an appropriate tool for the investigation of inelastic scattering of hadronic probes. Two different approaches have been used in these calculations. One approach is the deformed potential model [13] in which one assumes that the transition density of the excited nucleus is strongly peaked at the nuclear surface so that the full transition strength is probed in a peripheral collision. Although this assumption is reasonable for the excitation of heavy nuclei (e.g. ^{40}Ca, ^{208}Pb, etc.), it is rather crude for light nuclei, especially when the transition densities extend in a large radial space more than the nuclear size. This is the case for the soft multipole excitations since the transition densities have very long tail as is shown in Fig. 1. In these cases, the folding model for the reaction mechanism is more appropriate. One advantage of this approach is that the potentials which generate the distorted waves and the interaction potential are constructed by using the same microscopic nucleon-nucleus potential. For the scattering phase, we adopt the Eikonal approximation which is well justified at intermediate energy collisions, and renders the calculations simple and transparent.

The model that we describe here can be directly associated with the transition densities which are obtained by structure calculations like shell model or RPA. More-over it could have a predicting power for the differential cross sections of strongly excited states like giant resonances since ingredients of the calculations are all determined by other sets of experimental data at the same projectile energy. The Eikonal approximation is well justified for the description of the cross sections at forward angles, large beam energies and small energy transfers. This is sufficient for the present purpose since the peculiar features of the soft multipole excitations will appear under these conditions. Our model is first applied to the inelastic scattering of ^{208}Pb with a medium energy α-projectile to test whether the absolute magnitude and the angular distributions of the cross sections of giant resonances are properly described or not.

The ideal hadronic probe for our purpose is α or ^{12}C targets for radioactive beams. For such nuclei the nucleon-nucleus potential could be given by a gaussian form,

$$U_{NA}(r) = -(v_0 + w_0)\, e^{-r^2/a^2}\,, \tag{1}$$

where the potential depth v_0 and w_0 are isospin averaged values. The transition matrix element for the excitation of the nucleus A in peripheral collisions with another nucleus a is given by

$$\begin{aligned} T_{if} &= \int d^3R \int \Pi_j d^3r_j\, \Psi_{aA}^{(-)*}(\mathbf{R})\, \phi_f^*(\mathbf{r_1}, ..., \mathbf{r_A}) \\ &\times \sum_j U_{NA}\Big(|\mathbf{R} - \mathbf{r}_j|\Big)\; \Psi_{aA}^{(+)}(\mathbf{R})\, \phi_i(\mathbf{r_1}, ..., \mathbf{r_A}) \\ &= \int d^3R \int d^3r\, \Psi_{aA}^{(-)*}(\mathbf{R})\, \Psi_{aA}^{(+)}(\mathbf{R})\, U_{NA}\Big(|\mathbf{R} - \mathbf{r}|\Big)\; \delta\rho(\mathbf{r})\,, \end{aligned} \tag{2}$$

where $\mathbf{R}$ is the relative coordinate between the nuclei and $\mathbf{r}_j$ is the internal coordinate of the j-th nucleon in the nucleus A. The scattering and the intrinsic wave functions are denoted as $\Psi_{aA}(\mathbf{R})$ and $\phi(\mathbf{r_1}, ..., \mathbf{r_A})$, respectively. The transition density $\delta\rho(\mathbf{r})$ is defined by

$$\delta\rho(\mathbf{r}) \equiv \int \Pi_j d^3r_j \phi_f^*(\mathbf{r_1}, ..., \mathbf{r_A}) \sum_j \delta(\mathbf{r} - \mathbf{r_j}) \phi_f(\mathbf{r_1}, ..., \mathbf{r_A})\,. \tag{3}$$

Using the eikonal approximation for the scattering waves $\Psi^{(\pm)}$ and expanding $U_{NA}(|\mathbf{R} - \mathbf{r}|)$ and $\delta\rho(\mathbf{r})$ into multipoles, we find the amplitude to excite the λ multipole mode in the nucleus A as

$$T_{\lambda\mu} = 8\pi^{5/2}\, a\,(v_0+w_0)\, i^{\lambda+\mu}\, Y_{\lambda\mu}\Big(\theta=\frac{\pi}{2},0\Big)\int db\, b\, J_\mu\Big[2\sqrt{kk'}\, b\,\sin(\frac{\theta}{2})\Big]\, e^{i\chi(b)}\, \mathcal{O}_\lambda(b)\,, \tag{4}$$

where

$$\mathcal{O}_\lambda(b) \;=\; e^{-b^2/a^2}\int dr\, r^2\,\delta\rho_\lambda(r)\, e^{-r^2/a^2}\, j_\lambda\Big(i\frac{2rb}{a^2}\Big)\,, \tag{5}$$

and the Eikonal phase $\chi(b)$ is

$$\chi(b) = \frac{1}{\hbar v}\int_{-\infty}^{\infty} dz\, U_{opt}\Big(\sqrt{b^2+z^2}\Big) + 2\frac{Z_a Z_A e^2}{\hbar v}\,\ln(kb)\,. \tag{6}$$

In the equations above, $J_\mu(x)$ is the Bessel function of integer order, $j_\lambda(ix)$ is the spherical Bessel function of imaginary argument, and the optical potential U_{opt} is constructed by folding the nucleon-nucleus (N+a) potential with the nuclear density of A. A detailed derivation of the above calculations will be presented elsewhere. The differential cross section for the excitation to a given multipole is given by

$$\Big(\frac{d\sigma}{d\Omega}\Big)_\lambda = \Big(\frac{\mu_{aA}}{2\pi\hbar^2}\Big)^2 \sum_\mu |T_{\lambda\mu}|^2\,. \tag{7}$$

where μ_{aA} is the reduced mass.

Fig. 2 shows the differential cross sections of monopole and quadrupole giant resonances in ^{208}Pb excited by $\alpha-$projectile at E_{lab}= 172 MeV. Experimental data and parameters for the alpha-nucleon potential are taken from ref. [15]. The transition densities of both states are the Tassie ones calculated by using the H-F density of ^{208}Pb exhausting 100% of the energy-weighted sum rule values. It is remarkable that both the angular distributions and the absolute magnitudes of the cross sections at forward angles $\Theta_{cm} \leq 15^o$ are well described by using the established optical potentials for the nucleon-nucleus scattering. We show also the cross sections of the isovector dipole state in the lower part of Fig. 2. The cross sections for the excitation of isovector dipole mode are obtained by using eq. (4) with an isovector potential constructed by the difference between the neutron-alpha and the proton-alpha potentials [13], with parameters taken from ref. [16]. Because of isoscalar nature of the α-projectile, the cross section of the dipole state is much hindered compared with those of the monopole and the quadrupole states. This effect is known as an advantage of the isoscalar projectile to excite the isoscalar giant resonances excluding the isovector ones.

Our reaction model is applied to the soft multipole excitations of ^{11}Li with ^{12}C-target. The transition densities of the soft multipole excitations are calculated by using the self-consistent H-F + RPA method [18]. The dominant peaks of all multipoles appears at E_x=1~1.5MeV having narrow widths less that $\Gamma_{FWHM} \leq 1$ MeV. The transition strength for the each soft mode is calculated to be B(E0)= 61.4 $e^2 fm^4$, B(E1)= 0.82 $e^2 fm^2$ and B(E2)= 31.5 $e^2 fm^4$, respectively, exhausting 11 %, 2 % and 7 % of the EWSR values. Although the portion to the EWSR is small, the transition strengths of the soft multipoles are larger than those of the giant resonances in the same nucleus because of the very low excitation energies of the soft multipoles. In the dipole case, the calculated value is very close to the empirical value which is recently obtained by the triple coincidence experiments [3].

The differential cross sections are shown in Fig. 3 for the reaction of ^{11}Li on ^{12}C at the projectile energy E_{lab}=330 MeV (solid lines) and E_{lab}=660 MeV (dashed lines). The proton- and neutron-carbon potential parameters at these energies are taken from ref. [16]. There are substantial differences between the monopole and the quadrupole excitations at various aspects. The first crucial point is a steep slope of the monopole cross section at very forward angle $\Theta \sim 0^o$ than that of the quadrupole state; the difference is clearly seen in the ratio between the first and the second peaks of the cross sections which is almost 1 for L=2 while that is more than one order of magnitude for L=0. This difference is originated to the fact that three Legendre polynomials with different μ contribute to the cross section in the L=2 case as is seen in eq. (4), while only one Legendre polynomial appears in the L=0 case. This could be the key issue to distinguish the monopole excitation from the quadrupole in the experimental cross sections. The second point is the dips of the cross sections. The first deep minimum is found at $\Theta \simeq 1.6\ (1.0)\ ^o$ for the L=0 case, while the shallow one appears at $\Theta \simeq 1.0\ (0.7)\ ^{o}$ for the L=2 case at the projectile energy E_{lab} =330 (660) MeV. These differences was certainly an important clue to find the giant monopole states in many heavy nuclei[15]. For E_{lab}=660 MeV the dips of all multipole excitations occur in smaller intervals because of the larger wave numbers, but the magnitude of the cross sections do not change appreciably. This is because the nucleon-carbon potential are not much different at these two bombarding energies [16].

The average slope of the differential cross section is essentially governed by the surface diffuseness of the nuclei involved in the rection. It is interesting to compare the results in Figs. 2 and 3. Since the surface is sharp in ^{208}Pb, the slope decreases very slowly in the case of ^{208}Pb + α, while it drops quickly in the ^{12}C + ^{11}Li case because of a very loose surface of ^{11}Li. This behavior could be common in the reaction cross section with any halo nucleus[17]. It should be noticed that the absolute magnitude of the differential cross section in Fig. 3 is the order of 100 mb/sr for the monopole and quadrupole excitations which is almost the same as the observed magnitude of Pb + α reaction in Fig. 2. It is also seen that the soft dipole state has a less cross section and a different angular dependence than those of the other two multipoles. Although a secondary beam has always lower intensity than ordinary beams, the soft multipole excitations could be tested experimentally with modern high sensitive detector systems.

In conclusion, we studied the differential cross sections of the soft multipole excitations in ^{11}Li with ^{12}C target at the medium reaction energies by using the reaction model based on Eikonal approximation. Our model is proved to describe well the differential cross sections of the monopole and quadrupole giant resonances in ^{208}Pb excited by α-projectile with E_{lab}= 172 MeV, qualitatively and quantitatively at the forward angles $\Theta_{cm} \leq 15^{o}$. One of the advantages of our model is that the ingredients of the calculations are physically tested by other experiments. Thus, we have no adjustable parameters in the calculations which is important to make a prediction for future experiment. The inelastic differential cross sections of ^{11}Li with ^{12}C-target are studied by using the microscopic transition densities calculated by the H-F+RPA model. We find that there are substantial differences among the multipole excitations with L=0,1 and 2 in the angular distributions especially at forward angles which could be experimentally distinguished. The typical cross sections at forward angles are found to be 100 mb/sr for the monopole and the quadrupole which is almost the same as the observed cross sections of the giant resonances in ^{208}Pb with α-projectile.

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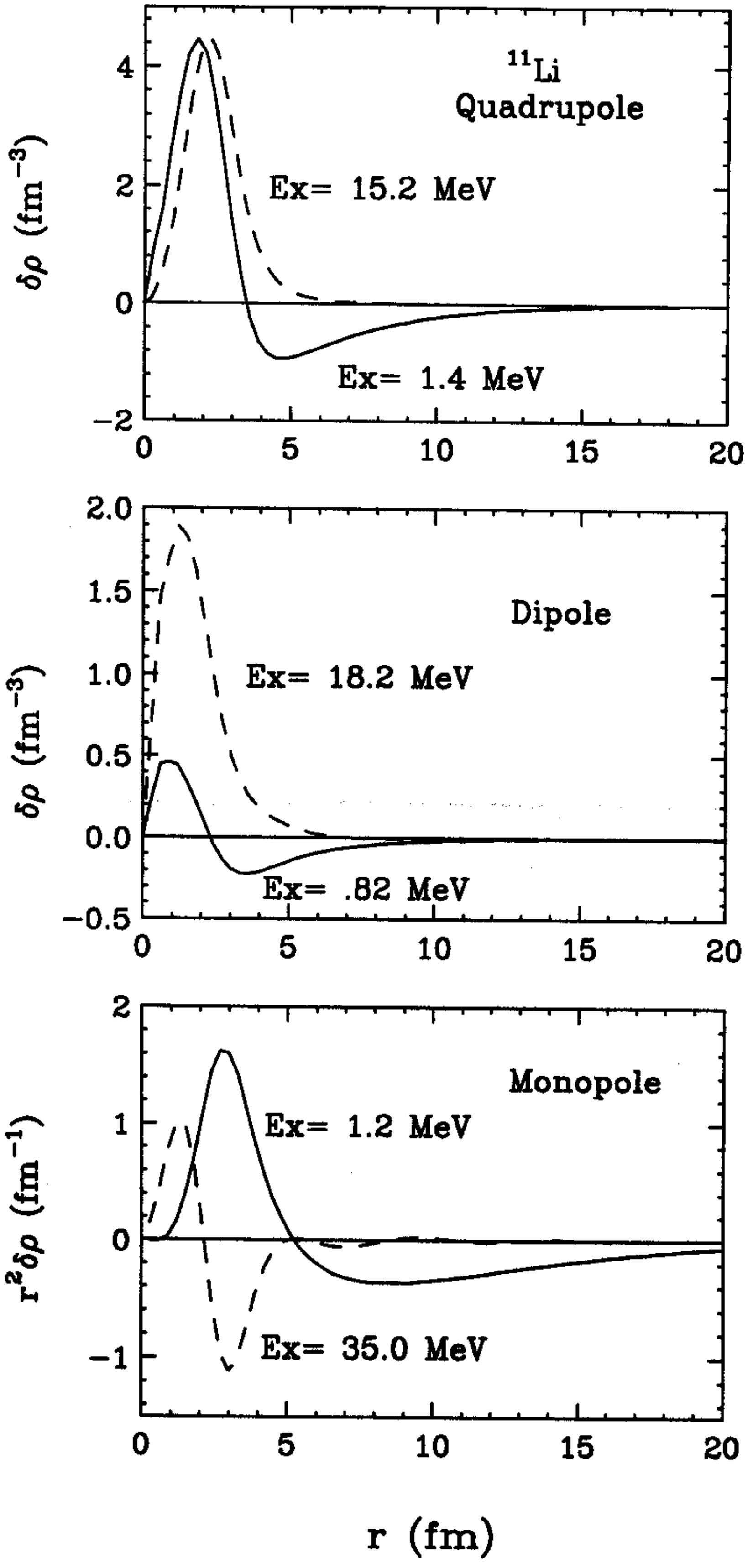
Figure Captions

Fig. 1. RPA transition densities of soft multipole states (solid curves) and giant resonances (dashed curves) in ^{11}Li. The ground state of ^{11}Li is obtained by a filling approximation of H-F single-particle orbits from the bottom of the potential in order. The spin-orbit coupling parameter W_0 of the Skyrme interaction SGII[14] is modified to be W_0 = 160 MeV.fm^5 in order to simulate loosely-bound halo neutrons in $1p_{1/2}$-orbit.

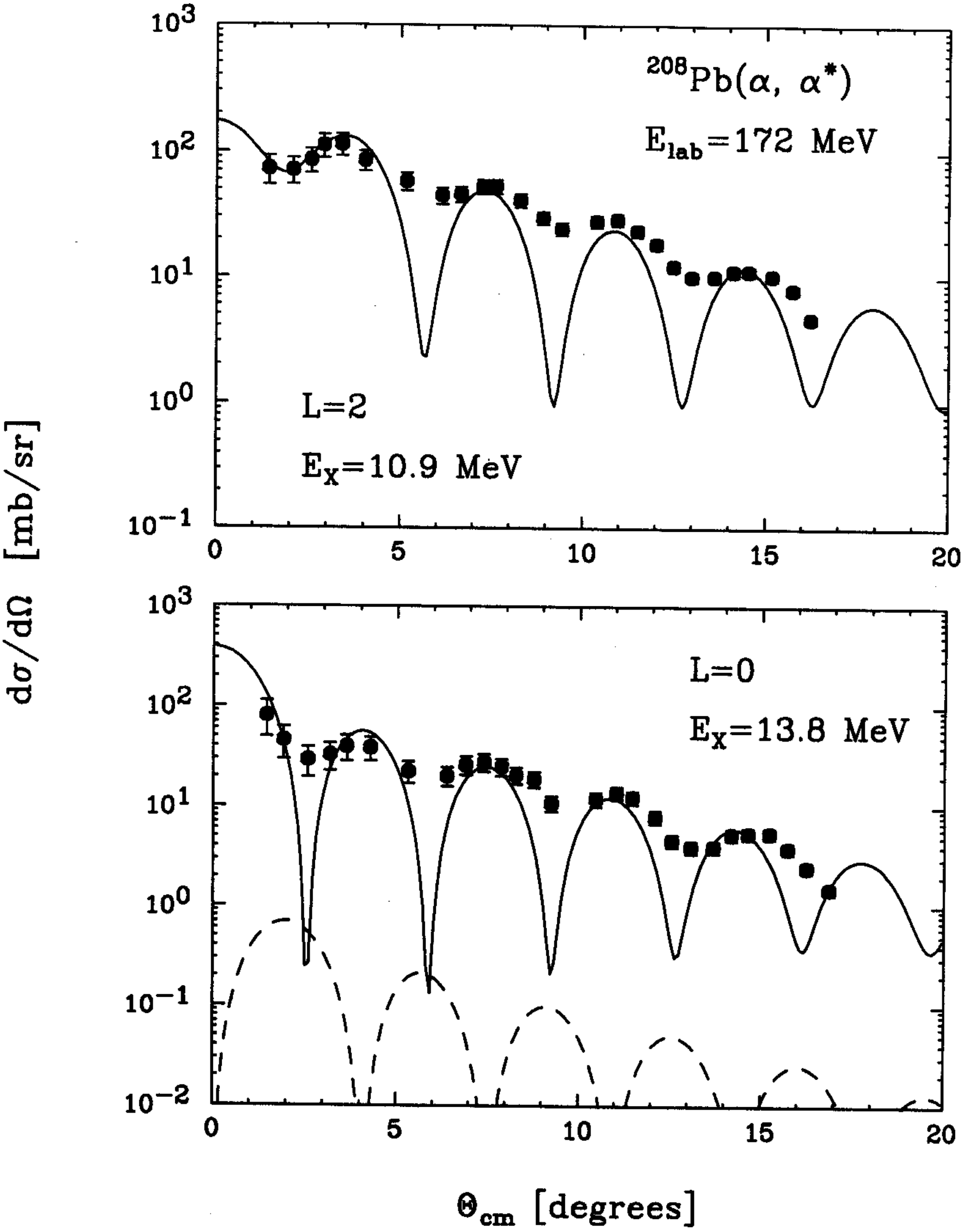
Fig. 2. Differential cross sections for giant resonances in ^{208}Pb excited by α-projectile at E_{lab}=172 MeV. The calculated cross sections are shown by the solid curves for L=0 and L=2 cases while the dashed curve in the lower part of the figure shows that of the giant dipole state. Experimental data are taken from ref. [15]).

Fig. 3. Differential cross sections of soft multipole states in ^{11}Li excited by the ^{12}C target at E_{lab}=330 MeV (solid curves) and at E_{lab}=660 MeV (dashed curves)



^{11}Li
Quadrupole
Ex= 15.2 MeV
Ex= 1.4 MeV
$\delta\rho$ (fm^{-3})
4
2
0
−2
0
5
10
15
20
Dipole
Ex= 18.2 MeV
Ex= .82 MeV
$\delta\rho$ (fm^{-3})
2.0
1.5
1.0
0.5
0.0
−0.5
Monopole
Ex= 1.2 MeV
Ex= 35.0 MeV
$r^2\delta\rho$ (fm^{-1})
2
1
0
−1
r (fm)

Figure 1



^{208}Pb(α, α*)
E_{lab}=172 MeV
L=2
E_X=10.9 MeV
L=0
E_X=13.8 MeV
dσ/dΩ [mb/sr]
Θ_{cm} [degrees]
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
0
5
10
15
20

Figure 2

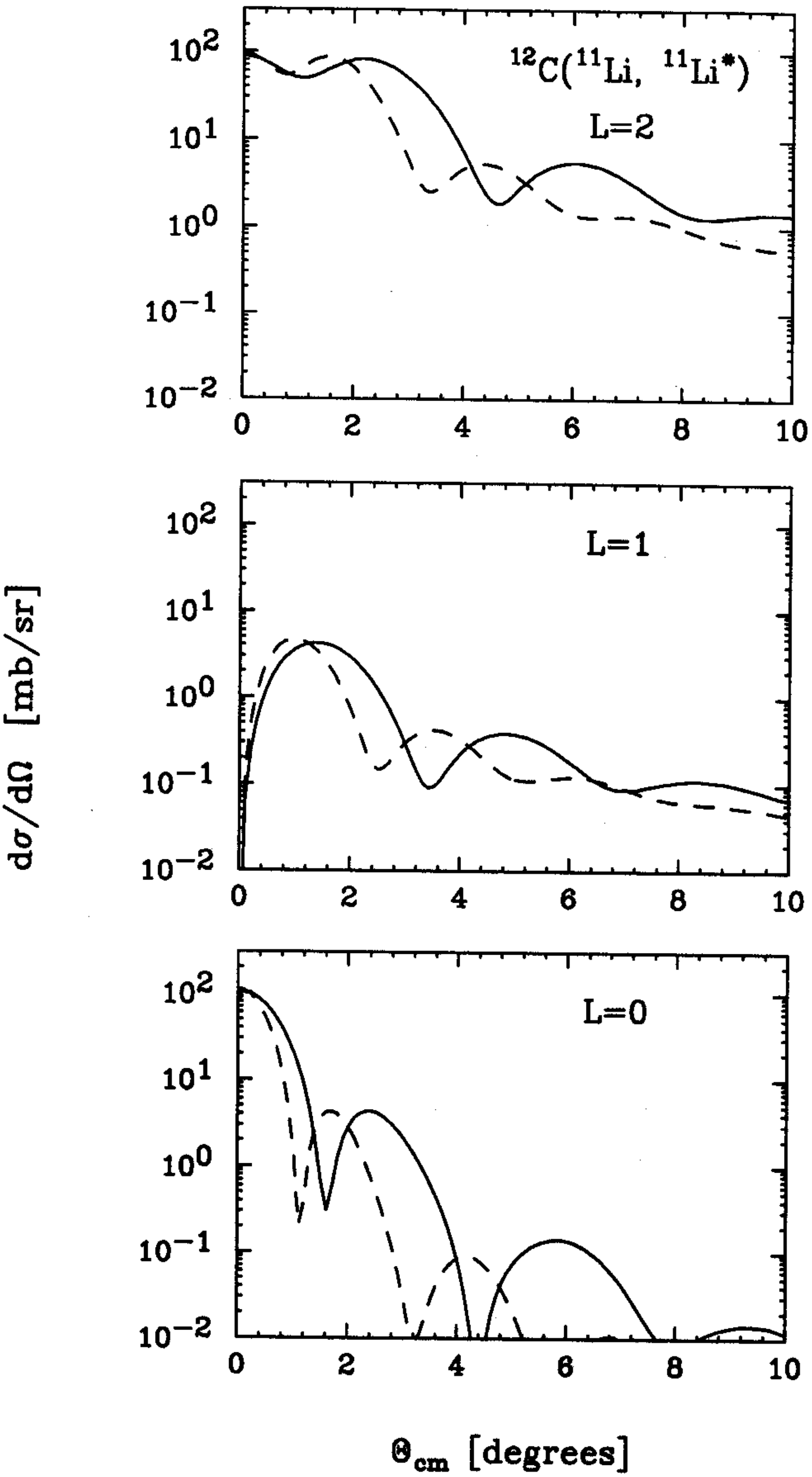

$^{12}C(^{11}Li,\ ^{11}Li^{*})$
L=2
L=1
L=0
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
0
2
4
6
8
10
$d\sigma/d\Omega$ [mb/sr]
Θ_{cm} [degrees]

Figure 3