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ENERGY CORRELATION DISTRIBUTIONS FROM THREE-BODY COULOMB BREAK-UP OF ^{11}Li

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Abstract

The existing data from a kinematically complete experiment of ^{11}Li fragmentation on a heavy target have been analysed in a direct electric dipole excitation model within a strict three body formulation. The $n-n$ and $^{9}Li-2n$ relative energy distributions have been calculated from measured ^{11}Li excitation energy distribution and compared with the experimental data.

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1 Introduction

The recent developments of the secondary nuclear beams have opened new possibilities for investigations of the structure of nuclei close to the neutron drip-line. Neutron halo phenomena have been found for loosely bound neutron rich nuclei e.g.^{11}Li, ^{11}Be, ^{8}He, ^{6}He [1, 2] during these investigations. The structure of ^{11}Li is one of the most intriguing problems in the family of neutron rich nuclei. A very narrow ^{9}Li momentum distribution have been found in fragmentation of ^{11}Li at high [3] and intermediate [4] energies. Extremely narrow forward peaked neutron angular distributions were observed for ^{11}Li break-up at low energies [5] . These results give an evidence that ^{11}Li to a first approximation can be considered as an inert core surrounded by two neutrons. The experimental fact that ^{9}Li and ^{11}Li have nearly the same electric quadrupole moments [6] tends support to this model. Large electromagnetic dissociation cross-sections observed in several measurements at different energies [5, 7, 8] can be attributed to the soft dipole mode of excitation (low-lying electric dipole mode) [9, 10] and gives additional support to the three body ^{11}Li structure. All experimental investigations mentioned above were inclusive measurements and for this reason physical information about neutron behavior in the neutron halo obtained from the experimental data was to some extent ambiguous. The simultaneous analysis of the transverse momentum distribution of ^{9}Li after fragmentation of ^{11}Li at high energies and neutron angular distribution obtained at low energies, shows a strong neutron-neutron correlation [2] while no indication of such correlation have been found experimentally [11]. Another important result is that the widths of ^{9}Li longitudinal momentum distribution [4] and neutron angular distributions [4] are roughly independent of the targets used in measurements in spite of obviously different reaction mechanism for low Z and high Z targets. For this reason kinematically complete experiments are needed to understand the reaction mechanism and neutron halo structure for neutron drip-line nuclei.

Kinematically complete measurements for dissociation of ^{11}Li in the three-body $^{9}Li + n + n$ channel were recently made at the energies tens of MeV/A [12, 13] for heavy targets and few hundreds MeV/A for light and heavy targets [14]. The excitation spectrum of ^{11}Li as well as relative energy spectra between two neutrons and between two neutron center of mass and ^{9}Li have been obtained for heavy targets [12, 13]. Here we discuss the experimental data [12] obtained with a 43 MeV/A ^{11}Li beam impinging on a lead target. It is known that at such energies the dominant contribution to the dissociation cross-section is coming from Coulomb E1 disintegration. The excitation spectrum [12, 13] of ^{11}Li shows a peak at an energy less than 1 MeV which seems to be attributed to a soft-dipole resonance prediction [15]. As pointed out in [13] Coulomb post acceleration of ^{9}Li contradicts such a model. It is not clear whether a strong n-n correlation in the ground state or the reaction mechanism is responsible for the fact that relative energy spectrum between two neutrons [12] is peaked much sharper than predicted from the n-n final state interaction model [12].

2 Outline of the direct Coulomb break-up model

At the energy of a few tens of MeV/nucleon on heavy targets nearly 80% of the two neutron removal cross-section may be accounted for by the Coulomb break-up, see e.g. [16]. Based on this fact one may suppose in the first approximation that the mechanism of ^{11}Li fragmentation at 43 MeV/nucleon on a lead target is due to pure Coulomb break-up (we neglect contributions of nuclear interactions in the break-up process). We analyze here the experimental energy distribution in a direct electric dipole excitation model within a strict three body formulation with the wave function expansion on hyperspherical harmonics (HH). Application of the HH method for A=6 nuclei are given in [17]. To simplify the formulae we omit all indices of no importance for the formulation of the model (e.g. spin of nucleon, $^{9}Li(3/2^-)$ core spin). In this case the ground state wave function (Φ_0) has quantum number 0^+ while the final state wave functions (Φ_f) have to be 1^- states . Since all measured [12] distributions are given in relative units we skip all coefficients that are not energy dependent.
The strength function for this kind of transition is proportional to:

$$dB(E1) \sim |\langle \Phi_0 | \hat{E}1 | \Phi_f \rangle|^2 d\tau \tag{1}$$

here $d\tau$ is the final state phase space volume, $\hat{E}1$ is the dipole operator. We need to determine a set of Jacobi momentum coordinates:

$$\vec{q_1} = (\frac{\mu_{12}}{2})^{\frac{1}{2}}(\frac{\vec{p_1}}{m_1} - \frac{\vec{p_2}}{m_2}); \; \vec{q_2} = (\frac{\mu_{3-12}}{2})^{\frac{1}{2}}(\frac{\vec{p_1}+\vec{p_2}}{m_1+m_2} - \frac{\vec{p_3}}{m_3}) \tag{2}$$

where m_i, μ_{ij}, μ_{k-ij}, $\vec{p_i}$ are masses, reduced masses and momenta of the particles in the ^{11}Li rest frame (indexes 1,2 are related to neutrons, 3 to the ^{9}Li core).We use hyperspherical coordinates $\vec{q} = \{q, \hat{q}_1, \hat{q}_2\ , \theta\}$ where besides common angular variables $\hat{q}_i$ a new angular variable θ is introduced. This variable describes the distribution of the total energy $E = q^2 = q_1^2 + q_2^2$ in the ^{11}Li center of mass system among two subsystems 1,2:

$$q_1 = q\sin\theta; \; q_1^2 = E_{12}; \; q_2 = q\cos\theta; \; q_2^2 = E_{3-12} \tag{3}$$

The phase space volume in selected coordinates can be written as:

$$d\tau = d\vec{q_1}d\vec{q_2} = q^5 dq d\hat{q}_1 d\hat{q}_2 (\sin\theta)^2(\cos\theta)^2 d\theta \tag{4}$$

The conjugated spatial coordinates were determined in the similar way and have the following form:

$$\vec{x_1} = (\mu_{12})^{\frac{1}{2}}(\vec{r_1} - \vec{r_2}); \; \vec{x_2} = (\mu_{3-12})^{\frac{1}{2}}(\frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1+m_2} - \vec{r_3}); \tag{5}$$
$$\rho^2 = x_1^2 + x_2^2; \; x_1 = \rho\sin\alpha; \; x_2 = \rho\cos\alpha.$$

For ground-state and final-state wave functions we expand (sometimes in implicit form) in series of hyperspherical harmonics, which give complete orthogonalized basis on a hypersphere with radius ρ (in spatial representation) or q (in momentum space). Hyperspherical harmonics have the explicit form (in spatial representation):

$$\mathcal{Y}^{l_1 l_2}_{K m_1 m_2}(\Omega_5^\rho) = \Psi_K^{l_1 l_2}(\alpha) Y_{l_1 m_1}(\hat{x_1}) Y_{l_2 m_2}(\hat{x_2}); \tag{6}$$
$$\Psi_K^{l_1 l_2}(\alpha) = N_K^{l_1 l_2} (\sin\alpha)^{l_1} (\cos\alpha)^{l_2} P^{l_1+\frac{1}{2}, l_2+\frac{1}{2}}_{\frac{K-l_1-l_2}{2}}(\cos 2\alpha)$$

here $P_N^{\nu_1,\nu_2}(\cos 2\alpha)$ is the Jacobi polynomial, $N_K^{l_1 l_2}$ is normalizing coefficient. The extra quantum number $K = l_1 + l_2 + 2n (n = 0, 1, 2, ...)$ is called the *hypermoment.* The value of this quantum number determines the effective centrifugal barrier in the three-body Schrödinger equation. The final state wave functions were chosen for the simplicity in the form of six-dimensional plane waves

$$\Phi_f \sim \exp \frac{i\sqrt{2}}{\hbar}(\vec{q_1}\vec{x_1} + \vec{q_2}\vec{x_2}) = \tag{7}$$
$$\frac{(2\pi)^3}{(\frac{\sqrt{2}}{\hbar} q\rho)^2} \sum_{K l_1 l_2 m_1 m_2} i^K J_{K+2}(\frac{\sqrt{2}}{\hbar}\rho q) \mathcal{Y}^{l_1 l_2}_{K m_1 m_2}(\Omega_5^\rho)^* \mathcal{Y}^{l_1 l_2}_{K m_1 m_2}(\Omega_5^q);$$
$$\Omega_5^\rho = \{\alpha, \hat{x_1}, \hat{x_2}\}; \ \Omega_5^q = \{\theta, \hat{q_1}, \hat{q_2}\}$$

where $J_{K+2}(\frac{\sqrt{2}}{\hbar}\rho q)$ are the Bessel functions. After integration over angular variables $\hat{q_1}, \hat{q_2}$ we can derive an expression for comparison with experimental distributions if we combine (7) with the formulae (1) and take into account that $d\sigma \sim N(E) dB(E1)$ (here $N(E)$ is spectrum of virtual photons):

$$\frac{d^2\sigma}{dE d\theta} \sim N(E) E^2 (\sin\theta)^2 (\cos\theta)^2 \sum_{KK' l_1 l_2} B_{KK' l_1 l_2}(E) \Psi_K^{l_1 l_2}(\theta) \Psi_{K'}^{l_1 l_2}(\theta)^*, \text{ where} \tag{8}$$
$$B_{KK' l_1 l_2}(E) = [\sum_{m_1, m_2} \langle \Phi_0 | \hat{E}1 | \frac{i^K}{(q\rho)^2} J_{K+2}(\frac{\sqrt{2}}{\hbar}\rho q) \mathcal{Y}^{l_1 l_2}_{K m_1 m_2}(\Omega_5^\rho)^* \rangle$$
$$\langle \Phi_0 | \hat{E}1 | \frac{i^K}{(q\rho)^2} J_{K'+2}(\frac{\sqrt{2}}{\hbar}\rho q) \mathcal{Y}^{l_1 l_2}_{K' m_1 m_2}(\Omega_5^\rho)^* \rangle^*]$$

It should be noted that the whole information about particle correlations in the ^{11}Li ground state wave function, is contained in $B_{KK' l_1 l_2}(E)$ coefficients. As it is seen from (8) these coefficients are obtained by the integration over all spacial coordinates and the information about neutron halo structure is therefore to the large extent washed out from $d^2\sigma / dE d\theta$.

3 Comparison with experimental data

To simplify (8) we use the well known property of the HH expansion which has an analog in the common spherical harmonic expansion for the two body case. This property is connected with the fact that centrifugal energy in a three body system is proportional to $\frac{(K+3/2)(K+5/2)}{\rho^2}$ ($\frac{L(L+1)}{r^2}$ in the two body system). For the low energy

case this implies that three body waves with the high value of K do not penetrate in the internal space region (where the ground state wave function is located) and do not contribute significantly to the overlap integral with the ground state wave function. We can thus expect small contributions to the expression (8) from terms with high values of K. It is worthwhile to remember that the final state should have quantum number 1^- and allowed values of K have to be odd (K=1,3,5....). The exact three body calculations [18] of the ground state wave function and dipole strength function in the HH method have also shown that for low total energy of the three body system we can neglect all terms in (8) except one with the minimal value of K ($K_{min} = 1$). The terms with K=0,2 give the dominant contribution to the ground state wave function as it was shown [18] for the different choices of $^9Li - n$ interactions. This means that in accordance with the selection rules for E1 transitions ($\Delta K = \pm 1$) waves with K=1,3 must give the dominant contribution to the final state wave function. It was namely shown in [18] that terms with K=1 give the dominant contribution to the dipole strength function for energies lower than 2 MeV even when all interactions for final state wave function have been taken into account properly. For these reasons we leave in (8) only terms with K=1, moreover we select only one of them with quantum numbers $l_1 = 0$ (between two neutrons) and $l_2 = 1$ (between c.m. of the two neutrons and the core), corresponding to the soft dipole mode of excitation (movement of the two neutrons with angular momentum between them $l_1 = 0$ relative to the ^9Li core with angular momentum $l_2 = 1$)).
In this approximation eq. (8) factorized upon variables E and θ and may be written as:

$$\frac{d^2\sigma}{dEd\theta} \sim N(E)E^2(\sin\theta)^2(\cos\theta)^2 B_{1101}(E)|\Psi_1^{01}(\theta)|^2. \qquad (9)$$

Taking into account that $J_3(x) \sim x^3$ (while $x \to 0$) one sees that for $E \to 0$ $d\sigma/dE \sim N(E)E^3$, which means that for low excitation energies we can expect a sharp increase of the excitation function. This effect corresponds qualitatively to the experimental results obtained in [12, 13]. From the factorized form (9) it is easy to get that $d\sigma/dE \sim N(E)E^2 B_{1101}(E)$. This expression for the excitation energy distribution can be inserted into (9) and then the formulae for the double differential cross-section in dependence upon excitation energy and the variable θ takes the form:

$$\frac{d^2\sigma}{dEd\theta} \sim (\frac{d\sigma}{dE})(\sin\theta)^2(\cos\theta)^2|\Psi_1^{01}(\theta)|^2 \sim (\frac{d\sigma}{dE})(\sin\theta)^2(\cos\theta)^4. \qquad (10)$$

since $\Psi_1^{01}(\theta) \sim \cos\theta$. This expression allows to construct a simple phenomenological model for fast estimations of the relative energy distributions. Such a model is based on (10), where instead of the theoretical $(d\sigma/dE)$ we use the experimental excitation function $(d\sigma/dE)_{exp}$. Changing variables in (10) from (E, θ) to (E, E_{3-12}) we have the following formula for the $^9Li - 2n$ (E_{3-12}) relative energy distribution [1]:

$$\frac{d\sigma}{dE_{3-12}} = \int_{E_{3-12}} (\frac{d\sigma}{dE})_{exp} \sqrt{(E - E_{3-12})E_{3-12}} \frac{E_{3-12}}{E^3} dE \qquad (11)$$

The square root in (11) describes the energy distribution for the isotropic break-up in the case of fixed total excitation energy where the final state density is proportional to the three-body phase space volume. A corresponding expression for the n-n relative energy distribution could be obtained by replacing E_{3-12} on $E - E_{12}$ under the integral and taking E_{12} as lower limit of the integration. But one extra factor, which has not yet been included, should be mentioned. This is the n-n final state interaction (FSI). As the final state wave functions were chosen as plane waves the n-n FSI was not taken into account. The final state interaction is the important factor that influences on n-n relative energy spectra because of the large scattering n-n length in the S-state and should therefore be added to the corresponding formula. This is done by multiplying $d\sigma/dE$ by a factor $F_{WM}(E_{nn})$ [19] corresponding to the well known Watson-Migdal model:

$$F_{WM} \sim \frac{(\sin\delta + bk)^2}{k^2}; \qquad (12)$$
$$\cot\delta = \frac{1}{k}(-\frac{1}{a} + r_0\frac{k^2}{2})$$

here k is the $n-n$ relative momentum, $a = -16.0 fm$ is n-n scattering length, $r_0 = 2.4 fm$ is effective radius and parameter b is equal to 1.4 fm [19].

Now we can write the final expression that may be compared with the experimental distribution:

$$\frac{d\sigma}{dE_{12}} = F_{WM}(E_{12}) \int_{E_{12}} (\frac{d\sigma}{dE})_{exp} \sqrt{(E - E_{12})E_{12}} \frac{(E - E_{12})}{E^3} dE \qquad (13)$$

For the calculations we have used the experimental excitation energy distribution from [12]. Note that we do not need any free parameters to get the shapes of the spectra.

The results are shown in Fig.1 and Fig.2. The points with the experimental errors on these pictures are the experimental data that were taken from [12]. The solid line in Fig.1 presents the calculations according to (13) while the dashed line corresponds to the same kind of calculation but with Watson-Migdal factor switched off. It should be noted that for n-n relative energy distribution specific correlations connected with the existence of hyperspherical harmonic with $K = 1, l_1 = 0$ and $l_2 = 1$ is important as well as effect of the n-n FSI to get an agreement with the experimental distributions as it is seen in Fig.1. The result of calculation of $^9Li - 2n$ relative energy distribution is shown in Fig.2 by the solid line. This calculation qualitatively reproduce the shape of

[1] It should be noted that when experimental data are presented as a function upon the ratio $\frac{E_{^9Li-nn}}{E}$ (which is same as $\cos\theta^2$) it avoids the influence of the integration over the excitation energy and helps to extract kinematical correlations described by the term with definite quantum numbers K, l_{nn} and $l_{^9Li-2n}$. For this reason such way of data presentations may be preferable for analysis.

experimental spectrum (note that we do not have any free parameters except normalization on absolute value). It is seen from Fig.2 that the maximum of the calculated curve is shifted down $\sim 150 - 250$ keV as compared with experimental data. This effect, which also has been found in [13] for 28 MeV/nucleon of ^{11}Li fragmentation on lead target, may be connected particularly with the Coulomb post acceleration of ^{9}Li in Coulomb field of the target after dissociation of ^{11}Li.

4 Summary

A simple phenomenological parameter-free model is suggested for rapid estimations of relative energy distributions from three-body Coulomb break-up of ^{11}Li. As an input the model needs only the ^{11}Li excitation energy distribution from the break-up. The model reproduces $d\sigma/dE_{nn}$ and $d\sigma/dE_{^9Li-nn}$ measured in [12] rather well. For $d\sigma/dE_{nn}$ both correlations in the final state three-body wave functions and n-n FSI are important to reproduce the experiment. The model may be useful for an analysis of Coulomb break-up of other neutron drip-line nuclei.

The analysis shows that the excitation energy spectrum of ^{11}Li is mainly sensitive to the ^{11}Li ground state wave function structure. The neutron halo correlations in the ground state also influence the relative energy distributions but not in a direct way. That is why the experimental data on ^{11}Li fragmentation at low energies on heavy targets, where Coulomb E1 transition makes a dominant contribution, are not very convenient to extract information on the ground state structure. For these reasons data obtained in kinematically complete experiments with high energy beams on *light* targets become very important. In this case the well known Serber mechanism (or sudden shaking) can be applied for the break-up reaction. In such cases ground state fragment correlations (see, for example, [20]) should manifest themselves in measured correlations in more pronounce form.

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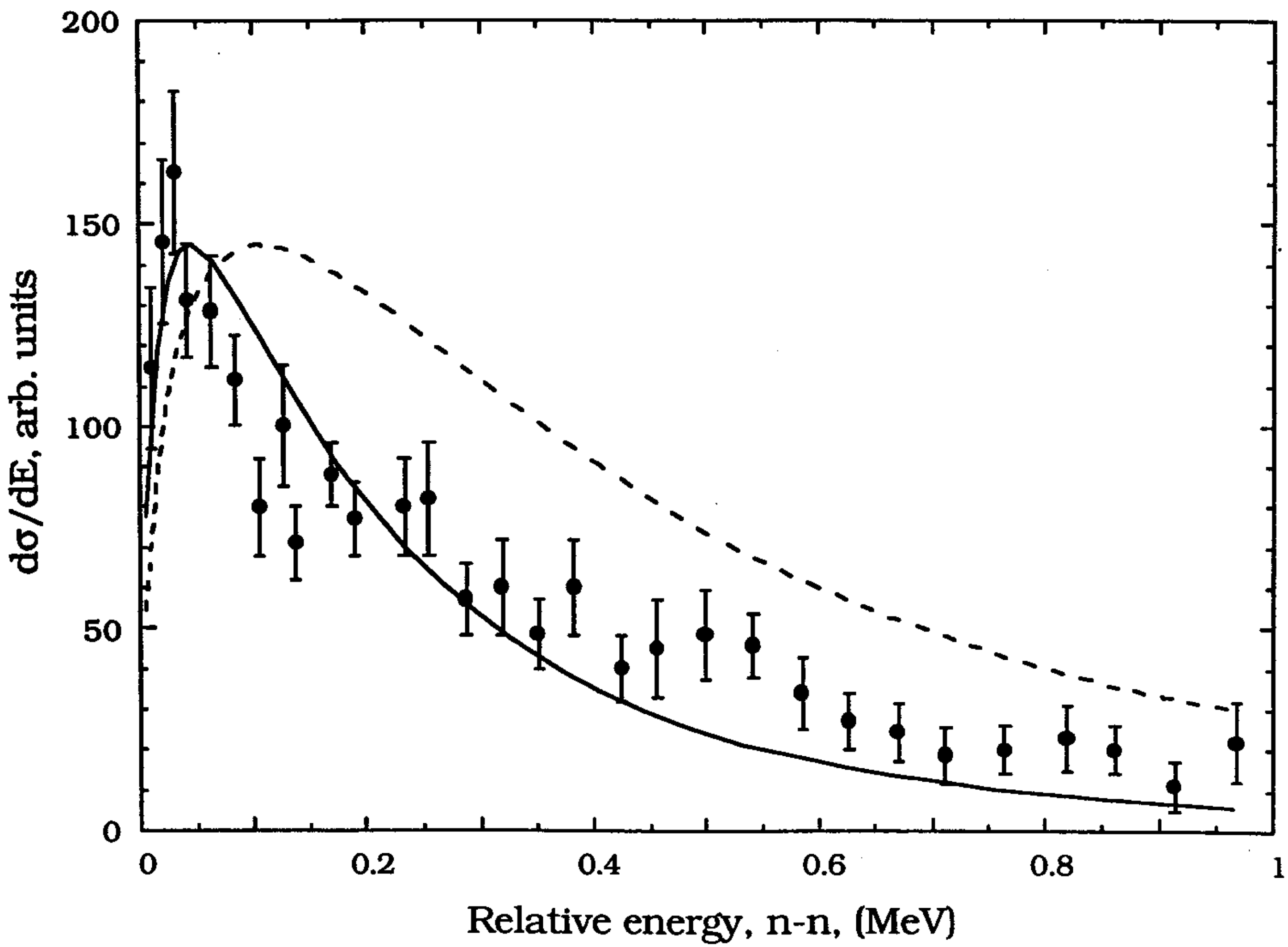
Figure captions

Fig. 1. The neutron-neutron relative energy distribution. The solid line is the results of the calculation according to the form.(13). The dashed line presents the calculation when final state interaction have not been taken into account.

Fig. 2. The $^9Li - 2n$ relative energy distribution. The solid line is the results of the calculation according to the form.(11).


dσ/dE, arb. units
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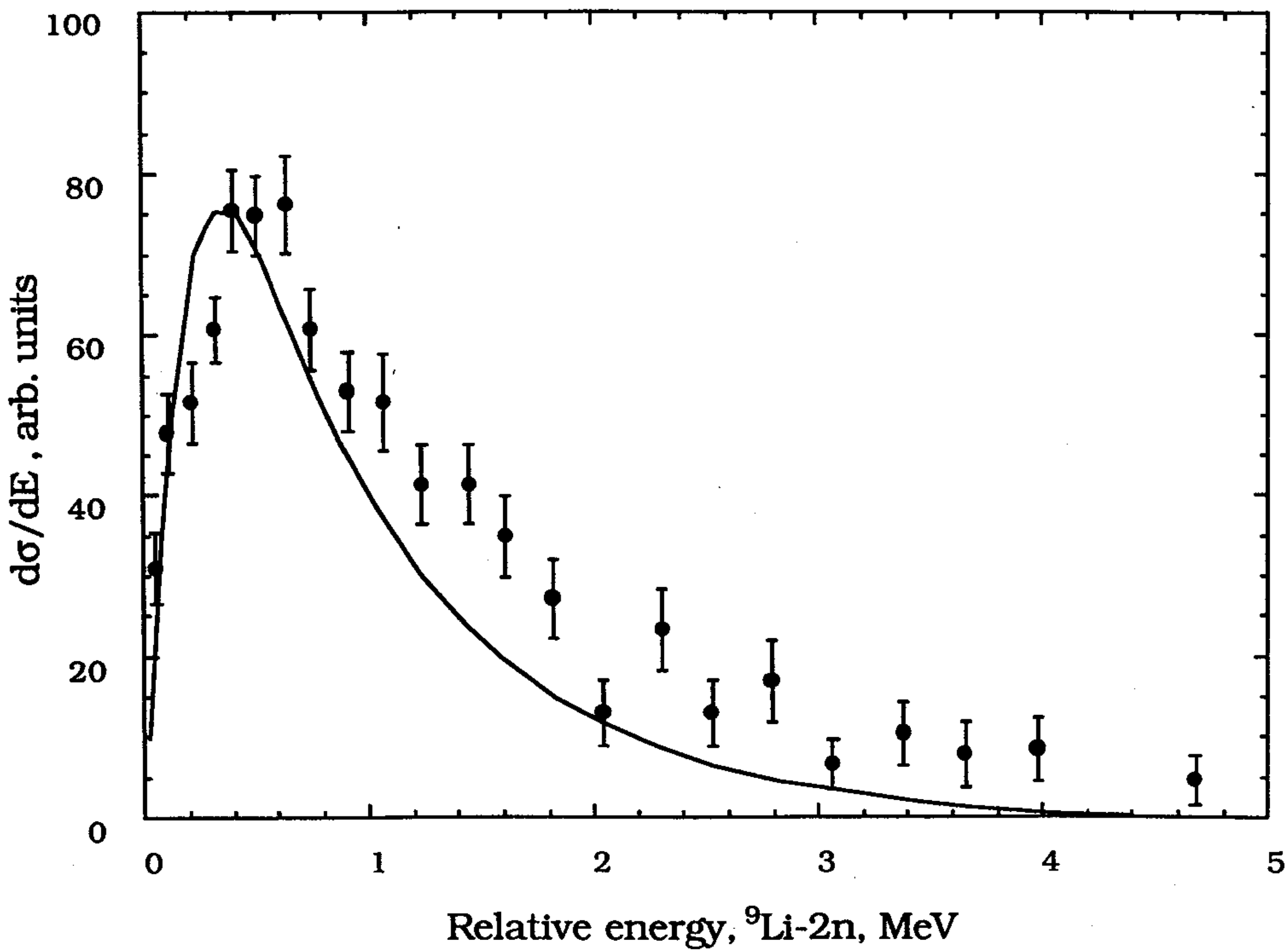
150
100
50
0
0
0.2
0.4
0.6
0.8
1
Relative energy, n-n, (MeV)



100
80
60
40
20
0
dσ/dE , arb. units
0
1
2
3
4
5
Relative energy, 9Li-2n, MeV