

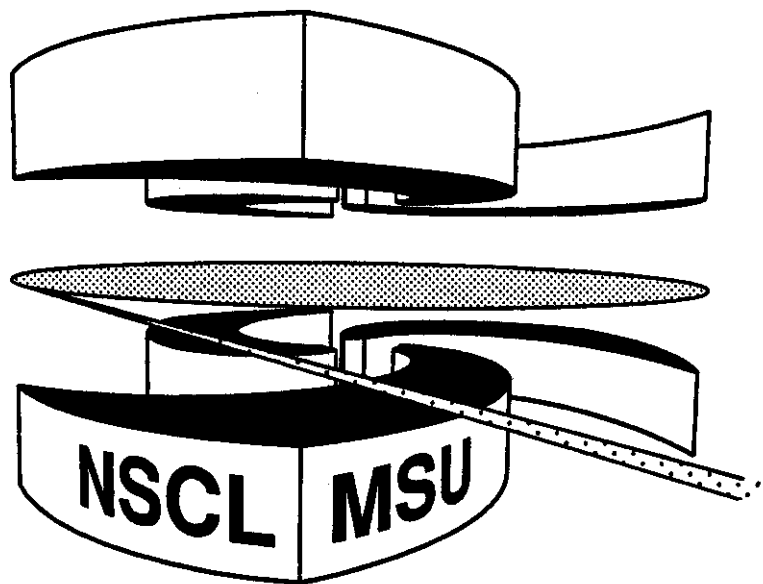


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**EXCITATION OF MULTIPHONON GIANT RESONANCE  
STATES IN RELATIVISTIC HEAVY ION COLLISIONS**

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## Excitation of Multiphonon Giant Resonance States

in Relativistic Heavy Ion Collisions

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We investigate the excitation of **multiphonon** states in relativistic heavy ion collisions. A semiclassical formalism is used to determine the one-step and two-step Coulomb excitation of the double-phonon GDR x GDR excitation. The excitation cross sections of the  $L = 0$  and  $L = 2$  states are determined separately with the use of sum rules for estimating corresponding matrix elements. The nuclear contributions to the excitation process are also calculated and shown to be small. The widths of final and intermediate states are taken into account; physical problems related to the spreading width of the double **phonon** states are discussed in detail. A comparison with recent experimental data shows a good agreement.

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## I. INTRODUCTION

The excitation of giant resonances (GR) in collisions with heavy ions was first investigated in cosmic ray experiments by Balasubrahmanyam *et al.* [1]. An analysis of the experiment concluded that Coulomb excitation was responsible for the dramatic increase of the total fragmentation cross section, as suggested by Artru and Yodh [2]. This occurs due to an increase of the strength of the electromagnetic field of a fast charged particle as its kinetic energy increases (for a review, see ref. [3] and references therein). Laboratory experiments to investigate this process began with the pioneer work of Heckman and Lindstrom [4]. Since then several experiments have aimed at the investigation of this process.

Due to the huge excitation cross sections for large  $Z$  nuclei, the possibility of excitation of multiphonon states of GR was suggested by Baur and Bertulani [5]. But, while the cross sections for the Coulomb excitation of GR in high energy heavy ion collisions can be as large as several barns, it was shown [5] that the cross sections for the excitation of double-phonon states, i.e. states formed by two GR, are smaller by a factor 10-100. The identification of these states by gamma-decay techniques was also shown to be feasible [5], but the cross sections would be even smaller by another factor 10-100. Thus, very elaborate measurements would be required to obtain a signature of the existence of these states.

Successful experiments on the Coulomb excitation of multiphonon states have been reported recently by Llope and Braun-Munzinger [6], Schmidt *et al.* [7] and Ritman *et al.* [8]. In the experiment of ref. [7] the excitation spectra of usual GR, e.g. the isovector giant dipole resonance (IVGDR), the isoscalar giant quadrupole resonance (ISGQR) and the isovector giant quadrupole resonance (IVGQR), were obtained. At about twice the energy of the IVGDR, a clear resonance was observed and identified as the double excitation of the IVGDR (a double-phonon state, in our terminology). This state was also identified in the experiments of refs. [6, 8].

The theory of Coulomb excitation of GR is well understood and has been studied extensively in the past (see, e.g. [3, 5, 9-11]). However, some points for investigation are still open. For example, in refs. [7, 8] it was found that the values of the experimental cross sections for the excitation of double phonon resonances are about a factor 2 larger than expected from theory [5]. The experimentally determined widths of the double resonances are close to  $\sqrt{2}$  times the width of a one-phonon state. This is also smaller than the prediction of ref. [5], namely  $\Gamma_2 = 2\Gamma_1$ .

In this article we address these and other problems related to the experimental measurements. We investigate possible mechanisms to explain the apparent discrepancy between experiment and theory. In Sect. 2 we derive basic formulas for the excitation amplitudes of one-phonon and two-phonon states. Nuclear matrix elements are estimated (Sect.3) under the assumption that the corresponding multipole sum rules are exhausted by the collective GR. The simple approach is developed to take into account the influence of the widths of GR onto the excitation cross sections.

One possibility of explaining the missing part of the theoretical cross section is by adding the nuclear contribution to the process. An eikonal calculation of the total cross sections for nuclear excitation of GR in high energy collisions is performed in Sect. 4. Explicit formulas are derived which can be useful for future studies. In Sect. 5 we perform the calculations for the double GDR states with angular momenta  $L = 0$  and  $L = 2$  and find the results in general agreement with the experiments of Schmidt *et al.* [7] and of Ritman *et al.* [8].

In Sect. 6 we discuss the origin of the width of the double-phonon state and how one can relate it to the width of a one-phonon state. This issue can shed light on

some aspects of the more general problem of damping of collective motion and onset of chaos in quantum many-body systems.

## II. COULOMB EXCITATION OF ONE- OR MULTI-PHONON STATES

The formalism for the excitation of GR in relativistic heavy ion collisions was developed in ref. [11] with semiclassical methods and in ref. [12] quantum-mechanically (see also, ref. [13]). We shall here derive simple formulas for relativistic Coulomb excitation including a calculation of higher order processes. We use the semiclassical formalism. As shown in ref. [3] the total cross sections are the same if one uses a quantum mechanical or a semiclassical approach (see also [13]). This is basically due to the very short wavelength of the relativistic heavy ions and to the weakness of the electromagnetic interaction.

Let us consider the excitation of a projectile in a collision with a target with charge  $Z_T$  at an impact parameter  $b$ . In first-order time dependent perturbation theory the excitation amplitude for the transition  $i \rightarrow f$ ;  $\omega_{fi} = \omega$ , is given by

$$a_{fi} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} e^{i\omega t} \left[ \rho_{fi}(\mathbf{r}) \phi(\mathbf{r}, t) - (\mathbf{v}/c) \cdot \mathbf{j}_{fi} \phi(\mathbf{r}, t) \right] d^3r dt, \quad (1)$$

where  $\rho_{fi}$  and  $\mathbf{j}_{fi}$  are the charge transition density and current matrix elements, respectively.  $\phi(\mathbf{r}, t)$  is the Lienard-Wiechert potential, given by

$$\phi(\mathbf{r}, t) = Z_T e \gamma \left[ (b-x)^2 + y^2 + \gamma^2(z-vt)^2 \right]^{-1/2}, \quad (2)$$

where  $\gamma = (1 - v^2/c^2)^{-1/2}$ , and  $v$  is the projectile velocity.

At relativistic energies, in contrast with Coulomb excitation at low energies, it is very important to include the magnetic interaction in eq. (1) even for pure electric excitations. In the particular case of the long-wavelength approximation,  $\omega r/c \ll 1$  (i.e.  $r/b \ll 1$ ), an easy derivation of the excitation amplitudes is possible. For example, in the dipole approximation

$$\phi(\mathbf{r}, t) - \phi(\mathbf{r} = 0, t) \simeq Z_T e \gamma \frac{(xb + \gamma^2 vtz)}{(b^2 + \gamma^2 v^2 t^2)^{3/2}} + \mathcal{O}(r^2/b^2). \quad (3)$$

Inserting the above result in eq. (1) the integrals over time are obtained in terms of the modified Bessel functions  $K_n$ . Using the continuity equation  $\nabla \cdot \mathbf{j}_{fi} = -i\omega \rho_{fi}$ , we obtain for the electric dipole excitations

$$a_{fi}^{(E1)} = -i \sqrt{\frac{8\pi}{3}} \frac{Z_T e}{\hbar v b} \xi \left\{ [D_{fi}^{(-1)} - D_{fi}^{(1)}] K_1(\xi) + i \frac{\sqrt{2}}{\gamma} D_{fi}^{(0)} K_0(\xi) \right\}, \quad (4)$$

where  $\xi = \omega b/\gamma v$ , and

$$D_{fi}^{(m)} = \int r Y_{1m}(\hat{\mathbf{r}}) \rho_{fi}(\mathbf{r}) d^3r \quad (5)$$

is the dipole matrix element for the nuclear excitation.

The magnetic interaction (second term of eq. (1)) is responsible for the factor  $1/\gamma$  in the term proportional to  $K_0$  in eq. (4), see e.g. [14]. Due to this factor, at relativistic energies the second term inside brackets of eq. (4),  $m = 0$ , is considerably reduced compared to the first. In such a situation only the transverse ( $m = \pm 1$ ) components of the interaction are important.

Using higher order terms of the expansion in eq. (3) and the continuity equation we can derive the amplitudes for the excitation of other multiplicities. For the electric quadrupole interaction one obtains

$$a_{fi}^{(E2)} = -i 2 \sqrt{\frac{\pi}{30}} \frac{Z_T e}{\gamma \hbar v} \left(\frac{\omega}{v}\right)^2 \left\{ -\frac{1}{\gamma} [Q_{fi}^{(2)} + Q_{fi}^{(-2)}] K_2(\xi) - i [Q_{fi}^{(1)} - Q_{fi}^{(-1)}] \left(2 - \frac{v^2}{c^2}\right) K_1(\xi) + \frac{\sqrt{6}}{\gamma} Q_{fi}^{(0)} K_0(\xi) \right\} \quad (6)$$

where

$$Q_{fi}^{(m)} = \int r^2 Y_{2m}(\hat{\mathbf{r}}) \rho_{fi}(\mathbf{r}) d^3 r \quad (7)$$

is the quadrupole matrix element for the nuclear excitation.

The formulas above have been derived under the assumption of the long-wavelength approximation. When this approximation is not valid the matrix elements given by eqs. (5) and (7) are to be replaced by the non-approximated matrix-elements for electromagnetic excitations [15]. However, the other factors do not change (see, e.g., [11]).

The amplitude for a two-step excitation to a state  $|2\rangle$  via intermediate states  $|1\rangle$  is given by

$$a_{20}^{2nd} = \sum_1 \frac{1}{(i\hbar)^2} \int_{-\infty}^{\infty} dt e^{i\omega_{21}t} V_{21}(t) \int_{-\infty}^t dt' e^{i\omega_{10}t'} V_{10}(t'), \quad (8)$$

where  $V_{21}(t)$  is a short notation for the interaction potential inside brackets of the integrand of eq. (1) for the transition  $|1\rangle \rightarrow |2\rangle$ .

Using the integral representation of the step function

$$\Theta(t - t') = - \lim_{\delta \rightarrow 0^+} \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{-iq(t-t')}}{q + i\delta} dq = \begin{cases} 1, & \text{if } t > t' \\ 0, & \text{if } t < t', \end{cases} \quad (9)$$

one finds [15]

$$a_{20}^{2nd} = \frac{1}{2} \sum_1 a_{21}^{1st}(\omega_{21}) a_{10}^{1st}(\omega_{10}) + \frac{i}{2\pi} \sum_1 \mathcal{P} \int_{-\infty}^{\infty} \frac{dq}{q} a_{21}^{1st}(\omega_{21} - q) a_{10}^{1st}(\omega_{10} + q), \quad (10)$$

where  $\mathcal{P}$  stands for the principal value of the integral. For numerical evaluation it is more appropriate to rewrite the principal value integral in eq. (10) as

$$\begin{aligned} \mathcal{P} \int_{-\infty}^{\infty} \frac{dq}{q} a_{21}^{1st}(\omega_{21} - q) a_{10}^{1st}(\omega_{10} + q) = \\ \int_0^{\infty} \frac{dq}{q} \left[ a_{21}^{1st}(\omega_{21} - q) a_{10}^{1st}(\omega_{10} + q) - a_{21}^{1st}(\omega_{21} + q) a_{10}^{1st}(\omega_{10} - q) \right]. \end{aligned} \quad (11)$$

To calculate  $a^{1st}(\omega)$  for negative values of  $\omega$ , we note that the interaction potential can be written as a sum of an even and an odd part. This, together with eq. (1), implies that  $a^{1st}(-\omega) = -[a^{1st}(\omega)]^*$ .

For three-phonon excitation we use the third term of the time-dependent perturbation expansion, and the same procedure as above (eqs. (9 - 11)).

The total cross sections for Coulomb excitation are obtained by integrating over impact parameters, from a minimum value  $b_{min}$ , consistent with pure Coulomb interaction, i.e.

$$\sigma^C = 2\pi \int_{b_{min}}^{\infty} db b |a_{fi}|^2. \quad (12)$$

Usually the minimum impact parameter is taken as a sum of the nuclear matter radii,  $b_{min} = 1.2(A_1^{1/3} + A_2^{1/3})$  fm. But, due to some uncertainty for the minimum impact parameter at which the strong interaction sets in, small changes in this value might be appropriate for a given system depending on the systematics of the experimental data.

For the excitation of single GR, the integration in eq. (12) can be done analytically as shown in ref. [11]. For the excitation of double and triple resonances however a numerical calculation has to be performed, using eqs. (8-12).

### III. SUM RULES AND SPREADING WIDTHS

The simplest way to determine the matrix elements of eqs. (5) and (7) is by means of sum rules under the assumption that those sum rules are exhausted by collective states. The conventional sum rules for the dipole and quadrupole transitions, derived without exchange and velocity-dependent corrections, are ( $\hbar = 1$ )

$$\sum_f \omega_{fi} |D_{fi}^{(m)}|^2 = \frac{3}{4\pi} \frac{1}{2m_N} \frac{NZ}{A} e^2; \quad (13)$$

$$\sum_f \omega_{fi} |Q_{fi}^{(m)}|^2 = 2 \frac{1}{2m_N} \frac{3R^2}{4\pi} e^2 \times \begin{cases} Z^2/A, & \text{isoscalar excitations,} \\ NZ/A, & \text{isovector excitations.} \end{cases} \quad (14)$$

We explain our procedure on the example of the dipole sum rule (13). The right hand side  $S_D$  of (13) being calculated for the fixed initial state  $|i\rangle$  in fact does not depend on the choice of  $|i\rangle$ . (This dependence is rather weak even if the exchange terms are taken into account). Since  $S_D$  does not depend on the projection  $m$  of the dipole operator  $D_1^{(m)}$  as well, it is convenient to introduce in usual way the reduced matrix elements of multipole operators,

$$\langle f; I_f M_f | \mathcal{O}_i^{(m)} | i; I_i M_i \rangle = \langle I_f M_f | I_i M_i; m \rangle (f; I_f | \mathcal{O}_i | i; I_i), \quad (15)$$

where  $f$  stands now for all quantum numbers except angular momentum ones,  $I$  and  $M$ , and to perform the additional summation of eq.(13) over  $m$ . In such a way one obtains

$$\sum_{f, I_f} \omega_{fi} (2I_f + 1) (f; I_f || D || i; I_i)^2 = 3(2I_i + 1) S_D. \quad (16)$$

Now let us take the ground state  $|0\rangle$  of an even-even nucleus with angular momentum  $I_0 = 0$  as an initial one  $|i; I_i\rangle$ . If we assume that the single GDR  $|1\rangle \equiv |1; 1\rangle$  is an isolated state saturating the corresponding sum rule, we just divide the right hand side of (16) by the excitation energy  $\omega_{10}$  to obtain the reduced matrix element

$$(1 || D || 0)^2 = \frac{S_D}{\omega_{10}}. \quad (17)$$

In order to be able to calculate the cross section of excitation of the double GDR, we have to take the single GDR state  $|1\rangle$  as an initial one. The corresponding sum in eq.(16), according to our assumption, is saturated by (i) "down" transition to the ground state  $|0\rangle$ , which has negative transition energy  $-\omega_{10}$  and, due to the symmetry properties of the Clebsch-Gordan coefficients, the strength which is 3 times larger than that of eq.(17), and (ii) "up" transitions to the double GDR states  $|2; I_2 = L\rangle$  where  $L$  can be equal to 0 and 2. The resulting sum rule for the up transitions is

$$\sum_{L=0,2} (2L + 1) \omega_{21}^{(L)} (2; L || D || 1)^2 = 12 S_D. \quad (18)$$

where  $\omega_{21}^{(L)} \equiv E_{2,L} - E_1$  is the energy of the second excitation. Actually, considering, instead of the sum over  $m$ , the original dipole sum rule (13) for fixed  $m$ , one can separate the two contributions to the sum (18) and find

$$(2; L || D || 1)^2 = 2 \frac{S_D}{\omega_{21}^{(L)}}. \quad (19)$$

Obviously, it is consistent with the sum rule (18).

Eqs. (17) and (19) imply the relation between the strengths of sequential excitation processes,

$$(2; L || D || 1)^2 = 2 \frac{\omega_{10}}{\omega_{21}^{(L)}} (1 || D || 0)^2. \quad (20)$$

For the equidistant vibrational spectrum this result is nothing but the standard Bose factor of stimulated radiation; our result is valid under more broad assumptions. The resulting enhancement factor includes, in addition, the ratio of transition frequencies which, according to the data, is slightly larger than 1. The generalization for the third and higher order excitation processes is straightforward.

The above assumption of saturation certainly does not account for the fact that the resonances are wide. In fact, this might be also relevant for the calculation of total

cross sections since the amplitudes given by eqs. (4) and (6) may vary strongly with the excitation energy. Therefore they might be sensitive to the shape of the strength function. The widths of the resonances can be taken into account in a simplified approach, as we describe next.

We assume that the damping of the collective modes is mostly due to the coupling to the background of complicated configurations in the vicinity of the resonance energy. Then the resonance state  $|\lambda\rangle$  gets fragmented acquiring the spreading width  $\Gamma_\lambda$ . The stationary final states  $|f\rangle$  in the region of the GR are superpositions (with the same exact quantum numbers as the collective mode) of the form

$$|f\rangle = C_\lambda^{(f)} |\lambda\rangle + \sum_\nu C_\nu^{(f)} |\nu\rangle, \quad (21)$$

where  $|\lambda\rangle$  is a pure GR state and  $|\nu\rangle$  are complicated many particle-many hole states. If the resonance component dominates in the excitation process as it should be for the one-body multipole operator, see eqs.(5) and (7), we find the first order amplitude  $a_{fi}^{(\lambda)}$  of the excitation of the individual state  $|f\rangle$  in the fragmentation region

$$a_{fi}^{(\lambda)} \simeq [C_\lambda^{(f)}]^* a_\lambda^{1st}(\omega_{fi}). \quad (22)$$

Here  $a_\lambda^{1st}$  stands for the original first order excitation amplitude of eq.(1). As a function of the transition energy, the probability for the one-phonon excitation is

$$P_\lambda^{1st}(\omega) = \sum_f \left[ |C_\lambda^{(f)}|^2 \delta(\omega - \omega_{fi}) \right] |a_\lambda^{1st}(\omega_{fi})|^2 \equiv \mathcal{F}_\lambda(\omega) |a_\lambda^{1st}(\omega)|^2 \quad (23)$$

where we introduced the strength function  $\mathcal{F}_\lambda(\omega)$ .

The traditional derivation of the strength function (see ref. [16]) is based on the rough assumptions concerning mixing matrix elements and the equidistant spectrum of complicated states. The matrix elements  $V_{\lambda\nu}$  which couple the collective mode to the background states are assumed to be of the same average magnitude for all remote states  $|\nu\rangle$  from both sides of the resonance. Under those conditions the resulting strength function has the Breit-Wigner (BW) shape

$$\mathcal{F}_\lambda(\omega) = \frac{1}{2\pi} \frac{\Gamma_\lambda}{(\omega - \omega_\lambda)^2 + \Gamma_\lambda^2/4}, \quad (24)$$

where  $\Gamma_\lambda$  is the spreading width of the collective resonance,

$$\Gamma_\lambda = 2\pi \frac{\langle V_{\lambda\nu}^2 \rangle_\nu}{d}, \quad (25)$$

$d$  is the mean level spacing of complicated states, coupling matrix elements are averaged over the states  $|\nu\rangle$  and  $\omega_\lambda$  is the energy centroid. At present, we used in our numerical calculations the BW strength function (24) with the empirical parameters  $\omega_\lambda$  and  $\Gamma_\lambda$ . However, the same procedure can be applied to any specific form of  $\mathcal{F}_\lambda(\omega)$ . Later (Sect.6) we come back to the question of justification of the model leading to eqs. (24) and (25).

The multiphonon states could also be reached by a direct excitation. Quite similarly, we can repeat the above arguments to calculate the probability for the direct excitation of a multiphonon state, with the appropriate spreading width and energy centroid of that state. The direct (or first-order) probabilities are then given by



$$P_\lambda^{1st}(\omega) = \mathcal{F}_\lambda(\omega) \left| a_\lambda^{1st}(\omega) \right|^2. \quad (26)$$

Let us now treat the case of the two-step excitation of GR (double-phonon). For simplicity, we denote the single-phonon state by  $|1\rangle$  and the double-phonon state by  $|2\rangle$ , the corresponding centroids being at  $\omega_1$  and  $\omega_2$  respectively. The total probability to excite the double-phonon state is obtained by

$$\begin{aligned} P(\omega) &= \sum_f \left| a_{fi}^{1st} + a_{fi}^{2nd} \right|^2 \delta(\omega - \omega_{fi}) \\ &\equiv P^{1st}(\omega) + P^{2nd}(\omega) + P^{int}(\omega), \end{aligned} \quad (27)$$

where  $P^{1st}$  is the direct (or first-order) excitation of the double-phonon state,  $P^{2nd}$  is the two-step (or second-order) excitation term, and the last term in eq.(27) is the interference between the two.

The second order amplitude is calculated from

$$a_{fi}^{2nd} = \frac{i}{2\pi} \int \frac{dq}{q + i0} \sum_k a_{fk}^{1st}(\omega_{fk} - q) a_{ki}^{1st}(\omega_{ki} + q). \quad (28)$$

Neglecting non-resonant terms,

$$\begin{aligned} a_{fi}^{2nd}(\omega) &\simeq \frac{i}{2\pi} \int \frac{dq}{q + i0} \sum_k [C_2^{(f)}]^* C_1^{(k)} a_{21}(\omega_{fk} - q) [C_1^{(k)}]^* a_{10}(\omega_{ki} + q) \\ &= [C_2^{(f)}]^* \int d\omega' \sum_k |C_1^{(k)}|^2 \delta(\omega_{ki} - \omega') \int \frac{i}{2\pi} \frac{dq}{q + i0} a_{21}(\omega_{fi} - \omega' - q) a_{10}(\omega' + q) \\ &= [C_2^{(f)}]^* \int d\omega' \mathcal{F}_1(\omega') \int \frac{i}{2\pi} \frac{dq}{q + i0} a_{21}(\omega_{fi} - \omega' - q) a_{10}(\omega' + q) \\ &\equiv [C_2^{(f)}]^* \tilde{a}_{20}(\omega_{fi}). \end{aligned} \quad (29)$$

Therefore, the second-order amplitude is given by

$$P^{2nd}(\omega) = \sum_f \left| a_{fi}^{2nd} \right|^2 \delta(\omega_{fi} - \omega) = \mathcal{F}_2(\omega) \left| \tilde{a}_{20}(\omega) \right|^2, \quad (30)$$

where  $\tilde{a}_{20}(\omega)$  is defined in the last step of eq. (29). Using similar derivations as above, we find for the interference term

$$P^{int}(\omega) = \mathcal{F}_2(\omega) \times 2 \operatorname{Re} \left\{ \left[ a_{20}^{1st}(\omega) \right]^* \left[ \tilde{a}_{20}(\omega) \right] \right\}. \quad (31)$$

#### IV. NUCLEAR EXCITATION

A possible origin of the observed discrepancy between experiment [7,8] and theory [5] could be due to the nuclear contribution to the excitation of GR. We feel appropriate to discuss this possibility in more details.

In heavy ion high energy collisions the nuclear excitation can be derived in the eikonal approximation [17]. The inelastic scattering amplitude is given by

$$T_{fi} = -\frac{4\pi^2 M}{\hbar^2} \langle \Psi^{(-)}(\mathbf{R})\phi_f(\mathbf{r})|U|\Psi^{(+)}(\mathbf{R})\phi_i(\mathbf{r}) \rangle \quad (32)$$

where  $M$  is the reduced mass of the system,  $\Psi^{(\pm)}$  are the c.m. scattering waves and  $\phi_{i,f}$  are the intrinsic wave functions of the nucleus in the ground state and final state, respectively.

In peripheral collisions we may assume that the projectile interacts with the target nucleus via vibrational fluctuations of the optical potential (deformed potential model). This assumption is based on the same spirit as the Bohr-Mottelson model for the coupling of intrinsic motion to nuclear deformations (see e.g. [18]). One obtains in this model

$$\begin{aligned} \langle \Psi^{(-)}(\mathbf{R})\phi_f(\mathbf{r})|U|\Psi^{(+)}(\mathbf{R})\phi_i(\mathbf{r}) \rangle &= -\langle \phi_f|\alpha_{\lambda\mu}|\phi_i \rangle \langle \Psi^{(-)}|k(R)Y_{\lambda\mu}^*(\hat{\mathbf{R}})|\Psi^{(+)} \rangle \\ &= -\left(\frac{\hbar}{2\omega_\lambda B_\lambda}\right)^{1/2} \langle \Psi^{(+)}|k(R)Y_{\lambda\mu}^*(\hat{\mathbf{R}})|\Psi^{(-)} \rangle, \end{aligned} \quad (33)$$

where  $k(R)$  is proportional to the derivative of the optical potential,

$$k(R) = R \frac{dU_{opt}}{dR}. \quad (34)$$

In eq. (33)  $B_\lambda$  is the mass parameter for the vibrational state. The term inside parentheses in eq. (33) can be related to the reduced matrix element (or deformation parameter) for the electromagnetic transition [16]. The differential cross section for the excitation of a vibrational mode becomes

$$\begin{aligned} \sigma_\lambda(\theta) &= \left(\frac{4\pi^2 M}{\hbar^2}\right)^2 \left(\frac{\lambda+3}{4\pi} Ze \langle r^\lambda \rangle_0\right)^{-2} B(E\lambda; \lambda \rightarrow 0) \sum_\mu \left| \langle \Psi^{(-)}|k(R)Y_{\lambda\mu}^*(\hat{\mathbf{R}})|\Psi^{(+)} \rangle \right|^2 \\ &= \left(\frac{4\pi^2 M}{\hbar^2}\right)^2 \left(\frac{3}{4\pi} Ze R_c\right)^{-2} B(E\lambda; \lambda \rightarrow 0) \sum_\mu \left| \langle \Psi^{(-)}|k(R)Y_{\lambda\mu}^*(\hat{\mathbf{R}})|\Psi^{(+)} \rangle \right|^2, \end{aligned} \quad (35)$$

where in the last step we assumed a uniform charge distribution with radius  $R_c$  for the ground state.

In the eikonal approximation, valid for high-energy collisions,

$$\left[\Psi^{(-)}\right]^* \Psi^{(+)} \simeq \frac{1}{(2\pi)^3} \exp\{i\mathbf{q}\cdot\mathbf{R} + i\chi(b)\}, \quad (36)$$

where  $\mathbf{q}$  is the momentum transfer in the reaction, and  $\chi(b)$  is the eikonal phase [17]. For the longitudinal momentum transfer we use the approximation  $q_z = k_f \cos \theta - k_i \simeq \Delta k \simeq \omega_\lambda/v$ , where  $v$  is the relative velocity. The azimuthal integration is straightforward and one obtains ( $\mathbf{R} \equiv (\mathbf{b}, Z)$ )

$$\langle \Psi^{(-)} | k(R) Y_{\lambda\mu}^*(\hat{\mathbf{R}}) | \Psi^{(+)} \rangle \simeq i^\mu \int db b J_\mu(q_t b) F_{\lambda\mu}(\omega_\lambda, b) e^{i\chi(b)}, \quad (37)$$

where

$$F_{\lambda\mu}(\omega_\lambda, b) = \frac{i^\mu}{(2\pi)^2} \int dZ e^{i\omega_\lambda Z/v} R \frac{dU_{opt}}{dR} Y_{\lambda\mu}(\theta, 0), \quad (38)$$

with  $R = \sqrt{b^2 + Z^2}$  and  $\cos \theta = Z/R$ . The integration over scattering angle may be done by using the high-energy approximation  $d\Omega \simeq 2\pi q_t dq_t/k^2$  and with the aid of the closure relation for the Bessel functions. We get

$$\sigma_\lambda^N = 2\pi \int db b \mathcal{P}_\lambda(b), \quad (39)$$

where

$$\mathcal{P}_\lambda = \left( \frac{4\pi^2}{\hbar^2 v^2} \right)^2 \left( \frac{3}{4\pi} ZeR \right)^{-2} B(E\lambda; \lambda \rightarrow 0) \exp\{-2 \text{Im} \chi(b)\} \sum_\mu |F_{\lambda\mu}(\omega_\lambda, b)|^2. \quad (40)$$

The function  $\mathcal{P}_\lambda(b)$  may be interpreted as the probability for the excitation of the vibrational mode  $\lambda$  in high-energy collisions at a given impact parameter.

For collisions at high energies we can assume that the optical potential is pure imaginary being related to the nucleon-nucleon cross sections and to the nuclear ground state matter distribution by [17]

$$U_{opt}(\mathbf{R}) = -\frac{1}{2} i\hbar v \sigma_{NN} \int \rho_T(\mathbf{R} - \mathbf{r}) \rho_P(\mathbf{r}) d^3r. \quad (41)$$

The imaginary part of the Glauber phase in eq. (36) is accordingly given by

$$2 \text{Im} \chi(b) = \sigma_{NN} \int dZ \int d^3r \rho_T(\mathbf{R} - \mathbf{r}) \rho_P(\mathbf{r}). \quad (42)$$

The nuclear excitation probability is peaked at grazing impact parameters. This occurs because the exponential factor in eq. (40) decreases with decreasing  $b$  and the function  $F_{\lambda\mu}$  decreases with increasing  $b$ .

## V. NUMERICAL RESULTS

The reactions  $^{136}\text{Xe} + ^{208}\text{Pb}$  at 0.69 GeV/nucleon and  $^{209}\text{Bi} + ^{208}\text{Pb}$  at 1 GeV/nucleon have been measured recently at GSI [7, 8]. We apply the formalism developed in the preceding sections to calculate the excitation probabilities and cross sections for these systems.

Cross sections (in mb) for the Coulomb excitation of the IVGDR, ISGQR and IVGQR in  $^{136}\text{Xe}$  incident on  $\text{Pb}$  at 0.69 GeV/nucleon are given in table 1. We have assumed that the IVGDR, ISGQR and the IVGQR are located at 15.3, 12.3 and 24 MeV, and that they exhaust 100%, 70% and 80% of the corresponding sum rules, respectively [19]. The contributions to various angular momentum projections of each state are shown separately. In the last column the total cross sections are calculated with the widths of the states taken into account. We use for the IVGDR, ISGQR and IVGQR the BW strength functions (24) with the resonance widths  $\Gamma = 4.8, 4$  and 7 MeV, respectively [19]. We see that states with higher angular momentum projections are more populated. The inclusion of the widths of the resonances in the calculation increases the cross sections by about 10-20%. These cross sections agree well with the values obtained by Schmidt *et al.* [7]. The experimental value for IVGDR is  $1.85 \pm 0.1$  b which is very close to ours. We use  $b_{min} = 1.2(A_T^{1/3} + A_P^{1/3})$  fm = 13.3 fm for the distance of closest approach in the reaction  $^{136}\text{Xe} + ^{208}\text{Pb}$ .

	$m = \pm 2$	$m = \pm 1$	$m = 0$	$\sigma_{total}$	$\sigma_{width}$
IVGDR	–	712	201	1630	1820
ISGQR	64	6.09	10.6	150	169
IVGQR	25.6	5.46	12.4	74.5	97

**Table I** - Cross sections (in mb) for the Coulomb excitation of the IVGDR, ISGQR and IVGQR in  $^{136}\text{Xe}$  incident on  $\text{Pb}$  at 0.69 GeV/nucleon. The contributions to the population of each angular momentum projection are shown separately. In the last column the total cross sections are calculated with the widths of the states taken into account.

Using the formalism developed in Sect. 4 we have also calculated the cross sections for the nuclear excitation of the ISGQR in the same reaction. The cross sections for the excitation of isovector modes are reduced by a factor  $[(N - Z)/A]^2$  since the isovector mode is excited due to the difference in strength of the nuclear interaction between the target and the protons and neutrons of the projectile [16]. This implies that the isovector excitations are strongly suppressed in nuclear excitations. Therefore, we do not consider them here. For the excitation of the ISGQR we find  $\sigma^N = 5.3$  mb, if we use the deformation parameter  $\beta R = 0.7$  fm for  $^{136}\text{Xe}$ . In the calculation of the nuclear potential (see eq. (41)) we used Fermi density distributions with parameters  $\rho_0 = 0.17 \text{ fm}^{-3}$  and  $R = 5.6$  (6.5) fm,  $a = 0.65$  (0.65) fm for  $\text{Xe}(\text{Pb})$ . The

nucleon-nucleon cross section used was 40 mb. A look at eqs. (38) and (40) explains the origin of the small value for the nuclear excitation cross sections. The exponential factor in the integrand of eq. (38) is irrelevant for excitation energies of order of the GR or less. This means that the Fourier transform of the nuclear interaction in high energy collisions,  $v \sim c$ , contains high frequencies. Thus, the magnitude of the function in eq. (38) is basically determined by the value of the deformation parameters (i.e. reduced matrix elements) for a given resonance. The magnitude of the nuclear excitation cross sections of GR in high energy collisions is given by the overlap in impact parameter between the last two terms of eq. (40), which is quite small. This contrasts with the Coulomb interaction which in high energy collisions contains high frequency components, but extends to large impact parameters.

The double dipole phonon state can couple to total angular momentum 0 or 2. As we mentioned in Sect. 2, for the state with  $L = 2$  there is the possibility of a direct quadrupole Coulomb excitation ( $L = 0$  states cannot be Coulomb excited [3]). For simplicity, we do not consider here the physics of the isospin coupling of the two GDR.

We calculated the direct and the two-step probabilities for the excitation of the double-phonon state according to the approach discussed in Sect. 2. The total cross sections obtained are shown in table 2. We found that the principal value term in eq. (10) contributes very little (less than 1%) to the GDR  $\times$  GDR cross section via a two-step process.

Double-phonon state	$m = \pm 2$	$m = \pm 1$	$m = 0$	$\sigma_{total}$	$\sigma_{width}$
L=0 (two-step)	-	-	22.8	22.8	28.4
L=2 (two-step)	23.3	13.4	51.4	124.8	154
L=2 (direct - 20% of SR)	3.27	0.86	2.12	10.3	11.8

**Table II** - Cross sections (in mb) for the Coulomb excitation of the double-GDR, in  $^{136}\text{Xe}$  incident on  $Pb$  at 0.69 GeV/nucleon. The contributions to the L=0 and the L=2 states are shown separately. In the last column the total cross sections are calculated with the widths of the states taken into account.

From table II we see that the inclusion of the widths of the final (GDR  $\times$  GDR) and the intermediate (GDR) state increase the cross sections by 10-20%. For the position and width of the GDR  $\times$  GDR state we took  $E = 28.3$  MeV and  $\Gamma = 7$  MeV, respectively [7] which corresponds to  $\omega_{10} = 15.3$  MeV and  $\omega_{21} = 13$  MeV in eqs. (8,10) and (13,14), both for  $L = 2$  and  $L = 0$ . For the calculation of the direct excitation we assumed that the resonance would exhaust 20% of the ISGQR sum rule. It is based on the hypotheses that the missing strength of the low-lying ISGQR could be located at the double dipole phonon state as a consequence of the anharmonic phonon coupling of the ( $Q\ddot{D}D$ )-type. Obviously, it should be considered as highly overestimated upper boundary of the direct excitation. Recently, Ponomarev and Voronov [21] have calculated the reduced transition probability for the excitation of

double-phonon states within the quasiparticle-phonon model. They find the value  $B(2^+, E2) = 4.2 e^2 fm^4$ . Using this value we obtained that the cross section for the direct excitation of the  $L = 2$  state is  $12 \mu b$ , much smaller than what we quote above. We conclude that even in the more optimistic cases the contribution of the direct mechanism to the total cross section for Coulomb excitation of the double-phonon state is much less than that of the two-step process.

Another conclusion drawn from the numbers of table 2 is that the excitation of the  $L = 2$  double-phonon state is much stronger than for the  $L = 0$  state. Adding the two contributions we find that the total cross section for the excitation of the double-phonon state (excluding the direct mechanism) in the reaction above is equal to 182 mb. The experimental value of ref. [7] is about  $215 \pm 50$  mb. As stated above, the nuclear contribution to the (direct) excitation of the double-phonon state is not relevant. If we assume again that about 20% of the sum rule strength is exhausted by this state (using e.g.  $\beta R = 0.1$  fm), we get 1.1 mb for the nuclear excitation of the  $L = 2$  double-phonon state.

We also compare our results with the experiment of Ritman et al. [8]. They measured the excitation of a  $^{208}Pb$  target by means of  $^{209}Bi$  projectiles at 1 GeV/nucleon and obtained  $770 \pm 220$  mb for the excitation cross section of the double resonance. We calculate the cross sections for the same system, using  $E_1 = 13.5$  MeV,  $\Gamma_1 = 4$  MeV,  $E_2 = 27$  MeV and  $\Gamma_2 = 6$  MeV for the energy position and widths of the GDR and the GDR  $\times$  GDR in  $^{208}Pb$ , respectively. Using the formalism developed in Sects. 2 and 3 and including the effects of the widths of the states, we find  $\sigma_1 = 5234$  b for the excitation of the GDR and  $\sigma_2 = 692$  mb for the excitation of the GDR  $\times$  GDR. Thus, while the cross section for the excitation of single phonons is a factor 2.8 larger than that of the experiment of ref. [7], the cross sections for the excitation of double phonons is larger by a factor 3.8. This is due to the larger value for the excitation probabilities caused by a larger  $B(E1)$  value for this reaction.

We found the ratio of  $(P_{m=+1} + P_{m=-1})/P_{m=0} = 9.4$  for the excitation of the GDR in the experiment of ref. [8]. They quote the value 28 in their calculations and fit the gamma-ray angular distribution according to this value. We think that this result could somewhat change the extracted value of the GDR  $\times$  GDR cross section which is quoted in ref. [8].

Using the formalism shown of Sects. 2 and 3 we find that the cross sections for the excitation of three-phonon states in the experiment of Schmidt et al. [7] is equal to 19.2 mb while it is equal to 117 mb for the experiment of Ritman et al. [8]. The identification of these resonances is therefore more difficult, but possible with the present experimental techniques. Using the same arguments leading to eq.(20) we find for the reduced matrix elements, in obvious notations,  $|D_{32}|^2 = 3(\omega_{10}/\omega_{32})|D_{10}|^2$ , which we used in our calculation. We assumed that  $\omega_{10}/\omega_{32} \simeq \omega_{10}/\omega_{21}$ . These enhancement factors for the excitation of higher phonon states are very important to explain the magnitude of the cross sections.

In summary, we find a good agreement between the experimental data and our calculations. We notice that the authors of refs. [7] have used the value  $b_{min} = 15.6$  fm for the distance of closest approach [20]. We find that this value is too high. As a function of impact parameter the GDR  $\times$  GDR excitation probability varies as  $1/b^4$ . Therefore, a small increase in the distance of closest approach decreases the cross section by a large amount. In fact, using the value quoted above (15.6 fm) we find that our calculated GDR  $\times$  GDR cross sections for the experiment of ref. [7] reduce by a factor 1.8. We think that the conventional use of  $b_{min} = 1.2(A_T^{1/3} + A_P^{1/3})$  fm is more appropriate. The nuclear interaction sets in at about this value. We

are able to explain both experiments [7, 8] using this parameterization. A systematic experimental study of the GDR  $\times$  GDR cross sections for various systems and energies is clearly appropriate at this stage in order to define an optimal parameterization for this quantity.

Another important question is related to the expected width of the multiphonon states. Early estimates [5] indicated that these widths should scale as  $\Gamma_n = n\Gamma_1$ . The experiments show however that a scaling as  $\Gamma_n = \sqrt{n}\Gamma_1$  is more appropriate, at least for the double GDR. We next address in detail different aspects of physics responsible for the width of the double phonon state.

## VI. WIDTH OF THE DOUBLE PHONON STATE

Here we discuss in qualitative terms the problem of the width of a collective state which can be thought of as being created by the excitation of two quanta in a complex many-body system. We assume that the genuine decay to continuum is of minor importance at the given excitation energy. Therefore we focus on the damping width which comes from the fact that the collective mode is a specific coherent superposition of simple configurations (for instance, of a particle-hole character) rather than a pure stationary state.

In the actual excitation process the predominant mechanism is that of the sequential one-phonon excitation. Under our assumption that the sum rule is saturated by the GR the intermediate states contribute to this process as far as they contain a significant collective component. Therefore the interference of many incoherent paths can be neglected so that we are interested in the shape  $P(E)$  of the excitation function at a given energy  $E = E_1 + E_2$  which can be obtained as a convolution of the single phonon excitation functions,

$$P(E) = \int dE_1 dE_2 P_1(E_1) P_2(E_2) \delta(E - E_1 - E_2). \quad (43)$$

The same shape should be revealed in the deexcitation process.

In this formulation the problem is different from what is usually looked at when one is interested, for example, in sound attenuation. In such classical problems the conventional exponential decrease of the wave intensity does not correspond to the decay of the state with a certain initial number of quanta. Contrary to that, here we have to compare the damping rates of individual quantum states with the fixed number of quanta, single- and double-phonon states in particular.

We have to mention also that in the nuclear GR case quantum effects are more pronounced since the temperature corresponding to the relevant excitation energy is less than  $\hbar\omega$  whereas in the measurements of the attenuation of the zero and first sound in the macroscopic Fermi-liquid [22] the situation is always inverse and the quantum limit is hardly attainable. (In nuclear physics the classical case can be studied with low-lying quadrupole vibrations).

Independently of specific features of nuclear structure (level density,  $A$ -dependence, shell effects, finiteness of the system leading to the linear momentum nonconservation and, therefore, to the estimate of the available phase space which could be different from that for infinite matter, and so on) we can try to make several comments of general nature.

If the anharmonic effects could be considered to be small we can assume that the phonons decay independently by what can be described, using the language of stationary quantum mechanics, as mixing to complicated background states. The

decay rate  $\Gamma_1(e)$  of an individual quasiparticle (elementary excitation) with energy  $e$  depends on the background level density and, whence, on the excitation energy. The decay of a state with  $n$  quasiparticles occurs as far as one of the constituents decays. It implies the simple estimate of the width  $\Gamma_n$  of the  $n$ -quantum state,  $\Gamma_n \simeq n\Gamma_1(E/n)$ . For the decay of typical many particle-many hole configurations [23–25] one usually takes the Fermi-liquid estimate  $\Gamma_1(e) \propto e^2$  which leads to  $\Gamma_n \propto T^3 \propto E^{3/2}$  since the average number of quasiparticles in a typical thermal configuration at temperature  $T$  is  $n \propto T$ . This estimate agrees with data. In the case of the pure  $n$ -phonon state  $E/n = \hbar\omega$  which results in the ratio  $r_n \equiv \Gamma_n/\Gamma_1 \simeq n$ .

Thus, the simplest line of reasoning favors the width of the double GR to be twice as big as the width of the single GR. At the first glance, this estimate is especially reasonable for the giant dipole since here the anharmonic effects, determining the whole pattern of low-lying vibrations, are expected to be very weak. Angular momentum and parity conservation forbids cubic anharmonicity which would mix single- and double-quantum states and influence both excitation cross sections and spreading widths. The main anharmonic term, apart from the mentioned in Sect.5 weak mixing of the giant quadrupole to the double dipole state with  $L = 2$ , probably corresponds to the phonon scattering similar to that in the IBM. It results in the shift of the double-phonon state from  $2\hbar\omega$  and splitting of  $L = 0$  and  $L = 2$  states hardly changing the decay properties. Experimentally, the energy shift seems to be rather small.

There are also other arguments for the width ratio  $r_2 = 2$ . In our calculation of cross sections we assumed the BW shape (24) of strength functions (23). If the sequential excitation is described by the BW functions  $P_1(E_1)$  with the centroid at  $e$  and the width  $\Gamma$ , and  $P_2(E_2)$  with corresponding parameters  $e'$  and  $\Gamma'$ , the convolution (42) restores the BW shape with the centroid at  $e + e'$  and the total width  $\Gamma + \Gamma'$ . For identical phonons it means that the width ratio  $r_2 = 2$ .

As we mentioned in Sect.3, the BW shape of the strength function is derived analytically within the simple model [16] of coupling between a phonon and complicated background states. One diagonalizes first the Hamiltonian in the subspace of those complicated states and get their energies  $\epsilon_\nu$ . If the underlying dynamics is nearly chaotic, the resulting spectrum will show up level repulsion and rigid structure similar to that of the Gaussian Orthogonal Ensemble (GOE), with the mean level spacing  $d$ . Roughly speaking, one can assume the equidistant energy spectrum. The collective phonon  $|1\rangle$  at energy  $E_1$  is coupled to those states and corresponding matrix elements  $V_{1\nu}$  are assumed to be of the same order of magnitude (much larger than the level spacing  $d$ ) for all states  $|\nu\rangle$  in the large energy interval around the collective resonance. Then the energies of the stationary states (final states  $|f\rangle$  in the notations of previous sections) are the roots  $E = E_f$  of the secular equation

$$F(E) \equiv E - E_1 - \sum_{\nu} \frac{V_{1\nu}^2}{E - \epsilon_{\nu}} = 0, \quad (44)$$

and the distribution of the collective strength, eq.(21),

$$|C_1^{(f)}|^2 = [dF/dE]_{E=E_f}^{-1} = [1 + \sum_{\nu} \frac{V_{1\nu}^2}{(E_f - \epsilon_{\nu})^2}]^{-1} \quad (45)$$

reveals the BW shape (24) and the "golden rule" expression (25) for the width  $\Gamma_1$ .



We can repeat the procedure for the double phonon state. Phonons of different kind would couple to different background states with different level spacing and coupling matrix elements. It corresponds to independent decay leading as we discussed above to  $\Gamma_2 = \Gamma + \Gamma'$ . For the identical phonons, we should take into account that the double phonon state  $|2\rangle$  is coupled to the states "single phonon + background" and the background states here are the same as those determining the width of the single phonon state  $|1\rangle$ . This picture is in accordance with the famous Axel-Brink hypotheses. Therefore the expression for the width, eq.(25), contains the same level density whereas all coupling matrix elements for the transition to a complicated state  $|\nu\rangle$  (plus a remaining phonon) have to be multiplied by the Bose factor,  $V_{2\nu} = \sqrt{2}V_{1\nu}$ . Thus, we come again to  $r_2 = 2$ .

The approach of the preceding paragraph can be slightly modified by introducing explicitly coupling via a doorway state [26] or GOE internal dynamics [27]. In both cases the Bose factor  $\sqrt{2}$  leads to the same result  $r_2 = 2$ .

In addition, the collective resonance might be further broadened by the coupling to low-lying collective vibrational or rotational modes. For example, in the simplest model where the dipole phonon radiates and absorbs low energy scalar quanta, it is easy to show that, in the stationary cloud of scalar quanta, their average number, which determines the fragmentation region of the dipole mode, is proportional to the squared number of dipole phonons. Hence it gives a large width ratio  $r_2 = 4$ . For the nuclei where actual data exist, this is not important since they are rather rigid spherical nuclei with no adiabatic collective modes.

On the other hand, one can present some arguments in favor of the width ratio  $r_2 = \sqrt{2}$  which apparently is preferred by the existing data.

First of all, this value follows from the convolution (43) of Gaussian distribution functions (instead of BW ones). Of course, this is the inconsistent approach since the experimentalists use BW or Lorentzian fit. But one can easily understand that the result  $r_2 = \sqrt{2}$  is not restricted to Gaussian fit. For an arbitrary sequence of two excitation processes we have  $\langle E \rangle = \langle E_1 + E_2 \rangle$  and  $\langle E^2 \rangle = \langle (E_1 + E_2)^2 \rangle$ ; for uncorrelated steps it results in the addition of fluctuations in quadrature,  $(\Delta E)^2 = (\Delta E_1)^2 + (\Delta E_2)^2$ . Identifying these fluctuations with the widths up to a common factor, we get for the identical phonons  $\Gamma_2^2 = 2\Gamma_1^2$ , or  $r_2 = \sqrt{2}$ .

The same conclusion will be valid for any distribution function which, as the Gaussian one, has a finite second moment, contrary to the BW or Lorentzian ones with the second moment diverging. In some sense we may conclude that, in physical terms, the difference between  $r_2 = 2$  and  $r_2 = \sqrt{2}$  is due to the different treatment of the wings of the distribution functions which reflect small admixtures of far remote states.

In the standard model of the strength function [16] all remote states are coupled to the collective mode equally strong. This is obviously an unrealistic assumption. The shell model (more generally, mean field) basis is the "natural" one [28] for estimating a degree of complexity of various states in a Fermi system at not very high excitation energy. In this representation matrix elements of residual interaction couple the collective state (coherent superposition of particle-hole excitations found for example in the framework of the RPA) only to the states of the next level of complexity (exciton class). Those states, in turn, become mixed with more complicated configurations. This process proliferates and each simple state acquires its spreading, or fragmentation, width  $2a = Nd$  where  $N$  stands for a typical number of stationary states carrying the noticeable weight of the ancestor state and the level spacing  $d$  is

basically the same as in the mean field approximation. Inversely,  $N$  can be viewed as the localization length of a stationary complicated state in the mean field basis.

In the stochastic limit [29] the local background dynamical properties can be modelled by those of the GOE with the semicircle radius  $a$ . This intrinsic spreading width  $a$ , which is expected to be of the order of magnitude of typical matrix elements of the original residual interaction between simple configurations, is the dynamical scale missed in the standard model which corresponds to the limit  $a \rightarrow \infty$ . The existence of this intrinsic scale can be associated [29] with the saturation [30] of the width of a single GR at high temperature.

The standard model supposedly is valid for the spreading width  $\Gamma$  small in comparison with  $a$ . Because of the relatively weak interaction leading to the isospin impurity, this is the case for the isobaric analog states (IAS) [31, 32] where typical spreading widths are less than 100 keV. This approach allows one to explain, at least qualitatively, small variations of the spreading widths of the IAS. The tunneling mixing of superdeformed states with the normal deformed background presents an extreme example of the small spreading width. However, in the case of GR the situation might be different.

To illustrate the new behavior in the opposite case of  $\Gamma \geq a$ , we can imagine the limit of the almost degenerate intrinsic states with very strong coupling to a collective mode. (The actual situation presumably is intermediate). Assuming that the unperturbed phonon state has an energy in the same region, one can easily see from eqs. (44) and (45) that the coupling results in the appearance of the two collective states sharing evenly the collective strength and shifted symmetrically from the unperturbed region by  $\Delta E = \pm \sqrt{\sum_{\nu} V_{\nu}^2}$ . The physical reason is evident: the interaction of the background states through the collective mode creates a specific coherent superposition which is hybridized with and repelled from the original collective state. The similar effect was discussed in different context in [33] and observed in numerical simulations [34]. The well known doubling of the resonance peak at the passage of a laser beam through a cavity containing a two-level atom is the simplest prototype of such a phenomenon.

In this limit one gets the effective width of collective response  $2\Delta E = 2\sqrt{N \langle V_{\nu}^2 \rangle} = 2\sqrt{a\Gamma_s/\pi}$  where  $\Gamma_s$  is the standard spreading width (25). This effective width is linearly proportional to the average coupling matrix element. Therefore it should increase by factor  $\sqrt{n}$  when applied to a  $n$ -phonon collective state. Thus, we anticipate in this limit  $r_2 = \sqrt{2}$ . One may say that the phonons do not decay independently being correlated via common decay channels. In the literature the similar result, due to apparently the same physical reasons, was mentioned in [35] referring to the unpublished calculations in the framework of the second RPA.

The precision of experimental observations is not sufficiently high to provide us with the accurate shape of the strength function for the double GR. Further experimental data along with the refined theoretical analysis are necessary.

## VII. CONCLUSIONS

We have performed a detailed investigation of the mechanisms for the excitation of multiphonon states in relativistic heavy ion collisions. The effects of the widths of the resonances as well as of the excitation due to the nuclear interaction have been included. We have seen that the inclusion of the widths of the resonances in the calculations of relativistic Coulomb excitation increases the cross sections by 10-20%.

The nuclear contribution to the cross sections is shown to be irrelevant when large- $Z$  nuclei are involved. We find that the experimental cross sections are well explained by our calculations.

We have used a sum rule approach in our calculations based on the assumption of saturation by collective GR. A microscopic RPA treatment for the excitation strengths would help to give more insight to this study. This is beyond the scope of this article. The problem of the width of multiphonon resonances also remains a challenge for our understanding of the damping mechanisms and onset of chaos. More theoretical work in this subject is highly desirable.

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