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PION LASERS FROM HIGH ENERGY COLLISIONS

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MSUCL-867

DECEMBER 1992

MSUCL-867
December, 1992

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Abstract: A technique is presented which allows rapid calculation of multi-particle Bose-Einstein interference to all orders for a finite system. This technique is used to illustrate how multi-particle interference can enhance the emission of pions in a heavy-ion collision. Corrections to multiplicity distributions, single-body spectra and two-body correlations are demonstrated. The possibility of creating a pion laser with the upcoming lead beam at CERN is discussed. The effects of symmetrization regarding isospin fluctuations are demonstrated in the context of a simple model. It is shown that these effects can play an important role in explaining anomalous (CENTAURO) cosmic ray events.

Ultrarelativistic hadronic and nuclear collisions provide the environment for creating dozens and in some cases hundreds of pions. Pions are bosons. Their bosonic nature should affect the single-body spectrum. For example, the Planck distribution of black-body radiation differs from the Boltzmann distribution. The two-body spectrum is also affected in that the emission of two-pions with small relative momentum is enhanced, as is well documented with correlation measurements for both hadronic and nuclear collisions. A laser is the most dramatic example of multi-particle symmetrization. The emission of bosons encourages the emission of more bosons, and if the particle density is sufficiently high, all sources emit simultaneously in a coherent fashion. Describing lasers requires understanding the full n-body symmetrization. We present a solution to the n-body symmetrization problem, and discuss the conditions for creating coherent behavior from incoherent sources.

For thermalized emission, multiparticle interference becomes important once the mean particle separation approaches the thermal wavelength. This is the case for high-multiplicity hadronic collisions and should be the case for relativistic heavy-ion collisions with sufficient energy, projectile size and target size. Accounting for multi-particle effects has proven difficult[1, 2]. The usual method is to 'Monte Carlo' events, then calculate the symmetrization weight for the event by squaring the symmetrized wave-function. This is numerically challenging because the weight approaches $N!$ as all N momenta become equal. We present a technique for calculating the many-body effects for an arbitrary emission function $S(p, x)$. For the case of gaussian emission probabilities, analytic results are obtained. For more complicated emission distributions, results can be obtained in a few minutes of Monte Carlo, as opposed to a few days which was the case with other methods. To illustrate these effects we show the correction to the one-body spectra, the two-body correlation and to the multiplicity distribution for simple gaussian geometries.

A high energy cosmic ray might produce a hundred particles in a volume of only a few cubic fermi. We show how multi-boson effects enhance high multiplicity events, and in a simple model, we illustrate how fluctuations in the isospin assymetry are greatly enhanced for very high multiplicity events. Previously unexplained fluctuations have been observed in emulsion, and are known as CENTAURO events[3].

For emission from uncorrelated chaotic sources, we assume that the T-matrices factorize. The matrix element for uncorrelated sources to emit N pions with momenta p_i is:

$$\int d^4x_1 d^4x_2 ... d^4x_N T_1(x_1) T_2(x_2) ... T_N(x_N) U(x_1, x_2, ... x_N; p_1, p_2, ... p_N) \tag{1}$$

The evolution matrix, U, has $N!$ terms. Before we show the result of squaring the entire matrix element, we first look at the the result of squaring $U(x_i; p_i)$. Ignoring the permutations over spatial-coordinate labels there are $N!$ terms in $|U|^2$. For $N = 4$, the terms are as follows:

$$\begin{aligned} & e^{i(p_1-p_1)\cdot x_a + i(p_2-p_2)\cdot x_b + i(p_3-p_3)\cdot x_c + i(p_4-p_4)\cdot x_d} \\ & + \frac{4!}{2!} \cdot \frac{1}{2} \cdot e^{i(p_1-p_1)\cdot x_a + i(p_2-p_2)\cdot x_b + i(p_3-p_4)\cdot x_c + i(p_4-p_3)\cdot x_d} \\ & + 4! \cdot \frac{1}{3} \cdot e^{i(p_1-p_1)\cdot x_a + i(p_2-p_3)\cdot x_b + i(p_3-p_4)\cdot x_c + i(p_4-p_2)\cdot x_d} \\ & + \frac{4!}{2!} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot e^{i(p_1-p_2)\cdot x_a + i(p_2-p_1)\cdot x_b + i(p_3-p_4)\cdot x_c + i(p_4-p_3)\cdot x_d} \\ & + 4! \cdot \frac{1}{4} \cdot e^{i(p_1-p_2)\cdot x_a + i(p_2-p_3)\cdot x_b + i(p_3-p_4)\cdot x_c + i(p_4-p_1)\cdot x_d} \end{aligned} \tag{2}$$

The factors in front of each term represent the number of similar terms which differ only through permuting the momenta indices.

To calculate a probability one must square the entire matrix element in equation 1. Since the coordinates x_i are integrated over, this brings about terms such as $T^*(x) \cdot T(x')$. We use the T-matrices to define an emission function $S(p, x)$ which is defined:

$$S(p,x) \propto \sum_i \int d^4y e^{ip\cdot y} T_i^*(x+y/2) T_i(x-y/2). \tag{3}$$

When evaluated at $p_0 = E_p$, $S(p,x)$ is the probability that an emitted particle has momentum $\mathbf{p}$ and originated from space-time point x, assuming symmetrization has been neglected. The emission function can be extracted from any dynamical or hydrodynamic simulation that theoretically describes the emission of pions. Performing the combinatorics and the algebra one can show that the probability of emitting 4 pions is:

$$w(4) = \frac{1}{Z} \cdot [\frac{1}{4!}C(1)^4 + \frac{1}{2!}C(1)^2 \cdot C(2)$$

$$+C(1)\cdot C(3) + \frac{1}{2!}C(2)^2 + C(4)]. \tag{4}$$

Here, Z is the normalization, $Z = \sum_i w(i)$. To explain the quantities $C(N)$ we write $C(4)$.

$$C(4) = \frac{1}{4} \int d^3p G_4(p,p). \tag{5}$$

$$G_4(p,q) = \eta^4 \cdot \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \; S(\frac{p+k_1}{2}, x_1) e^{(p-k_1)\cdot x_1} d^3k_1$$

$$\cdot S(\frac{k_1+k_2}{2}, x_2) e^{(k_1-k_2)\cdot x_2} d^3k_2 \cdot S(\frac{k_2+k_3}{2}, x_3) e^{(k_2-k_3)\cdot x_3} d^3k_3 \cdot S(\frac{k_3+q}{2}, x_4) e^{(k_3-q)\cdot x_4}.$$

We take the emission function to be normalized such that $\int d^4x d^3p S(p,x) = 1$. A factor $\frac{\eta^N}{N!}$ has entered which entails assuming there are many more sources than pions, and that their random emission would lead to a Poisson distribution with an average multiplicity of η if symmetrization were neglected. This equation can be represented with diagrams where $C(n)$ is a ring diagram with n dots connected with n lines. Each dot represents a spatial coordinate x_i and each line represents a momenta k_i. Then $G(p,q)$ can be considered a line formed by connecting n dots, where the incoming and outgoing segments are labeled p and q, and all the unlabeled interior lines and dots represent coordinates over which are integrated. The quantity $w(n)$ is the sum of all diagrams, connected or disconnected, of order n. The rules for prefactors are the same as used in perturbation theory for particles interacting with an external field.

The corrected single and two-body spectra can also be calculated from these diagrams. To calculate the single-body spectrum for events with fixed N one begins with $w(N)$, the sum of all diagrams of order N, and then make one cut, in all possible ways, labelling the cut segment, $\mathbf{p}$. The probability of measuring a pion with momentum $\mathbf{p}$ becomes:

$$P(\mathbf{p}) = \frac{1}{w(N)}[G_1(p,p)\cdot w(N-1) + G_2(p,p)\cdot w(N-2) + G_3(p,p)\cdot w(N-3) + ...] \quad (6)$$

The two-particle probability is the sum of all diagrams where there have been two cuts. If we consider events with all numbers of pions, the single and two-particle probabilities take on a particularly simple form.

$$P(\mathbf{p}) = G_1(p,p) + G_2(p,p) + G_3(p,p) + ... \quad (7)$$

$$P_2(\mathbf{p},\mathbf{q}) = [G_1(p,p) + G_2(p,p) + G_3(p,p) + ...]\,[G_1(q,q) + G_2(q,q) + G_3(q,q) + ...]$$
$$+[G_1(p,q) + G_2(p,q) + G_3(p,q) + ...]\,[G_1(q,p) + G_2(q,p) + G_3(q,p) + ..]$$

It is straight forward to calculate all the quantities $G_n(p,q)$ and therefore $C(n)$, either through Monte-Carlo integrations or by analytically performing the integration as can be done for the case of a Gaussian distribution. The missing piece is calculating $w(N)$ from $C(n), n < N$. Here we present the algorithm for finding the sum of all possible diagrams of order N in terms of ring diagrams of order n. First we define $\sigma(i,n)$, the sum of all diagrams of order n made up of ring diagrams where no ring diagram is of order greater than i. Once we have found $\sigma(N,N)$ which equals $w(N)$ we have accomplished our mission. The sum of all direct diagrams $\sigma(1,n)$ is:

$$\sigma(1,n) = \frac{1}{n!}\cdot C(1)^n. \quad (8)$$

Then we find $\sigma(i,n)$ for successively higher i.

$$\sigma(2,n) = \sigma(1,n) + C(2)\cdot\sigma(1,n-2) + \frac{1}{2!}C(2)^2\cdot\sigma(1,n-4) + \frac{1}{3!}C(2)^3\cdot\sigma(1,n-6)...$$

$$\sigma(3,n) = \sigma(2,n) + C(3)\cdot\sigma(2,n-3) + \frac{1}{2!}C(3)^2\cdot\sigma(2,n-6) + ...$$

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$$w(N) = \sigma(N,N) = \sigma(N-1,N) + C(N)$$

It is worthwhile to work with an example where $G_n(p,q)$ can be found analytically. This will allow rapid calculation so that physical consequences of the multi-particle effects can be studied more readily. Such an example is a non-relativistic thermal gaussian distribution.

$$S(p,x) = \frac{1}{(2\pi R^2 mT)^{(3/2)}} \cdot e^{-\frac{p_0}{T}-\frac{x^2}{2R^2}} \cdot \delta(x_0) \tag{9}$$

Successive convolutions over gaussians yield more gaussians, and the functions $G_n(p,q)$ can be expressed as:

$$G_n(p,q) = \alpha(n) \cdot exp(-\frac{\beta_n(p^2+q^2) + |\mathbf{p}-\mathbf{q}|^2 R_n^2}{2}) \tag{10}$$

The quantities α, p_n, and R_n are found iteratively.

$$\alpha_{n+1} = \alpha_n \cdot \eta \cdot \frac{1}{\Delta^3} \cdot \frac{1}{[R_n^2 + R^2 + \gamma_n]^{3/2}} \tag{11}$$

$$R_{n+1}^2 = \frac{R_n^2 \cdot R^2}{R_n^2 + R^2 + \gamma_n}$$

$$\beta_{n+1} = \beta_n + \frac{R_n^2 \cdot \gamma_n}{R_n^2 + R^2 + \gamma_n}$$

$$\gamma_{n+1} = \frac{1}{\Delta^2} + \frac{R_n^2 \cdot \gamma_n}{R_n^2 + R^2 + \gamma_n}$$

where,

$$\beta_1 = \gamma_1 = \frac{1}{\Delta^2}, \quad R_1 = R, \quad \alpha_1 = \frac{\eta}{\Delta^3 (2\pi)^{3/2}}.$$

To demonstrate the effect of the symmetrization on the multiplicity distribution, we consider a small system of size, $R = 1.5\ fm$, and $\Delta = 250\ MeV/c$. This Δ would yield a mean p_T of 313 MeV/c, roughly corresponding to the mean p_T of non-jet pions in high energy collisions. We consider a distribution which if symmetrization were neglected would be poissonian with a mean of $\eta = 14$. Figure 1 shows the multiplicity distribution corrected for symmetrization according to the symmetrization weights as in equation 4. The mean has increased from 14 to 31.3. The symmetrization enhancement increases for higher multiplicity events. The variance of the distribution has increased to make the distribution super-poissonian, meaning the variance has become larger than the square root of the mean.

The increase of the mean depends on η. For fewer particles, the effect of symmetrization is small. In figure 2, the resulting average multiplicity is shown as a function of η, the average multiplicity when symmetrization is ignored. This is done for a source of size $R = 3.5fm$ and $\Delta = 250MeV$, reasonable sizes and momenta for relativistic heavy-ion reactions. Symmetrization does not affect the multiplicity for small η. But as η approaches a critical value, 122.2, the resulting average multiplicity goes to infinity. This is because the sum of diagrams for the single-particle probability diverges, when η becomes large. We label this critical value η_c. By carefully inspecting the form of $G_n(p,p)$ for $p=0$ at large n, one can see that the sum diverges when η exceeds

$$\eta_c = \left((\Delta R)^2 + \sqrt{(p_0 R)^2 + \frac{1}{4}} \right)^{\frac{3}{2}}. \tag{12}$$

The critical multiplicity η_c does not scale as R^3 until R is large. Figure 3 demonstrates these finite-size effects. The critical density is plotted as a function of the radius R for a fixed value of Δ. The critical density is the density at the origin when η_c pions are distributed with a gaussian distribution characterized by R. For large R the critical density approaches a constant, in this case .145 fm^{-3}. For smaller sources the required density is higher because η_c approaches a constant as R goes to zero. As long as the source size is much larger than the thermal wave length, $1/\Delta$, the condition for this discontinuity is that the phase space occupancy at the origin exceeds unity when symmetrization is neglected.

Heavy-ion collisions at high energy create systems where the size of the excited region is larger than the thermal wavelength. The number of possible pion emissions also significantly exceeds the number of pions actually emitted. The assumption of unconstrained Poissonian emission is therefore reasonable. The natural step is to compare the estimated pion density that occurs in heavy ion collisions to the critical density for large sources. The highest pion density currently achieved is with the SPS at CERN using a $200A \cdot GeV$ sulphur beam incident on a fixed lead target. Here roughly 100 like-charge pions are produced in a central collision. These pions are distributed over roughly three units of rapidity. Pions emitted with a rapidity difference greater than unity are usually from spatially disjoint regions of phase space. Pions emitted from long-lived resonances do not contribute either. To make a crude estimate of the pion density we consider one fourth of the pions to be emitted from a static gaussian source of a size slightly larger than the sulphur nucleus. Dividing 25 pions into a volume described by a gaussian distribution of $R = 2.5\ fm$, the resulting π_- density is .10 fm^{-3}. This represents a significant fraction of the critical density. The size of $R = 2.5 fm$ is small enough that finite size effects increase the critical multiplicity such that 25 pions is exactly half the critical number. When the lead beam at the SPS is available in 1994, the pion density should roughly double. The larger source size will reduce finite-size effects as well. Critical multiplicity might then be attained. Stimulated emission would then become unstable and nearly all pion emission channels available should emit pions into low momentum states. Two characteristics of such emission would be anomalous pion multiplicities and stopping. We refer to such behavior as a pion laser. As we discuss the effects on single-body and two-body measurements, the similarities to photon lasers will become clear.

Figure 4 shows the symmetrization-corrected single-pion probability divided by the single-pion probability when symmetrization is neglected. The ratio is plotted against the pion's momentum. We have used the same gaussian distributions with $R = 3.5fm$ and $\Delta = 250MeV/c$ and have fixed the number of pions to 100, 200 and 400 for the three lines shown on the graph. The symmetrization brings about a low-p_T enhancement. This is significant in the case of multiplicities of 100, but spectacular for multiplicities of 200 and 400. A significant fraction of the pions are in the same phase space-cell. The momentum of the cell is zero to within the inverse source radius. In other words, the wavelength of the pion laser corresponds to zero momentum. In analogy to liquid helium the zero momentum pions are in a condensate. One can perform the same diagrammatic calculation for particles-in-a-box geometry that we presented here for gaussian geometry. For large boxes, the correction to the single-particle spectra can be shown to be precisely the Bose-Einstein distribution divided by the Boltzmann distribution. The enhancement in figure 4 has two momentum scales. For lower multiplicities the p_T enhancement has a characteristic width of Δ, while once the condensate is formed a narrow peak occurs at zero momentum with a width of $1/R$. Low p_T enhancement has been searched for at CERN with mixed results [4–7].

The modified two-particle correlation functions are shown in figure 5. These are constructed by dividing the two-particle probabilities by the product of the single-particle probabilities. The total momentum of the pair has been integrated over in both the numerator and the denominator, and the correlation functions are shown as a function of the relative momentum only. Again, this is for the gaussian case of $R = 3.5fm$ and $/Delta = 250MeV/c$. The upper part of figure 5 shows the correlation function for the case of unconstrained emission. The two-pion and single-pion probabilities are constructed by using events with all different multiplicities. For the three correlation functions shown in the upper portion of figure 5, η was adjusted to yield average pion multiplicities of 200 for the dotted line, 125 for the light line, and the heavy line shows the correlation function assuming only a few pions were emitted. The correlation function broadens significantly once the multiplicity goes beyond 150. The intercept at $k = 0$ is unchanged. As η approaches the laser threshold the width becomes infinite and the correlation function becomes flat. The lower portion of figure 5 shows the correlation function where the two-pion and single-pion probabilities are constructed using events of a fixed multiplicity. The behavior is strikingly different from that shown in the upper portion of figure 5. The heavy line again corresponds to the correlation function for small multiplicities where higher order effects are negligible. The light line and the dotted line correspond to multiplicities of 200 and 400 respectively. The intercept of the correlation function approaches unity as the multiplicity goes to infinity. This is characteristic of a coherent state. This is remarkable, as the sources were assumed to be incoherent. Whereas quantum mechanics textbooks refer to coherent states as those produced by coherently summing states of different multiplicities, this behavior appears for fixed multiplicity.

A pion condensate is not the ground state for pions at a fixed energy density. The ground state has zero chemical potential unless there is an isospin imbalance. If a condensate were formed by over-populating the pion states at low momentum there would be a tendency to return to equilibrium through annihilating pions and using the energy to raise the temperature. Pion annihilation is a slow process in a baryon-free region, because it requires at least 3 incoming incoming pions to produce an outgoing on-shell state with fewer pions. Rates of pion annihilation have been studied by several authors [8–11]. In a thermodynamic context, several authors have introduced effective chemical potentials to account for the overpopulation of pions. For an infinite system, condensation occurs when the chemical potential reaches the pion mass.

Photon lasers require an overpopulation of the excited state or population inversion in order to cascade coherently. In heavy-ion collisions the analog to the population inversion is the initial energy of the colliding nucleons. This collective energy can be considered the excited state which would prefer to become thermal energy through colliding and stopping. In ultrarelativistic heavy-ion collisions a great deal more energy is in collective flow than in pions. Once laser behavior has started it should continue until the driving collective energy is depleted and the nucleons are at equilibrium with the pions. This limits the number of pions, but the limit is several times larger than the number of pions usually produced in an ultrarelativistic collision. Anomalous stopping is one signal of a pion laser.

Proton induced reactions can at times produce a large multiplicity, but the emission is usually spread over several units of rapidity, and multiparticle effects are negligible. However, if a high energy proton were incident on a nuclear target, they could collide up to a half dozen times, thus raising the density of like-pions to near the lasing threshold. Large particle accelerators and cosmic rays can both serve as a sources for high energy projectiles. One would look for lasing behavior in the highest multiplicity events, where the largest number of partons would have interacted. To test whether the laser-like behavior has occurred is difficult because one would be looking at the tail of a broad multiplicity distribution. Looking at events with large multiplicity biases both the transverse energy spectra and the rapidity distribution due to energy conservation. Since the size of the system would be on the order of 1 fermi, the peak in the transverse momentum spectrum would be so broad, it would meld into the background. Looking at two-particle correlations is also difficult with only a few events. An independent signal would be large isospin fluctuations, especially for events where the pions are spread over only a few units of rapidity. Events with all the pions of the same charge should be preferentially enhanced by symmetrization. This behavior has already been seen in cosmic ray measurements. Events with an anomalous isospin imbalance are known as CENTAURO events.

To illustrate the effects of symmetrization on the isospin distribution, we use the following simple model. We consider an emitter of pions that has isospin $\frac{1}{2}$. We assume that it has emitted 75 pions. Angular momentum algebra says that the emitter has a 2/3 probability of emitting a charged pion and therefore reversing its isospin and a 1/3 probability of emitting a neutral pion. This model conserves isospin which is known to be important for describing multiplicity distributions[12], so the net charge of the pions must be either -1, 1 or 0. Neglecting symmetrization, the distribution is binomial and the average number of $\pi_0 s$ is 25. Including symmetrization, the probability of emitting n_0 $\pi_0 s$ is:

$$P(n_0) = \frac{1}{N} \cdot \frac{75!}{n_0!\,(75-n_0)!} \cdot w(n_0)\; w(n_+)\; w(n_-) \tag{13}$$

The normalization is N and the symmetrization weights for the three charge states are $w(n)$. We calculate the symmetrization weights using the same gaussian model as before with $\Delta = 250MeV/c$. Figure 6 shows the isospin distribution for three different sizes, R. For large sources the distribution is binomial, peaked at 25. For smaller sources the distribution broadens.

This model is certainly oversimplified, but the qualitative effect of the symmetrization rests on one assumption - that the source size is small. Many more exotic explanations of CENTAURO behavior are present in the literature. Before one believes explanations based on misaligned chiral condensates or quark nuggets, one must thoroughly discount symmetrization as the cause. This requires performing careful phenomenological calculations that match the rapidity distribution and transverse energy spectra as well as the isospin imbalance.

An important ingredient which is missing in all the calculations here is the coulomb interaction between the pions. The three-body quantum-mechanical coulomb problem is not yet solved, and it is unlikely that the N-body problem will be solved any time soon. For proton-induced reactions this is probably not so important. Symmetrization enhances the correlation for relative momenta of the order of $1/R$, which for 1 fm sources is much larger than the characteristic momentum of the Gamow coulomb correction. However, for large sources, the symmetrization enhancement in two-particle correlation functions is significantly nullified by the Gamow correction. Given the fact that both positive and negative particles might be in a condensate, it is difficult to predict exactly what effect the coulomb repulsion will have. Luckily, $\pi_0 s$ are not affected. It may well turn out that the only coherent behavior seen in heavy-ion collisions is in the π_0 sector. Coherent pion emission has been alluded to several times in the literature[13, 14]. However, there lacked an explanation as to how and when it should occur, given that the sources of the pions are incoherent. In this paper we have presented a formalism which allows rapid calculation of symmetrization weights from incoherent sources. Symmetrization effects to all orders are now calculable, even using complicated simulations to generate the source functions. Even if critical densities are not obtained for lasing the pions, these methods will be crucial for understanding the large multi-pion distortions to the single and two-body pion measurements to be made in upcoming experiments. The gaussian models used here, although overly simple, were sufficient to illustrate the wide variety of fascinating phenomena that might occur in the next generation of relativistic heavy-ion experiments and may already be present in cosmic ray data.

This behavior is not predicated on any exotic physical assumptions, only on the fact that pions are bosons and that the systems are attempting to emit a large number of them in a small amount of phase space. It seems inescapable that once energy densities of the order of .5 GeV/fm^3 are produced in heavy ion reactions, using collisions that emit pions with the p_T spectra of proton-proton collisions, that laser-like behavior will take place. From viewing figure 4 one concludes that the laser behavior turns on sharply. Doubling the number of sources changes a nearly negligible symmetrization effect into an unstable coherent cascade. Energy densities of precisely this order are expected to be achieved with the upcoming lead beam at CERN. At the relativistic heavy ion collider to be built at Brookhaven (RHIC), the energy densities will be much higher. Although the state of matter at higher density should be that of the quark-gluon plasma, the matter will have to return to the hadronic state. During the rapid cooling of the plasma, supercooling might well take place. In that event the system might find itself in a situation where it is trying to convert a large amount of energy to pions, using sources (quark droplets or tubes) which are at a lower temperature than an equilibrated hadronic gas at that energy density. These conditions would be ripe for producing a pion condensate. Certainly, a great deal of speculation can take place regarding behavior in upcoming experiments. Unlike many aspects of detecting the quark-gluon plasma, the consequences of coherent pion emission: anomalous stopping, isospin fluctuations, damping of the two-particle correlation function and low p_T enhancement, are unambiguous and directly measurable. This aspect should foster a fruitful interaction between theory and experiment.

ACKNOWLEDGMENTS

This work was supported by W. Bauer's Presidential Faculty Fellow Award, NSF grant no. PHY-925355, and from NSF grant no. PHY-9017077.

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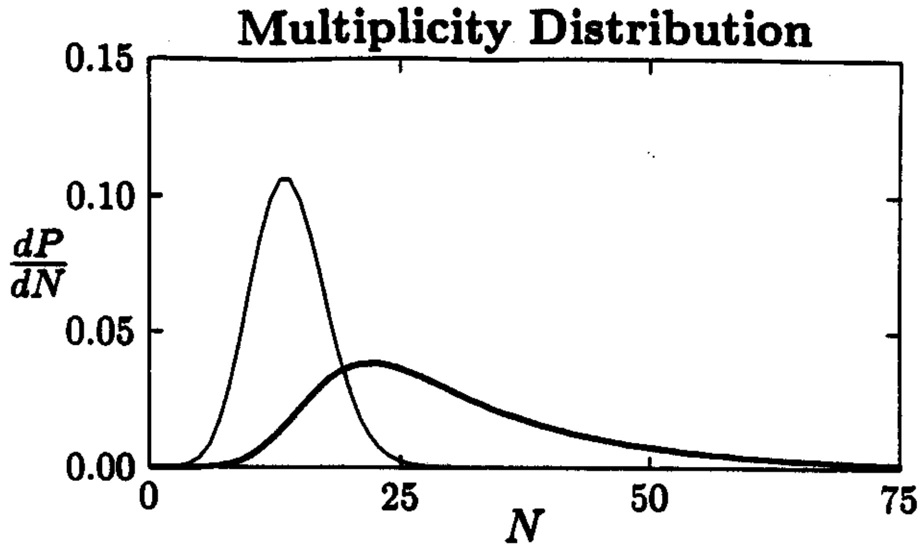

Figure 1. The light line represents a Poissonian multiplicity distribution with a mean of 14. The heavy line shows the same multiplicity distribution corrected for Bose-Einstein symmetrization. The source functions were assumed to be gaussian with a size $R = 1.5 fm$ and a momentum spread $\Delta = 250 MeV/c$.

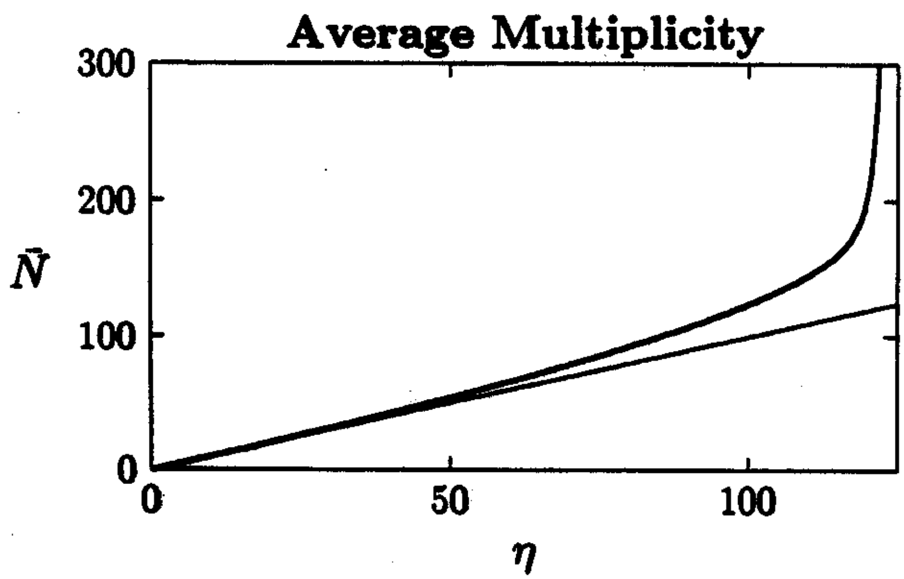

Figure 2. The heavy line represents the mean multiplicity for a system that would emit with Poissonian weights characterized by η, if symmetrization were neglected. The source function was assumed to be gaussian with $R = 3.5 fm$ and $\Delta = 250 MeV/c$. As η approaches η_c the sums in equation (7) no longer converge and the average multiplicity goes to infinity. If the source were large, symmetrization would not be important and the result would be the light line, $\bar{N} = \eta$.

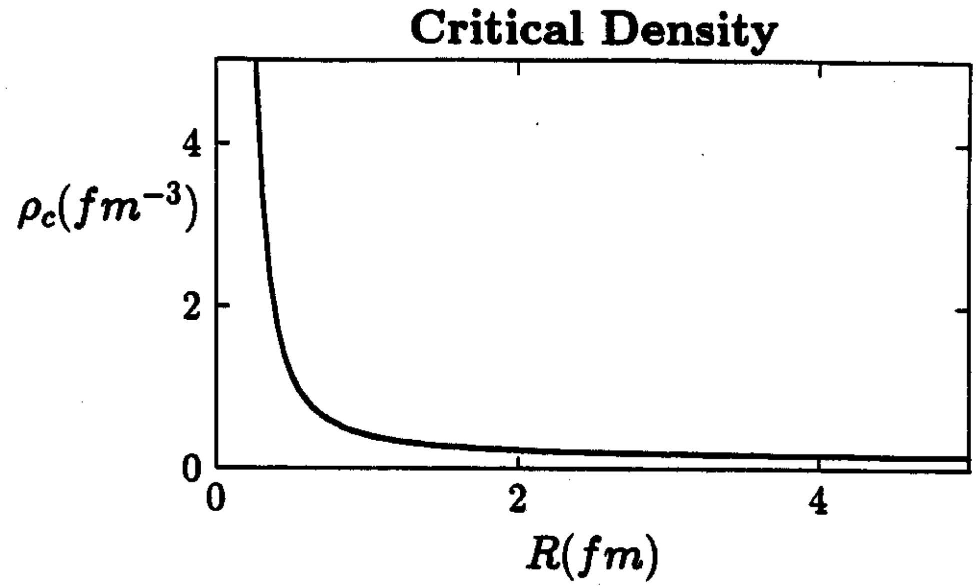

Figure 3. For a gaussian source of size R, the critical density is the density at the origin required for lasing. If the emission of pions is random (Poissonian) and one expects a pion density greater than the critical density when symmetrization is neglected, the inclusion of symmetrization leads to infinite multiplicity. Finite-size effects require a higher critical density for small sources.

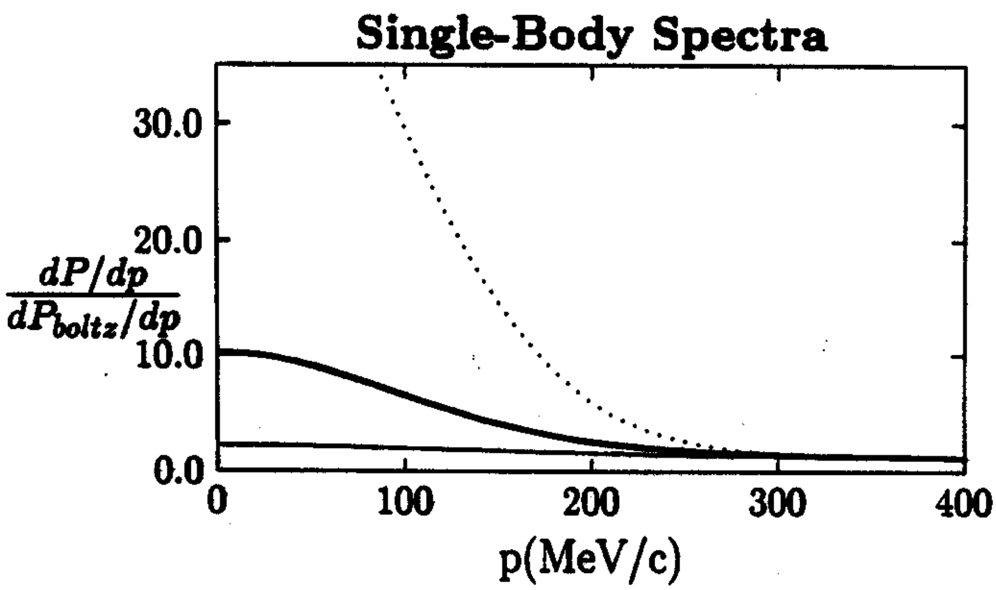

Figure 4. The single-particle spectra divided by what the spectra would be if symmetrization were neglected is shown as a function of the momenta for a 3.5fm source where the particle number is fixed at 125 (light line), 200 (heavy line) and 400 (dotted line). For low densities this is the analog to the Bose-Einstein spectrum divided by the Boltzmann spectrum, but for large particle number, a significant fraction of the pions populate the lowest energy state which has a width $1/R$.

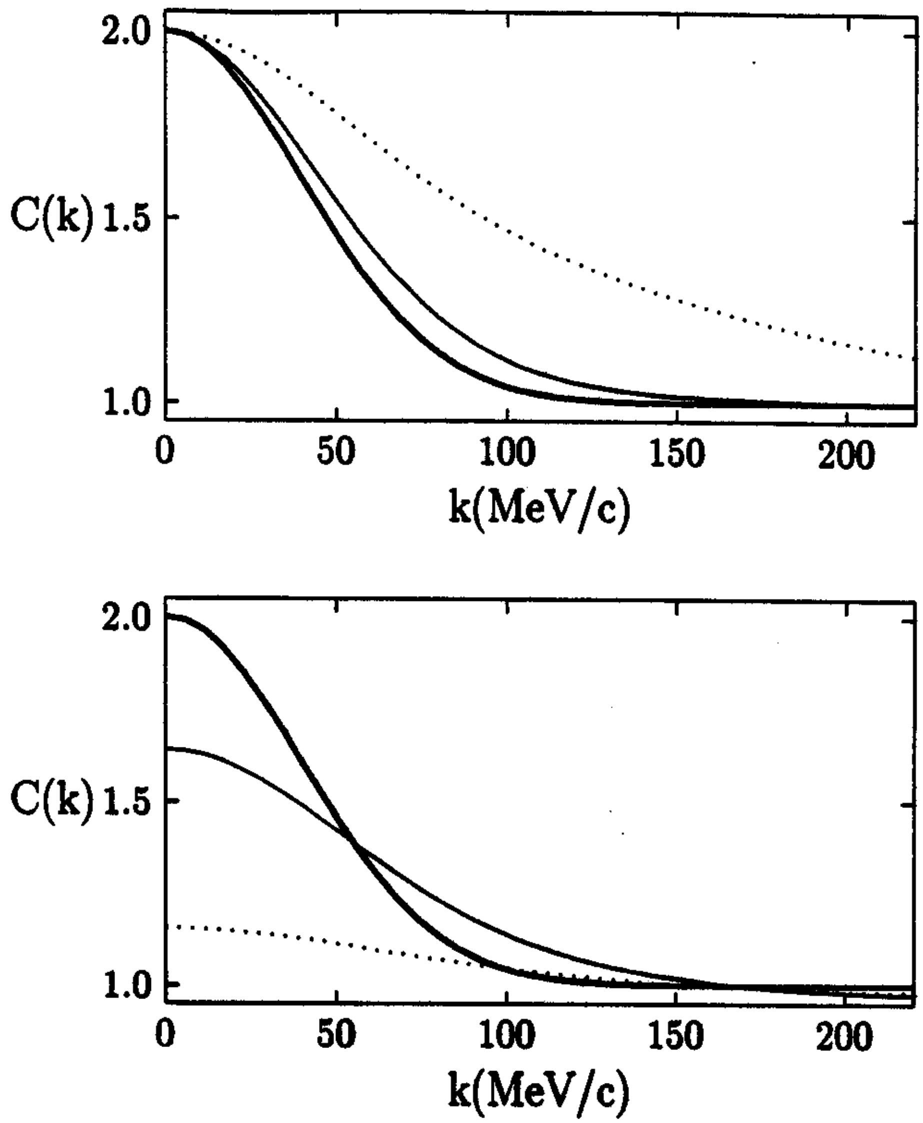

Figure 5. The top figure shows the two-pion correlation function for an unconstrained (Poissonian) 3.5*fm* source. Here the correlation functions are constructed from events of all multiplicities where the average number of pions from the event vary from just a few (heavy line) to 125 (light line) to 200 (dotted line). The intercept does not change, but the width increases. The bottom figure shows the correlation function when the multiplicity of the events used to construct the correlation function is fixed at a few (heavy line), 200 (light line) and 400 (dotted line). The intercept then falls for high multiplicities.

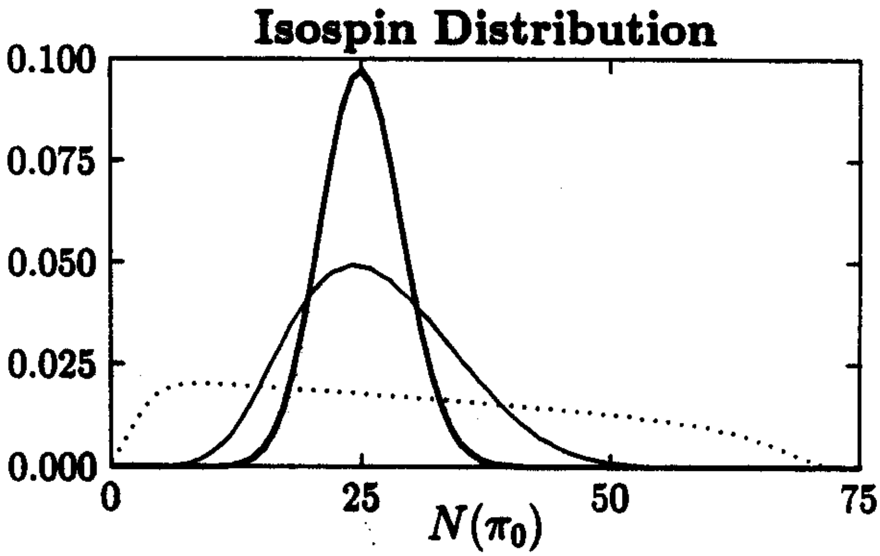

Figure 6. The probability of emitting N π_0 s given that 75 pions were emitted is shown for gaussian source sizes of .7 fm (dotted line), 1.5 fm (light line) and large R (heavy line). The effect of the symmetrization is to enhance the events where all the pions are of the same flavor, which broadens the distribution when R is small.