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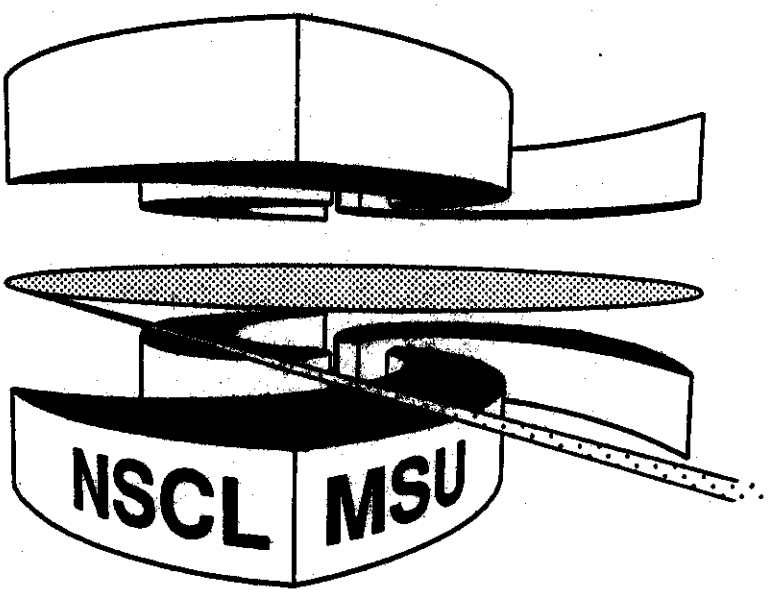

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Coherent States Description of the α Decay

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The Glauber coherent states are considered as possible candidates for the wave functions of the nucleons clustered in the α particle before decay. It is shown that the center of mass wave function may be easily separated, and the result is in agreement with the previous models. This new approach is applied to discuss the geometric effect of the nuclear shape on the preformation probability.

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The semi-microscopic models of the α-decay give essentially the decay rate as a product between a factor representing the "preformation" probability P inside the potential barrier ,the collision rate with the surface C_s, and a "penetrability" factor T given by the channel relative wave function, accounting for the escape probability. For the ground state α-decay of the heavy nuclei the escape probability factor is dominant , but also is extremely sensitive to the parameters of the potential barrier separating the internal and external regions. Between these the radius parameter R is especially important, and it may be related both to the geometric picture of two touching spheres representing the daughter nucleus and the α particle, as well as to the shell model parameters.

In a recent series of papers [1-3] it was proposed a procedure to fix R from a Bohr-Sommerfeld quantization condition, relating R with the potential depth and the values of the harmonic oscillator principal quantum number G of the first state above the Fermi surface in the daughter nucleus. For the L=0 transitions this procedure assumes that before the decay the α particle performs a purely radial vibration in the initial nucleus. Considering a Cartesian frame with the Z-axis along the direction of the radial vibration, the result may be related also to the independent particle picture developed by Harvey [4-5] in order to understand the effects of the Pauli principle on the shell structure at fission. According to the Harvey model, if the Z-axis is chosen along the direction of motion between the final nuclei then the X and Y number of internal quanta remain constant, and only the number of quanta on the Z direction must change. In the present case this means that the nucleons forming the α particle originate in the occupied orbitals at the Fermi level of the initial nucleus carrying a total number G of oscillator quanta on the Z axis.

The analysis of the experimental data within this model has shown a general good agreement, but a strong decrease of the extracted preformation factor P was observed

when the neutron number approaches the magic value N=126. Calculating P from the ratio of the experimental decay rate and the theoretical estimate C_sT this was interpreted as due to a variation in the radius R, generated either by a change in the value of the quantum number G ,[1], or as effect due to the increase of the rms radius for the valence neutrons ,[6]. Worth noting is that a similar variation appears for the preformation factors extracted from reaction data ,[7],[8], suggesting that it is not an artificial effect produced by a wrong calculation of the transmission coefficient T.

The aim of the present work is to discuss the qualitative change of the orbit as a possible source for the observed discontinuity. As it was proved in [9-10] for nuclei close to the double magic nucleus ^{208}Pb a new large amplitude collective motion may appear, consisting in surface soliton waves. These solitons should represent heavy nucleon clusters, but by contrast to the pure radial vibration assumed in the Harvey model, their motion is qualitatively different, as restricted to the surface.

For the α decay a microscopic corespondent of this hydrodynamic picture may be obtained approximating the nuclear potential by a simple harmonic oscillator term and selecting the exact solutions of the time dependent Schrödinger equation which are stable with respect to the time evolution. These wave packets are less used than the standard eigenstates of the time - independent problem, but are well known as Glauber coherent states. Now, within a kinematical approach to the soliton description will be assumed that each nucleon ν of the α particle is placed in a Glauber state:

$$\zeta^{\nu}_{(\vec{r}_\nu)} = (\frac{c}{\pi})^{\frac{3}{4}} e^{-\frac{c}{2}(\vec{r}_\nu - \vec{R})^2 + \frac{i}{\hbar}\vec{P}(\vec{r}_\nu - \frac{\vec{R}}{2})} \tag{1}$$

with $c = \frac{m\omega}{\hbar}$ the size parameter for a single nucleon. This state is a gaussian wave packet centered on the average values of the coordinate and momentum operators:

$$< \zeta^{\nu} \mid \vec{r}_{\nu} \mid \zeta^{\nu} >= \vec{R} \tag{2}$$

$$< \zeta^{\nu} \mid \vec{p}_{\nu} \mid \zeta^{\nu} >= \vec{P} \tag{3}$$

As required by the condition of coherent time evolution by the harmonic oscillator hamiltonian, these average values should be time dependent as given by:

$$\vec{R}_{(t)} = cos(\omega t)\vec{R}_{(0)} + \frac{sin(\omega t)}{m\omega}\vec{P}_{(0)} \tag{4}$$

$$\vec{P}_{(t)} = cos(\omega t)\vec{P}_{(0)} - m\omega sin(\omega t)\vec{R}_{(0)} \tag{5}$$

with $\vec{R}_{(0)}$ and $\vec{P}_{(0)}$ the initial conditions. In particular when $\vec{R}_{(0)} \times \vec{P}_{(0)} = 0$ the Glauber packet has a radial vibration as in the Harvey model, while if $\vec{R}_{(0)} \cdot \vec{P}_{(0)} = 0$ and $\mid \vec{P}_{(0)} \mid = m\omega \mid \vec{R}_{(0)} \mid$ then it rotates on the surface as a soliton wave.

If each of the four nucleons from the α particle is placed in such a nonstationary Glauber packet, the orbital cluster wave function is :

$$\Psi^{\alpha}(\vec{r_1}, \vec{r_2}, \vec{r_3}, \vec{r_4}) = \prod_{\nu=1}^{4} \zeta^{\nu}_{(\vec{r}_{\nu})} \tag{6}$$

Defining as usual the intrinsic coordinates $\vec{\rho}, \vec{\rho_1}, \vec{\rho_2}$ by :

$$\vec{\rho_1} = \frac{\vec{r_1} - \vec{r_2}}{\sqrt{2}} \tag{7}$$

$$\vec{\rho_2} = \frac{\vec{r_3} - \vec{r_4}}{\sqrt{2}} \tag{8}$$

$$\vec{R_1} = \frac{\vec{r_1} + \vec{r_2}}{\sqrt{2}} \tag{9}$$

$$\vec{R_2} = \frac{\vec{r_3} + \vec{r_4}}{\sqrt{2}} \tag{10}$$

$$\vec{\rho} = \frac{\vec{R}_1 - \vec{R}_2}{\sqrt{2}} \tag{11}$$

, and the center of mass coordinate

$$\vec{R}_{cm} = \frac{\vec{R}_1 + \vec{R}_2}{2\sqrt{2}} \tag{12}$$

then :

$$\Psi^{\alpha}(\vec{r_1}, \vec{r_2}, \vec{r_3}, \vec{r_4}) = \Phi^{\alpha}(\vec{\rho_1}, \vec{\rho_2}, \vec{\rho}) \cdot \frac{1}{2\sqrt{2}} Z^{cm}(\vec{R}_{cm}) \tag{13}$$

with :

$$\Phi^{\alpha}(\vec{\rho_1}, \vec{\rho_2}, \vec{\rho}) = (\frac{c}{\pi})^{\frac{9}{4}} e^{-\frac{c}{2}(\vec{\rho_1}^2 + \vec{\rho_2}^2 + \vec{\rho}^2)} \tag{14}$$

and

$$Z^{cm}(\vec{R}_{cm}) = (\frac{C}{\pi})^{\frac{3}{4}} e^{-\frac{C}{2}(\vec{R}_{cm} - \vec{R})^2 + \frac{i}{\hbar} 4\vec{P} \cdot (\vec{R}_{cm} - \frac{\vec{R}}{2})} \tag{15}$$

with $C = 4c$. Denoting by $\Psi^{\alpha}(\vec{\rho_1}, \vec{\rho_2}, \vec{\rho}, \vec{R}_{cm})$ the α cluster wave function normalized with respect to the volume element in the new coordinates, then

$$\Psi^{\alpha}(\vec{\rho_1}, \vec{\rho_2}, \vec{\rho}, \vec{R}_{cm}) = \Phi^{\alpha}(\vec{\rho_1}, \vec{\rho_2}, \vec{\rho}) \cdot Z^{cm}(\vec{R}_{cm}) \tag{16}$$

appears as a product between the well known intrinsic function of the α particle Φ^{α} and a Glauber wave packet for the center of mass with a half-width of the probability distribution $\sqrt{\frac{ln(2)}{C}}$. The average values of the center of mass coordinate and momentum are :

$$< Z^{cm} \mid \vec{R}_{cm} \mid Z^{cm} >= \vec{R} \tag{17}$$

$$< Z^{cm} \mid \vec{P}_{cm} \mid Zcm >= 4\vec{P} \tag{18}$$

with $\vec{R}$ and $\vec{P}$ time dependent as given by the equations (4), (5). Thus $Z^{cm}(\vec{R}_{cm})$ represent a straightforward generalization of the static gaussian centered at the touching radius considered in [11]. The real vectors (17),(18) may be combined into a single complex vector $\vec{z}$ defined by:

$$\vec{z} = \sqrt{\frac{C}{2}}(< Z^{cm} \mid \vec{R}_{cm} \mid Z^{cm} > + \frac{i}{\hbar C} < Z^{cm} \mid \vec{P}_{cm} \mid Z^{cm} >) \tag{19}$$

Its time dependence is simply given by $\vec{z}_{(t)} = e^{-i\omega t}\vec{z}_{(0)}$, and using this parameterization the center of mass coherent state may be written as:

$$\mid Z^{cm} >= e^{(\vec{z}\cdot\vec{B}^{+} - \vec{z}^{*}\vec{B})} \mid 0 > \tag{20}$$

with $\vec{B} = (B_1, B_2, B_3)$ the center of mass boson operator

$$\vec{B} = \sqrt{\frac{C}{2}}(\vec{R}_{cm} + \frac{i}{\hbar C}\vec{P}_{cm}) \tag{21}$$

$\vec{P}_{cm} = -i\hbar\vec{\nabla}_{cm}$ and $\mid 0 >$ the vacuum of $\vec{B}$.

The classical orbit given by (4) is in general an ellipsis in the plane determined by the vectors $\vec{R}_0$ and $\vec{P}_0$. Choosing the Y and Z coordinates along its principal axis, the trajectory may be characterized by the corresponding principal radii R_2 and R_3, as well as by the deformation parameter [12]:

$$\delta = \frac{3}{2}\frac{R_3^2 - R_2^2}{R_3^2 + 2R_2^2} \tag{22}$$

In particular the linear Harvey vibrations and the circular soliton rotations are limit cases when $\delta = 1.5$ and 0 respectively. Choosing in the first case the vibration direction along the Z-axis , then $\vec{z}_{(0)}$ has a single non vanishing component z_3 which may be fixed real and equal to $\sqrt{\frac{C}{2}}$ times the vibration amplitude $R_3 = R_{max}$. The Bohr-Sommerfeld quantization of these orbits selects for R_{max} the values $\sqrt{\frac{2G}{C}}$ with G the associated quantum number. Denoting by $\mid Z_v^{cm} >$ the function (20) with these initial conditions, the associated stationary state $\mid G >$ is given by the time average requantization procedure [13],[14]:

$$| G >= \frac{1}{T} \oint dt e^{iG\omega t} | Z_v^{cm} > \tag{23}$$

After normalization, this function become a standard harmonic oscillator state with G quanta on the Z-axis,

$$| G; vib >= \frac{1}{\sqrt{G!}} (B_3^+)^G | 0 > \tag{24}$$

In the general case, new boson operators B_{ell}^+ may be defined by:

$$B_{ell}^+ = \frac{1}{| \vec{z}_{(0)} |} (\vec{z}_{(0)} \cdot \vec{B}^+) \tag{25}$$

with

$$| \vec{z}_{(0)} | = \sqrt{\sum_{i=1}^{3} z_{i(0)}^* z_{i(0)}} \tag{26}$$

and the state (20) associated to this particular orbit at t=0 may be written as

$$| Z_{ell}^{cm} >= e^{\rho(B_{ell}^+ - B_{ell})} | 0 > \tag{27}$$

with $\rho = | \vec{z}_{(0)} | = \sqrt{\frac{C}{2}(R_2^2 + R_3^2)}$. The Bohr-Sommerfeld condition relate now the radii R_2, R_3 to the quantum number G by the relation $R_2^2 + R_3^2 = \frac{2G}{C}$, and the associated stationary state given as before by time average and normalization is:

$$| G; ell >= \frac{1}{\sqrt{G!}} (B_{ell}^+)^G | 0 > \tag{28}$$

It is worth noting that these considerations are valid strictly for a spherical harmonic oscillator potential. For a deformed oscillator the only periodic orbits are along the principal axes, while the other trajectories will be in general open, covering the whole nuclear volume. Thus, for a prolate well-deformed nucleus the Harvey states (24) are the most favoured candidates to the preformed states of the α particle, as they are associated to the unique closed orbit with the highest R_{max}, thus with the smallest barrier. However near the spherical nuclei the elliptic orbits become almost stable,

having only a low frequency precession around the X axis, and should be considered in the study of preformation.

In general the preformation probability P of the α particle would be given by the amplitude of the final configuration in the initial state [11]. Thus the preformation probability of the states obtained above from an initial $L = 0$ state is reflected by the modulus squared of their overlap with the exact spherical harmonic oscillator states $\mid N, L, M >$ with L=0, M=0, and N an even integer. To calculate this overlap the state $\mid N, 0, 0 >$ was expressed in terms of the boson operators (21) by the formula:

$$\mid N,0,0 >= \frac{1}{\sqrt{(N+1)!}}[(B_3^+)^2 + (B_1^+)^2 + (B_2^+)^2]^{\frac{N}{2}} \mid 0 > \tag{29}$$

Using this expression P may be easily estimated as:

$$|< G; ell \mid G,0,0 >|^2 = \frac{1}{G+1} \mid \frac{3\delta}{3+\delta} \mid^G \tag{30}$$

In terms of the radius variation

$$\varepsilon = \frac{R_{max} - R_3}{R_{max}} \tag{31}$$

P take also the simple form:

$$P = \frac{(1 - 4\varepsilon + 2\varepsilon^2)^G}{G+1} \tag{32}$$

The quantum number G appearing in this formula may be chosen according to the Harvey prescription, summing the oscillator quantum numbers for the occupied orbitals at the Fermi level, or it may be estimated as an average over a group of nuclei from the model fits [1]. In the region of ^{208}Pb the average value is $\sim 22, 24$ and correspondingly the preformation probability of the pure radial vibrational states ($\varepsilon = 0$) is about 0.04, in agreement with the values determined in [6-7] for the nuclei with a neutron number far from 126.

The parameter ε should be related to the relative variation in the size of the nucleus,

assumed further to be the same as the relative variation of the rms charge radius. At N=126 the experimental rms charge radii has a decrease ϵ of about 2% ,[6],[15], and for this value (32) predict a decrease of P by a factor ~ 0.15. This result is consistent with the observed variation of the extracted preformation probability ,[6-7] and according to the previous discussion might indicate a change of the favoured cluster orbits from a pure radial vibration to an elliptic trajectory with a large deformation, $\delta = 1.33$. It is important to remark that beside this geometric effect, an additional influence on the preformation may arise from the particular shell structure of the parent nucleus. Therefore further calculations are necessary to separate the contribution of each of these factors.

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