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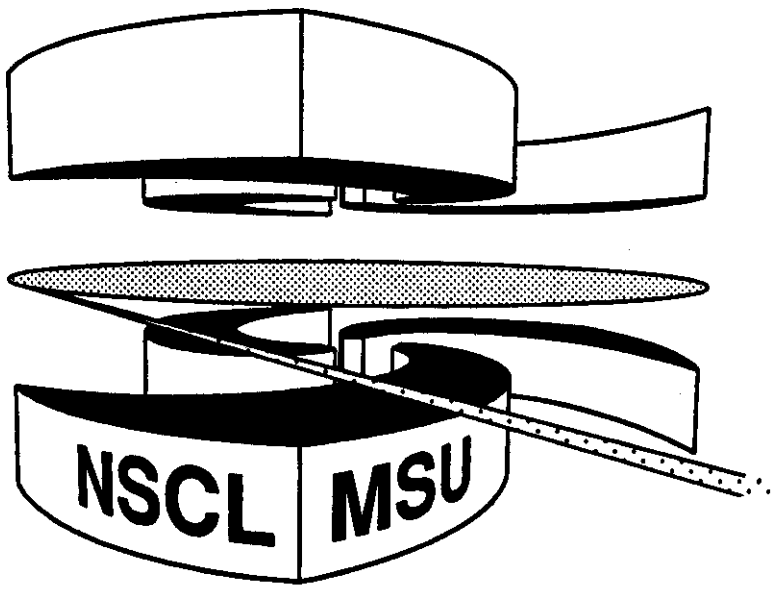

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Coupling Between Slow and Fast Degrees of Freedom in Systems with Complex Spectra : Driven Systems

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Abstract

We consider many-body systems which display slow/collective modes and have complex spectra of intrinsic states, such as atomic nuclei, atomic clusters, deformable cavities, and so forth. The effects of the coupling between the fast and the slow degrees of freedom is analyzed, by assuming random matrix properties for the intrinsic degrees of freedom, and that the time evolution of the slow degrees of freedom modifies the intrinsic configuration of the system. By neglecting the reaction of the intrinsic degrees of freedom on the slow modes, we derive evolution equations for intrinsic state population probabilities, the average excitation energy and its fluctuations. These evolution equations are characterized by strong memory effects, and in the long time limit becomes Markovian.

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1. Introduction

A common feature to complex many body systems is that their behaviour can be treated and understood in terms of elementary modes, whose energy and times scales are roughly characterized as slow/shape/collective/extrinsic and fast/noncollective/intrinsic. Typical examples of such systems are atomic nuclei, atomic clusters, molecules, perhaps nanoparticles under certain conditions, some field theoretical models of elementary particles, deformable cavities, and so forth. The distinction between the two types of modes is not always obvious, nor easy to characterize quantitatively, and often the analysis of the properties of such systems require more artistry than scientific technique. For example, the entire history of nuclear physics can be recanted as the study of the interplay of slow and fast degrees of freedom, even though the rigorous definition of the collective degrees of freedom still awaits its satisfactory solution (see Ref. [1] and references therein). In contrast, similar situations encountered in molecular and atomic physics are much less ambiguous, since the collective coordinates are more easily identified. Other classes of systems to which the approach we develop in this paper might be relevant, and which to a certain extent can be thought of as simplified models of atomic nuclei or atomic clusters, are deformable cavities and drums interacting with excited sound waves or electromagnetic fields. Imagine an elastic cavity/drum with perfectly reflecting walls and excited electromagnetic waves inside or a stadium shaped cavity with a free metallic sphere inside [2]. The motion of the the metallic sphere is completely ergodic (except some unstable trajectories of measure zero of course). If one fills the cavity with electromagnetic fields, the coupling between these systems will induce chaotic behaviour of the waves. The walls of the drum in the case of the deformable cavity (or the metallic sphere) play the role of the mean field while the waves inside are the analogue of the single particle degrees of freedom. If one taps the drum, a low frequency mode is generated. Since the shape of the drum changes slowly in time (with respect to the characteristic period of the electromagnetic wave inside) there is a coupling between the two types of motion and consequently an energy exchange. In spite of the apparent simplicity of such a system, the dynamical evolution can be quite complex, and questions like the time scales and the character of the energy exchange between the two types of modes are not trivial and as of yet unresolved.

There are several general questions we have to clarify before proceeding any further: What do we mean by systems with complex spectra? Is there a domain where a general approach to the coupling between fast and slow degrees of freedom is likely to be generic and the details of the underlying microscopic Hamiltonian are unimportant? Moreover, why is this problem important?

Physical systems of interest have as a rule complex spectra. For example, unless the shape of the cavity/drum is regular (as in the case of a sphere or ellipsoid, a cube, etc.), the spectrum has GOE (Gaussian Orthogonal Ensemble) characteristics which seem to be generic [3 – 6]. Unless an atomic nucleus or an atomic cluster is magic, in its ground state or only slightly excited (this includes also states in the vicinity of so called yrast line in nuclei) there are no reasons to expect a regular excitation spectrum (e.g. a clean

rotational or vibrational spectrum). Many–body systems undergoing large amplitude collective motion (e.g. fission, finite many–body systems at finite temperature, relatively slow collisions between two such systems, etc.) are characterized by an irregular spectrum of intrinsic excitations. A great wealth of experimental evidence can be understood in terms of directly exciting mostly slow/shape/collective degrees of freedom in these cases. It is unquestionable however, that these slow degrees of freedom are coupled to the intrinsic modes and their motion is relatively quickly degraded. Can one understand this as some kind of dissipation? Since one is dealing with finite systems one might expect that such an approach is not quite legitimate. In the event that such an approach is possible, does dissipation have any peculiar nature in finite systems? A recent analysis performed by Wilkinson [7] seems to indicate that in the strict adiabatic regime the diffusion constant is $D \sim V^{3/2}$ rather than the expected $D \sim V^2$, where V is the velocity of the slow degrees of freedom. This implies that the friction force is $\sim V^{1/2}$ and consequently it is much larger than expected ($\sim V$) at very low velocities. Nemes and Weidenmüller [8] argue that in the adiabatic limit the friction force should behave like $\sim V^3$. In a numerous number of other theoretical analyses it has been assumed implicitly that the effective friction has the usual behaviour, i.e. $\sim V$, see the review [9] and references therein and Refs. [10]. In particular all phenomenological approaches to nuclear fission, based on the Fokker–Planck equation, see review [9], imply that the friction/dissipation has a classical/standard character. Since the landmark paper of Hill and Wheeler [11] the problem of dissipation in large amplitude collective motion has been approached in a variety of ways [7 – 21] and one can hardly reconcile the different theoretical assumptions on which different derivations are based.

The problem we shall be dealing with in this paper is how the energy is transferred from the slow to the fast degrees of freedom in a generic system with a complex spectrum (a generic example will be described in the next section and illustrated in Fig. 4), and how the dynamics is influenced by the coupling between the two types of modes. We shall implicitly assume that our system has some well defined slow/shape/collective degrees of freedom, which undergo large amplitude collective motion. Recent analyses [22 – 24] suggest that level crossings and dynamical generated gauge fields can play a rather unexpected role in large amplitude collective motion and our initial motivation was to study a system with a large number of level crossings and the emergence of chaotic behaviour and dissipation in a relatively complex system. (A formally similar system to the ones analysed in Refs. [2, 22, 23] has been studied in Ref. [25], an ensemble of N two level atoms coupled to the electromagnetic field, which under certain circumstances becomes chaotic as well.) As a first step towards that goal we shall be dealing with a driven system only in the present paper and relegate the problem of coupled slow and fast motion to future publications. The term large amplitude collective motion requires some clarification. A rather simple and intuitive definition could be the following: the slow/shape/collective coordinates change to such an extent that the intrinsic instantaneous excitation spectrum is largely modified and there are essentially no correlation whatsoever between such intrinsic configurations.

2. Collective–Intrinsic Interactions

2.1 GOE Fluctuations

The physical system we have in mind has some slow/shape/collective variables, which we shall denote with X, Y, Z, ... and the corresponding canonical conjugate momenta P_X, P_Y, P_Z ..., coupled to a set of fast variables. We treat the slow variables as classical, while the fast variables will be treated quantum mechanically. In principle one can quantize these classical degrees of freedom as well, if there is such a need, but the analysis becomes much more complicated and it seems that the main physics can be extracted at the classical level. This mixed system will be described by a Hamiltonian

$$H = T_{kin} + V(X,\ Y,\ Z,\ ...) + H(X,\ Y,\ Z,\ ...), \tag{1}$$

where T_{kin} and $V(X)$ are the kinetic and potential energies for the classical degrees of freedom, and $H(X)$ is the quantum Hamiltonian, which depends parametrically on the slow variables. The intrinsic degrees of freedom play the role of a thermostat for the collective degrees of freedom, since their number is large. Therefore one expects that once the energy has been transferred to these intrinsic degrees of freedom, for all practical purposes it cannot be retrieved, i.e. the Poincaré return time is essentially infinite. The spectrum of essentially any many–body system is so complicated that about the only things one can infer is the mean level density and the level spacing fluctuations.

For any given spectrum $\{E_i, i = 1, ..., N\}$, the level density can be written as a sum of two pieces, the first describing the average shape of the spectrum, and the second describing the local fluctuations from the mean:

$$\rho(E) = \sum_{i=1}^{N} \delta(E - E_i) = \rho_{ave}(E) + \rho_{fluc}(E). \tag{2}$$

The universal property of statistical systems refer to the nature of the fluctuations, rather than the mean. For this reason, one usually unfolds the spectrum (i.e. removes the average behavior ρ_{ave}, and examines ρ_{fluc}). This can be done achieved by defining a unit energy level density as follows:

$$\epsilon_i = \int^{E_i} dE \rho_{ave}(E), \tag{3}$$

$$s_i = \epsilon_{i+1} - \epsilon_i \simeq (E_{i+1} - E_i)\rho_{ave}(\frac{E_i + E_{i+1}}{2}). \tag{4}$$

In the case of a complex system, the level spacing distribution $P(s)$ seems to have an almost universal character [3–6] which is well reproduced by the Wigner surmise for a GOE ensemble

$$P(s) = \frac{\pi}{2} s \exp\left[-\frac{\pi s^2}{4}\right]. \tag{5}$$

The properties of ρ_{ave} will depend on the physical system we consider. In general, the mean level density $\rho(E,X)$ for a given shape X of the system, is a rapidly increasing function of the excitation energy E. For a classical system with N degrees of freedom $\rho(E,X)$ has a power law behaviour for very low or very high energies (away from phase transitions)

$$\rho_{classical}(E) = \int d^N q d^N p \delta(E - H(q,p)) \sim E^n. \tag{6}$$

(For a system of free particles $n = N/2 - 1$, while for N harmonic oscillators, $n = N - 1$.) For a quantum many–fermion system the Bethe formula is a good approximation at relatively high excitation energies (but not yet in the classical regime)

$$\rho_{Bethe} \sim E^{-5/4} \exp\left[\sqrt{aE}\right], \tag{7a}$$

while for low excitation energies [26]

$$\rho(E) \sim E^2. \tag{7b}$$

(This is essentially what one will obtain by taking into account only 2p–2h configurations.) The corresponding specific heats for such systems are

$$C_{classical}(T) = (n+1), \tag{8}$$

$$C_{quantum}(T) \sim T^2. \tag{9}$$

This last formula is only valid for fermion systems at low temperatures with GOE fluctuations. See also the discussion in Refs. [3, 27], concerning small metallic particles with Poisson versus GOE fluctuations spectra.

We would like to consider a model Hamiltonian, which reproduces a desired average level density and is characterized by a GOE level spacing distribution. If we do not superimpose the GOE fluctuations on any average background ρ_{ave}, we obtain the pure GOE limit, which gives a well known, and for our purposes unphysical, GOE average density of states known as the Wigner semicircle. Consider a standard GOE matrix, consisting of an $N \times N$ real symmetric matrix, whose matrix elements are Gaussian random numbers with zero mean and variance

$$\overline{H_{kl}} = 0, \quad \overline{H_{ij}H_{kl}} = \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} = \Delta_{ijkl}. \tag{10}$$

The overline stands for the ensemble average. In the limit $N \to \infty$, the average level density is given by

$$\rho(E) = \begin{cases} \frac{1}{2\pi}\sqrt{4N - E^2}, & |E| \le 2\sqrt{N} \\ 0, & |E| > 2\sqrt{N} \end{cases} \tag{11}$$

We can now compute the thermodynamic for this GOE limit. The partition function for such a system is

$$Z(\beta) = \int dE \exp(-\beta E)\rho_{average}(E) \tag{12}$$

As a function of the temperature the specific heat is maximum at $T = 0$, when $C(0) = 3/2$ and decreases at large T as $\approx N/T^2$. (The specific heat will not change if the Hamiltonian H is multiplied by an arbitrary nonvanishing constant.) This behaviour of the specific heat, which corresponds to the classical limit, was obtained because the ensemble average was taken for the partition function before computing $C(T)$. One should first compute the specific heat and take the average afterwards. This will modify $C(T)$ only at very low temperatures, when, due to the discrete character of the spectrum, $C(T) \to 0$ for $T \to 0$, in a relatively small temperature range near zero. Because of such a behaviour of the specific heat a straight GOE choice (which is supposed to describe the fluctuation properties of spectra only) for $H(X)$ is clearly unrealistic. It means that the effective number of degrees of freedom in the quantum system is at most 3 and smaller than 1 at high temperatures. Consequently, the quantum system cannot serve as a big reservoir or sink of energy and one cannot study dissipation for the collective motion. In particular, when either driving a pure GOE system or coupling it to a slow variable the following behaviour emerges. The average energy of the quantum system tends to zero, irrespective of the initial conditions and the variance of the energy also tends to a constant $\sim N$, i.e. the maximum possible for such a system. The system saturates rather quickly and after a while nothing "interesting" happens. The system can both absorb or give energy on the initial stages, depending on whether the initial energy of the quantum system is negative or positive. The final state of "thermal equilibrium", in this case $T = \infty$, has zero average energy and a vanishing specific heat. Even though such a behaviour is "natural" for this type of Hamiltonian, it allows the study of the coupling between the internal (i.e. $H_1(X)$ and external systems only in the region of almost constant level density, when, strictly speaking, the internal system is already at $T = \infty$.

The more resonable approach is to use a deformed GOE [3], in which we superimpose the GOE fluctuations on our desired average density of states. This is realized by choosing the Hamiltonian $H(X)$ in the following form

$$H(X) = H_0(X) + \lambda H_1(X), \tag{13}$$

where $H_0(X)$ is a diagonal matrix, λ a constant and $H_1(X)$ a GOE matrix as above. If we take the diagonal elements such that $H_{0\,kk}(X) \sim k^{1/(n+1)}$, the resulting spectrum is characterized by an average level density $\rho(E) \sim E^n$. As a result the specific heat for such a system behaves in the high temperature limit as $C(T) = n+1$. An alternative way is to follow Balian [28] and to minimize the information for a given average level density. However, the statistical properties of such an ensemble will be rather complicated and more difficult to handle analytically. The high temperature or classical behaviour sets in relatively quickly and corrections to it arise only at relatively low temperatures. In general $C_{classical} \geq C_{quantum}$, approaching only at high temperatures or when $N \to \infty$. By choosing in an appropriate way the diagonal matrix elements of $H_0(X)$ one can very likely model a realistic many–body system. (One can recover the Wigner surmise for parameter values with $\rho\lambda \sim 1$, in which case the averaged spectrum is continuous [29, 30].)

The Hamiltonian (13) has a deficiency in that $H_1(X)$ connects has matrix elements between all states. (This leads to some difficulties in the study of nonequilibrium properties

of such a system, which we discuss below.) One solution is to choose $H_1(X)$ as a banded GOE matrix. This will amount to the introduction of more parameters: the average number of states at a given excitation energy effectively coupled by $H_1(X)$ - $N_{effective}$ - and the energy spreading of these states - $\Delta E = N_{effective}/\rho(E)$. Using a loose but intuitive language one can say that if $H_0 \gg H_1$, the instantaneous spectrum is quasi-discrete and consists of slightly perturbed levels of H_0 (the average level density has the shape of a set of very narrow Lorentzians, or rather Gaussians, around the unperturbed levels). In the limit $H_0 \ll H_1$, the instantaneous spectrum is essentially a pure GOE spectrum, with a density of states given by Eq. (11) (see Refs. [29, 30] for the case of a full GOE matrices, Refs. [30, 31] for banded GOE matrices, and Appendix A. Appendix B contains a nontrivial exception to this rule). This particular case ($\rho \approx$ *constant*), although not of direct interest for nuclei and other complex systems undergoing large amplitude collective motion, is nevertheless of interest. It corresponds to systems like a 2-dimensional Bunimovich stadium or Sinai oscillating billiard filled with sound waves or electromagnetic fields, or a small metallic object inside a Bunimovich stadium or Sinai billiard filled with electromagnetic fields [2], since the average density of states for a 2-dimensional system is approximately constant. Another reason for considering such a spectrum is that one can study the role of the average level density in dissipation by keeping it under control and varying other "relevant" parameters.

2.2 Intrinsic–Collective Correlations

In constructing the interaction Hamiltonian $H_1(X)$, we are have to define how the intrinsic states are correlated for different values of the collective coordinate X. We shall follow Wilkinson's suggestion [7] and define the GOE $N \times N$ matrix as follows

$$[H_1(X)]_{kl} = \int f(X-Y) w_{kl}(Y) dY, \tag{14}$$

where $f(x)$ is some sufficiently smooth function and $w_{kl}(Y)$ is a Gaussian white noise

$$\overline{w_{kl}(X)} = 0, \tag{15a}$$

$$\overline{w_{kl}(X) w_{mn}(Y)} = \Delta_{klmn} \delta(X-Y). \tag{15b}$$

In the above formulas the overline stands for the ensemble average. The rationale for such a choice is rather simple. If the collective variables change relatively slowly, one naturally expects that at each instant in time the intrinsic configuration of the system is almost quasistatic and the instantaneous spectrum is a continuous or even rather smooth function of the collective variables. This is not the only possible way to generate such Hamiltonians, which locally (for each fixed value of the parameter X) are members of the GOE, and we shall mention other alternatives below.

From the above relations one can easily show that

$$\overline{[H_1(X)]_{kl}[H_1(Y)]_{mn}} = \Delta_{klmn} \int f(X-Z) f(Y-Z) dZ = \Delta_{klmn} F(X-Y). \tag{16}$$

As a result, for each value of X, the spectrum of $H_1(X)$ is GOE. From Eq. (16) it follows also that $F(X-Y) = F(Y-X)$. This can certainly be generalized by changing $f(X-Y)$ to $f(X,Y)$ and/or by choosing a position dependent strength $\lambda(X)$. But for our purposes here, these additional complications are not necessary. The numerical evaluation of the integral in Eq. (14) requires some care, since it is clearly not a continuous function, and a discretization of the integral as a standard Riemann integral is not valid. The standard measure one implements for such a stochastic integral is due to Wiener and Ito. (Essentially, these fluctuations imply that $\Delta x^2 \sim \Delta t$, rather than Δt^2 in normal discretization, resulting in a square root in the measure.) The simplest discretized form of the integral (14) is then

$$[H_1(X)]_{kl} = \sum_i f(X - Y_i) w_{kli} \sqrt{\Delta Y}, \tag{17}$$

where ΔY is the mesh size, $Y_i = i\Delta Y + Y_0$ are the discrete mesh points and w_{kli} are Gaussian random numbers with zero mean and variance

$$\overline{w_{kli} w_{mnj}} = \Delta_{klmn} \delta_{ij}. \tag{18}$$

Numerically, we have checked that in the case of a Gaussian interpolating function

$$f(X) = \left[\frac{\alpha}{\pi}\right]^{1/4} \exp\left[-\frac{\alpha X^2}{2}\right], \tag{19a}$$

$$F(X-Y) = \int_{-\infty}^{\infty} f(X-Z) f(Y-Z) dZ = \exp\left[-\frac{\alpha (X-Y)^2}{4}\right], \tag{19b}$$

a mesh size satisfying $2\alpha \Delta Y^2 \leq 1$ ensures a relative accuracy of at least 10^{-6} of the Eq. (16), when the corresponding ensemble average is evaluated numerically.

2.3 Dimensional Analysis of the Time–dependent Schödinger Equation

Our central object of study is the time–dependent Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = H(X)\psi = [H_0 + \lambda H_1(X)]\psi. \tag{20}$$

In the case of a driven system X is a given function of time. The Hamiltonian has a regular and a GOE part. Let us assume that $X = V_0 t$, i.e. the system is driven with constant velocity, the average level density ρ_0 is constant and the characteristic distance over which the GOE part changes is X_0 (for the Gaussian interpolating function discussed in the previous section $X_0 \approx 1/\sqrt{\alpha}$). In terms of a dimensionless time $\tau = V_0 t / X_0$ (this implies $X = X_0 \tau$), the Schrödinger equation becomes

$$i\frac{\partial \psi}{\partial \tau} = [\varepsilon \tilde{H}_0 + \kappa H_1(\tau)]\psi = \varepsilon[\tilde{H}_0 + \rho_0 \lambda H_1(\tau)]\psi = \tilde{H}(\tau), \tag{21}$$

where the tildes indicate the dependence on the scaled variables, and where we have taken

$$\varepsilon = \frac{X_0}{\hbar\rho_0 V_0} = \frac{T_0}{\tau_0}, \quad T_0 = \frac{X_0}{V_0}, \quad \tau_0 = \hbar\rho_0, \quad \kappa = \lambda\rho_0\varepsilon. \tag{22}$$

Here τ_0 is the characteristic frequency of the quantum system (which can also be thought of, under appropriate circumstances, as its Poincaré time), T_0 is the period or characteristic time of the slow mode and ρ_0 is the average level density (assumed constant) for H_0. If $\rho_0\lambda \sim O(1)$ (which is the case we are mostly interested in) then $\varepsilon \sim O(\kappa)$ and the spectrum has an average density ρ_0.

As previous analyses showed, such a Hamiltonian displays GOE fluctuations when the averaged spectrum is continuous [29, 30]. We shall not use explicitly in our derivations the fact that $\varepsilon \sim O(\kappa)$ and in principle one can use our equations in order to study different other limits, e.g. $\varepsilon \gg \kappa$ or $\varepsilon \ll \kappa$, as well, if such a need arises. One can write formally the solution of the Schrödinger equation as

$$\psi(\tau) = T\exp\left[-i\varepsilon\int_0^\tau ds\tilde{H}(s)\right]\psi(0) = U(\tau)\psi(0), \tag{23}$$

where T stands for the usual time–ordering operator. The tilde in the definition of the rescaled Hamiltonians in Eqs. (21)–(23) will henceforth be dropped, since from here on we shall work with these rescaled Hamiltonians only. Written as such, the problem displays almost all relevant dependencies. The Hamiltonian is an operator of order one, which changes significantly with time over an interval of order one. One can consider several separate problems. In the limit of very slow collective motion $\varepsilon \to \infty$ and even for relatively small intervals of time τ the propagator (23) is a very rapidly oscillating function. This feature might seem unexpected, since one would expect essentially very small changes in the strict adiabatic limit. In spite of its formal appearance as a time, actually τ is a measure for distances ($X = X_0\tau$). Since we are interested in large amplitude collective motion, likely the most interesting physical problem is the evolution of the system over an interval $X \sim O(X_0)$, i.e. one period of the large amplitude collective motion, $\tau \sim O(1)$ and one has to study the time evolution as a function of only one remaining parameter ε. Another interesting aspect is the behaviour of the system in the long–time limit $\tau \to \infty$, i.e. over many periods of the large amplitude collective motion, for different values of ε.

2.4 Ensemble Averaging: Uncorrelated States

One can take advantage of the fact that H_1 is a GOE matrix and perform the ensemble average. We shall be interested in computing matrix elements of the following type

$$\langle\psi(\tau)|A(\tau)|\psi(\tau)\rangle = \langle\psi(0)|U^\dagger(\tau)A(\tau)U(\tau)|\psi(0)\rangle, \tag{24}$$

where $A(\tau) = 1,\ H(\tau),\ H(\tau)^2,\ \partial H(X)/\partial X,\ \ldots$. For the time being we shall consider these types of matrix elements, for the case when the initial state $\psi(0)$ is not correlated

with the Hamiltonian, in particular $\psi(0)$ is not an eigenstate of $H(0)$ and such that the following ensemble average vanishes identically

$$\overline{H_1(\tau)_{kl}\psi_m(0)} \equiv 0, \quad \text{for all} \quad \tau > 0. \tag{25}$$

Here k, l and m correspond to arbitrary eigenstates of H_0, which will form the basis we shall work in. In section 3.2 we develop a more general method to treat correlated initial states, relaxing the condition (25).

To apply the simplifications provided by the GOE nature of $H_1(X)$, it is convenient to make a perturbation expansion of the full propagator $U(\tau)$ of Eq. (23):

$$\begin{aligned} U(\tau) &= S_0(\tau) + \int_0^\tau ds_1 S_0(\tau - s_1)[-i\kappa H_1(s_1)]S_0(s_1) \\ &\quad + \int_0^\tau ds_2 \int_0^{s_2} ds_1 S_0(\tau - s_2)[-i\kappa H_1(s_2)]S_0(s_2 - s_1)[-i\kappa H_1(s_1)]S_0(s_1) \\ &\quad + \ldots \end{aligned} \tag{26}$$

where

$$S_0(\tau) = \exp[-i\varepsilon H_0 \tau]. \tag{27}$$

We will assume that H_0 is time independent and diagonal, with eigenvalues e_k. This will not reduce the generality of our results, and can be easily lifted. As a result the free propagator is diagonal

$$[S_0(\tau)]_{kl} = \delta_{kl} \exp[-i\varepsilon e_k \tau] = \delta_{kl} S_{0\,k}(\tau). \tag{28}$$

In taking the ensemble average of the matrix element (26) we shall work out at first the average of the propagator $U(\tau)$, which we shall denote by $S(\tau)$. Upon taking the average all terms with an odd number of H_1 give a vanishing contribution. Due to the Gaussian character of the matrix elements of H_1, the ensemble average of the terms containing an even number of H_1 can be expressed through second moments only, by using Eq. (16). Graphically, a free propagator $S_0(\tau)$ can be represented by a thin oriented line and a contraction of two operators H_1, as an effective exchange of a phonon, by a wavy line. The first nontrivial average in the perturbation series involves four H_1 operators. Denoting the interaction generically as V, and the free propagator as S_0, one can easily check that diagrams involving contractions which cross, of the form

$$S_0 V S_0 V S_0 V S_0 V S_0$$

are suppressed by $O(1/N)$ with respect to the uncrossed diagrams of the type

$$S_0 V S_0 V S_0 V S_0 V S_0 \,, \qquad S_0 V S_0 V S_0 V S_0 V S_0 \,.$$

This is true in all orders, that diagrams containing crossed phonon lines are at least $1/N$ smaller than the corresponding diagrams without crossed phonon lines [3, 21, 29]. The

diagrams with crossed phonon lines amount to a renormalization of the vertex. Consequently, in the large N limit, which is of most interest in the study of complex systems, only diagrams without crossed phonon lines should be retained in the leading $1/N$ approximation. In each $2n$ order of the perturbation expansion the number of these leading diagrams is given by the Catalan number $(2n)!/[n!(n+1)!]$ [3], for the averaged propagator and by $(2n+1)!/[n!(n+1)!]$ for the matrix element (24) if $A(\tau)$ is statistically independent of H_1.

In evaluating the matrix element (24), each term of the perturbation expansion (26) can be represented as a thin line starting at time 0 up to time τ, the operator $A(\tau)$ (the measurement) as a fat dot and one more thin line going from τ to 0 again. The operators H_1 at different intermediate times can be represented as crosses. Upon taking the average, one links all possible pairs of crosses (and the fat dot in the middle if $A(\tau) = H_1(\tau)$, etc.) with a wavy line, see Fig. 1. In the leading $1/N$ approximation, only noncrossing contractions are retained [3, 21, 29]. If one sums up all such diagrams on one side of the central fat dot, it follows that the averaged propagator $S(\tau)$ is diagonal, and that each diagonal matrix element satisfies the following equation ($0 \le s_2 \le s_1 \le \tau$),

$$\begin{aligned} S_k(\tau) &= S_{0\,k}(\tau) - \int_0^\tau ds_1 \int_0^{s_1} ds_2 S_{0\,k}(s_2)\kappa^2 F(s_1 - s_2) \\ &\quad \times \{S_k(s_1 - s_2) + \sum_{n=1}^{N} S_n(s_1 - s_2)\} S_k(\tau - s_1) \qquad (29) \\ &= S_{0\,k(}\tau) - \int_0^\tau ds_1 \int_0^{s_1} ds_2 S_{0\,k}(s_2)\Sigma(s_1 - s_2) S_k(\tau - s_1), \end{aligned}$$

where $S_{0\,k}(\tau) = \exp[-i\varepsilon e_k \tau]$ and $\Sigma(s)$ is the proper self–energy. This is indicated graphically in Fig. 2. In the leading $1/N$ approximation, the $S_k(s_1 - s_2)$ term under the integral can be neglected, in which case the proper self–energy is identical for all states. The above equation describes a particle which propagates in an elastic medium and emits and absorbs phonons. It can be simplified somewhat by computing the ratio of the dressed and free propagators $\sigma_k(\tau)$

$$\begin{aligned} \sigma_k(\tau) &= 1 - \int_0^\tau ds_1 \int_0^{s_1} ds_2 \kappa^2 F(s_1 - s_2) \sum_{n=1}^{N} \exp[-i\varepsilon(e_n - e_k)(s_1 - s_2)] \\ &\quad \times \sigma_n(s_1 - s_2)\sigma_k(\tau - s_1), \qquad (30) \end{aligned}$$

which yields the interaction picture representation of the propagator.

In the particular simple case when $H_0 \equiv 0$, $e_k \equiv 0$ and $F(s) \equiv 1$ these equations can be solved explicitly (either by iterating Eqs. (29)–(30) or by going to the energy representation, as discussed in Appendix A). Since all diagonal matrix elements are equal, we can drop the subscript without ambiguity, and arrive at the ensemble averaged propagator:

$$S(\tau) = \sigma(\tau) = 1 - \int_0^\tau ds_1 \int_0^{s_1} ds_2 \kappa^2 (N+1) S(s_1 - s_2) S(\tau - s_1) = \frac{J_1(2\kappa\sqrt{N+1}\tau)}{\kappa\sqrt{N+1}\tau}, \quad (31)$$

where $J_1(X)$ is the cylindrical Bessel function of first order. Since we are working in the large N approximation, we can replace $N+1$ with N in all of the above formulas.

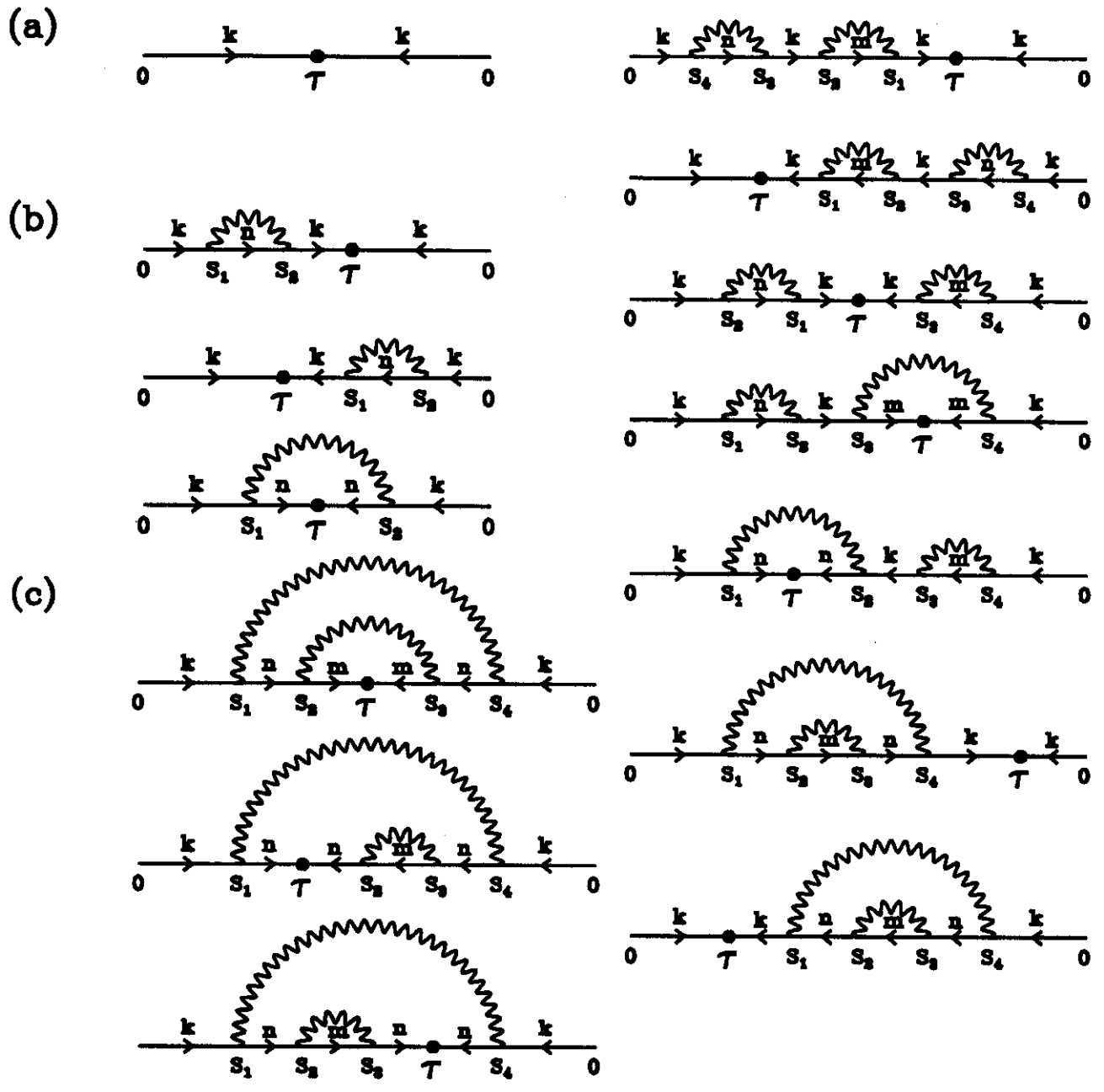

Figure 1: A few low order diagrams in the leading $1/N$ expansion for the matrix element in Rel. (24). The thin lines represent the free propagators $S_{0\,k}(s_2 - s_1)$, the wavy lines the correlator $\kappa^2 F(s_1, s_2)$ and the fat dot the measurement operator $A(\tau)$ in Rel. (24), here assumed to be uncorrelated with the Hamiltonian $H_1(s)$ at all times.

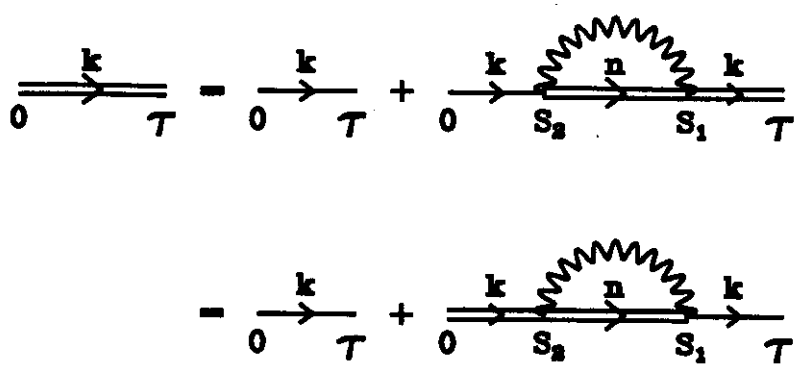

Figure 2: Diagrammatic representation of the equation (29) for the averaged propagator $S_k(s)$, represented as a double line.

This extra 1 originates from the fact that the variance of a diagonal matrix element of a GOE matrix is twice the variance of a nondiagonal element and consequently a diagonal transition $k \to k$ has twice the weight of a nondiagonal one $k \to j \neq k$. In particular, in Eq. (29) one can drop the term $S_k(s_1 - s_2)$ under the integral without any loss of accuracy. (This has already been done in Eq. (30).) By going to the energy representation of the propagator (31), it is simple to obtain the average level density of a pure GOE matrix given in Eq. (11).

This particular example of a time–independent, pure GOE matrix ($H_0 \equiv 0$), is certainly not relevant for case of the large amplitude collective motion. It implies an infinite correlation time or infinite correlation length in collective space, and that the intrinsic spectra corresponding to two vastly different shapes of the system are strongly correlated. A more realistic type of correlation might be $F(s) = \exp(-\alpha|s|)$, with $\alpha \geq 0$, or some other similar function, since there is no reason to expect that the intrinsic spectra, corresponding to different shapes might be correlated. In the limit $\alpha \to \infty$, one can easily find the averaged propagator $S(\tau)$ in Eq. (29), since the full propagator $S_n(s_1 - s_2)$ under the integral can be replaced by the free propagator $S_n(s_1 - s_2) = S_{0\,n}(s_1 - s_2)$, and then demonstrate that $S(\tau)$ decays exponentially as $\tau \to \infty$. This last approximation corresponds to considering not only the $N \to \infty$ limit, but also the $\tau \gg \tau_{correlation}$ limit. We consider this further in the next sections. This limit has been considered by other authors [32 – 34], but without assuming $N \to \infty$.

3. Statistical Properties and Correlations

Before proceeding any further with the computation of the ensemble average in the large N approximation of the general matrix element (24), it is instructive to consider the particular cases when $A(\tau) \equiv 1$ and $A(\tau) = |n\rangle\langle n|$, where n denotes an arbitrary eigenstate of H_0. As it will become evident form our analysis, this will clarify the formal structure of our approach and at the same time will shed light on the meaning and importance of different possible diagrams. For the particular case when $A(\tau) \equiv 1$, the matrix element of A must remain unity, independent of the ensemble averaging and τ, since we are computing the norm of the time–dependent wave function for a Hermitian (time–dependent) Hamiltonian. As a consistency check, we examine this to leading order in $1/N$.

3.1 Conservation of the Total Flux

One thing which seems disturbing is the fact that the averaged (leading order) propagator is not unitary; one specific example is Eq. (31), which decays as $\tau^{-3/2}$ when $\tau \to \infty$. This can be understood in terms of a particle in a medium. Before the "final measurement" at the time τ, the particle can emit and absorb phonons. Consequently, an instantaneous state of the system is generally a particle–n–phonon state: a particle and no phonons, a particle and one phonon and so on. The total probability, which includes all these states, would give unity as expected. However, at the time of measurement τ, the particle can be in a very complex state with several emitted phonons, and such a state will have a nonvanishing overlap only with a state with an equal number of phonons present. Diagrammatically, this amounts to adding all rainbow diagrams, which connect both sides of the fat dot (the measurement) at the time τ.

One can show, simply by computing order by order the contribution of the rainbow diagrams, that they restore the unitarity of the time evolution as expected. The rainbow diagrams correspond to the leading $1/N$ expansion terms, and any diagram with crossed phonon lines is a correction of higher order. The $N \to \infty$ limit for the rainbow diagrams amounts to absorbing the phonons in exactly the reverse order they have been emitted. For example, if two phonons are emitted, the second phonon should be absorbed first and only after that the absorption of the first phonon follows. By emitting a phonon the phase of the wave function is changed and coherence can be preserved only if emission and absorption occurs in such a manner, otherwise the phase coherence is lost and an additional $1/N$ weight appears. The set of diagrams which remains to be summed up are shown in Fig. 1, up to two loops.

The zeroth order contribution in Fig. 1(a) is equal to

$$S_{0\,k}(\tau) S^{\dagger}_{0\,k}(\tau) \equiv 1 \,. \tag{32}$$

The first order diagrams are shown in Fig. 1(b). Their combined contribution vanishes

identically, as one can check by inserting the free propagator (28):

$$
\begin{aligned}
0 \;=\; & -\int_0^\tau ds_1 \int_{s_1}^\tau ds_2 S_{0\,k}(s_1) \sum_n S_{0\,n}(s_2-s_1)\kappa^2 F(s_2,s_1) S_{0\,k}(\tau-s_2) S^\dagger_{0\,k}(\tau) \\
& -S_{0\,k}(\tau)\int_0^\tau ds_1 \int_{s_1}^\tau ds_2 S^\dagger_{0\,k}(s_1) \sum_n S^\dagger_{0\,n}(s_2-s_1)\kappa^2 F(s_2,s_1) S^\dagger_{0\,k}(\tau-s_2) \qquad (33) \\
& +\int_0^\tau ds_1 \int_0^\tau ds_2 S_{0\,k}(s_1) \sum_n S_{0\,n}(\tau-s_1)\kappa^2 F(s_2,s_1) S^\dagger_{0\,n}(\tau-s_2) S^\dagger_{0\,k}(s_2)\,.
\end{aligned}
$$

Here we have used a slightly more general type of correlator, which does not necessarily depend on the time difference, but satisfies however the time–reversal symmetry $F(s_2,s_1) = F(s_1,s_2)$. Each interaction vertex has a factor of $-i$ (or i for the conjugate propagator), so that the top two diagrams in Fig. 1(b) have an overall minus sign, while the bottom diagram does not. Notice also that in the diagram with a phonon line across the fat dot (the last of Eqs. (33) and the bottom diagram in Fig. 1(b)), the two intermediate times S_1 and S_2 are not ordered. This results in the cancelation of these diagrams.

In the next order in the leading $1/N$ expansion, there are the ten different diagrams indicated in Fig. 1(c). The number of topologically different diagrams – which is given by $(2n+1)!/[n!(n+1)!]$, where n is the number of phonon lines – increases very fast thereafter. It is relatively straightforward to check that order by order the total contributions vanishes exactly.

One more simplification occurs in the $\tau \gg \tau_{corr}$ limit, which is also formally equivalent to the $\tau \to \infty$ limit. One can show that in each order the diagrams with n phonons at the same time are $1/\tau^{n-1}$ smaller than the diagrams with only one phonon. For example, in the second order, this corresponds to neglecting the last two diagrams in Fig. 1(c). In any two phonon diagram one has to perform four time integrations. The correlator $F(s_1,s_2)$ restricts the time integration to an interval $|s_1-s_2| \approx \tau_{corr}$. If there are two phonons at the same time, such a diagram will behave like $\sim \tau$ in the $\tau \to \infty$ limit, since $|s_1-s_2| \approx |s_3-s_4| \approx |s_1-s_3| \approx \tau_{corr}$ and only one integration is over an interval of length $\approx \tau$. The remaining diagrams with only one phonon at any time behave like τ^2, since there is only one restriction, namely $|s_1-s_2| \approx |s_3-s_4| \approx \tau_{corr}$, while there are two integrations over a large interval $\approx \tau$. If a phonon line surrounds the fat dot, however, diagrams with two or more phonons at the same time are allowed, such as the first three second order diagrams in Fig. 1(c). This is because the time arguments for such a phonon line are not ordered, so that there is always an extra 'long' integration time interval, as in diagrams with only one phonon line at any time. Hence, in the $\tau \gg \tau_{corr}$ limit, the equation for the dressed propagator $S(\tau)$ simplifies to

$$
S_k(\tau) = S_{0\,k}(\tau) - \int_0^\tau ds_1 \int_0^{s_1} ds_2 S_{0\,k}(s_2)\kappa^2 F(s_1-s_2) \sum_{n=1}^{N} S_{0\,n}(s_1-s_2) S_k(\tau-s_1). \qquad (34)
$$

While the propagator can be simplified, the evaluation of matrix elements still includes the rainbow diagrams involve contraction between the two propagators. Essentially they

have to be summed up as before, and under a rainbow one has to include the partially dressed propagator (33). It can be checked that this reduced class of diagrams conserves probability as well up to corrections of order τ_{corr}/τ. Physically, this limit corresponds to an expansion in τ_{corr}/τ, where τ_{corr} is the characteristic correlation time of the "phonon propagator" $F(s)$, consequently it is defined by the highest phonon frequency $\omega_{high} \sim 2\pi/\tau_{corr}$. Even though it seems that in each order one can select a subclass of dominant diagrams in this combined leading $1/N$ and τ_{corr}/τ expansion, and show that they cancel (except zeroth order of course), this is by no means a rigorous proof. One should not forget the other important parameter in the model – ε – which can change the physical properties rather dramatically. The adiabatic limit, which corresponds formally to $\varepsilon \to \infty$, is very similar to the semiclassical limit, which has its own rather nontrivial mathematical intricacies.

The correlation time τ_{corr} does not determine the only relevant frequency ($\omega_{high} \sim 2\pi/\tau_{corr}$) in this problem. A cursory look at the integrand in Eq. (31)

$$\kappa^2 F(s_1 - s_2) \sum_{n=1}^{N} \exp[-i\varepsilon(e_n - e_k)(s_1 - s_2)]\sigma_n(s_1 - s_2), \tag{35}$$

indicates that the cutoff in time can be determined not only by the phonon spectrum, i.e. by $F(s_1 - s_2)$, but by the sum over the intermediate states n as well. If all the states are coupled to each other through H_1, and if the spectrum of H_0 has a large extension, the sum in Eq. (34) vanishes on average for $(s_1 - s_2) \neq 0$ and can be essentially replaced (with some care) by $N\delta(s_1 - s_2)$. As we mentioned earlier, this feature of the GOE matrix H_1 is unphysical. One can make the model more realistic by replacing the full GOE matrix with a bordered matrix, by replacing $[H_1(X)]_{kl}$ with $V_{kl}[H_1(X)]_{kl}$, where $V_{kl} = V_{lk} \approx 1$ if $|k - l| \leq N_{effective}$ and $V_{kl} = V_{lk} \approx 0$ if $|k - l| \gg N_{effective}$. This would modify the equation for the averaged propagator $S(\tau)$ in a simple manner:

$$\begin{aligned} S_k(\tau) &= S_{0\,k}(\tau) - \int_0^\tau ds_1 \int_0^{s_1} ds_2 S_{0\,k}(s_2)\kappa^2 F(s_1 - s_2) \sum_{n=1}^{N} V_{kn}^2 S_n(s_1 - s_2) S_k(\tau - s_1) \\ &= S_{0\,k}(\tau) - \int_0^\tau ds_1 \int_0^{s_1} ds_2 S_{0\,k}(s_2)\Sigma_k(s_1 - s_2) S_k(\tau - s_1). \end{aligned} \tag{30'}$$

As a result, the proper self–energy becomes state dependent. In this way one more parameter will enter the game: the effective number of states coupled by H_1 at a given average excitation energy ($N_{effective} = \rho_0 \Delta E$) where ΔE is the energy range of the states so coupled. $N_{effective}$ will replace N, and in a complex system one can expect this to be rather large and consequently the same arguments used above can be carried on. ΔE will compete with the highest phonon frequency mentioned above and depending on the particular physical situation one or the other will determine the extent of the memory of the system. Using the analogy with the particle propagating in an elastic medium, the memory effects can be interpreted as simply the action of the excited waves generated by the moving particle in the medium on the particle. As in the case of a boat on an initially quiet lake, the waves produced by the moving boat shake the boat as well.

There is another physical time scale, the so called spreading width $\Gamma^{\downarrow} = 2\pi\rho_0\lambda^2 = 2M_2$ (see Ref. [29] and Appendix A), where M_2 is the average value of the imaginary part of the proper self–energy of the averaged propagator (29) in the energy representation. It characterizes the "life–time" $\tau^{\downarrow} = \hbar/\Gamma^{\downarrow}$ of an eigenstate of the Hamiltonian H_0. In our "natural units" $\tau^{\downarrow} \approx \tau_0$, within an order of magnitude or so, providing $\rho_0\lambda \sim 1$.

The different correlations times we have discussed seem to be independent, in spite of the fact that the dressed propagator is defined in terms of the phonon lines. This can be illustrated by considering the familiar situations for $F(s)$ being either time–independent, or a delta function of time $F(s) \sim \delta(s)$. In these limits, the characteristic time of the dressed propagator is neither infinite nor zero, respectively.

The rather complex nonlinear character of the problem we consider in this article may conspire in such a way that new expansion parameters have to be considered. It is not clear at the moment if this is indeed the case, and we will proceed with the analysis assuming it is not. As a matter of fact, at least one new such scale indeed arises, if one considers the so called two–point fluctuations, or in other words the average level density–level density, which is expressible through the average of a product of two propagators in the energy representation. In the time representation the average of two propagators is directly related with the quantity (24). It has been shown [29, 30] that for GOE matrices $\overline{\rho(E_1)\rho(E_2)} - \overline{\rho(E_1)}\;\overline{\rho(E_2)} \sim 1/N^2(E_1 - E_2)^2$, which consequently implies that for this quantity the expansion parameter is not $1/N$ but rather $1/N^2(E_1 - E_2)^2$. This is likely to be important only when considering the very long time evolution and might of not direct relevance to us yet. Nevertheless, this aspect needs further study, especially in connection with our procedure of introducing correlated initial conditions, described in a section below.

3.2 Equation For Occupation Number Probabilities

It is of certain interest to study the following situation. Assume that initially the system described above is in some eigenstate of the regular Hamiltonian H_0 and the system is driven. One can imagine a slightly more general situation, when the initial state is an arbitrary superposition of eigenstates of H_0 or even in some mixed state. We shall derive here a system of equations, which describe the evolution of the occupation number probabilities, when the system is externally driven.

We are therefore interested in computing the following set of quantities

$$n_k(\tau) = \sum_l n_l(0)\overline{\langle l|U^{\dagger}(\tau)|k\rangle\langle k|U(\tau)|l\rangle} = \sum_l n_l(0)P_{lk}(\tau), \qquad (36)$$

where the quantum numbers for the bra and ket states correspond to eigenstates of H_0 and the overline stands as before for the ensemble average. Since the initial occupation number probabilities do not enter the averaging, we have assumed here that condition (25) is fulfilled. Later on we shall analyse a more general situation. Eq. (36) is not the whole story, since we must include not only the initial occupation number probabilities, but also cross terms with $l \neq l'$. However, it will become clear from our analysis, that

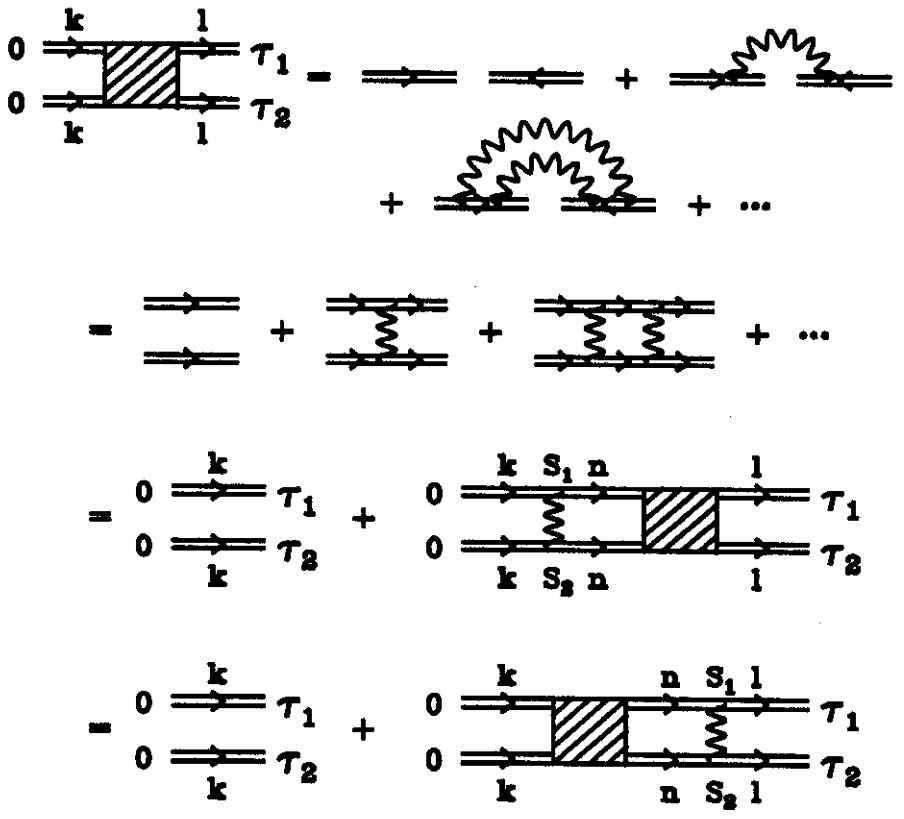

Figure 3: Diagrammatical representation of the Eq. (39) for the transition amplitudes $T_{kl}(\tau_1, \tau_2)$.

such terms are suppressed by at least $1/N$, and in the leading order of the $1/N$ expansion, can be safely ignored (see discussion of the Eq. (39) and Fig. 3 below).

Let us define a more general transition probability $T_{kl}(\tau_1, \tau_2)$ as

$$T_{lk}(\tau_1, \tau_2) = T^*_{lk}(\tau_2, \tau_1) = T^*_{lk}(-\tau_1, -\tau_2) = \overline{\langle l|U^\dagger(\tau_1)|k\rangle\langle k|U(\tau_2)|l\rangle}, \qquad (37)$$

which are related with the transition probabilities $P_{lk}(\tau) = P_{kl}(\tau) = T_{lk}(\tau, \tau)$. The derivation of a system of equations for T_{kl} will parallel the methods used in the previous sections. We begin with a formal expansion of the propagator $U(\tau)$ in powers of $H_1(\tau)$ and then perform the ensemble average on this expansion. As before, we shall sum up all the diagrams on the left and on the right of the measurement operator $A(\tau) = |k\rangle\langle k|$ and subsequently sum up the remaining diagrams, with phonon lines surrounding the measurement operator. Such a summation procedure implies that the renormalization effects for the propagator are more important than the rest, and for that reason, are taken care of first. We already know that this is certainly not the case and likely a more appropriate procedure would be to sum at the same time diagrams with phonon lines on both sides of the measurement operator and surrounding the measurement operator. The

are two heuristic arguments why such a procedure is not chosen: *i)* the formal structure of the emerging equations seem much more complicated, although still tractable to some extent (e.g. in the limit $\tau \gg \tau_{corr}$ or $\tau \to \infty$) and *ii)* in a time dependent picture, the interaction time, (i.e. the effective time integration), might provide a "natural" cutoff and subsequently ensure the relative stability of the evolution equations.

The transition probabilities satisfy two sum rules

$$\sum_k P_{kl}(\tau) = \sum_l P_{kl}(\tau) \equiv 1. \tag{38}$$

These sum rules are obviously satisfied by the nonaveraged quantities, but also by the ensemble averaged quantities in the leading order of the $1/N$ expansion and also in the combined $1/N$ and $1/\tau$ expansion as well. The second sum rule above follows simply from the time–reversal invariance of our model (all Hamiltonians are Hermitian).

The quantities $T_{lk}(\tau_1, \tau_2)$ satisfy the following evolution equations

$$\begin{aligned} T_{lk}(\tau_1,\tau_2) &= S_k(\tau_1)S_k^*(\tau_2)\delta_{kl} + \int_0^{\tau_1} ds_1 \int_0^{\tau_2} ds_2 S_l(s_1)S_l^*(s_2)\kappa^2 F(s_1-s_2) \\ &\quad \times \sum_n V_{ln}^2 T_{nk}(\tau_1 - s_1, \tau_2 - s_2) \qquad (39a) \\ &= S_k(\tau_1)S_k^*(\tau_2)\delta_{kl} + \int_0^{\tau_1} ds_1 \int_0^{\tau_2} ds_2 \sum_n T_{ln}(s_1,s_2)V_{nk}^2\kappa^2 \\ &\quad \times F(s_1-s_2)S_k(\tau_1-s_1)S_k^*(\tau_2-s_2), \qquad (39b) \end{aligned}$$

which can be derived easily following the same line of argument as in the previous sections in the leading order of the $1/N$ expansion. One can also derive it diagrammatically, by summing up all rainbow diagrams, as shown in Fig. 3. In Fig. 3, we did not display the measurement operator and the rainbow diagrams alluded to here are actually ladder diagrams. In these equations, we have included the vertices V_{kl}, which effectively transform the Hamiltonian $H_1(s)$ into a generalized bordered GOE matrix, for the reasons we discussed earlier. The existence and relative simplicity of such a system of evolution equations, which defines the final state occupation number probabilities only in terms of the initial occupation number probabilities, is due to the special character of the GOE ensemble, in particular to the form of the correlator (16). As we have mentioned above in connection with Eq. (36), the fact that in the measurement operator $|k\rangle\langle k|$ both bra and ket vectors carry the same quantum numbers implies that the initial bra and ket vector have identical quantum numbers as well. The easiest way to see this is by analysing each term of the infinite perturbation series for $T_{lk}(\tau_1, \tau_2)$ in Fig. 3.

The set of quantities $T_{kl}(\tau_1, \tau_2)$ contain too much information and one can significantly reduce the amount of calculations by defining the generalized occupation number probabilities

$$\mathcal{N}_k(\tau_1,\tau_2) = \sum_l T_{kl}(\tau_1,\tau_2)n_l(0), \tag{40}$$

which satisfy the evolution equation

$$\mathcal{N}_k(\tau_1,\tau_2) = S_k(\tau_1)S_k^*(\tau_2)n_k(0) + \int_0^{\tau_1} ds_1 \int_0^{\tau_2} ds_2$$

$$\times \sum_l \mathcal{N}_l(s_1,s_2)V_{lk}^2\kappa^2 F(s_1-s_2)S_k(\tau_1-s_1)S_k^*(\tau_2-s_2), \qquad (41)$$

which is readily derived using the second form of the evolution equations (39). One then has

$$n_k(\tau) = \mathcal{N}_k(\tau,\tau). \qquad (42)$$

In the particular case when $\tau_{corr} \to 0$, the correlator $F(s_1-s_2)$ can be replaced with a properly chosen Dirac $\delta(s_1-s_2)$ function. By considering the equal time transitions $T_{lk}(\tau,\tau) = P_{lk}(\tau)$, and the simplifications for the dressed propagator $S_k(\tau)$, we can obtain simple dynamical equations. Explicitly, make the following replacements

$$F(s_1-s_2) \to \delta(s_1-s_2)\int_0^\infty ds F(s) = \gamma\delta(s_1-s_2) \qquad (43)$$

in the equation for the propagator $S_k(\tau)$ and

$$F(s_1-s_2) \to \delta(s_1-s_2)\int_{-\infty}^\infty ds F(s) = 2\gamma\delta(s_1-s_2) \qquad (44)$$

in the equation for the transition probabilities $T_{kl}(\tau_1,\tau_2)$. The quantity γ can be interpreted as a new correlation time, which should be of the same order of magnitude with τ_{corr}. The corresponding equations are then

$$\begin{aligned} S_k(\tau) &= S_{0\,k}(\tau) - \gamma\kappa^2\int_0^\tau ds S_{0\,k}(s)\sum_n V_{kn}^2 S_k(\tau-s) \\ &= S_{0\,k}(\tau)\exp[-\gamma\kappa^2\sum_n V_{kn}^2\tau], \qquad (45) \\ P_{kl}(\tau) &= \exp[-2\gamma\kappa^2\sum_n V_{kn}^2\tau]\delta_{kl} \\ &\quad + \int_0^\tau ds \exp[-2\gamma\kappa^2\sum_m V_{km}^2 s]2\gamma\kappa^2\sum_n V_{kn}^2 P_{nl}(\tau-s) \\ &= \exp[-2\gamma\kappa^2\sum_n V_{kn}^2\tau]\delta_{kl} \qquad (46) \\ &\quad + \int_0^\tau ds\sum_n P_{kn}(s)2\gamma\kappa^2 V_{nl}^2\exp[-2\gamma\kappa^2\sum_m V_{lm}^2(\tau-s)]. \end{aligned}$$

Using the second form of the Eq. (46), and $n_k(s) = \sum_l n_l(0)P_{lk}(s)$, the equation for the occupation number probabilities becomes

$$\begin{aligned} n_l(\tau) &= \exp[-2\gamma\kappa^2\sum_n V_{kn}^2\tau]n_l(0) \\ &\quad + \int_0^\tau ds\sum_n n_n(s)2\gamma\kappa^2 V_{nl}^2\exp[-2\gamma\kappa^2\sum_m V_{lm}^2(\tau-s)]. \qquad (47) \end{aligned}$$

In this limit the evolution equations become linear and can be solved in a straightforward manner using simple matrix operations. A differential form for these equations can be easily derived as well:

$$\frac{dn_l(\tau)}{d\tau} = \sum_n\{2\gamma\kappa^2 V_{nl}^2 - \delta_{nl}\sum_m 2\gamma\kappa^2 V_{ml}^2\}n_n(\tau). \qquad (48)$$

If $V_{kl} \equiv 1$ one can easily verify that

$$P_{kl}(\tau) = \exp[-2N\gamma\kappa^2\tau]\delta_{kl} + \frac{1}{N}\{1 - \exp[-2N\gamma\kappa^2\tau]\}, \tag{49}$$

$$n_k(\tau) = \exp[-2N\gamma\kappa^2\tau]n_k(0) + \frac{1}{N}\{1 - \exp[-2N\gamma\kappa^2\tau]\}. \tag{50}$$

In the case when $V_{kl} \not\equiv 1$ there will be in general $N-1$ different relaxation times, but the same asymptotic state. (Because the total probability has to be conserved, see Eq. (38), there always is a zero eigenvalue for the matrix on the rhs of Eq. (48).) On this explicit solution one can see a characteristic feature of the transition probabilities. Strictly speaking, the nondiagonal term is of the next to the leading order in the $1/N$ expansion. The meaning of the different terms in the equation for the transition probabilities can be easily interpreted. The first diagonal term describes the depletion/loss of the initial populations while the other terms correspond to the gain. It is remarkable that in this approximation, the nonperturbed spectrum has completely disappeared from the final answer: the unperturbed energies e_k do not appear explicitly. Even though we have implicitly assumed that the level density is constant, the reader can easily verify that this result does not depend on that form of the spectrum. This limit seems to correspond to a rather unphysical situation, since it implies that $\tau_{corr}|e_n - e_{n\pm1}| \ll 1$, or in other terms that $T_0 \ll \tau_0$ (see Eq. (22)), while one would expect just the opposite. However, such a limit have been considered before [32 – 34], and is likely of interest as well in problems such as the so called motional narrowing effects.

The asymptotic state in this limit corresponds to equal population probabilities for the entire spectrum – $n_k(\infty) = 1/N$ – and an infinite temperature. In the case of an equidistant spectrum $e_k = k$, the average $\overline{H_0}$ has the time dependence

$$\begin{aligned}\overline{E(\tau)} &= \sum_k e_k n_k(\tau) \\ &= \exp[-2N\gamma\kappa^2\tau]\sum_k e_k n_k(0) + \frac{N+1}{2}\{1 - \exp[-2N\gamma\kappa^2\tau]\}.\end{aligned} \tag{51}$$

For any finite times the process is a Markov process as well. It is easy to check that

$$P_{kl}(\tau_1 + \tau_2) = \sum_n P_{kn}(\tau_1)P_{nl}(\tau_2), \tag{52}$$

$$n_k(\tau_2) = \sum_l P_{kl}(\tau_2 - \tau_1)n_l(\tau_1). \tag{53}$$

In the case of a finite correlation time τ_{corr} such relations, which define a Markov process, do not exist. For any finite τ the occupation number probabilities $n_k(\tau)$ are correlated with the Hamiltonian $H_1(s)$ if $|\tau - s| \sim \tau_{corr}$, which implies that the system has a finite memory time. Thus the above type of solution is best suited for the description of very long time behaviour of a system with a finite correlation time in the limit $\tau/\tau_{corr} \to \infty$. Similar evolution equations have been considered many times before, in connection with different problems [9, 21, 35]. A problem which has not been addressed by earlier studies

is the nature of initial conditions. Since these equations are characterized by memory effects, the problem of initial conditions is non–trivial. In transport theories, one usually invokes Boltzmann's hypothesis of molecular chaos, as we did here. If the emerging evolution equations are Markov in character one can find such an assumption satisfactory. Since our evolution equations display memory effects, one has to analyse correlated initial conditions and we shall discuss such types of initial conditions in a section below.

3.3 Non-Adiabatic Properties: Absence of Landau-Zenner Behavior

In the above discussion we have checked that the approach is consistent in several limits. A cursory analysis of the derived evolution equations reveals a rather startling conclusion. The model so far seems to be a reasonable way to model the temporal behaviour of a complex system. The common wisdom is that an "irreversible" loss of energy during an adiabatic (and may be even nonadiabatic to some extent) evolution in large amplitude collective motion occurs mainly through the excitations of the intrinsic degrees of freedom mostly at level crossings, through the Landau–Zener mechanism [7, 9, 11 – 18, 20]. Similar ideas have been invoked also for the description of the excitation mechanism of electrons in quantum wires, threaded by a variable magnetic flux [36]. This amounts to essentially an incoherent sum over separate transitions at different level crossings. (This has some obvious shortcomings, in that level crossings are no longer well defined when one has $N \gg 1$ levels.) There are several implicit assumptions, not always spelled out explicitly in such analyses. The first assumption is obviously that during a very slow (adiabatic) evolution the system jumps from one adiabatic level to another one only in the neighborhood of a level crossing, via the Landau–Zener mechanism. In the limit when the collective velocity is vanishing $V_0 \to 0$, i.e. $\varepsilon \to \infty$, the Landau–Zener transitions are exponentially small. Another implicit assumption is that subsequent transitions are well separated in time and no interference effects are present.

The above analysis points seemingly to a completely different picture, a coherent excitation mechanism of the *whole* system, or at least of some big portion of its spectrum, always. In all the diagrams we have summed up so far, there always is the *big* factor N (or N_{eff} to be more precise). Therefore, instead of at most one transition at a time, as in the Landau–Zener picture, there are N transitions at the same time. There is almost no doubt (except maybe the very long time behaviour, see remarks above concerning two–point fluctuations) that such a factor emerges, and that summing the leading order diagrams in the $1/N$ expansion should lead to the proper description. The effective number of intermediate states always multiplies the phonon line. In any diagram the main contribution comes from such a process: a transition from one state to all others in a coherent way and afterwards a recombination into the same state, and not from only one state to another single one. In discussing the Landau–Zener mechanism of the excitation of the intrinsic modes, everybody seems to be concerned with the presence of small level spacings. On the other hand, since the GOE part of the Hamiltonian H_1 seems to couple a large number of states, the rigidity of the spectrum, the so called Δ_3 statistics, and/or

the two–level cluster function Y_2 (which describes the corelation between two arbitrary levels) are very likely to play an essential role (if not the essential role) as well. It will be extremely interesting to see whether Hamiltonians with different Δ_3 statistics and/or different Y_2 cluster functions, but with similar nearest–neighbor level spacing distributions behave differently or not. In spite of the random character of the interaction H_1 at all times, the system is in a highly correlated state of one type or another. At the same time the transition is spread over a time interval of the order of the correlation time τ_{corr}. This last feature looks particularly puzzling for the following reasons. In Fig. 4 we have plotted two spectra, as a function of the running time τ for two Hamiltonians: the first one constructed according to Eqs. (14) and (19), and the second one as once suggested by Wilkinson [7]

$$H(s) = H_0 + \cos(\omega s)H_1 + \sin(\omega s)H_2, \tag{54}$$

where $H_{1,2}$ are two constant GOE matrices. The correlator in these two cases are obviously qualitatively different, in the first case is given by Rel. (19b) while for the above Hamiltonian is $F(s) = \cos\omega(s_1 - s_2)$ and therefore the spectra at different points in the ensemble are essentially strongly correlated (or anticorrelated). On the other hand these two particular Hamiltonians have qualitatively similar appearances. One would expect for that reason, as the common wisdom based on the Landau–Zener picture seems to suggest, that the time evolution in these two cases, and therefore the "dissipation" in particular, has essentially the same character. It is reasonable to expect that in the first case the system has a shorter memory (if one assumes that the memory is determined to some extent by the phonon, i.e. $\tau_{corr} = 2\pi/\omega$), while in the second case long time correlations are present in the time evolution of the system, (Appendix B). Strictly speaking, the averaged spectrum is discrete in this case and recurrences during the time evolution appear, at least to some extent.

In the Landau–Zener mechanism the transitions are assumed to be uncorrelated. At the same time one can easily see that within one correlation time ($\tau_{corr} \approx 2\pi/\omega$) for the Hamiltonian (54) one can find several level crossings, i.e. the average distance between two consecutive level crossings is smaller than the correlation length. One has therefore a rather confusing situation. Either the obvious parameters of the theory are completely irrelevant and the actual dynamics of the system is governed by a new set of emerging parameters, as we have alluded to earlier, or at least some of the assumptions of the commonly accepted point of view are inconsistent with the true dynamical behaviour of a system with a complex excitation spectrum.

The evolution equations (30') and (39) seem to have an even wider region of applicability than that for which they were derived. In particular, one can consider also the case of a GUE (Gaussian Unitary Ensemble) instead of GOE. The GUE Hamiltonians are characterized by a stronger level repulsion (smaller level spacing distribution at small separations) than GOE and consequently the number of level crossings is significantly smaller. One might expect that the Landau–Zener excitation mechanism is strongly suppressed in the strict adiabatic limit for GUE in comparison with GOE [7]. However, the only change in the formalism we have described above amounts only to $F(s) \to 2F(s)$ in the leading order of the $1/N$ expansion, or in other words to a slight rescaling of the

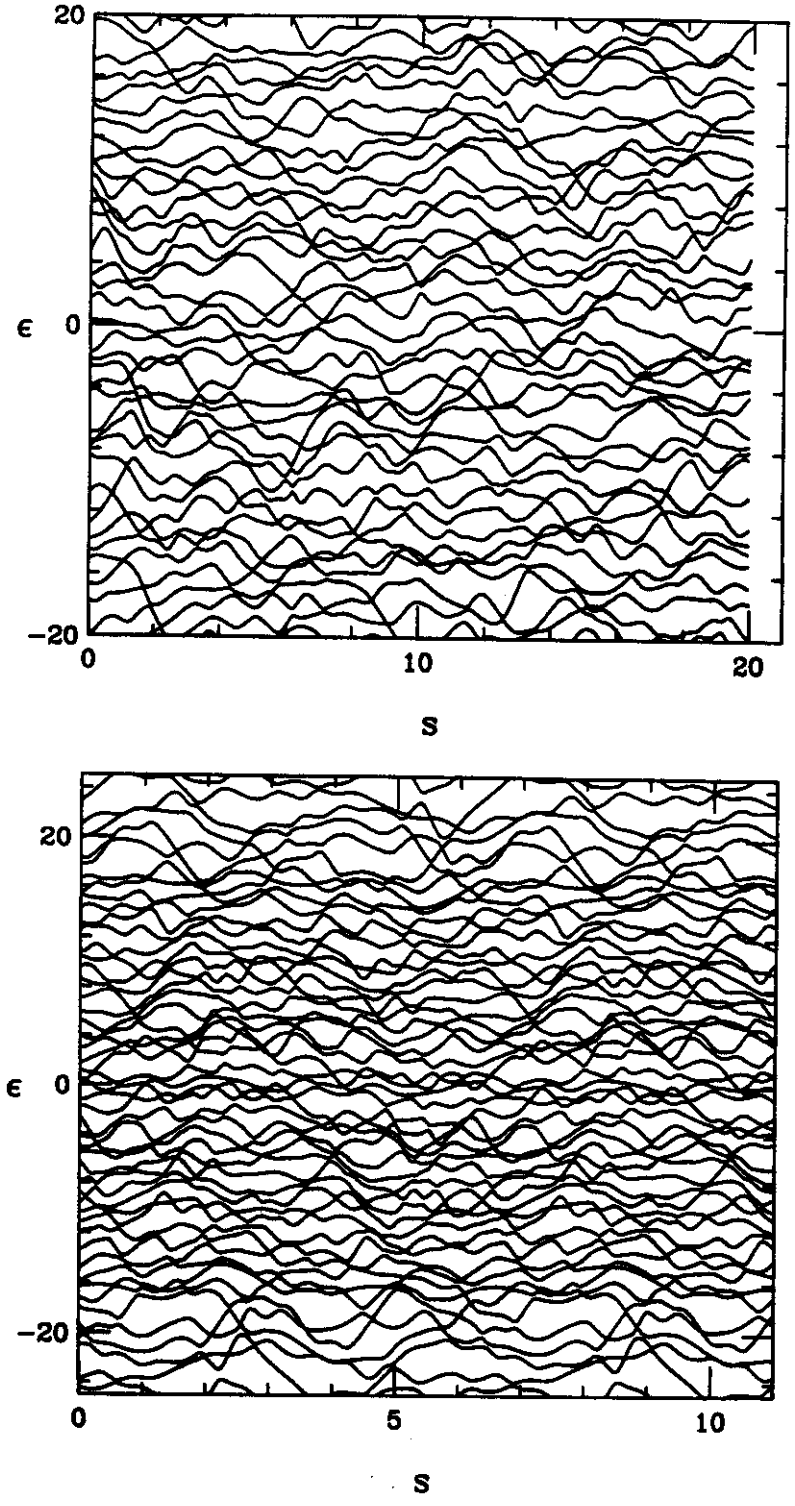

Figure 4: Two characteristic level schemes for the Hamiltonians constructed according to Eqs. (13, 14), with $\alpha = 1$, Fig. 4a upper plot, and Eq. (54), with $H_0 \equiv 0, \quad \omega = 1$, Fig. 4b, lower plot.

Hamiltonian H_1 ($\lambda \rightarrow \sqrt{2}\lambda$). Consequently, one can safely conclude then that the dissipation in both cases is essentially similar, which once more implies that the Landau–Zener excitation mechanism has nothing to do with the collective energy degradation, at least in this particular model.

We have assumed that as a function of the running time τ the Hamiltonian is a smooth function. However, it is easy to realize that exactly the same equations can be derived also if we are dealing with colored noise and in particular with white noise as well. In such a case the Hamiltonian is only a continuous function of the running time τ but not necessarily smooth [37]. After taking the ensemble average, there is no difference between these obviously different types of Hamiltonians. In the case of colored or white noise one can hardly identify level crossings and moreover justify the adiabatic hypothesis. If so, this seems to be simply the killing blow for the commonly accepted Landau–Zener mechanism for the excitation of the intrinsic degrees of freedom, at least if one includes the GOE character of the Hamiltonian as one of the initial premises. This line of reasoning suggests one more way of generating GOE Hamiltonians, to which the present approach should apply, and which might be used for modeling complex systems. Dyson [38] suggested a Brownian model for the ensembles of random matrices. He considered both stationary and nonstationary processes with however time independent diffusion coefficients and a time independent potential force. It is natural to consider along with this type of Brownian processes also Brownian ensemble of Hamiltonians in either nonstationary environments and/or some external fields and moreover the case of colored noise as well. This opens quite wide opportunities for the modeling of transport phenomena in many–body systems.

The assumption of the GOE character of the spectrum and therefore of the ensemble average is basically equivalent to some form of ergodicity: one replaces the time average over a long trajectory with an ensemble average. The ergodicity argument has been invoked many times in connection with the random matrix approach [3 – 5, 39]. During a long time evolution the system will traverse many level crossings and at any point along the trajectory the instantaneous spectrum seems to have always a GOE character. If the GOE properties of the system have a certain transitive character, as in the example described by us, and the initial conditions are arbitrary, i.e. the system can start its evolution at any point along the time axis τ or space axis X respectively, this ergodic hypothesis makes sense. In the case when the system does not have such transitiveness one merely has to slightly modify in a more or less obvious fashion the ensemble averaging. (Since we are not going to analyse this more general situation in the present paper we shall defer this aspect to a future publication.) Consequently the acceptance of the ergodic hypothesis seems legitimate. For many the introduction of ensembles of random Hamiltonians still looks like a rather artificial hypothesis, since any real system is described by one given Hamiltonian and not an ensemble. There are many arguments [3 – 6] why it is useful to introduce random matrices, and perhaps the best argument is still the simplicity of the ensuing formalism. At the same time this hypothesis can be accepted much easier if one realizes that there is a certain analogy with similar many–body techniques, like Hartree–Fock type of approximations or canonical and grand canonical ensembles, when one essentially does a similar thing, mixes several different systems and as a result a much more transparent

formalism arises. Since the matrix elements of many–body systems seem to be GOE in character, see e.g. Ref. [39], one can use alternatively ensemble averages either over the spectrum of a given Hamiltonian or over an ensemble of such Hamiltonians with equal results, which is exactly another way of stating that the GOE has ergodic properties.

Besides the enumerated difficulties with the Landau–Zener picture mentioned above, there are other less evident problems. A closer analysis of these types of spectra, see Fig. 4, shows that the identification of a level crossing is sometimes a rather ill defined procedure. One can encounter quite often a situation where three levels come close to each other and the simple inspection of the level scheme shows that one has rather a three level crossing. More complicated crossings are also easy to find (e.g. the distance between two levels is almost constant for quite a while and the avoided level crossing cannot be assimilated with a point but rather with a short segment), even though with a relatively low frequency. A straightforward implementation of the Landau–Zener mechanism becomes rather ambiguous. The same type of difficulties will arise if one will attempt to extract from computer generated spectra statistical properties of level crossings, since the algorithms used so far are blind to such situations.

The adiabaticity parameter $\varepsilon \sim \kappa$ formally plays a dual role: of the transition strength and of an effective $1/\hbar$. As we have mentioned earlier, the adiabatic limit corresponds strictly speaking to $\varepsilon \to \infty$ and one might expect that in this case something similar to the semiclassical approximation can be derived. The role of momentum and coordinate are played by the energies e_n and time τ respectively. One can derive such an approximation scheme in the limit of a continuous spectrum, when the average level spacing tends to zero, i.e. $\overline{|e_n - e_{n\pm1}|} \to 0$. Formally one has to replace sums with integrals according to the simple prescription

$$\sum_n C_n \to \int dE \rho(E) C(E) \,, \tag{55}$$

where $\rho(E)$ is the average level density and $C_n = C(e_n)$ are some arbitrary quantities. The effective potential $F(s_1 - s_2)$ in this case will be nonlocal, however methods dealing with such situations exists and a series of rather unusual characteristics of the propagation might emerge [40]. We shall not attempt such an approach in the present paper, even though this is highly desirable. A different type of semiclassical limit is also possible. In the evolution equations (39), (40) one can introduce the sum and difference of the two time arguments, namely $\tau = (\tau_1 + \tau_2)/2$ and $s = \tau_1 - \tau_2$ and perform a Wigner type of transformation on the time variable s, (perhaps followed as well by a Husimi transform). Such a procedure is appropriate if the characteristic time scale for the τ–time variable is much larger than the characteristic time scale for the s–time variable and it has been used in Ref. [21]. It implies that the memory of the system is rather short and in the canonically conjugated variable to s the generalized transition amplitudes T_{kl} or the generalized occupation number probabilities $\mathcal{N}_k$ are rather smooth functions. For complex systems, where the level densities are high, a combined Wigner type of approach (i.e. both version described here) is likely to be very effective.

3.4 Correlated Initial Conditions

We have not analysed yet the case when the initial state is an eigenstate of the instantaneous Hamiltonian $H(0)$. In order to do this one has to learn how to compute correlators of the type $\overline{\langle\phi_n(0)|H_1(\tau)|\phi_n(0)\rangle}$ (and more complicated ensemble averages with several H_1), where $\phi_n(0)$ is the $n-$th instantaneous eigenfunction at $\tau = 0$ of the *total* Hamiltonian $H(0)\phi_n(0) = [H_0 + H_1(0)]\phi_n(0) = E_n(0)\phi_n(0)$. However, if one assumes that the system has a short memory, one might expect that the initial state of the system should not be very important after a while and the previously described formalism should probably be valid. This is a very subtle issue. Even though the initial conditions might decorrelate relatively quickly, it is not appropriate yet to simply invoke our formalism. At any moment, even at much later times, there well exist correlations with the Hamiltonian in the near past and near future, relative to that moment in time. Consequently, a procedure to deal with correlated initial conditions is more than desirable.

We need a slight generalization of our formalism developed so far for the case when the initial state is an eigenstate of the Hamiltonian $H(0)$. Let us introduce the projectors

$$\sum_{E_n(0)=E} |\phi_n(0)\rangle\delta(E - E_n(0))\langle\phi_n(0)| = \delta(E - H(0)) \tag{56a}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} dt \exp[i(E - H(0))t],$$

$$\sum_{E_1\leq E_n(0)\leq E_2} |\phi_n(0)\rangle\langle\phi_n(0)| = \rho(E_1, E_2, H(0)) \tag{56b}$$

$$= \int_{E_1}^{E_2} dE \frac{1}{2\pi}\int_{-\infty}^{\infty} dt \exp[i(E - H(0))t]$$

and use them in order to construct initial conditions, which correspond to either a sharp value of the energy for the total Hamiltonian $H(0)$ or a superposition of such states in a given energy interval (E_1, E_2). One can introduce other types of initial density matrices in terms of the initial Hamiltonian $H(0)$, e.g. assume that initially the system is at some temperature. Instead of the matrix element (24) we shall compute now the following quantities

$$\overline{\mathrm{Trace}\{\delta[E - H(0)]U^\dagger(\tau)A(\tau)U(\tau)\}}, \tag{57a}$$

$$\overline{\mathrm{Trace}\{\rho(E_1, E_2, H(0))U^\dagger(\tau)A(\tau)U(\tau)\}}, \tag{57b}$$

but, as before, we shall assume that the measurement operator $A(\tau)$ is uncorrelated with the Hamiltonian $H(s)$ for the time being and for simplicity consider the case when $A(\tau) = |n\rangle\langle n|$, i.e. the projector into one arbitrary eigenstate of the Hamiltonian H_0. If we compute the occupation number probability of the quantum state $|n\rangle$ at the time τ in these initial ensembles (57), we have for the two cases,

$$N_n(\tau) = \begin{cases} \frac{1}{2\pi}\int_{-\infty}^{\infty} dt e^{iEt}\overline{\mathrm{Trace}\{\exp[-iH(0)t]U^\dagger(\tau)|n\rangle\langle n|U(\tau)\}}, \\ \\ \frac{1}{2\pi}\int_{E_1}^{E_2} dE \int_{-\infty}^{\infty} dt e^{iEt}\overline{\mathrm{Trace}\{\exp[-iH(0)t]U^\dagger(\tau)|n\rangle\langle n|U(\tau)\}}, \end{cases} \tag{58}$$

For the sake of simplicity of the ensuing formulas we shall consider only the ensemble average in Eq. (58) and not write down explicitly the integral over the variables t and E. It is convenient to continue the Hamiltonian $H(\tau)$ to negative times according to $H(\tau) \equiv H(0)$ if $\tau \leq 0$. The correlator (16) becomes now

$$F(s_1, s_2) = \begin{cases} F(s_1 - s_2), & \text{if } s_1, s_2 \geq 0; \\ F(s_1), & \text{if } s_1 \geq 0 \text{ and } s_2 \leq 0; \\ F(-s_2) = F(s_2), & \text{if } s_1 \leq 0 \text{ and } s_2 \geq 0; \\ F(0), & \text{if } s_1, s_2 \leq 0; \end{cases} \tag{16'}$$

This correlator satisfies the symmetry condition $F(s_1, s_2) = F(s_2, s_1)$, which we have used in proving the conservation of probability. Our task is to calculate the ensemble average in Eq. (58) for both positive and negative times t. We can rewrite the ensemble average as

$\underline{t \geq 0}$:

$$\begin{aligned} W_n(\tau, t) &= \overline{\text{Trace}\{\exp[-itH(0)]U^\dagger(\tau)|n\rangle\langle n|U(\tau)\}} \\ &= \overline{\text{Trace}\left\{\exp\left[-i\int_{-t}^{0} dsH(s)\right] U^\dagger(\tau)|n\rangle\langle n|U(\tau)\right\}} \\ &= \overline{\text{Trace}\left\{T_a \exp\left[i\int_0^\tau dsH(s)\right] |n\rangle\langle n| T\exp\left[-i\int_{-t}^{\tau} dsH(s)\right]\right\}} \\ &= \sum_k \overline{\langle k|T_a \exp\left[i\int_0^\tau dsH(s)\right] |n\rangle\langle n| T\exp\left[-i\int_{-t}^{\tau} dsH(s)\right] |k\rangle} \end{aligned} \tag{59a}$$

and

$\underline{t \leq 0}$:

$$\begin{aligned} W_n(\tau, t) &= \overline{\text{Trace}\{\exp[-itH(0)]U^\dagger(\tau)|n\rangle\langle n|U(\tau)\}} \\ &= \overline{\text{Trace}\left\{\exp\left[i\int_t^{0} dsH(s)\right] U^\dagger(\tau)|n\rangle\langle n|U(\tau)\right\}} \\ &= \overline{\text{Trace}\left\{T_a \exp\left[i\int_t^\tau dsH(s)\right] |n\rangle\langle n| T\exp\left[-i\int_{0}^{\tau} dsH(s)\right]\right\}} \\ &= \sum_k \overline{\langle k|T_a \exp\left[i\int_t^\tau dsH(s)\right] |n\rangle\langle n| T\exp\left[-i\int_{0}^{\tau} dsH(s)\right] |k\rangle} \end{aligned} \tag{59b}$$

In these formulas T_a stands for the time anti-time-ordering and T for the time-ordering operators, respectively. It should be obvious that

$$W_n(\tau, t) = W_n^*(\tau, -t), \tag{60}$$

which ensures that the occupation number probabilities $N_n(\tau)$ in Eq. (58) are real. The rules we have developed earlier for the computation of the matrix element (24) can now

be easily generalized in order to compute the quantities (59). There are only a few rather trivial changes. When computing the averaged propagator S_k or S_k^* one has to consider now a slightly different one, which describes propagation from $-|t|$ to τ and use the correlator (16'). The same applies for the diagrams involving a phonon line over the measurement operator $A(\tau)$. In this case, one of the particle lines in the corresponding diagrams will go from $-|t|$ to τ, while the second line, as before, from 0 to τ only. In an analogous way one can prove that the total probability is conserved in the leading $1/N$ expansion, by summing the rainbow diagrams. The $\tau \to \infty$ or $\tau \gg \tau_{corr}$ limit now becomes somewhat tricky. The reason is that for negative times the correlation time is essentially infinite and no simplification of the diagrams, similar to the one described earlier, is valid in this region. Nevertheless, the corresponding averaged propagators can be evaluated rather easily, see Appendix A. However, for any intermediate positive times one can as before consider a reduced class of diagrams and as a result the time evolution becomes a Markovian process.

At this point we like to remind the reader about the possible importance of diagrams beyond the $1/N$ leading order. As we have mentioned in a previous section, in the case of the average of two pure GOE propagators in the energy representation, the expansion parameter under certain circumstances seems to be $1/N|E_1 - E_2|$ rather than $1/N$ [5, 29, 30]. Since for negative times we are dealing with pure GOE matrices, such corrections might be relevant and should be introduced as well. If that is the case, since the corresponding formula have already been derived, it is possible to introduce them. However, in the present case we have to deal with only one negative time, and the correlator (16') $F(s_1, s_2) \equiv 1$ for $s_{1,2} < 0$ never actually enters into the calculations, so we can neglect such refinements.

In spite of the fact that this correlated initial condition formalism is equivalent to the one we have analysed earlier, the actual formulae are much more cumbersome to deal with. The main cause of this is the fact that the correlator is no longer a function of only the difference of the two times. From the physical point of view, we cannot expect realistic systems to always have this symmetry, and a more general correlator will most likely have to be considered. The cost is that we have lost a certain symmetry with respect to the two particle lines, and as a result the corresponding quantity of interest $W_n(\tau, t)$ (which replaces $P_{kl}(\tau)$ now) is complex. The advantage is that once these quantities are calculated one can consider essentially any type of initial state, in an arbitrary part of the spectrum. This last feature of the theory reflects the fact that the system retains for some time certain information about the initial state. One can imagine now that similar modifications of the formalism will emerge if one has to consider a measurement operator correlated somehow with the Hamiltonian, e.g. if one intends to compute the average of $H(\tau)$, $H^2(\tau)$ or $\partial H(\tau)/\partial\tau$. As it turns out these quantities are relatively easy to estimate and we shall do this below.

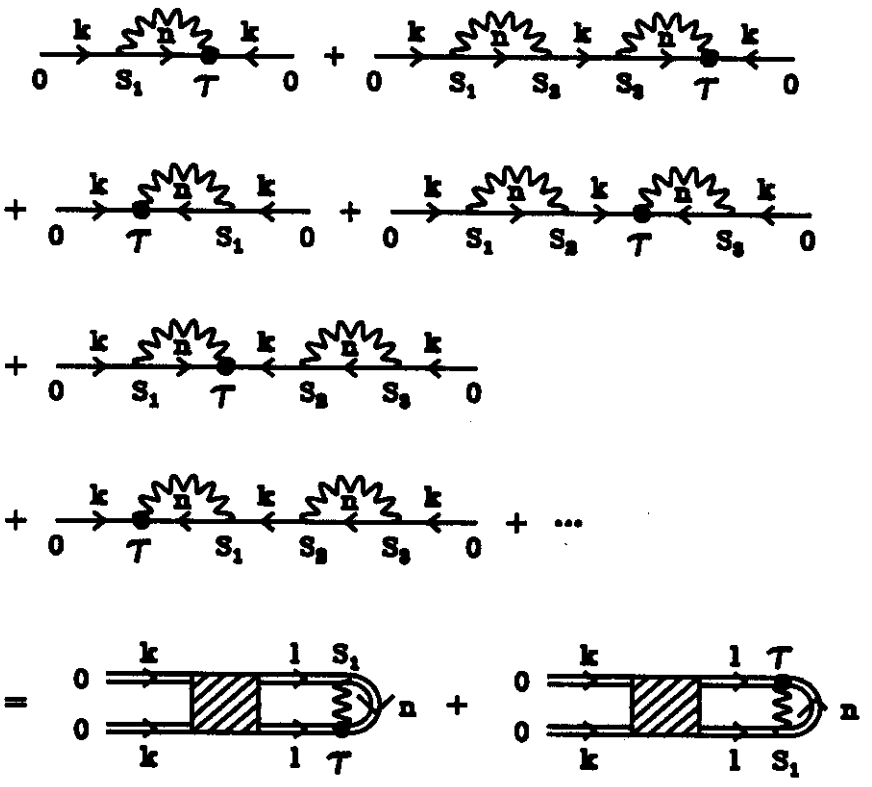

Figure 5: Diagrammatic representation of the matrix element (64).

4. Average Excitation Energy and Hellman–Feynman Forces

If the initial state is uncorrelated with the Hamiltonian one can show that

$$\overline{\langle\psi(0)|U(\tau)^\dagger H(\tau)U(\tau)|\psi(0)\rangle} = \overline{\langle\psi(0)|U(\tau)^\dagger[\epsilon H_0 + \lambda H_1(\tau)]U(\tau)|\psi(0)\rangle}$$
$$= \sum_k \epsilon_0 e_k n_k(\tau) + \overline{\langle\psi(0)|U(\tau)^\dagger \lambda H_1(\tau)U(\tau)|\psi(0)\rangle}, \qquad (61)$$

where $\epsilon_0 = 1/\rho_0$. In the particular case when $H_0 \equiv 0$, this ensemble average is identically vanishing. One can also show by inspecting the diagrammatic expansion of this quantity, shown in Fig. 5, that in the leading order of the $1/N$ expansion

$$\overline{\langle\psi(0)|U(\tau)^\dagger \lambda H_1(\tau)U(\tau)|\psi(0)\rangle}$$
$$= -i\lambda\kappa \sum_{kln} n_k(0) V_{ln}^2 \int_0^\tau ds F(\tau - s)\{T_{kl}(s,\tau)S_n(\tau - s) - T_{kl}(\tau, s)S_n^*(\tau - s)\}, \qquad (62)$$

which is generally non–zero. Note that $T_{kl}(\tau, s) = T_{kl}^*(s, \tau)$ (see Eq. (37)). The ensemble average of the Hellman–Feynman force, acting on the classical degrees of freedom, can be

evaluated to leading order in the $1/N$ expansion as

$$\begin{aligned}\mathcal{F}(t) &= -\overline{\langle\psi(\tau)|\frac{\partial H(X)}{\partial X}|\psi(\tau)\rangle} = -\frac{\lambda}{X_0}\overline{\langle\psi(\tau)|\frac{\partial H_1(\tau)}{\partial \tau}|\psi(\tau)\rangle} \\ &= -i\frac{\lambda\kappa}{X_0}\sum_{kln} n_k(0)V_{ln}^2\int_0^\tau ds\frac{\partial F(\tau-s)}{\partial\tau}[T_{kl}(s,\tau)S_n(\tau-s) - T_{kl}(\tau,s)S_n^*(\tau-s)].\end{aligned} \tag{63}$$

An alternative way to calculate the force is by taking the time derivative of the average energy (61)–(62). (In one case one calculates the derivative of the average while in the other is the average of the derivative.) It is tempting to interpret this quantity as the friction force, however one should not forget that $\kappa \sim 1/V_0$ and therefore the force seems to be actually inversely proportional to the velocity, unless the integral term is quadratic in velocity in some regime. One can estimate easily this force in the first order of the perturbation in the interaction and obtain that

$$\mathcal{F}(t) = \frac{2\lambda\kappa}{X_0}\sum_{kln} n_k(0)V_{ln}^2\int_0^\tau ds\frac{\partial F(s)}{\partial s}\sin[\varepsilon(e_n-e_k)s]. \tag{64}$$

Since $F(s)$ is maximum for a vanishing argument, its derivative should be negative near zero. If the initial state is the ground state, i.e. $n_k(0) = \delta_{k0}$, the energy difference is positive and for the case of a Gaussian correlator, the integral is negative. In the strict adiabatic limit, $\varepsilon \to \infty$, and the force will be negative, but will decay exponentially as $V_0 \to 0$. For the case $F(s) = \exp(-\alpha|s|)$, the force is essentially independent of velocity (neglecting some fast and small oscillating terms) in this approximation. On the other hand, one cannot retain only the first order perturbation term when $\varepsilon \sim \kappa \to \infty$. Such an approximation is legitimate in the opposite limit, of very large velocities. In this case, as $\varepsilon \sim \kappa \to 0$ and the force behaves like $1/V_0^2$. As before, in the particular case when $H_0 \equiv 0$, this force vanishes identically. The ensemble average (62), and therefore the ensemble average of the Hellman–Feynman force, also vanishes identically in the limit $\tau_{corr} \to 0$, which, as we have argued before, is similar to the limit $\tau \gg \tau_{corr}$. One has simply to replace the correlator by $\gamma\delta(\tau - s)$ in Eq. (62) and the transition probabilities amplitudes $T_{kl}(\tau, s)$ by $P_{kl}(\tau)$ (which are real) and this follows immediately. As one sees, the introduction of a friction coefficient is somewhat problematic.

One can write down now explicit formulas for the case when the initial state is either an eigenstate of the initial Hamiltonian or some superposition of such states. Since the principle is already clear and in order to avoid cluttering the paper with too many formulas (which by now should be rather easy to derive if needed) we shall not display them here.

The second moment of the excitation energy can be computed as well

$$\begin{aligned}&\overline{\langle\psi(0)|U(\tau)^\dagger H(\tau)^2 U(\tau)|\psi(0)\rangle} \\ &= \overline{\langle\psi(0)|U(\tau)^\dagger\{\epsilon_0^2 H_0^2 + +\epsilon_0\lambda[H_0H_1(\tau) + H_1(\tau)H_0] + \lambda^2 H_1(\tau)^2\}U(\tau)|\psi(0)\rangle} \\ &= \sum_k[\epsilon_0^2 e_k^2 + \sum_n \lambda^2 V_{kn}^2]n_k(\tau) - i\epsilon_0\lambda\kappa \\ &\quad\times\sum_{kln} n_k(0)V_{ln}^2 e_n\int_0^\tau ds F(\tau-s)\{T_{kl}(s,\tau)S_n(\tau-s) - T_{kl}(\tau,s)S_n^*(\tau-s)\}\end{aligned}$$

$$
\begin{aligned}
&- \lambda^2\kappa^2 \sum_{lknm} n_l(0) V_{kn}^2 V_{nm}^2 \int_0^\tau ds_1 \int_{s_1}^\tau ds_2 F(\tau - s_1) F(\tau - s_2) \\
&\quad \times \{T_{lk}(s_1, \tau) S_n(\tau - s_1) S_m(\tau - s_2) + T_{lk}(\tau, s_1) S_n^*(\tau - s_1) S_m^*(\tau - s_2)\} \\
&+ \lambda^2\kappa^2 \sum_{lknm} n_l(0) V_{kn}^2 V_{km}^2 \int_0^\tau ds_1 \int_0^\tau ds_2 T_{lk}(s_1, s_2) F(\tau - s_1) \\
&\quad \times F(\tau - s_2) S_n(\tau - s_1) S_m^*(\tau - s_2). \qquad (65)
\end{aligned}
$$

In the limit $\tau_{corr} \to 0$ or $\tau \gg \tau_{corr}$, only the first term survives, while the terms involving integrals identically cancel.

It is of extreme interest to compute the fluctuation properties of the Hellman–Feynman force (63). Even though this force is in general nonvanishing, it does vanish in the long time limit and one might expect that its variance is different from zero. Indeed

$$
\begin{aligned}
&\frac{\lambda^2}{X_0^2} \overline{\langle \psi(\tau) | \frac{\partial H_1(\tau)}{\partial \tau} \frac{\partial H_1(\tau)}{\partial \tau} | \psi(\tau) \rangle} = - \frac{\lambda^2}{X_0^2} \sum_{kn} n_k(\tau) V_{kn}^2 \frac{\partial^2 F(0)}{\partial \tau^2} \\
&\quad - \frac{\kappa^2\lambda^2}{X_0^2} \sum_{lknm} n_l(0) V_{kn}^2 V_{nm}^2 \int_0^\tau ds_1 \int_{s_1}^\tau ds_2 \frac{\partial F(\tau - s_1)}{\partial \tau} \frac{\partial F(\tau - s_2)}{\partial \tau} \\
&\qquad \times [T_{lk}(s_1, \tau) S_n(\tau - s_1) S_m(\tau - s_2) + T_{lk}(\tau, s_1) S_n^*(\tau - s_1) S_m^*(\tau - s_2)\} \\
&\quad + \frac{\kappa^2\lambda^2}{X_0^2} \sum_{lknm} n_l(0) V_{kn}^2 V_{km}^2 \int_0^\tau ds_1 \int_0^\tau ds_2 T_{lk}(s_1, s_2) \\
&\qquad \times \frac{\partial F(\tau - s_1)}{\partial \tau} \frac{\partial F(\tau - s_2)}{\partial \tau} S_n(\tau - s_1) S_m^*(\tau - s_2). \qquad (66)
\end{aligned}
$$

Strictly speaking this quantity is not equal to the average of the square of the Hellman–Feynman force, however it should give a good estimate of the fluctuation properties of the Hellman–Feynman force. The calculation of $\overline{\mathcal{F}(t)^2}$ will require the introduction of new quantities into the theory, whose ensemble evaluation is a quite involved procedure. One can easily compute the two times correlator of this quantity and of the Hamiltonian as well. The corresponding expressions, not given here, are only slightly more complicated.

Again, in the $\tau_{corr} \to 0$ or $\tau \gg \tau_{corr}$ limit, the integral terms cancel, and only the first term survives. In this limit it looks like the reaction of the quantum system can be modeled by a Langevin type of force, characterized by colored noise, since there is no Dirac delta function in this correlator. One might suspect that when $\tau_{corr} \to 0$, this correlator diverges, which will imply a Dirac delta function. For the simple case when $F(s) = \exp[-s^2/4]$, $\alpha X_0^2 = 1$, $\tau_{corr} = X_0$, (see Eq. (19b)), one can easily check that this correlator behaves like $1/X_0^2$, if one retains only the first nonintegral term. This divergence is stronger than one might have expected ($\sim 1/X_0$) and suggests that this limit should be taken more carefully. In particular, it could amount to the fact that one cannot replace the dressed propagator $S_k(s)$ with the free one $S_{0\,k}(s)$, as we have described earlier, when we discussed the $\tau \to \infty$ or $\tau \gg \tau_{corr}$ limits, and one has to consider the next order terms in this expansion.

These results are a bit puzzling, since we did not obtain a dissipative force (friction) but only a fluctuating force. On the other hand, one cannot use directly this result in a

combined treatment of the coupled quantum and classical system, since from the outset we have assumed that the quantum system is driven. If only a fluctuating force would act on the classical variable, the total energy of the system would not be conserved, even though we have a closed system. One can reinforce by hand the conservation of the total energy, but such a procedure seems to us as quite arbitrary. Obviously one has to rederive the coupled equations of motion, while considering the ensemble average at the same time. The corresponding theory will be more complicated, since it will involve the evaluation of mixed correlators, among classical and quantum quantities. The momenta and the coordinates of the classical system at any arbitrary time are correlated with the quantum propagators. Special assumptions have to be made again about the character of the initial conditions. We shall address these questions in a future publication. In a consistent theory, the total energy of the system has to be conserved. We might expect therefore that the equations of motion described here have a limited region of validity, as it is usually the case in the analysis of driven systems, if one intends to study the influence of the driven system on the driving one. Whether in a complete treatment a dissipative force arises, at least at some approximation level, it is certainly a very challenging question, even in the framework of such a simplified quantum model. One should remark that, as a rule, we used to trying to derive a friction force and a diffusion constant, based on the usual assumption that the Einstein–Langevin description of the Brownian motion is the only possible alternative. While the Einstein–Langevin approach to Brownian motion is an extremely nice phenomenological model, and at the same time (due to the concerned effort of several generations of physicists and mathematicians) the mathematical apparatus is extremely powerful, one should not forget that this is simply a phenomenological model and not a law of nature. There is no reason to expect it to be valid under all circumstances, and in the literature there are a multitude of generalizations and other developments. In particular in Ref. [41] a completely different approach to Brownian motion has been introduced, which, similarly to what seems to be happening in the present model, has only a fluctuating force. Moreover, the approach of Ref. [41] is both deterministic and time–reversal invariant, which is completely at odds with the standard Langevin model of the Brownian motion, which is dissipative and stochastic.

5. Conclusions

Our final goal is to study coupled slow and fast degrees of freedom (with complex spectra) in systems like atomic nuclei, atomic clusters, deformable cavities, etc. Due to the continuous energy exchange among these degrees of freedom, the dynamics of the slow modes is strongly modified and the energy stored initially in the slow degrees of freedom will slowly be degraded. Whether one can analyse dynamics of the slow degrees of freedom only and introduce a phenomenological type of dissipation and to devise a scheme in which

to compute it reliably seems to be still an open question. Wilkinson suggested recently that the Landau–Zener excitation of the internal degrees of freedom might lead to a very peculiar type of dissipation in the extreme adiabatic approximation. It was argued that the diffusion coefficient for the slow degrees of freedom should be proportional to $V_0^{3/2}$ rather than V_0^2 as one might have expected. This will amount to the fact that the loss of energy from the slow to the intrinsic degrees of freedom is significantly enhanced at small collective velocities. His conclusions are based on some rather "natural" assumptions: the spectrum of the intrinsic excitations has a GOE character, the system is externally driven and excitation proceeds through incoherent Landau–Zener transitions. While the analysis in the present paper is based on a similar Hamiltonian for the intrinsic degrees of freedom, we did not make any assumptions about the dynamical evolution and excitation mechanism, but rather we have followed the philosophy of the GOE theory and derived exact equations of motion in the leading order of the $1/N$ expansion. Even though we have been able to solve exactly these equations of motion only in some unphysical limits, they do not seem to imply an usual diffusive behaviour. In particular the Hellman–Feynman force does not seem to have a dissipative and a stochastic component (i.e. friction and Langevin force, white noise). There are strong memory effects and the interplay among different times scales of the problem seem to be rather complex. However, the ensuing equations of motion are rather simple and amenable for a straightforward numerical solution and further analytical study as well. Surprisingly enough, the presence of level crossings and the role of the Landau–Zener excitation mechanism do not seem to play any special role. One might be tempted to use these results and attempt a description of a system of coupled slow and fast degrees of freedom. One can use our evolution equations only for driven systems, when the reaction of the intrinsic degrees of freedom on the slow modes is absent (e.g. when deforming by hand the shape of a cavity). If the slow degrees of freedom are dynamical variables, the corresponding equations of motion have to be rederived by taking into account the correlations among the slow and fast modes, due to the inherent fluctuations of the slow modes, induced by the coupling to the fast modes.

Even though our theoretical approach is not applicable directly to atomic nuclei and atomic clusters in particular, it will be extremely interesting to apply this approach to deformable cavities. Experimentalists focused their attention to static cavities only, however the study of a deformable cavity should be extremely interesting in itself and at the same time shed light on the energy transfer mechanism from slow to fast degrees of freedom in systems with complex intrinsic spectra. One can try to deform a superconducting cavity filled with microwaves, like the one used Refs. [42, 43], using maybe a piezoelectric crystal. The most difficult experimental problem is likely to be able to attain large deformations of the cavity and relatively fast at the macroscopic scale. One can try however to perform a similar experiment with sound waves instead. The question is if it is possible to create a cavity with relatively small absorption and ensure relatively long lived sound waves inside the cavity somehow. One can think as well of magnetic bottles for electrons at very low temperatures. Trapping of plasma is a subject with a venerable history and means to trap charges with magnetic fields are rather refined. The only difference would be trapping very cold electrons (instead of hot, as is the case of plasma studies) and at

very low densities. The linear dimensions of the magnetic bottle should be comparable with the wavelength of the captured electrons inside (this is why very cold electrons are likely candidates). Another possibility could be the ballistic microstructures of the type studied in Refs. [44, 45], to which a variable magnetic fields might be added, which will play the role of the time–dependent driving force.

A very promising type of experiment could be done on molecules, similar to the one described in Refs. [46] and analysed in Refs. [47]. In these experiments a singlet state of a molecule is excited with a laser. A static magnetic field is applied in order to bring triplet states in the same energy region with the excited singlet state and as a result they will strongly mix. The triplet states have a very high level density and essentially a GOE type of spectrum. A slight modification of this type of experiments could be the application of both a static and time–dependent magnetic field at the same time. The triplet states will be effectively describe by a Hamiltonian of the type

$$H(t) = H_x B_x(t) + H_y B_y(t) + H_z B_z(t), \tag{67}$$

where $H_{x,y,z}$ are independent GOE Hamiltonians for the triplet states. Let us assume that the magnetic fields has a static and a time dependent orthogonal components

$$\mathbf{B}(t) = \mathbf{B}_0 + \mathbf{B}_1(t), \quad \mathbf{B}_0 \cdot \mathbf{B}_1(t) = 0. \tag{68}$$

It is obvious then that the time correlator of the Hamiltonian (69) has the following form, if the time–dependent component is rotating with the frequency ω

$$\mathbf{B}_0 = (0, 0, B_0), \quad \mathbf{B}_1(t) = B_1(\cos\omega t, \sin\omega t, 0), \tag{69}$$

$$\overline{H(t_1)H(t_2)} \sim B_0^2 + B_1^2 \cos\omega(t_1 - t_2), \tag{70}$$

i.e. of the same type as the Hamiltonian (54) we have discussed above. Other magnetic field configurations, e.g. $\mathbf{B}_0 || \mathbf{B}_1$, can be considered as well, which will lead to slightly different correlators. One can think of different modifications of the experiment described in Ref. [46]. To study as a function of time the excited spectrum of the molecule one can use even femtoseconds techniques, as described in Ref. [48] and therefore very small time scales. By varying not only B_0, B_1 but also ω one can study rather diverse properties of the spectrum and of the energy transport and also rather unusual features, like the appearance of a discrete averaged spectrum for this type of correlator (see Appendix B), which seems to be a new aspect of random matrix ensembles. The range of possible "clean" experiments seems to be rather diverse and we do not doubt that one can propose other types of systems as well. Here, we only tried to suggest some, which might not prove to be the best candidates, from an experimental point of view.

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Appendix A: Propagators for Time–Independent Hamiltonians

In the particular case when the GOE part of the Hamiltonian in Eq. (1) is time independent, one can derive a relatively simple equation for the averaged propagator $S_k(t)$

$$\begin{aligned} S_k(t) &= \overline{\langle k|\psi(t)\rangle} = \overline{\langle k|\exp[-iHt]|k\rangle} \\ &= -\int \frac{dE}{2\pi i}\exp(-iEt)\frac{1}{E-\epsilon_0 e_k - M(E)} \\ &= -\int \frac{dE}{2\pi i}\exp(-iEt)\frac{[E-\epsilon_0 e_k - M_1(E)] - iM_2(E)}{[E-\epsilon_0 e_k + M_1(E)]^2 + M_2^2(E)} \\ &= \int \frac{dE}{\pi}\exp(-iEt)\frac{M_2(E)}{[E-\epsilon_0 e_k + M_1(E)]^2 + M_2^2(E)} \\ &= \int dE\exp(-iEt)\rho_k(E), \end{aligned} \tag{A.1}$$

where $M(E) = M_1(E) + iM_2(E)$ is the proper self–energy and $\epsilon_0 = 1/\rho_0$ is the average level spacing for the regular part of the Hamiltonian. One can show that

$$\rho_k(E) = \langle k|\delta(E-H)|k\rangle = -\frac{1}{\pi}\mathrm{Im}\, S_k(E) = \frac{1}{\pi}\frac{M_2(E)}{[E-\epsilon_0 e_k - M_1(E)]^2 + M_2^2(e)}, \tag{A.2}$$

where

$$\int dE\rho_k(E) = 1, \qquad \int dE\rho(E) = N,$$

$$\rho(E) = \mathrm{Trace}\,\delta(E-H) = \sum_k \rho_k(E) = \frac{M_2(E)}{\pi\lambda^2}. \tag{A.3}$$

In the middle of the spectrum the magnitude of the imaginary part of the proper self–energy is given approximately by

$$M_2(\epsilon_0 e_k) \approx \pi\lambda^2\rho_0, \quad \text{for small} \quad \lambda \tag{A.4a}$$

$$M_2(E) \approx \lambda\sqrt{N}, \quad \text{for large} \quad \lambda. \tag{A.4b}$$

The transition between the two regimes occurs for $\lambda\rho_0 \approx \sqrt{N}$. For the small values of λ the averaged spectrum looks like a collection of well separated Lorentzians, centered approximately at the energies of the unperturbed Hamiltonian, with a width equal to $2M_2(E)$. In the opposite limit the spectrum is essentially similar to a pure GOE one, see Eq. (11). The spectrum changes from a quasidiscrete one, at small values for λ, to a continuous one when the spreading width becomes comparable to the mean level spacing, i.e. $M_2(E) \approx \epsilon_0$, when $\lambda\rho_0 \approx 1$.

The proper self–energy satisfies a relatively simple equation, known as Pastur equation,

$$M(E) = \lambda^2 \sum_k \frac{1}{E-\epsilon_0 e_k - M(E)}. \tag{A.5}$$

We shall introduce the following notations

$$M_1(E) = x, \quad M_2^2(E) = y, \quad E - x = z. \tag{A.6}$$

If $M_2 \equiv 0$, then one can find the real part of the proper self–energy by solving the following equations

$$M_1(E) = x = \lambda^2 \sum_k \frac{1}{E - \epsilon_0 e_k - x} = \lambda^2 \sum_k \frac{1}{z - \epsilon_0 e_k}, \tag{A.7a}$$

$$E = z + \lambda^2 \sum_k \frac{1}{z - \epsilon_0 e_k}, \quad \frac{dE}{dz} = 1 - \lambda^2 \sum_k \frac{1}{[z - \epsilon_0 e_k]^2}. \tag{A.7b}$$

It is convenient for numerical purposes to use z as an independent variable and find E and $M_1(E)$ as a function of z starting from a large negative value $(-\infty)$ for z. When

$$\lambda^2 \sum_k \frac{1}{[z_0 - \epsilon_0 e_k]^2} = 1, \tag{A.8}$$

the imaginary part of the proper self–energy starts increasing and one has to switch to the following system of equations

$$\lambda^2 \sum_k \frac{1}{[z - \epsilon_0 e_k]^2 + y} = 1, \tag{A.9}$$

$$x = \lambda^2 \sum_k \frac{z - \epsilon_0 e_k}{[z - \epsilon_0 e_k]^2 + y}, \tag{A.10}$$

$$E = z + \lambda^2 \sum_k \frac{z - \epsilon_0 e_k}{[z - \epsilon_0 e_k]^2 + y}. \tag{A.11}$$

As a result one has to solve numerically only one equation, Eq. (A.9), in order to find y as a function of z. Afterwards the corresponding energy and the real part of the proper self–energy are computed in a straightforward manner. (At $z = z_0$ one obviously has $y = 0$.)

One can derive a differential equation for $y(z)$

$$\frac{dy}{dz} = -\frac{A(y,z)}{B(y,z)}, \quad A(y,z) = 2 \sum_k \frac{z - \epsilon_0 e_k}{[z - \epsilon_0 e_k]^2 + y}, \quad B(y,z) = \sum_k \frac{1}{[z - \epsilon_0 e_k]^2 + y}. \tag{A.12}$$

Obviously this equation is valid only if $y \geq 0$. When y becomes zero one has to switch again to solving the equation for the real part of the proper self–energy only until the condition (A.8) is fulfilled again. One can easily show that $dE/dz > 0$ too, which ensures the validity of this numerical approach.

If one will replace the pure GOE matrix H_1 with a banded one, as was explained in the main text (by introducing the quantities V_{kl}), the equations derived here will be slightly

more complicated, because instead of a single proper self–energy one has to introduce one for every level k, i.e. N in total. Eqs. (A.1) and (A.5) will now read

$$\begin{aligned} S_k(t) &= \overline{\langle k|\psi(t)\rangle} = \overline{\langle k|\exp[-iHt]|k\rangle} \\ &= -\int \frac{dE}{2\pi i}\exp(-iEt)\frac{1}{E-\epsilon_0 e_k - M_k(E)} \\ &= -\int \frac{dE}{2\pi i}\exp(-iEt)\frac{[E-\epsilon_0 e_k - M_{1\,k}(E)] - iM_{2\,k}(E)}{[E-\epsilon_0 e_k + M_{1\,k}(E)]^2 + M_{2\,k}^2(E)} \qquad \text{(A.13)} \\ &= \int \frac{dE}{\pi}\exp(-iEt)\frac{M_{2\,k}(E)}{[E-\epsilon_0 e_k + M_{1\,k}(E)]^2 + M_{2\,k}^2(E)} \\ &= \int dE\exp(-iEt)\rho_k(E), \\ M_l(E) &= \lambda^2\sum_k V_{lk}^2\frac{1}{E-\epsilon_0 e_k - M_k(E)}. \qquad \text{(A.14)} \end{aligned}$$

Appendix B: Periodic Correlators

We shall describe here some features of the averaged Green functions for the case when the correlator (16) is a periodic function, as is the case for the Hamiltonian (54) proposed by Wilkinson [7]. As in the case described in Appendix A it is convenient and possible to solve the equation for the averaged propagator in the energy representation. For the Hamiltonian (54) the equation for the averaged propagator reads

$$S_k(E) = \frac{1}{E - \epsilon_0 e_k} + \kappa^2 \frac{1}{E - \epsilon_0 e_k} \sum_l [S_l(E+\omega) + S_l(E-\omega)] S_k(E), \qquad (B.1)$$

where ω is the angular frequency of the correlator (16), i.e. $F(s) = \cos(\omega s)$. For the sake of simplicity of the argument we have assumed here that $V_{kl} \equiv 1$. We shall first analyse a simpler version of the equation (B.1) (similar to the one considered in Ref. [49], where the case $e_k \equiv 0$ was considered), namely

$$S_k(E) = \frac{1}{E - \epsilon_0 e_k} + \kappa^2 \frac{1}{E - \epsilon_0 e_k} \sum_l S_l(E-\omega) S_k(E). \qquad (B.2)$$

There are two different interpretations of this equations, depending on the context. It corresponds to the averaged propagator for a GOE Hamiltonian of type (54) and in the limit $\omega \to 0$ one recovers the Pastur equation for an usual GOE matrix, as described in Appendix A. At the same time it corresponds to a particle interacting with a single phonon of frequency ω. It is remarkable that in the limit $\omega \to 0$ one should recover the GOE spectrum. A detailed study of the properties of such propagators will be reported elsewhere [50]. One can introduce the proper self–energy

$$M(E) = \kappa^2 \sum_l [S_l(E+\omega) + S_l(E-\omega)], \qquad (B.3)$$

$$M(E) = \kappa^2 \sum_l S_l(E-\omega), \qquad (B.4)$$

for propagators (B.1, B.2) respectively and obtain relatively easily the following equations

$$M(E) = \kappa^2 \sum_l \left[\frac{1}{E - \epsilon_0 e_k - M(E+\omega)} + \frac{1}{E - \epsilon_0 e_k - M(E-\omega)} \right], \qquad (B.5)$$

$$M(E) = \kappa^2 \sum_l \frac{1}{E - \epsilon_0 e_k - M(E-\omega)}. \qquad (B.6)$$

In the particular case $e_k \equiv 0$ Eq. (6) have been solved in Ref. [49] and there it was shown that

$$S(E) = \frac{1}{\sqrt{N}\kappa} \frac{J_p(x)}{J_{p+1}(x)}, \quad \text{where} \quad p = \frac{E}{\omega} \quad \text{and} \quad x = \frac{2\sqrt{N}\kappa}{\omega}. \qquad (B.7)$$

Here $J_p(x)$ is the cylindrical Bessel function. One can easily see that the spectrum in this case is discrete for any finite ω and is given by the zeros of the Bessel function $J_{p+1}(x)$. The poles are situated approximately in the interval $(-2\sqrt{N}\kappa, 2\sqrt{N}\kappa)$ and their number is approximately given by $2\sqrt{N}\kappa/\omega$. In the limit $\omega \to 0$ one recovers the Wigner semicircle. The propagator in the time representation decays slower with time, in comparison with the pure GOE case, see Eq. (31), and at times $t \approx 2\pi n/\omega, \quad n = 1, 2, 3, \ldots$ is significantly greater than the pure GOE propagator and a beating pattern is recognizable. In the case of an arbitrary spectrum e_k, the spectrum is still discrete, even though rather dense. One can easily construct a continuous fraction representation for the proper self–energy (B.6) and find the averaged spectrum, see Ref. [50] for a detailed analysis. The spectrum of the propagator (B.1, B.3, B.5) has similar features. It is a remarkable fact that such a relatively small modification of the GOE propagator ($\omega \neq 0$) changes drastically the character of the averaged spectrum. In the present case ($\omega \neq 0$), strictly speaking, we are dealing with a time dependent Hamiltonian and one cannot introduce eigenvalues, but either its instantaneous spectrum or the Floquet spectrum of its quasienergies. At the same time, the averaged spectrum is stationary.

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