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CLASSICAL PHASE SPACE STRUCTURE INDUCED BY SPONTANEOUS SYMMETRY BREAKING

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Abstract: The symmetry breaking ground states are naturally joined within a local phase space structure distinguished by the low energy quantum dynamics. This structure is constructed explicitly for the case of the nuclear deformed ground states and a single angle coordinate. As application, is presented a semiclassical treatment of the nuclear isovector angle vibrations which lead to a boson-like excitation operator.

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The study of the phase portrait in Hilbert space for Hamiltonian quantum dynamical systems is faced with considerable difficulties, and initially the attention was concentrated on the stationary states. However, for the realistic many body systems encountered in chemistry, nuclear or condensed matter physics even the ground state cannot be found exactly, and approximate solutions are obtained constraining the quantum dynamics from the Hilbert space $\mathcal{H}$ to some finite dimensional trial manifold M. This is usually a symplectic manifold (M,ω^M), with ω^M defined naturally restricting from $\mathcal{H}$ to M the symplectic form $\omega^{\mathcal{H}}\in\Omega^2(\mathcal{H})$, $\omega^{\mathcal{H}}_\psi(X,Y)= 2\mathrm{Im}\langle X_\psi|Y_\psi\rangle$, $X_\psi,Y_\psi\in T_\psi\mathcal{H}$,[1]. If the constrained dynamics has an unique critical point of minimum energy, than it is invariant to the symmetry group R of the Hamiltonian operator $\hat{H}$ and it approximates the ground state.But often this is not the case, because the variational equation $\delta h_M=0$, $h_{M(Z)}=\langle Z|\hat{H}|Z\rangle$, $|Z\rangle\in M$, may have a solution $|g\rangle$ which is not invariant to the action of R. As important examples may be mentioned the deformed nuclear ground states obtained by Hartree-Fock calculations, or the BCS superfluid states. These are not invariant to the action of the rotation group $SO(3,\mathbb{R})$, respectively of the "gauge" group U(1) generated by the particle number operator. The low-lying spectra show in these cases a simple structure, described in terms of a small number of so called collective degrees of freedom. But the connection between the original quantum dynamics and the collective behaviour remained a challenging problem of the many body theory.

The purpuse of this letter is to present the collective dynamics as a special case of low-energy quantum dynamics, occuring when the quantum ground state break a continous symmetry of the Hamiltonian. This situation appears usually for the approximate ground states, but may also be expected for the exact ones , at the classical limit.

Consider R to be a connected Lie group represented in $\mathcal{H}$ by the unitary operators $\{\hat{U}_r, r\in R\}$, and $M\subseteq\mathcal{H}$ an R-invariant trial manifold.If $|g\rangle\in M$ is a symmetry breaking ground state, than a whole critical submanifold $Q\subset M$,$Q=\{\hat{U}_r|g\rangle, r\in R\}$ can be generated by the action of R on $|g\rangle$.

Suppose that ω^M vanishes on the tangent space TQ, namely Q is an isotropic submanifold of M. When the algebra r of R is semisimple, this means that the representation operators for r have vanishing ground state expected values. In particular, for the deformed ground states the average of the angular momentum operators $\hat{L}_x, \hat{L}_y, \hat{L}_z$ should be zero, and the isotropic submanifold Q appears as the coordinate space for the rotational "collective" degrees of freedom.

If Q is isotropic, than at any $q\in Q$ the tangent space T_qQ has a coisotropic ω^M-orthogonal complement F_q defined by $F_q=\{X\in T_qM|\omega^M(X,Y)=0 \text{ for all } Y\in T_qQ\}$,[2], and the quotient $E_q= F_q/T_qQ$ is a symplectic vector space. Consider P_q to be a complement of F_q , that is $T_qM=P_q\oplus F_q$.Than P_q is isotropic, with the same dimension as T_qQ, and such that ω^M restricted to $P_q\times T_qQ$ is nondegenerate. Thus locally, at every point q of Q one has a classical phase space structure, with P_q representing the space of the momenta canonically conjugate to the collective coordinates. The remaining "intrinsic" variables are represented within E_q. However, this local phase space structure is not unique, and in general it cannot be integrated to a classical phase space manifold.

At this point dynamical considerations may start to play an important role, eliminating the ambiguities and selecting an integrable structure. To get insight into this role of the dynamics it is instructive to study the non-trivial case of the canonical momentum associated to a single angle coordinate φ .

Consider φ to be the rotation angle around the X-axis, I the associated momentum coordinate, and $|g\rangle$ a deformed ground state. Suppose that exists a symplectic submanifold $\mathcal{P}$ containing $|g\rangle$, parameterized by φ and I, such that $\mathcal{P}=\{|Z\rangle_{(\varphi,I)}\in M\,|\,|Z\rangle_{(0,0)}=|g\rangle, |Z\rangle_{(\varphi,I)}=\exp(-i\varphi\hat{L}_x)|Z\rangle_{(0,I)}, \varphi\in[0,2\pi], I\in\mathbb{R}\}$. The condition as φ and I to be canonically conjugate shows that generally $|Z\rangle_{(0,I)}$ belongs to the manifold $\mathcal{F}_I=J^{-1}(I)$ with $J:M\longrightarrow\mathbb{R}$, $J_{(Z)}=\langle Z|\hat{L}_x|Z\rangle$. Denoting $\mathcal{F}_I\cap\mathcal{P}$ by $\mathcal{P}_I$, than $\mathcal{P}_0\equiv Q$ and at any $|Z\rangle\in\mathcal{P}_I$ the coisotropic complement of $T_{|Z\rangle}\mathcal{P}_I$ is $T_{|Z\rangle}\mathcal{F}_I$.

Due to the intrinsic variables, the dimension of $\mathcal{F}_I$ is high, and the definition of $|Z\rangle_{(0,I)}$, which fix the space P_q, requires additional chriteria. Assuming that $|Z\rangle_{(0,I)}$ represent a state with no "intrinsic" excitation , a first selection can be made retaining only an absolute minimum for the restriction of h_M to $\mathcal{F}_I$. If this minimum is unique up to a rotation $\hat{R}_x=\exp(-i\varphi\hat{L}_x)$, than a representative joined smoothly with $|g\rangle$ will correspond to $|Z\rangle_{(0,I)}$. This constrained variational problem reduces to a normal one for the modified Hamiltonian $\hat{H}^*=\hat{H}-\omega\hat{L}_x$, intensively used within the self consistent cranking model of the nuclear rotation ,[3].Thus, denoting by $|Z\rangle_\omega$ the minima of $\langle Z|\hat{H}^*|Z\rangle$ and by ω_I the value determined from $I={}_\omega\langle Z|\hat{L}_x|Z\rangle_\omega$, the result is $|Z\rangle_{(0,I)}=|Z\rangle_{\omega_I}$.

As it is known ,[3], if $\hat{H}$ is a nuclear Hamiltonian consisting of a single-particle spherical oscillator term and a quadrupole-quadrupole interaction, and M is the manifold of the Hartree-Fock states, than $|Z\rangle_\omega$ is a Slater determinant constructed with the cranked anisotropic oscillator eigenstates. These eigenstates have a simple expression, and are connected with the spherical harmonic oscillator eigenfunctions by unitary transformations.

Consider $H=\sum_{k=1}^{3}\omega_k(a_k^+a_k+\frac{1}{2})$, $a_k^+=\sqrt{m\omega_k/2}\,(x_k-\frac{i}{m\omega_k}p_k)$,to be the deformed oscillator Hamiltonian,and denote by $b_k^+=\sqrt{m\omega_o/2}(x_k-\frac{i}{m\omega_o}p_k)$,k=1,3, the Dirac-Fock operators for the spherical oscillator having $\omega_o^2=(\omega_2^2+\omega_3^2)/2$. Than, the angular momentum operator is $L_x=i(b_2b_3^+-b_3b_2^+)$, and it can be shown that if $\omega/\omega_o<\sqrt{3}/2$, than $H^*=H-\omega L_x=UH_oU^{-1}$ with :

$$U=\exp[-i\lambda(b_2^+b_3+b_3^+b_2)]\exp[\sum_{k=1}^{3}\vartheta_k(b_k^{+2}-b_k^2)/2\quad] \tag{1}$$

and $H_o=\sum_{k=1}^{3}\Omega_k(b_k^+b_k+\frac{1}{2})$.

Here $\Omega_1=\omega_1$, while $\lambda,\vartheta_k,\Omega_2,\Omega_3$ are explicitly related to the "deformation" parameter $\eta=(\omega_2^2-\omega_3^2)/2\omega_o^2$ and the cranking frequency ω by the formulas:

$$\tan 2\lambda=2\omega/(\omega_o\eta)\ ,\ \sinh 2\vartheta_k=\omega_o(1-\omega_k^2/\omega_o^2)/2\Omega_k\ ,k=1,2,3 \tag{2}$$

$$\Omega_k^2=(\omega_o+\varepsilon_k)^2-(\omega_o\eta)^2/4\quad ,k=2,3 \tag{3}$$

$$\varepsilon_k=s_k\omega_o\eta/(2\cos 2\lambda),s_k=(-1)^k,k=2,3. \tag{4}$$

Now, if c_a^+ ,(c_a), $a\equiv(\Phi,\sigma)$, creates ,(annihilates), a fermion in the orbital eigenstate Φ of H^*, with spin projection $\sigma=\pm\frac{1}{2}$ than for a system of N fermions,the state $|Z\rangle_\omega=c_{a_1}^+c_{a_2}^+\ldots c_{a_N}^+|0\rangle$, may be expressed as $|Z\rangle_\omega=\hat{U}_\omega|HF\rangle_o$, where: $\hat{U}_\omega=\exp[-i\lambda\hat{C}_x]\exp[-i\sum_{k=1}^{3}\vartheta_k\hat{S}_k/2]$, with:

$$\hat{C}_x=\sum_{\alpha,\beta}\langle\alpha|b_2^+b_3+b_3^+b_2|\beta\rangle c_\alpha^+c_\beta \tag{5}$$

$$\hat{S}_k=i\sum_{\alpha,\beta}\langle\alpha|b_k^{+2}-b_k^2|\beta\rangle c_\alpha^+c_\beta \tag{6}$$

$$|HF\rangle_o=c_{\alpha_1}^+c_{\alpha_2}^+\ldots c_{\alpha_N}^+|0\rangle,\ \alpha=(\psi,\sigma) \tag{7}$$

and ψ eigenfunction of H_o.

The action of $\hat{S}_1$ shows easily that the transformations generated by $\hat{S}_k$ correspond to the transition from a "spherical" to a "deformed" basis. By construction, the operator $\hat{C}_x$ has the main role in generating the shift of the angular momentum average, and thus it appears as an "angle" operator conjugate to $\hat{L}_x$. However, by contrast to the angle operators defined by the kinematic approach of the commutation relations, [4],[5], its relevance is restricted to the particular dynamics investigated here.

Consider $\hat{C}_y$,$\hat{C}_z$ to be the conjugate operators obtained when $\hat{L}_y$, respectively $\hat{L}_z$ are used as cranking terms instead $\hat{L}_x$. Then the sets $\{\hat{L}_k,\hat{C}_k\}$,$\{\hat{L}_k,\hat{S}_k\}$, k=x,y,z generate an su(3), respectively gl(3) Lie algebras. Both are subalgebras of the symplectic Lie algebra sp(3,$\mathbb{R}$) intensively studied for the algebraic description of the collective dynamics ,[6].

A simple, but suggestive application of the phase space $\mathcal{P}$ constructed here concern the microscopic correspondent for the classical isovector angle vibrations considered in many recent studies on the nuclear magnetism ,[7]. Therefore, the appropriate excitation operator for the magnetic dipole states must be a nonhermitian combination including both "coordinate" and "momentum" operators ,[8] ,and this is fixed only if $\hat{U}_\omega$ is known. Consider two different submanifolds $\mathcal{P}_p$, $\mathcal{P}_n$ containing the proton, respectively the neutron functions: $|Z\rangle^p_{(\varphi^p,I^p)}$, $|Z\rangle^n_{(\varphi^n,I^n)}$ .Within this space, the low energy nuclear angle vibration with frequency Ω may be represented by the product $|Z\rangle^{pn}(\varphi^p,I^p,\varphi^n,I^n)=|Z\rangle^p_{(\varphi^p,I^p)}|Z\rangle^n_{(\varphi^n,I^n)}$ with φ^p,φ^n, time-dependent as given by:$\varphi^p_{(t)}= a^p\sin\Omega t$, $\varphi^n_{(t)}= -a^n\sin\Omega t$.

For the first excited state the oscillation amplitudes a^p, a^n are completely determined by the non-rotation condition $I^p + I^n = 0$, and by the Bohr-Sommerfeld quantization formula $\oint(\dot{\varphi}^p I^p + \dot{\varphi}^n I^n)dt = 2\pi$. In addition, if a^p, a^n are small, the requantization procedure employed before ,[9], give the associated stationary state by the time-average $|\Omega_x\rangle = T^{-1}\oint \exp(i\Omega t)|Z\rangle^{pn} = \hat{B}^+|gd\rangle$, with $T = 2\pi/\Omega$, $|gd\rangle = |Z\rangle^{pn}(0,0,0,0)$, and :

$$\hat{B}^+ = \frac{1}{2}\left[a^p \hat{L}_x^p - a^n \hat{L}_x^n - \frac{i\Omega}{\omega_2 - \omega_3}(a^p \hat{C}_x^p - a^n \hat{C}_x^n) \right] \qquad (8)$$

The presence of the $\hat{C}$-dependent terms in $\hat{B}^+$ lead now to the boson-like relation: $\langle gd|[\hat{B}, \hat{B}^+]|gd\rangle = 1$ as required for the excitation operators within the random phase approximation. If the particle-hole excitations between different oscillator shells are neglected, than $\mathrm{sgn}(\eta) i\hat{C}_x|gd\rangle \cong -\hat{L}_x|gd\rangle$,and $\hat{B}^+$ will be replaced by a hermitian operator. The same hermitian operator is obtained if the crank i ng term $-\omega\hat{L}_x$ is considered "small" and $|Z\rangle_\omega$ is estimated in the first order of the perturbation expansion. After normalization the state $|\Omega_x\rangle$ reduces in this last case to the term independent on spin and superfluidity from the state |ROT> constructed previously ,[10], to represent the microscopic correspondent for the collective angle vibration.

This example show clearly the importance of using the whole manifold $\mathcal{P}$, rather than only the "coordinate space" Q generated by $\hat{R}_x$. Worth noting is that although within a different algebraic framework, the same approach was successfully applied to the case of the nuclear isovector Josephson-like oscillations in the BCS gauge space ,[11]. Nevertheless, beside the cranking model or the angle vibrations , the closed expression for $\hat{U}_\omega$ presented here may have a wide range of applications

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