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Abstract

Correlators of the octet baryons in the hot pion gas are studied in the framework of the QCD sum rule. The condensates appearing in the OPE side of the correlators become T-dependent through the interaction with thermal pions. We present an explicit demonstration that the $O(T^2)$-dependence of the condensates is completely compensated by the change of the pole residue and the $\pi + B \to B'$ scattering effect in the spectral functions. Therefore the baryon masses are constant to this order, although $\langle \bar{u}u \rangle_T \simeq \langle \bar{u}u \rangle_0 (1 - T^2/8f_\pi^2)$, which is consistent with the chiral symmetry constraint by Leutwyler and Smilga.

1 Introduction

Modification of the hadronic properties at finite temperature has been receiving greater attention in connection with the relativistic heavy ion collisions such as RHIC and LHC. Although the finite-T properties of various mesons have been extensively studied in the literature using QCD Sum Rules (QSR) [1, 2, 3] and effective theories for QCD [4, 5, 6, 7], baryon properties have not been seriously investigated except for the nucleon [8, 9]. In principle, the change of baryon masses should manifest itself in the yield modification and the threshold behaviour of lepton pairs coming from the baryon-antibaryon annihilation in the relativistic heavy ion collisions. Therefore it is a pressing issue to have a QCD prediction for the finite-T behavior of the baryon correlators.

Since the work by Shifman, Vainstein and Zakharov [10], the QCD sum rule method has been extensively used as a systematic tool to study various resonance properties based on QCD (see [11] for a review). Especially, the masses of octet and decuplet baryons have been well reproduced in terms of the vacuum condensates [12, 13, 14, 15]. Hence it is worth while to extend this strongly QCD motivated phenomenology to study finite-T behavior of baryon correlators. In this paper we shall analyze the correlators of the octet baryons (N, Λ, Σ and Ξ) at finite temperature. The source currents we will investigate are the following interpolating fields which have been used in QSR:

$$\eta^N(x) = \epsilon^{abc}u^a(x)C\gamma_\mu u^b(x)\gamma_5\gamma_\mu d^c(x), \tag{1.1}$$

$$\eta^\Lambda(x) = \sqrt{\frac{2}{3}}\epsilon^{abc}\left[u^a(x)C\gamma_\mu s^b(x)\gamma_5\gamma_\mu d^c(x) - d^a(x)C\gamma_\mu s^b(x)\gamma_5\gamma_\mu u^c(x)\right], \tag{1.2}$$

$$\eta^\Sigma(x) = \epsilon^{abc}u^a(x)C\gamma_\mu u^b(x)\gamma_5\gamma_\mu s^c(x), \tag{1.3}$$

$$\eta^\Xi(x) = -\epsilon^{abc}s^a(x)C\gamma_\mu s^b(x)\gamma_5\gamma_\mu u^c(x), \tag{1.4}$$

where a, b, c are the color indices and C is the charge conjugation matrix. Although one can choose other sets of source currents for the octet baryons, it has been known that the above combination gives the best description of the octet baryon masses.

A useful quantity for studying the finite-T behavior of baryons is the retarded correlation function for the above fields [16]:

$$\Pi^R_{\alpha\beta}(\omega,\mathbf{q},T) = i\int d^4x\, e^{iqx}\theta(x_0)\langle\eta_\alpha(x)\overline{\eta}_\beta(0) + \overline{\eta}_\beta(0)\eta_\alpha(x)\rangle_T, \tag{1.5}$$

where $q = (\omega, \mathbf{q})$ is the external four momentum, $\langle\mathcal{O}\rangle = Tr\left(\mathcal{O}e^{-H/T}\right)/Z$ $(Z = Tr(e^{-H/T}))$ is the thermal average for the operator $\mathcal{O}$ with H being the QCD hamiltonian, and α and β are the spinor indices of the interpolating fields (1.1)–(1.4). The retarded correlator satisfies the dispersion relation with an appropriate spectral function $\rho(\omega, \mathbf{q}, T)$:

$$\Pi^R_{\alpha\beta}(\omega,\mathbf{q},T) = \int_{-\infty}^{\infty} du\, \frac{\rho_{\alpha\beta}(u,\mathbf{q},T)}{u-\omega-i\epsilon}. \tag{1.6}$$

In the QCD sum rule method, one applies the operator product expansion (OPE) to the left hand side of the correlator in the deep Euclidean region $Q^2 = -q^2 \to \infty$ and adjusts the resonance parameters (resonance masses, pole residues, width of the resonances, continuum threshold, etc) in the spectral function ρ so as to reproduce the OPE side of the correlator. This procedure provides us with an expression for the resonance parameters in terms of the condensates. Therefore, in the QSR approach, the change of the condensates at finite-T naturally causes a response of the baryons as a change of their properties at finite-T. For example, one might naively expect that the nucleon mass will drop at finite temperature as the chiral order parameter decreases.

On the other hand, Leutwyler and Smilga [8] considered the nucleon correlator in a thermal pion gas and showed that the nucleon mass does not have $O(T^2)$-dependence: In the thermal pion gas, the T-dependence of the nucleon correlator is associated with the pion-nucleon forward scattering amplitude. It becomes zero in the soft pion limit (Adler's zero) and thus the real part of the self-energy becomes zero at the pole position of $T = 0$. The same statement is also true for all the octet baryons.

The purpose of this paper is to demonstrate how this seeming contradiction can be reconciled in the framework of the finite-T QCD sum rules. We organize the QSR at $T \neq 0$ for the nucleon in the dilute pion gas [2]. This approximation should be valid at relatively low temperature ($T \leq 150$ MeV) well inside the confined phase. In this framework, the condensates appearing in the OPE side of the correlators receive T-dependence through the pion matrix elements of the same operators. We pay particular attention to the effect of the $\pi + N \to N$ scattering in the phenomenological side of the QSR as was suggested by Eletsky [9]. Then it can be shown that the $O(T^2)$-dependence of the condensates is exactly compensated by this scattering and the mass does not have $O(T^2)$-dependence as was found by Leutwyler and Smilga. Thus the naive expectation motivated by the Ioffe's mass formula for the nucleon, $M_N(T) \simeq (-2(2\pi)^2\langle\overline{u}u\rangle_T)^{1/3}$, does not work at finite-T. The situation is analogous for the other octet baryons.

The construction of this paper is the following: In section 2, after briefly summarizing our finite-T QSR for the case of the nucleon along the line of [2], we present an explicit demonstration that the nucleon mass does not have a $O(T^2)$-dependence in the QSR. The discussion for the hyperons is similar. So we will not repeat it, but present some of the formulas in the Appendix. In section 3, we shall analyze the baryon correlators in the pion gas in terms of PCAC without using QCD sum rules. We shall see that the octet baryon correlators at finite-T can be written in terms of the correlators at $T = 0$ with T-dependent coefficients and our OPE expression derived in section 2 is consistent with those relations. This explains why the pole positions of the octet baryon correlators do not have $O(T^2)$-dependence. Section 4 is devoted to a brief summary and outlook.

2 QCD Sum Rules for Nucleon at $T \neq 0$

2.1 OPE at $T \neq 0$

We shall first discuss the QCD side of the correlator. For simplicity, we study the nucleon at rest, i.e. $\boldsymbol{q} = \boldsymbol{0}$. In the region, $Q^2 = -q^2 = -\omega^2 > 0$, the retarded correlator (1.5) is identical to the causal correlator:

$$\Pi_{\alpha\beta}(\omega, \boldsymbol{q}, T) = i \int d^4x\, e^{iqx} \langle T\left(\eta_\alpha(x)\overline{\eta}_\beta(0)\right)\rangle_T. \tag{2.1}$$

At $T \neq 0$, $\Pi_{\alpha\beta}(q,T)$ can be decomposed into the three scalar components:

$$\Pi_{\alpha\beta}(q,T) = \Pi_1(q,T)\delta_{\alpha\beta} + \Pi_2(q,T)\not{q}_{\alpha\beta} + \Pi_3(q,T)\not{u}_{\alpha\beta}, \tag{2.2}$$

where u_μ is the average four-flow velocity of the medium equal to $u_\mu = (1,0,0,0)$ in the rest frame. In the deep Euclidean region $Q^2 \to \infty$, one can apply OPE to $\Pi(q,T)$, which, for Π_2 as an example, can be schematically written as,

$$\begin{aligned}\Pi_2(q,T) &= \sum_i C_i^{\mu_1\cdots\mu_s}(q,\mu)\mathcal{O}^{i(d,s)}_{\mu_1\cdots\mu_s}(\mu) \\ &= \frac{-1}{64\pi^4}q^4\ln(Q^2) + \left(\langle\mathcal{O}^{(4,0)}\rangle_T + \frac{q^\mu q^\nu}{q^2}\langle\mathcal{O}^{(4,2)}_{\mu\nu}\rangle_T\right)\ln(Q^2) \\ &+ \frac{1}{q^2}\left(\langle\mathcal{O}^{(6,0)}\rangle_T + \frac{q^\mu q^\nu}{q^2}\langle\mathcal{O}^{(6,2)}_{\mu\nu}\rangle_T + \frac{q^\mu q^\nu q^\lambda q^\sigma}{q^4}\langle\mathcal{O}^{(6,4)}_{\mu\nu\lambda\sigma}\rangle_T\right) + \cdots,\end{aligned} \tag{2.3}$$

where the i-th local operator $\mathcal{O}^{i(d,s)}_{\mu_1\cdots\mu_s}(\mu)$ renormalized at the scale μ has dimension-d and spin-s with s Lorentz indices and $C_i^{\mu_1\cdots\mu_s}(q,\mu)$ is the corresponding Wilson coefficient. As a complete set of the local operators in (2.3), one can always choose symmetric and traceless operators with respect to all Lorentz indices. We will hereafter assume this symmetry condition for all nonscalar operators. In the above equation the following two features peculiar to $T \neq 0$ are implemented [2]:

(i) T-dependence of the correlators appears only as a thermal average of the local operators in the OPE as a consequence of the QCD factorization. This is indeed natural if we note that such a soft effect should be ascribed to the condensates $\langle\mathcal{O}^i\rangle_T$ as long as the temperature is low enough compared to the separation scale μ, i.e. $T \ll \mu \ll Q$.

(ii) At $T \neq 0$, there is no Lorentz invariance due to the presence of the thermal factor $e^{-H/T}$, and hence nonscalar operators survive as condensates:

$$\langle\mathcal{O}^i_{\mu_1\cdots\mu_s}(\mu)\rangle_T = (u_{\mu_1}\cdots u_{\mu_s} - \text{traces})a^i(\mu,T). \tag{2.4}$$

At relatively low temperature in the confined phase, the system can be regarded as a non-interacting gas of Goldstone bosons (pions). In this approximation, T-dependence of the condensates can be written as

$$\langle\mathcal{O}^i\rangle_T \simeq \langle\mathcal{O}^i\rangle + \sum_{a=1}^{3}\int\frac{d^3p}{2\varepsilon(2\pi)^3}\langle\pi^a(\boldsymbol{p})|\mathcal{O}^i|\pi^a(\boldsymbol{p})\rangle n_B(\varepsilon/T), \tag{2.5}$$

where $\varepsilon = \sqrt{\boldsymbol{p}^2 + m_\pi^2}$, a denotes the isospin index, $n_B(x) = [e^x - 1]^{-1}$ is the Bose-Einstein distribution function, and $\langle\cdot\rangle$ is the usual vacuum average. Here we have used the covariant normalization for the pion state: $\langle\pi^a(\boldsymbol{p})|\pi^b(\boldsymbol{p}')\rangle = 2\varepsilon(2\pi)^3\delta^{ab}\delta^3(\boldsymbol{p}-\boldsymbol{p}')$. Thus we need pion matrix elements of the local operators appearing in the OPE to carry out the finite-T sum rule. For the scalar operators $\mathcal{O}^{(d,0)}$, we can apply the soft pion theorem

$$\langle\pi^a(\boldsymbol{p})|\mathcal{O}|\pi^b(\boldsymbol{p})\rangle = \frac{-1}{f_\pi^2}\langle 0|\left[\mathcal{F}_5^a,\left[\mathcal{F}_5^b,\mathcal{O}\right]\right]|0\rangle + O\left(\frac{m_\pi^2}{\Lambda_{HAD}^2}\right), \tag{2.6}$$

where Λ_{HAD} is a typical hadronic scale of $O(1\,\mathrm{GeV})$ and $\mathcal{F}_5^a$ is the isovector axial charge defined by

$$\mathcal{F}_5^a = \int d^3x\, \overline{q}(x)\gamma_0\gamma_5 \frac{\tau^a}{2} q(x). \tag{2.7}$$

Pion matrix elements of the nonscalar operators are associated with the pion structure functions measurable in the hard processes (deep inelastic scattering, Drell-Yan, direct photon production etc.). Experimentally, however, only twist-2 part of these matrix elements are known to some extent, and therefore we have to await future precise measurements of the twist-4 pion structure functions to carry out the satisfactory QSR at finite-T. Pion matrix elements of these operators read

$$\langle \pi(p)|\mathcal{O}^i_{\mu_1\cdots\mu_s}|\pi(p)\rangle \sim (p_{\mu_1}\cdots p_{\mu_s} - \mathrm{traces})A_i(\mu), \tag{2.8}$$

and therefore the contribution of nonscalar condensates becomes the effect of $O(T^4)$ or higher in T ($m_\pi \sim T$ is assumed) as is easily seen by inserting (2.8) into (2.5). In [2] we found that these effects can be neglected below $T = 160$ MeV. We shall consistently ignore these condensates in this work. For the same reason, $\Pi_3(q,T)$ also becomes higher order with respect to temperature. The effect of the heavier resonances (K, η etc) was also found to be negligible at $T \leq 160$ MeV because of the suppression coming from the distribution function $\sim e^{-m_K/T}$ [2].

Applying the soft pion theorem to scalar operators appearing in the OPE of the nucleon correlators, we get

$$\Pi_1^N(q,T) = \frac{1}{4\pi^2}\langle\overline{u}u\rangle\left(1-\frac{\zeta}{8}\right)q^2\ln(Q^2), \tag{2.9}$$

$$\Pi_2^N(q,T) = \frac{-1}{64\pi^4}q^4\ln(Q^2) - \frac{1}{32\pi^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\ln(Q^2) - \frac{2\langle\overline{u}u\rangle^2}{3q^2}, \tag{2.10}$$

where $\zeta = (T^2/f_\pi^2)B_1(m_\pi/T)$ with

$$B_1(z) = \frac{6}{\pi^2}\int_z^\infty dy\, \sqrt{y^2-z^2}\frac{1}{e^y-1}. \tag{2.11}$$

In (2.9) and (2.10) we note the following points:

(i) Although we discarded the terms proportional to m_u and m_d because of their smallness, we included the pion mass correction coming from the Bose-Einstein distribution, since it appears in the form of m_π/T in B_1. B_1 approaches 1 at $m_\pi \ll T$, while it is strongly suppressed as $B_1 \sim e^{-m_\pi/T}$ at $m_\pi \gg T$.

(ii) The chiral condensate $\langle\overline{u}u\rangle_T$ changes as $\langle\overline{u}u\rangle_0(1-\zeta/8)$. Its T-dependence was calculated by the chiral perturbation theory including up to $O(T^6)$ effects [17], which gives the same coefficient as above for the $O(T^2)$ effect. In the temperature range $T \leq 160$ MeV, both calculations agree well within 5 %.

(iii) By using the QCD trace anomaly, T-dependence of $\langle\frac{\alpha_s}{\pi}G^2\rangle_T$ can be estimated [2]. It gives a negligible change of the condensate at finite T (0.5% at $T = 200$ MeV). We thus ignored its T-dependence in (2.10).

(iv) The four-quark condensate in the nucleon channel turned out to be T-independent, which is quite different from the behavior of the square of the chiral order parameter. Here we used the vacuum saturation assumption, i.e., $\langle(\overline{u}\Gamma u)^2\rangle \to \langle\overline{u}u\rangle^2$ *after* applying the soft pion theorem, as is usually adopted in the vacuum QSR. The calculation of the four-quark condensates is somewhat tedious, so we shall present a nonfactorized form of the four-quark condensates for the octet baryons in the Appendix. There one sees that the T-dependence of the four-quark condensates is different in different channels. For example, the four-quark condensate in the nucleon channel is T-independent, while it is proportional to $1-\zeta/6$ in the Π_1 structure in the Σ-channel (See also (A.11)–(A.15) in the Appendix).

2.2 Spectral function

In the pion gas, the spectral function for the nucleon current acquires a contribution from the scattering with the thermal pions: $\pi + N \to N, \Delta$ etc. The effect of these scatterings has to be taken into account in the spectral function before the change of the condensates is ascribed to the shift of the pole position. For the purpose of identifying these contributions, we introduce the expression for the spectral function[16]:

$$\begin{aligned}\rho_{\alpha\beta}(\omega,\boldsymbol{q},T) &= \frac{1}{\pi}\mathrm{Im}\Pi^R_{\alpha\beta}(\omega,\boldsymbol{q},T)\\ &= \frac{1}{Z}(2\pi)^3\sum_{n,m}\langle n|\eta_\alpha(0)|m\rangle\langle m|\overline{\eta}_\beta(0)|n\rangle\\ &\qquad\times(e^{-\varepsilon_n/T}+e^{-\varepsilon_m/T})\delta(\omega-\omega_{mn})\delta^{(3)}(\boldsymbol{q}-\boldsymbol{p}_{mn}),\end{aligned}\tag{2.12}$$

where the states $|m\rangle$ and $|n\rangle$ have the four momentum $(\varepsilon_m,\boldsymbol{p}_m)$ and $(\varepsilon_n,\boldsymbol{p}_n)$, respectively, and $\omega_{mn}=\varepsilon_m-\varepsilon_n$, $\boldsymbol{p}_{mn}=\boldsymbol{p}_m-\boldsymbol{p}_n$. If we put $|n\rangle=|0\rangle$, $|m\rangle=|N(p)\rangle$ ($|m\rangle=|0\rangle$, $|n\rangle=|\overline{N}(p)\rangle$) in (2.12), this is the contribution from the nucleon (anti-nucleon) at $T=0$. By introducing the nucleon and the anti-nucleon spinor by the relation

$$\langle 0|\eta_\alpha(0)|N(p)\rangle=\lambda_N u_\alpha(p),\qquad \langle 0|\overline{\eta}_\alpha(0)|\overline{N}(p)\rangle=\lambda_N\overline{v}_\alpha(p)\tag{2.13}$$

with the normalization $\overline{u}(p)u(p)=2M_N$ and $\overline{v}(p)v(p)=-2M_N$, we get for this contribution

$$\rho(\omega,\boldsymbol{q})=\frac{\lambda_N^2}{2p_0}(\not{q}+M_N)\Big(\delta(\omega-p^0)-\delta(\omega+p^0)\Big)\tag{2.14}$$

with $p^0=\sqrt{\boldsymbol{q}^2+M_N^2}$. Using the Borel sum rule method, we can study the T-dependence of the mass $M_N(T)$, of the pole residue $\lambda_N(T)$, and of the continuum threshold $S_0(T)$. However, it is difficult to incorporate the effects of the width and the scattering contribution induced in the pion gas. Thus we shall first list up the effects which should not be associated with the change of the above three resonance parameters. Then we put these additional structures at $T\neq 0$ in the spectral function when we carry out the Borel sum rule.

(i) $\pi+N\to N$ ($\pi+\overline{N}\to\overline{N}$) contribution; $|n\rangle=|\pi(k)\rangle$, $|m\rangle=|N(p)\rangle$ ($|n\rangle=|\overline{N}(p)\rangle$, $|m\rangle=|\pi(k)\rangle$): By taking into account the Bose symmetrization among pions which equally fill both $|n\rangle$ and $|m\rangle$, but do not interact with the nucleon current, we arrive at

$$\rho^{\pi+N\to N}(q,T) = (2\pi)^3\int\frac{d^3k}{(2\pi)^3 2k^0}n_B(k^0/T)\int\frac{d^3p}{(2\pi)^3 2p^0}$$

$$\times \left[\sum_{\text{spin}=\pm 1/2} \sum_a \langle \pi^a(k)|\eta_\alpha(0)|N(p)\rangle\langle N(p)|\overline{\eta}_\beta(0)|\pi^a(k)\rangle \right.$$
$$\times \left(\delta(\omega - p^0 + k^0)\delta^{(3)}(\boldsymbol{q} - \boldsymbol{p} + \boldsymbol{k}) + \delta(\omega - p^0 - k^0)\delta^{(3)}(\boldsymbol{q} - \boldsymbol{p} - \boldsymbol{k})\right)$$
$$+ \sum_{\text{spin}=\pm 1/2} \sum_a \langle \overline{N}(p)|\eta_\alpha(0)|\pi^a(k)\rangle\langle \pi^a(k)|\overline{\eta}_\beta(0)|\overline{N}(p)\rangle$$
$$\left. \times \left(\delta(\omega + p^0 + k^0)\delta^{(3)}(\boldsymbol{q} + \boldsymbol{p} + \boldsymbol{k}) + \delta(\omega + p^0 - k^0)\delta^{(3)}(\boldsymbol{q} + \boldsymbol{p} - \boldsymbol{k})\right)\right]. \tag{2.15}$$

Here we discarded the thermal factor for the nucleon (Fermi distribution function), since $1/(e^{M_N/T} - 1) \simeq 0$ at $T \leq 200$ MeV. We have to include two kinds of contribution to the matrix element $\langle \pi^a(k)|\eta_\alpha(0)|N(p)\rangle$ [18].

(a) Direct coupling of η^N to the pion (Fig. 1): This contribution can be calculated by applying the soft pion theorem:

$$\langle \pi^a(k)|\eta^{\rm p}_\alpha(0)|N(p)\rangle = \frac{-i}{f_\pi}\langle 0|\,[\mathcal{F}^a_5, \eta^{\rm p}_\alpha(0)]\,|N(p)\rangle, \tag{2.16}$$

$$[\mathcal{F}^a_5, \eta^{\rm p}_\alpha(0)] = \left(\frac{\tau^a}{2}\right)_{22} \gamma_5 \eta^{\rm p} - \left(\frac{\tau^a}{2}\right)_{12} \gamma_5 \eta^{\rm n}, \tag{2.17}$$

where $\eta^{\rm p}$ is the proton current defined in (1.1) and $\eta^{\rm n} = -\epsilon^{abc} d^a(x) C\gamma_\mu d^b(x)\gamma_5\gamma_\mu u^c(x)$ is the neutron current. Using (2.16) and (2.17) in (2.15) and putting $\boldsymbol{q} = \boldsymbol{0}$, we arrive at [1]

$$\rho^{\pi+N\to N}_{(0)}(q,T) = \left(\frac{-3\lambda_N^2}{4f_\pi^2}\right) \int \frac{d^3k}{(2\pi)^3 2k^0 2p^0} n_B(k^0/T) \times$$
$$\left[\gamma_5(\not{p} + M_N)\gamma_5 \left(\delta(\omega - p^0 + k^0) + \delta(\omega - p^0 - k^0)\right)\right.$$
$$\left. + \gamma_5(\not{p} - M_N)\gamma_5 \left(\delta(\omega + p^0 + k^0) + \delta(\omega + p^0 - k^0)\right)\right]. \tag{2.18}$$

Since we used the soft pion theorem in calculating the matrix element in (2.16), it suffices to consider the soft pion limit $k = (k^0, \boldsymbol{k}) \to 0$ in (2.18). Then (2.18) becomes

$$\rho^{\pi+N\to N}_{(0)}(q,T) = \left(\frac{\lambda_N^2 \zeta}{32M_N}\right)\left(\delta(\omega - M_N) - \delta(\omega + M_N)\right)(\not{q} - M_N). \tag{2.19}$$

Equation (2.18) was derived by Eletsky [9] taking the imaginary part of the corresponding retarded correlator.[2] There it was shown that (2.18) has a localized structure around the nucleon and anti-nucleon poles, which can be well approximated by (2.19).

[1] Here $\boldsymbol{p}$ has $\boldsymbol{k}$ or $-\boldsymbol{k}$ corresponding to two $\delta^{(3)}$-functions in (2.15).

[2] The result given in eq. (11) of [9] is two times larger than the one given in (2.18) by mistake. I thank V. L. Eletsky for correspondence to clarify this point.

(b) Coupling of η^N to the nucleon which interacts with π (Fig. 2): This contribution can be calculated by using the vertex

$$\langle\pi(k)|\eta(0)|N(p)\rangle = \lambda_N\langle\pi(k)|\psi^N(0)|N(p)\rangle = \frac{\lambda_N g_{\pi NN}}{2M_N}\frac{1}{\not p - \not k - M_N}\not k\gamma_5 u(p), \tag{2.20}$$

where we assumed the $\pi - N - N$ interaction lagrangian as

$$\mathcal{L}_{\pi NN} = \frac{g_{\pi NN}}{2M_N}\overline{N}\gamma_5\gamma_\mu\vec{\tau}N\partial^\mu\vec{\pi}. \tag{2.21}$$

Inserting (2.20) into (2.15), one obtains in the $\boldsymbol{q} = \mathbf{0}$ limit:

$$\begin{aligned}
\rho_{(1)}^{\pi+N\to N}(q,T) \;=\; & \int \frac{d^3k}{(2\pi)^3 2k^0 2p^0} n_B(k^0/T)\, 3\left(\frac{\lambda_N g_{\pi NN}}{2M_N}\right)^2 \frac{1}{(q^2 - M_N)^2} \\
& \times \left[(\not q + M_N)\not k\gamma_5\,(\not p + M_N)\not k\gamma_5\,(\not q + M_N)\right. \\
& \qquad \times \left(\delta(\omega - p^0 + k^0) + \delta(\omega - p^0 - k^0)\right) \\
& + (-\not q + M_N)\not k\gamma_5\,(\not p - M_N)\not k\gamma_5\,(-\not q + M_N) \\
& \qquad \times \left.\left(\delta(\omega + p^0 + k^0) + \delta(\omega + p^0 - k^0)\right)\right]
\end{aligned} \tag{2.22}$$

$$= \left(\frac{g_A{}^2\lambda_N^2\zeta}{32M_N}\right)(\delta(\omega - M_N) - \delta(\omega + M_N))\,(\not q + M_N)\,. \tag{2.23}$$

In (2.23), we used the Goldberger-Treiman relation $g_{\pi NN}/M_N = g_A/f_\pi$. Note that in (2.19) the two structures proportional to $\mathbf{1}$ and $\not q$ have opposite signs while they have the same sign in (2.23).

One can easily check that the crossing term between (a) and (b) disappears.

In the above (a) and (b), we have obtained the $\pi + N \to N$ scattering contribution to the spectral function at $\boldsymbol{q} = \mathbf{0}$ in the form:

$$\begin{aligned}
\rho^{\pi+N\to N}(\omega,T) &= \rho_{(0)}^{\pi+N\to N}(q,T) + \rho_{(1)}^{\pi+N\to N}(q,T) \\
&= \left(\frac{\lambda_N^2\zeta}{32M_N}\right)(\delta(\omega - M_N) - \delta(\omega + M_N))\left\{\left(1 + g_A^2\right)\not q - \left(1 - g_A^2\right)M_N\right\}.
\end{aligned} \tag{2.24}$$

We note that up to $O(T^2)$ the use of the nucleon mass and the pole residue of $T = 0$ in (2.24) is consistent with our treatment of the OPE side because of the presence of the factor ζ in (2.24). (From a physical ground, one might wish to replace them by those at $T \neq 0$. But these two procedures cause only an $O(T^4)$ difference in the final result.)

In (2.24), the $\pi + N \to N$ scattering term eventually becomes an effective delta function at the pole position of $T = 0$. However, it contributes differently to the $\mathbf{1}$ and $\not q$ structures of the correlators. Therefore its effect must be taken into account in the spectral function when we make use of the Borel sum rule method. Otherwise, an erroneous mass shift of the nucleon would occur.

(ii) $\pi+N \to \Delta$ contribution: Using the lowest order piece of the chiral invariant $\pi - N - \Delta$ interaction lagrangian $\mathcal{L}_{\pi N\Delta} \sim g_{\pi N\Delta}\overline{\Delta}_\mu\gamma_5 N\partial^\mu\pi$ [19], we can calculate the vertex:

$$\langle\pi(k)|\eta(0)|\Delta(p)\rangle \sim \frac{\lambda_N g_{\pi N\Delta}}{\not p - \not k - M_N}k^\mu\Delta_\mu(p), \tag{2.25}$$

where $\Delta_\mu(p)$ is the Rarita-Shwinger spinor for Δ. The sum over spin for Δ gives the projection operator [19]

$$\sum_{spin}\Delta_\mu(p)\Delta_\nu(p) = \mathcal{P}_{\mu\nu} = \left[g_{\mu\nu} - \frac{2}{3M_\Delta^2}p_\mu p_\nu - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3M_\Delta}(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\right](\not p + M_\Delta). \tag{2.26}$$

In the soft pion limit, the $\pi + N \to \Delta$ scattering term appears at $\omega \sim M_\Delta$ where the nucleon propagator in (2.25) becomes proportional to $1/(M_\Delta^2 - M_N^2)$ and the vertex contribution is $k^\mu k^\nu \mathcal{P}_{\mu\nu} \sim k^2$. Therefore the $\pi + N \to \Delta$ scattering contribution becomes $O(T^4)$ and we shall discard it.

One can easily repeat the same steps as above (i) (a) (b) and (ii) for the other octet baryons. By the same reason as (ii), the effect of the transition to the decuplet baryons ($\pi + \Sigma \to \Sigma^*(1385)$, $\pi + \Xi \to \Xi^*(1530)$) is $O(T^4)$. Even among octet baryons, (i)(b) type scattering between Λ and Σ is $O(T^4)$ because of their mass difference.

2.3 Borel sum rule

We are now ready to perform the Borel sum rule analysis for the octet baryons. We assumed that the finite-T medium is the dilute pion gas which has zero chemical potential, and thus the charge conjugation symmetry is preserved, i.e., there appears no splitting between the baryon and anti-baryon poles.[3] Therefore the spectral function for the nucleon reads at $\boldsymbol{q} = 0$

$$\begin{aligned}\rho^N(u,T) &= \lambda_N^2(T)\delta(u^2 - M_N^2(T))(\not q + M_N(T))\mathrm{sign}(u) + \rho^{\pi+N\to N}(u,T) \\ &\quad +(\text{continuum by step function}),\end{aligned} \tag{2.27}$$

where $\rho^{\pi+N\to N}$ is defined in (2.24). Corresponding to (2.2), we decompose the spectral function as $\rho^N(q) = \rho_1(q) + \not q\rho_2(q)$. Then both ρ_1 and ρ_2 satisfy the relation $\rho_i(-\omega, \boldsymbol{q} = 0) = -\rho_i(\omega, \boldsymbol{q} = 0)$ $(i = 1, 2)$ as was obtained in the previous subsection. In the deep Euclidean region, the dispersion relation (1.6) can be written as

$$\Pi_i^R(\omega^2 = -Q^2, \boldsymbol{q} = 0, T) = \Pi_i(Q^2, T) = \int_0^\infty du \frac{2u\rho_i(u, \boldsymbol{q} = 0)}{u^2 + Q^2}. \quad (i = 1,\ 2) \tag{2.28}$$

[3]This is in contrast to the system with a finite baryon number such as the nuclear matter. In order to organize sum rules for baryons in such a system, the dispersion relation needs some modification. See [20] for the detail.

Applying the Borel transform

$$\begin{aligned}
\Pi_i(M^2,T) &\equiv \hat{B}_M \Pi_i(Q^2,T) \\
&\equiv \lim_{\begin{pmatrix} Q^2, n\to\infty \\ Q^2/n = M^2:\text{fixed}\end{pmatrix}} \frac{1}{(n-1)!}(Q^2)^n \left(\frac{-d}{dQ^2}\right)^n \Pi_i(Q^2,T) &(2.29)\\
&= \int_0^\infty 2udu\, e^{-u^2/M^2}\rho_i(u,T) &(2.30)
\end{aligned}$$

to (2.9) and (2.10) with the spectral function (2.27), we get the following relation:

$$2a\left(1-\frac{\zeta}{8}\right)M^4 = \tilde{\lambda}_N^2(T)M_N(T)e^{-M_N^2(T)/M^2} - \frac{\tilde{\lambda}_N^2(0)(1-g_A{}^2)M_N(0)\zeta}{16}e^{-M_N^2(0)/M^2}, \quad (2.31)$$

$$M^6 + M^2 b + \frac{4}{3}a^2 = \tilde{\lambda}_N^2(T)e^{-M_N^2(T)/M^2} + \frac{\tilde{\lambda}_N^2(0)(1+g_A{}^2)\zeta}{16}e^{-M_N^2(0)/M^2}, \quad (2.32)$$

where

$$\begin{aligned}
a &= -(2\pi)^2\langle\bar{u}u\rangle, &(2.33)\\
b &= \pi^2\langle\frac{\alpha_s}{\pi}G^2\rangle, &(2.34)\\
\tilde{\lambda}_N^2(T) &= 2(2\pi)^4\lambda_N^2(T). &(2.35)
\end{aligned}$$

From (2.31) and (2.32) one gets the expression for the nucleon mass:

$$M_N(T) = \frac{2a\left(1-\frac{\zeta}{8}\right)M^4 + \left(\tilde{\lambda}_N^2(0)(1-g_A{}^2)M_N(0)\zeta/16\right)e^{-M_N^2(0)/M^2}}{M^6 + M^2 b + \frac{4}{3}a^2 - \left(\tilde{\lambda}_N^2(0)(1+g_A{}^2)\zeta/16\right)e^{-M_N^2(0)/M^2}}. \quad (2.36)$$

In (2.36), we omitted the correction due to the continuum contribution for brevity. The formula which include the correction is given by the following replacement:

$$\begin{aligned}
M^6 &\to M^6\left\{1-\left(1+\frac{S_0}{M^2}+\frac{S_0^2}{2M^4}\right)e^{-S_0/M^2}\right\},\\
M^4 &\to M^4\left\{1-\left(1+\frac{S_0}{M^2}\right)e^{-S_0/M^2}\right\},\\
M^2 &\to M^2\left(1-e^{-S_0/M^2}\right). &(2.37)
\end{aligned}$$

An important consequence from (2.36) is that $M_N(T)$ is completely T-independent. In fact, if we replace $\tilde{\lambda}_N^2(0)$ in the numerator of (2.36) by the one obtained from (2.31) by setting

$T = 0$, and replace $\tilde{\lambda}_N^2(0)$ in the denominator of (2.36) by the one obtained from (2.32), we can easily see that the T-dependence disappears from $M_N(T)$. If we did not include the scattering term, the nucleon mass would behave as $M_N(T) = M_N(0)(1 - \zeta/8)$ as can be seen from (2.36). This means that the T-dependence of the condensates caused through the interaction with the thermal pions is completely compensated by the $\pi - N$ scattering term and the pole position does not move at least to order $O(T^2)$. This is consistent with the statement of Leutwyler and Smilga based on the chiral lagrangian [8]: If we calculate the self energy of the nucleon using the πNN effective lagrangian (2.21), we can easily check that the real part of the self energy is zero at the $T = 0$ pole position. This is a direct consequence of the Adler's consistency condition required for the use of PCAC [18] (in another word, (2.21) has a derivative coupling).

The pole residue changes as $\tilde{\lambda}_N^2(T) = \tilde{\lambda}_N^2(0)(1-(1+g_A^2)\zeta/16)$ as is seen from (2.31) and (2.32); the same result obtained by [8] in the chiral lagrangian approach.

From the above demonstration, it is clear that we have to take into account the new structure in the spectral function consistently with the T-dependence in the OPE side of the correlator. Otherwise the usual procedure in the sum rule leads to an artificial change of the resonance parameters.

In [21], the nucleon mass was calculated by a finite-T QCD sum rule method. The authors found a dropping nucleon mass even in the low temperature region as $\langle \overline{u}u\rangle_T$ decreased at finite-T. They did not take into account the $\pi + N \to N$ scattering effect and assumed that the T-dependence of the four-quark condensate is the same as $(\langle \overline{u}u\rangle_T)^2$. Although their calculation was not based on the pion gas approximation, the present consideration shows it is crucial to treat both the OPE side and the phenomenological side consistently. Correct treatment of the T-dependence of all the condensates is also required.

3 Analysis of Correlators using PCAC

In this section we shall examine the octet baryon correlators starting from the pion gas approximation without using OPE:

$$\Pi(q,T) \simeq \Pi(q,0) + i\int d^4x\, e^{iqx} \int \frac{d^3k}{(2\pi)^3 2k^0} n_B(k^0/T)\langle \pi^a(k)| T\left(\eta(x)\overline{\eta}(0)\right)|\pi^a(k)\rangle. \quad (3.1)$$

Applying the LSZ reduction formula for the pion, then using the PCAC relation $\partial^\mu A_\mu^a(x) = f_\pi m_\pi^2 \phi^a(x)$ for the pion field $\phi^a(x)$ and taking the soft pion limit, one arrives at

$$\begin{aligned}
\Pi(q,T) \;\simeq\; & \Pi(q,0) - \frac{i\delta^{ab}\zeta}{24}\int d^4x\, e^{iqx} \times \\
& \Big\{ \langle T\left(\left[\mathcal{F}_5^a(x^0), \left[\mathcal{F}_5^b(x^0), \eta(x)\right]\right]\overline{\eta}(0)\right)\rangle + \langle T\left(\eta(x)\left[\mathcal{F}_5^a(0), \left[\mathcal{F}_5^b(0), \overline{\eta}(0)\right]\right]\right)\rangle \\
& + \langle T\left(\left[\mathcal{F}_5^a(x^0), \eta(x)\right]\left[\mathcal{F}_5^b(0), \overline{\eta}(0)\right]\right)\rangle + \langle T\left(\left[\mathcal{F}_5^b(x^0), \eta(x)\right]\left[\mathcal{F}_5^a(0), \overline{\eta}(0)\right]\right)\rangle \Big\} \\
& + \cdots, \qquad (3.2)
\end{aligned}$$

where $+\cdots$ denotes the terms associated with the axial charges carried by the octet baryons (terms with g_A in the previous section), which becomes $O(T^4)$ or higher except at the pole position of $T=0$.[4] Utilizing the formula (2.17) together with

$$\delta^{ab}\left[\mathcal{F}_5^a,\left[\mathcal{F}_5^b,\eta^N\right]\right]=\frac{3}{4}\eta^N, \tag{3.3}$$

we obtain the following expression for the nucleon correlator at $T\neq 0$:

$$\Pi^N(q,T) = \left(1-\frac{\zeta}{16}\right)\Pi^N(q,0)-\frac{\zeta}{16}\gamma_5\Pi^N(q,0)\gamma_5+\cdots. \tag{3.4}$$

The second term of the r.h.s. of (3.4) corresponds to the $\pi+N\to N$ scattering term in the sum rule approach in the previous section and the first term of the r.h.s. of (3.4) is the modification of the residue of the nucleon current. The above analysis of the current simply tells us that the nucleon correlator at $T\neq 0$ can be written as a superposition of the same correlator at $T=0$ with T-dependent coefficients and there is no $O(T^2)$ shift of the pole position, which is consistent with the observation of [8]. (As was shown in [8] using (2.21), the contribution to the real part of the self-energy from $+\cdots$ in (3.4) becomes zero at the pole position but induces an $O(T^2)$ wave function renormalization.)

Equation (3.4) reads $\Pi_1^N(q,T)=\Pi_1^N(q,0)(1-\zeta/8)+\cdots$ and $\Pi_2^N(q,T)=\Pi_2^N(q,0)+\cdots$. The T-dependence of these two equations is the same as the OPE given in (2.9) and (2.10). This is the reason we observed no mass shift in the Borel sum rule. We remind the readers once again that the factorization of the four-quark condensate in the medium level $\langle(\overline{q}\Gamma q)^2\rangle_T\to\langle\overline{q}q\rangle_T^2$ is not justified. If we adopted this procedure in section 2, the T-dependence of the OPE side of the correlator would be different from (3.4).

It is easy to extend the above analysis to other octet baryon correlators. For this purpose we need the following commutators:

$$\left[\mathcal{F}_5^a,\eta^\Lambda\right] = \sqrt{\frac{2}{3}}\gamma_5\left[-\left(\frac{\tau^a}{2}\right)_{21}\eta^{\Sigma^+}+\left(\frac{\tau^a}{2}\right)_{12}\eta^{\Sigma^-}+\frac{1}{\sqrt{2}}\left(\left(\frac{\tau^a}{2}\right)_{11}-\left(\frac{\tau^a}{2}\right)_{22}\right)\eta^{\Sigma^0}\right], \tag{3.5}$$

$$\left[\mathcal{F}_5^a,\eta^{\Sigma^+}\right] = \frac{\tau_{12}^a}{\sqrt{2}}\gamma_5\eta^{\Lambda'}, \tag{3.6}$$

$$\left[\mathcal{F}_5^a,\eta^{\Xi^0}\right] = \left(\frac{\tau^a}{2}\right)_{11}\gamma_5\eta^{\Xi^0}+\left(\frac{\tau^a}{2}\right)_{12}\gamma_5\eta^{\Xi^-}, \tag{3.7}$$

$$\delta^{ab}\left[\mathcal{F}_5^a,\left[\mathcal{F}_5^b,\eta^\Lambda\right]\right] = -\sqrt{3}\eta^{\Lambda'}, \tag{3.8}$$

$$\delta^{ab}\left[\mathcal{F}_5^a,\left[\mathcal{F}_5^b,\eta^\Sigma\right]\right] = \eta^\Sigma, \tag{3.9}$$

$$\delta^{ab}\left[\mathcal{F}_5^a,\left[\mathcal{F}_5^b,\eta^\Xi\right]\right] = \frac{3}{4}\eta^\Xi, \tag{3.10}$$

where we introduced a new current $\eta^{\Lambda'}$ defined by

$$\eta^{\Lambda'}=\sqrt{2}\epsilon^{abc}u^aC\gamma_5\gamma_\mu d^b\gamma_\mu s^c, \tag{3.11}$$

[4]In the sum rule, we needed to integrate over the spectral functions, which is why the $O(T^2)$ contribution appeared.

in (3.6) and (3.8). By using these relations in (3.2), we obtain the following relations among the correlators:

$$\Pi^{\Lambda}(q,T) = \Pi^{\Lambda}(q,0) + \frac{\sqrt{3}\zeta}{24}\Pi^{\Lambda\Lambda'}(q,0) - \frac{\zeta}{12}\gamma_5\Pi^{\Sigma}(q,0)\gamma_5 + \cdots, \tag{3.12}$$

$$\Pi^{\Sigma}(q,T) = \left(1 - \frac{\zeta}{12}\right)\Pi^{\Sigma}(q,0) - \frac{\zeta}{12}\gamma_5\Pi^{\Lambda'}(q,0)\gamma_5 + \cdots, \tag{3.13}$$

$$\Pi^{\Xi}(q,T) = \left(1 - \frac{\zeta}{16}\right)\Pi^{\Xi}(q,0) - \frac{\zeta}{16}\gamma_5\Pi^{\Xi}(q,0)\gamma_5 + \cdots, \tag{3.14}$$

with

$$\Pi^{\Lambda\Lambda'}_{\alpha\beta}(q,0) = i\int d^4x\, e^{iqx}\langle T\left(\eta^{\Lambda}_{\alpha}(x)\overline{\eta}^{\Lambda'}_{\beta}(0) + \eta^{\Lambda'}_{\alpha}(x)\overline{\eta}^{\Lambda}_{\beta}(0)\right)\rangle. \tag{3.15}$$

The current $\eta^{\Lambda'}$ is anti-symmetric under the exchange between u and d quarks and thus $\eta^{\Lambda'}$ (or $\gamma_5\eta^{\Lambda'}$) should have some overlap with isoscalar strangeness=–1 baryons such as $\Lambda(1115)$, $\Lambda^*(1405)$ ($J^P = 1/2^-$) as well as the $\pi - \Sigma$ continuum contribution. As is shown in [15], there are 5 independent interpolating fields without derivatives for Λ. One can see by the Fierz rearrangement between s and u (or d) that $\eta^{\Lambda'}$ consists of η^{Λ} and those others. We tried to identify its structure by the vacuum QSR including the operators up to dimension-6, but it does not seem to have a dominant pole contribution. In any case, (3.13) tells us that the finite-T Σ-correlator can be written as the modification of the residue and the mixing with Λ' correlator. In principle, the second term of (3.12) also describes the modification of the residue. As for Ξ, the situation is completely parallel with the nucleon. The T-dependence of the OPE expressions (A.15) and (A.16) is consistent with (3.14) as they should.

4 Summary and Outlook

In this paper we have presented an explicit demonstration that the $O(T^2)$ dependence of the condensates which appear in the OPE of the octet baryon correlators is totally absorbed by the scattering terms $\pi + B \to B'$ and the modifications of the pole residues, in the framework of the QCD sum rules. This result is consistent with the statement by Leutwyler and Smilga [8]. The result stems from the fact that the baryon correlators in the thermal pion gas can be written as a superposition of the correlators of $T = 0$ with T-dependent coefficients up to $O(T^2)$. The procedure for achieving consistency with this relation is somewhat intricate in the QSR. Therefore one has to pay particular attention to the consistency between the assumption made to estimate the T-dependence of the condensates and the new structure appearing in the phenomenological spectral function.

In the chiral lagrangian approach, the $O(T^2)$ mass shift of baryons is caused by tadpole interactions such as $m_s KK\overline{B}B/f_\pi^2$ (K is the kaon field) [22]. In the $m_u = m_d = 0$, $m_s \neq 0$ limit studied in this work, the kaon or η field always accompanies the tadpole contribution as in the case of the above interaction. Thus without including kaons or η in the heat bath, baryons do not receive a $O(T^2)$ mass shift. However, the presence of those massive excitations is suppressed as $\sim e^{-m_K/T}$. For example, the above term in the effective lagrangian causes the mass shift of the order of $m_s T^2/(24 f_\pi^2) B_1(m_K/T) \sim 1$ MeV at $T = 150$ MeV. If we included the kaons and η's in the heat bath together with the nonzero strange-quark mass, the T-dependence of the OPE for the hyperons shown in (A.11)–(A.16) would be different from (3.12)–(3.14). This would lead to the $O(T^2)$ mass shift of the octet baryons, although it should be tiny because of $B_1(m_K/T)$ (~ 0.06 at $T = 150$ MeV).

To go beyond $O(T^2)$, one needs more information on the pion structure functions (twist-4 effects), $O(T^4)$ or higher T-dependence of the scalar condensates and the analysis of the structure Π_3 in (2.2) as well as more involved treatment for the phenomenological side (such as octet $\rightarrow$ decuplet transitions). These issues are beyond the scope of the present study. I hope the lesson we learned through the demonstration in this work will be useful for more advanced studies on these higher order effects.

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Appendix

Here we will present the nonfactorized form of the four-quark operators which appear in the OPE for the octet baryon currents. Nonscalar four-quark operators are associated with the twist-4 contribution in the pion structure function. We ignore them since they become $O(T^4)$ effect in the QCD sum rules at $T \neq 0$. We decompose the contribution of the four-quark condensate into the two pieces corresponding to the two structures of the correlators:

$$\Pi^{N,\Lambda,\Sigma,\Xi}_{4-\mathrm{quark}}(q) = \tilde{\Pi}_1^{N,\Lambda,\Sigma,\Xi} + \tilde{\Pi}_2^{N,\Lambda,\Sigma,\Xi}\not{q}. \tag{A.1}$$

(i) N:

$$\tilde{\Pi}_1^N(q) = 0, \tag{A.2}$$

$$\begin{aligned}\tilde{\Pi}_2^N(q) = \frac{-1}{q^2}\Bigg[&\left(1-\frac{1}{N_c}\right)\left\{\frac{5}{2}\langle\overline{u}\gamma_\mu u\overline{d}\gamma_\mu d\rangle + \frac{3}{2}\langle\overline{u}\gamma_\mu\gamma_5 u\overline{d}\gamma_\mu\gamma_5 d\rangle\right\} \\ &-\frac{1}{2}\left\{\frac{5}{2}\langle\overline{u}\gamma_\mu\lambda^a u\overline{d}\gamma_\mu\lambda^a d\rangle + \frac{3}{2}\langle\overline{u}\gamma_\mu\gamma_5\lambda^a u\overline{d}\gamma_\mu\gamma_5\lambda^a d\rangle\right\} \\ &+\left(1-\frac{1}{N_c}\right)\left\{\frac{1}{2}\langle(\overline{u}u)^2\rangle - \frac{1}{2}\langle(\overline{u}\gamma_5 u)^2\rangle - \frac{1}{4}\langle(\overline{u}\gamma_\mu\gamma_5 u)^2\rangle + \frac{1}{4}\langle(\overline{u}\gamma_\mu u)^2\rangle\right\} \\ &-\frac{1}{2}\left\{\frac{1}{2}\langle(\overline{u}\lambda^a u)^2\rangle - \frac{1}{2}\langle(\overline{u}\gamma_5\lambda^a u)^2\rangle - \frac{1}{4}\langle(\overline{u}\gamma_\mu\gamma_5\lambda^a u)^2\rangle + \frac{1}{4}\langle(\overline{u}\gamma_\mu\lambda^a u)^2\rangle\right\}\Bigg],\end{aligned} \tag{A.3}$$

where λ^a is the SU(3) color matrix and we explicitly kept the $N_c(=3)$ dependence. As is seen from (A.3), the four-quark operators always appear in the form of

$$\left(1-\frac{1}{N_c}\right)\{\overline{q}\Gamma q\overline{q'}\Gamma q' + \cdot\cdot\} - \frac{1}{2}\{\overline{q}\Gamma\lambda^a q\overline{q'}\Gamma\lambda^a q' + \cdot\cdot\}.$$

We will henceforth use the abbreviation $-\frac{1}{2}\{\text{with } \lambda^a\}$ to denote the second contribution.

(ii) Λ:

$$\begin{aligned}\tilde{\Pi}_1^\Lambda(q) = &\frac{-m_s}{3q^2}\left[\left(1-\frac{1}{N_c}\right)\left\{4\langle\overline{u}u\overline{d}d\rangle - \langle\overline{u}\sigma_{\mu\nu}u\overline{d}\sigma_{\mu\nu}d\rangle - 4\langle\overline{u}d\overline{d}u\rangle + \langle\overline{u}\sigma_{\mu\nu}d\overline{d}\sigma_{\mu\nu}u\rangle\right\}\right. \\ &\left.-\frac{1}{2}\{\text{with } \lambda^a\}\right] \\ &+\frac{m_s}{3q^2}\left[\left(1-\frac{1}{N_c}\right)\left\{\langle\overline{u}u\overline{s}s\rangle + \langle\overline{d}d\overline{s}s\rangle - \frac{1}{2}\langle\overline{u}\gamma_\mu u\overline{s}\gamma_\mu s\rangle - \frac{1}{2}\langle\overline{d}\gamma_\mu d\overline{s}\gamma_\mu s\rangle\right.\right. \\ &\left.\left.-\frac{1}{2}\langle\overline{u}\gamma_\mu\gamma_5 u\overline{s}\gamma_\mu\gamma_5 s\rangle - \frac{1}{2}\langle\overline{d}\gamma_\mu\gamma_5 d\overline{s}\gamma_\mu\gamma_5 s\rangle\right\}\right.\end{aligned}$$

$$-\frac{1}{2}\left\{\text{with } \lambda^a\right\}\Big], \tag{A.4}$$

$$\begin{aligned}
\tilde{\Pi}_2^{\Lambda}(q) &= \frac{-2}{3q^2}\left[\frac{1}{2}\left(1-\frac{1}{N_c}\right)\left\{\frac{5}{2}\langle\overline{u}\gamma_\mu u\overline{d}\gamma_\mu d\rangle + \frac{3}{2}\langle\overline{u}\gamma_\mu\gamma_5 u\overline{d}\gamma_\mu\gamma_5 d\rangle + \frac{5}{4}\langle\overline{u}\gamma_\mu u\overline{s}\gamma_\mu s\rangle\right.\right. \\
&\qquad \left. + \frac{3}{4}\langle\overline{u}\gamma_\mu\gamma_5 u\overline{s}\gamma_\mu\gamma_5 s\rangle + \frac{5}{4}\langle\overline{d}\gamma_\mu d\overline{s}\gamma_\mu s\rangle + \frac{3}{4}\langle\overline{d}\gamma_\mu\gamma_5 d\overline{s}\gamma_\mu\gamma_5 s\rangle\right\} \\
&\quad -\frac{1}{4}\left\{\text{with } \lambda^a\right\} \\
&\quad +\left(1-\frac{1}{N_c}\right)\left\{\langle\overline{u}u\overline{s}s\rangle - \langle\overline{u}\gamma_5 u\overline{s}\gamma_5 s\rangle + \frac{1}{2}\langle\overline{u}\gamma_\mu u\overline{s}\gamma_\mu s\rangle - \frac{1}{2}\langle\overline{u}\gamma_\mu\gamma_5 u\overline{s}\gamma_\mu\gamma_5 s\rangle\right. \\
&\qquad \left. + \langle\overline{d}d\overline{s}s\rangle - \langle\overline{d}\gamma_5 d\overline{s}\gamma_5 s\rangle + \frac{1}{2}\langle\overline{d}\gamma_\mu d\overline{s}\gamma_\mu s\rangle - \frac{1}{2}\langle\overline{d}\gamma_\mu\gamma_5 d\overline{s}\gamma_\mu\gamma_5 s\rangle\right\} \\
&\quad -\frac{1}{2}\left\{\text{with } \lambda^a\right\} \\
&\quad \left. -\frac{1}{2}\left(1-\frac{1}{N_c}\right)\left\{\frac{5}{2}\langle\overline{u}\gamma_\mu d\overline{d}\gamma_\mu u\rangle + \frac{3}{2}\langle\overline{u}\gamma_\mu\gamma_5 d\overline{d}\gamma_\mu\gamma_5 u\rangle\right\} + \frac{1}{4}\left\{\text{with } \lambda^a\right\}\right].
\end{aligned} \tag{A.5}$$

(iii) Σ:

$$\begin{aligned}
\tilde{\Pi}_1^{\Sigma}(q) &= \frac{m_s}{q^2}\left[\left(1-\frac{1}{N_c}\right)\left\{-\langle(\overline{u}u)^2\rangle + \langle(\overline{u}\gamma_5 u)^2\rangle - \frac{1}{2}\langle(\overline{u}\gamma_\mu u)^2\rangle + \frac{1}{2}\langle(\overline{u}\gamma_\mu\gamma_5 u)^2\rangle\right\}\right. \\
&\qquad -\frac{1}{2}\left\{\text{with } \lambda^a\right\} \\
&\quad \left. +\left(1-\frac{1}{N_c}\right)\left\{\langle\overline{u}\gamma_\mu u\overline{s}\gamma_\mu s\rangle - \langle\overline{u}\gamma_\mu\gamma_5 u\overline{s}\gamma_\mu\gamma_5 s\rangle\right\} - \frac{1}{2}\left\{\text{with } \lambda^a\right\}\right],
\end{aligned} \tag{A.6}$$

$$\begin{aligned}
\tilde{\Pi}_2^{\Sigma}(q) &= \frac{-1}{q^2}\left[\left(1-\frac{1}{N_c}\right)\left\{\frac{5}{2}\langle\overline{u}\gamma_\mu u\overline{s}\gamma_\mu s\rangle + \frac{3}{2}\langle\overline{u}\gamma_\mu\gamma_5 u\overline{s}\gamma_\mu\gamma_5 s\rangle\right\} - \frac{1}{2}\left\{\text{with } \lambda^a\right\}\right. \\
&\quad +\left(1-\frac{1}{N_c}\right)\left\{\frac{1}{2}\langle(\overline{u}u)^2\rangle - \frac{1}{2}\langle(\overline{u}\gamma_5 u)^2\rangle - \frac{1}{4}\langle(\overline{u}\gamma_\mu\gamma_5 u)^2\rangle + \frac{1}{4}\langle(\overline{u}\gamma_\mu u)^2\rangle\right\} \\
&\qquad \left. -\frac{1}{2}\left\{\text{with } \lambda^a\right\}\right].
\end{aligned} \tag{A.7}$$

(iv) Ξ:

$$\begin{aligned}
\tilde{\Pi}_1^{\Xi}(q) &= \frac{m_s}{q^2}\left[\left(1-\frac{1}{N_c}\right)\left\{-3\langle\overline{u}u\overline{s}s\rangle + \langle\overline{u}\sigma_{\mu\nu}u\overline{s}\sigma_{\mu\nu}s\rangle + \frac{1}{2}\langle\overline{u}\sigma_{\mu\nu}\gamma_5 u\overline{s}\sigma_{\mu\nu}\gamma_5 s\rangle\right\}\right. \\
&\qquad \left. -\frac{1}{2}\left\{\text{with } \lambda^a\right\}\right],
\end{aligned} \tag{A.8}$$

$$\tilde{\Pi}_2^{\Xi}(q) = \frac{-1}{q^2}\left[\left(1-\frac{1}{N_c}\right)\left\{\frac{5}{2}\langle\overline{u}\gamma_\mu u\overline{s}\gamma_\mu s\rangle + \frac{3}{2}\langle\overline{u}\gamma_\mu\gamma_5 u\overline{s}\gamma_\mu\gamma_5 s\rangle\right\} - \frac{1}{2}\left\{\text{with } \lambda^a\right\}\right.$$

$$+\left(1-\frac{1}{N_c}\right)\left\{\frac{1}{2}\langle(\overline{s}s)^2\rangle - \frac{1}{2}\langle(\overline{s}\gamma_5 s)^2\rangle - \frac{1}{4}\langle(\overline{s}\gamma_\mu\gamma_5 s)^2\rangle + \frac{1}{4}\langle(\overline{s}\gamma_\mu s)^2\rangle\right\}$$

$$\left.-\frac{1}{2}\left\{\text{with } \lambda^a\right\}\right]. \tag{A.9}$$

Factorizing these four-quark operators into the square of the chiral order parameter, one can easily get the form of (2.9), (2.10) and the following (A.11)–(A.16) at $T=0$. To get the T-dependence of the four-quark condensates, we need to calculate the double commutators of (A.2)–(A.9) with the isovector axial charge. The calculation is tedious but straightforward. The following formulas are useful in carrying out the calculation ($q=(u,d)$):

$$\begin{aligned}
[\mathcal{F}_5^a, \overline{q}q] &= -\overline{q}\gamma_5\tau^a q, \\
[\mathcal{F}_5^a, \overline{q}\gamma_5 q] &= -\overline{q}\tau^a q, \\
[\mathcal{F}_5^a, \overline{q}\sigma_{\mu\nu}q] &= -\overline{q}\sigma_{\mu\nu}\gamma_5\tau^a q, \\
[\mathcal{F}_5^a, \overline{q}\sigma_{\mu\nu}\gamma_5 q] &= -\overline{q}\sigma_{\mu\nu}\tau^a q, \\
[\mathcal{F}_5^a, \overline{q}\gamma_\mu q] &= [\mathcal{F}_5^a, \overline{q}\gamma_\mu\gamma_5 q] = 0, \\
\left[\mathcal{F}_5^a, \overline{q}\tau^b q\right] &= -\delta^{ab}\overline{q}\gamma_5 q, \\
\left[\mathcal{F}_5^a, \overline{q}\gamma_5\tau^b q\right] &= -\delta^{ab}\overline{q}q, \\
\left[\mathcal{F}_5^a, \overline{q}\sigma_{\mu\nu}\tau^b q\right] &= -\delta^{ab}\overline{q}\sigma_{\mu\nu}\gamma_5 q, \\
\left[\mathcal{F}_5^a, \overline{q}\sigma_{\mu\nu}\gamma_5\tau^b q\right] &= -\delta^{ab}\overline{q}\sigma_{\mu\nu}q, \\
\left[\mathcal{F}_5^a, \overline{q}\gamma_\mu\tau^b q\right] &= i\epsilon^{abc}\overline{q}\gamma_\mu\gamma_5\tau^c q, \\
\left[\mathcal{F}_5^a, \overline{q}\gamma_\mu\gamma_5\tau^b q\right] &= i\epsilon^{abc}\overline{q}\gamma_\mu\tau^c q.
\end{aligned} \tag{A.10}$$

After calculating the double commutators, we end up with other four-quark operators. Applying the factorization to these four-quark operators, we eventually got (2.9), (2.10) and the following (A.11)–(A.16):

$$\Pi_1^\Lambda(q,T) = \frac{1}{12\pi^2}\left(4\langle\overline{u}u\rangle\left(1-\frac{\zeta}{8}\right) - \langle\overline{s}s\rangle\right)q^2\ln(Q^2) + \frac{m_s}{96\pi^4}q^4\ln(Q^2)$$

$$-\frac{4m_s}{3q^2}\langle\overline{u}u\rangle^2\left(1-\frac{\zeta}{6}\right) + \frac{4m_s}{9q^2}\langle\overline{u}u\rangle\langle\overline{s}s\rangle\left(1-\frac{\zeta}{8}\right), \tag{A.11}$$

$$\Pi_2^\Lambda(q,T) = \frac{-1}{64\pi^4}q^4\ln(Q^2) + \frac{m_s}{12\pi^2}\left(4\langle\overline{u}u\rangle\left(1-\frac{\zeta}{8}\right) - 3\langle\overline{s}s\rangle\right)\ln(Q^2)$$

$$-\frac{1}{32\pi^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\ln(Q^2)+\frac{2}{9q^2}\langle\overline{u}u\rangle^2\left(1-\frac{\zeta}{2}\right)-\frac{8}{9q^2}\langle\overline{u}u\rangle\langle\overline{s}s\rangle\left(1-\frac{\zeta}{8}\right) \tag{A.12}$$

$$\Pi_1^{\Sigma}(q,T) = \frac{1}{4\pi^2}\langle\overline{s}s\rangle q^2\ln(Q^2)-\frac{m_s}{32\pi^2}q^4\ln(Q^2)-\frac{4m_s}{3q^2}\langle\overline{u}u\rangle^2\left(1-\frac{\zeta}{6}\right), \tag{A.13}$$

$$\begin{aligned}\Pi_2^{\Sigma}(q,T) &= \frac{-1}{64\pi^4}q^4\ln(Q^2)-\frac{m_s}{4\pi^2}\langle\overline{s}s\rangle\ln(Q^2)\\ &\quad -\frac{1}{32\pi^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\ln(Q^2)-\frac{2\langle\overline{u}u\rangle^2}{3q^2}\left(1-\frac{\zeta}{8}\right),\end{aligned} \tag{A.14}$$

$$\Pi_1^{\Xi}(q,T) = \frac{1}{4\pi^2}\langle\overline{u}u\rangle\left(1-\frac{\zeta}{8}\right)-\frac{2m_s}{q^2}\langle\overline{u}u\rangle\langle\overline{s}s\rangle\left(1-\frac{\zeta}{8}\right), \tag{A.15}$$

$$\Pi_2^{\Xi}(q,T) = \frac{-1}{64\pi^4}q^4\ln(Q^2)-\frac{1}{32\pi^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\ln(Q^2)-\frac{2\langle\overline{s}s\rangle^2}{3q^2}. \tag{A.16}$$

The T-dependence of (A.15) and (A.16) is the same as (3.14). We also note the T-dependence of the four-quark operators is different in the different channels.

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Figure Captions

Fig. 1 $\pi + N \rightarrow N$ scattering term in which η^N couples to π directly.

Fig. 2 $\pi + N \rightarrow N$ scattering term in which η^N couples to the nucleon that interacts with π.

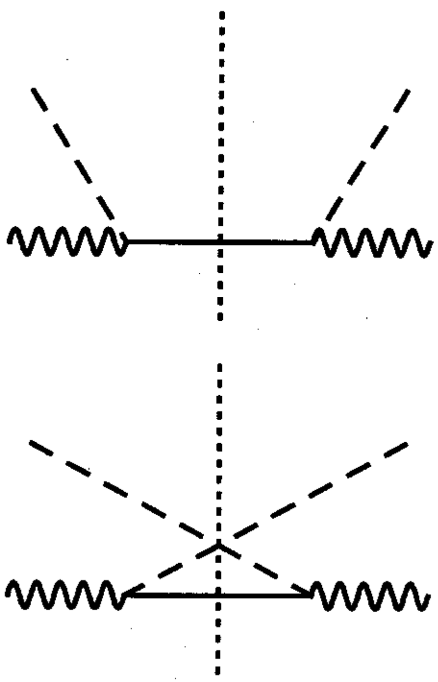

Fig. 1

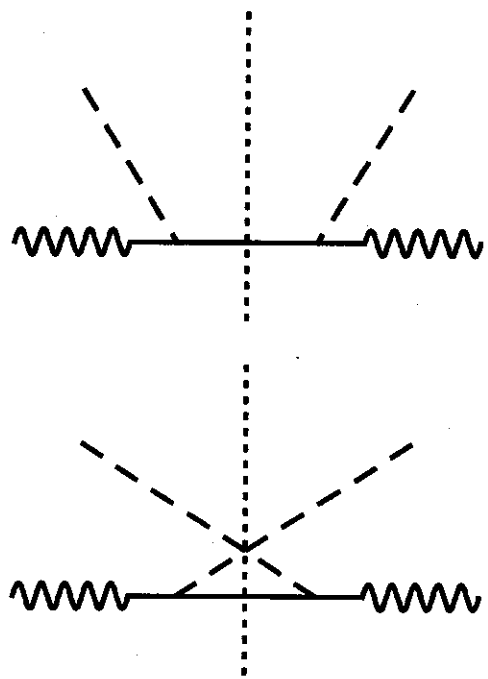

Fig. 2