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ρ, ω, φ-Nucleon Scattering Lengths from QCD Sum Rules

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Abstract

The QCD sum rule method is applied to derive a formula for the ρ, ω, ϕ meson-nucleon spin-averaged scattering lengths $a_{\rho,\omega,\phi}$. We found that the crucial matrix elements are $\langle \bar{q}\gamma_\mu D_\nu q\rangle_N$ ($q = u$, d) (twist-2 nucleon matrix element) for $a_{\rho,\omega}$ and $m_s\langle \bar{s}s\rangle_N$ for a_ϕ, and obtained $a_\rho = 0.14 \pm 0.05$ fm, a, $=$ 0.11 $\pm$ 0.05 fm and $a_\phi = 0.035 \pm 0.015$ fm. This result implies a slight increase (< 60 MeV for ρ, ω, and < 15 MeV for ϕ) of the effective mass of these vector mesons in the nuclear matter.

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The operator product expansion (OPE) provides us with a convenient tool to decompose a variety of correlation functions into perturbatively calculable c-number coefficients and the nonperturbative matrix elements. In its application to the QCD sum rules (QSR), the OPE supplies an expression for the resonance parameters in terms of the vacuum condensates representing the nonperturbative dynamics in the correlators. In the application to the deep inelastic scattering (DIS), the OPE isolates the quark-gluon distribution functions of the target from the short distance cross sections. In this paper, we investigate the vector meson (ρ, ω, ϕ)-nucleon scattering length utilizing these two aspects of the OPE. A similar idea was presented in a recent work [4] on the nucleon-nucleon scattering length. We shall also discuss the mass shift of these vector mesons in the nuclear medium using the result on the scattering length.

Let's start our discussion with the forward scattering amplitude of the vector current J_μ^V ($V = \rho, \omega, \phi$) off the nucleon target with the four momentum $p = (p^0, \boldsymbol{p})$ and the polarization s:

$$T_{\mu\nu}(\omega, \boldsymbol{q}) = i \int d^4x \, e^{iq\cdot x} \langle ps| T\left(J_\mu^V(x) J_\nu^V(0)\right) |ps\rangle, \tag{1}$$

where $q = (\omega, \boldsymbol{q})$ is the four-momentum carried by J_μ^V and the nucleon state is normalized covariantly as $\langle p|p'\rangle = (2\pi)^3 2p^0 \delta(\boldsymbol{p} - \boldsymbol{p}')$. We set $p = (m_N, \boldsymbol{0})$ throughout this work and suppress the explicit dependence on p and s. The vector current J_μ^V is defined as $J_\mu^{\rho,\omega}(x) = (1/2)(\bar{u}\gamma_\mu u(x) \mp \bar{d}\gamma_\mu d(x))$, $J_\mu^\phi(x) = \bar{s}\gamma_\mu s(x)$. Near the pole position of the vector meson, $T_{\mu\nu}$ can be associated with the T-matrix for the forward $V - N$ helicity amplitude, $\mathcal{T}_{hH,h'H'}(\omega, \boldsymbol{q})$ ($h(h')$ and $H(H')$ are the helicities of the initial (final) vector meson and the initial (final) nucleon, respectively, and they take the values of $h, h' = \pm 1, 0$ and $H, H' = \pm 1/2$) as

$$\epsilon_\mu^{(h)}(q) T_{\mu\nu}(\omega, \boldsymbol{q}) \epsilon_\nu^{(h')*}(q) \simeq -\frac{f_V^2 m_V^4}{(q^2 - m_V^2 + i\eta)^2} \mathcal{T}_{hH,h'H'}(\omega, \boldsymbol{q}), \tag{2}$$

where we introduced the coupling f_V and the mass m_V of the vector meson V by the relation $\langle 0|J_\mu^V|V^{(h)}(q)\rangle = f_V m_V^2 \epsilon_\mu^{(h)}(q)$ with the polarization vector $\epsilon_\mu^{(h)}(q)$ normalized as $\sum_{\rm pol.} \epsilon_\mu^{(h)}(q)\epsilon_\nu^{(h)}(q) = -g_{\mu\nu} + q_\mu q_\nu/q^2$. As is well known in DIS, $T_{\mu\nu}$ can be decomposed into the four scalar components respecting the current conservation and the invariance under parity and time-reversal. (Two of them correspond to the spin-averaged structure functions W_1 and W_2, and the other two to the spin-dependent ones G_1 and G_2.) Correspondingly, there are 4 independent helicity amplitudes for the vector current-nucleon scattering; $\mathcal{T}_{1\frac{1}{2},1\frac{1}{2}}$, $\mathcal{T}_{1\frac{-1}{2},1\frac{-1}{2}}$, $\mathcal{T}_{0\frac{1}{2},0\frac{1}{2}}$, $\mathcal{T}_{1\frac{-1}{2},0\frac{1}{2}}$, all the rest being obtained by time-reversal and parity from these four. Since the information on G_1 and G_2 is still lacking, we shall focus on the combination $T = T_1 + (1 - (p\cdot q)^2/m_N^2 q^2) T_2$ ($\mathrm{Im}\, T_i \sim W_i$, $i = 1, 2$), which projects the $V - N$ spin-averaged T-matrix, $\mathcal{T}(\omega, \boldsymbol{q})$. In the low energy limit ($\boldsymbol{q} \to \boldsymbol{0}$), $\mathcal{T}$ is reduced to the $V - N$ spin-averaged scattering length $a_V = (1/3)(a_{1/2} + 2a_{3/2})$ ($a_{1/2}$ and $a_{3/2}$ are the scattering lengths in the spin-1/2 and 3/2 channels, respectively) as $\mathcal{T}(m_V, \boldsymbol{0}) = 24\pi(m_N + m_V)a_V$ [5]. A useful quantity for the dispersion analysis is the retarded correlation function defined as

$$T_{\mu\nu}^R(\omega, \boldsymbol{q}) = i \int d^4x \, e^{iq\cdot x} \theta(x^0) \langle ps| \left[J_\mu^V(x), J_\nu^V(0)\right] |ps\rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} du \, \frac{\mathrm{Im}\, T_{\mu\nu}^R(u, \boldsymbol{q})}{u - \omega - i\eta}, \tag{3}$$

which is analytic in the upper half ω-plane with a fixed $\boldsymbol{q}$. Noting the crossing symmetry, the contribution to the spin-averaged spectral function from the two particle intermediate state at $\boldsymbol{q}=\mathbf{0}$ (V and N) can be written as

$$\begin{aligned}&\frac{1}{\pi}\mathrm{Im}\left[T^R(\omega,\mathbf{0})\right]\\&\quad=-24\pi f_V^2 m_V^4(m_N+m_V)a_V\left[\theta(\omega)\delta'(\omega^2-m_V^2)-\theta(-\omega)\delta'(\omega^2-m_V^2)\right]+(\mathrm{S.P.}),\end{aligned} \tag{4}$$

where $\delta'(x)$ is the first derivative of the δ-function (double pole term) and (S. P.) denotes the simple pole term representing the off-shell energy dependence of the T-matrix. Using this form of the spectral function in eq. (3), and noting that the retarded correlation function $T^R_{\mu\nu}$ becomes identical to the causal correlation function $T_{\mu\nu}$ in the deep Euclidean region $\omega^2=-Q^2\to-\infty$, one gets

$$T(\omega^2=-Q^2)=-24\pi f_V^2 m_V^4(m_N+m_V)a_V\frac{1}{(m_V^2+Q^2)^2}+\mathcal{R}(Q^2), \tag{5}$$

where we have used the fact that T becomes a function of ω^2 in this limit. In eq. (5), we assumed the spectral function is saturated by the $VN\to VN$ scattering and the continuum contribution, and $\mathcal{R}(Q^2)$ denotes the sum of the simple pole term ($\sim 1/(m_V^2+Q^2)$) and the continuum contribution with its threshold S_0' ($\sim 1/(S_0'+Q^2)$).

We now proceed to the OPE side of $T(Q^2)$ (l.h.s. of eq. (5)). Unlike in DIS, our OPE is the short distance expansion and hence all the operators with the same dimension contribute in the same order with respect to $1/Q^2$. The complete OPE expression for $T(Q^2)$ including the operators up to dimension=6 is given in ref. [6] in the context of the finite temperature QSR. For the ρ and ω mesons, it reads from eq. (2.13) of [6] as ($-$ for ρ and $+$ for ω)

$$\begin{aligned}T^{\rho,\omega}(Q^2)&=\frac{1}{4}\left[-\frac{2m_q}{Q^2}\langle\bar{u}u+\bar{d}d\rangle_N-\frac{1}{6Q^2}\langle\frac{\alpha_s}{\pi}G^2\rangle_N+\frac{2\pi\alpha_s}{Q^4}\left(\langle\mathcal{Q}_5^{\mp}+\frac{2}{9}\mathcal{Q}^+\rangle_N\right)\right]\\&\quad-\frac{m_N^2}{2Q^2}A_2^{u+d}+\frac{5m_N^4}{6Q^4}A_4^{u+d}-\frac{m_N^2}{2Q^4}\left(B_{1\mp}+\frac{1}{4}B_2+\frac{5}{8}B_3\right),\end{aligned} \tag{6}$$

where $\langle\cdot\rangle_N$ denotes the spin-averaged nucleon matrix element, and $\mathcal{Q}_5^{\mp}$ and $\mathcal{Q}^+$ are the scalar four-quark operators familiar in the QSR for the ρ and ω mesons; $\mathcal{Q}_5^{\mp}=(\bar{u}\gamma_\mu\gamma_5\lambda^a u\mp\bar{d}\gamma_\mu\gamma_5\lambda^a d)^2$, $\mathcal{Q}^+=\left(\bar{u}\gamma_\mu\lambda^a u+\bar{d}\gamma_\mu\lambda^a d\right)\sum_q^{u,d,s}\bar{q}\gamma_\mu\lambda^a q$. In eq. (6), $A_n^{u+d}\equiv A_n^u+A_n^d$ ($n=2,4$) are related to the twist-2 operators and are given as the n-th moment of the parton distribution function ($q=u,d,s$); $\langle\mathcal{ST}\left(\bar{q}\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_n}q(\mu)\right)\rangle_N=(-i)^{n-1}A_n^q(\mu)(p_{\mu_1}\cdots p_{\mu_n}-\text{traces})$, $A_n^q(\mu)=2\int_0^1 dx\,x^{n-1}\left(q(x,\mu)+(-1)^n\bar{q}(x,\mu)\right)$ with the renormalization scale μ. B_i ($i=1\mp,2,3$) are associated with the twist-4 matrix elements as $\langle\mathcal{O}^i_{\mu\nu}(\mu)\rangle_N=(p_\mu p_\nu-m_N^2 g_{\mu\nu}/4)B_i(\mu)$ with $\mathcal{O}^{1\mp}_{\mu\nu}=(g^2/4)\mathcal{ST}\left((\bar{u}\gamma_\mu\gamma_5\lambda^a u\mp\bar{d}\gamma_\mu\gamma_5\lambda^a d)(\mu\to\nu)\right)$, $\mathcal{O}^2_{\mu\nu}=(g^2/4)\mathcal{ST}\left((\bar{u}\gamma_\mu\lambda^a u+\bar{d}\gamma_\mu\lambda^a d)\sum_q^{u,d,s}\bar{q}\gamma_\nu\lambda^a q\right)$, $\mathcal{O}^3_{\mu\nu}=ig\mathcal{ST}\left(\bar{u}\{D_\mu,{}^*G_{\nu\lambda}\}\gamma^\lambda\gamma_5 u+(u\to d)\right)$, where the color matrix λ^a is normalized as $\mathrm{Tr}(\lambda^a\lambda^b)=2\delta^{ab}$ and the symbol $\mathcal{ST}$ makes the operators symmetric and traceless with respect to the Lorentz indices.

To get an expression for the $V-N$ scattering length, we first make a Borel transform of eqs. (5) and (6) with respect to Q^2, and then eliminate the simple pole term, using the sum rule obtained by taking the derivative with respect to the Borel mass M^2. We also eliminate the unknown coupling constant f_V by taking the ratio between the obtained sum rule and the QSR expression for the vector current correlators in the vacuum. We thus get an expression for the spin-averaged scattering length a_V as

$$a_{\rho,\omega} = \frac{\pi M^2}{3m_{\rho,\omega}^2(m_N + m_{\rho,\omega})} \frac{r\beta/(\alpha M^2) + t\gamma/(\alpha M^4)}{(1+\frac{\alpha_s}{\pi})(1-e^{-S_0/M^2}) + b/M^4 - c/M^6}, \tag{7}$$

with

$$\begin{aligned}
r &= m_N\Sigma_{\pi N} - \frac{2}{27}m_0^2 + \frac{m_N^2}{2}A_2^{u+d}, \\
t &= -\frac{112\pi\alpha_s}{81}\left(\langle\bar{u}u\rangle\langle\bar{u}u\rangle_N + \langle\bar{d}d\rangle\langle\bar{d}d\rangle_N\right) - \frac{5}{6}m_N^4 A_4^{u+d} + \frac{m_N^2}{2}\left(B_{1\mp} + \frac{1}{4}B_2 + \frac{5}{8}B_3\right), \\
b &= 4\pi^2 m_q\langle\bar{u}u + \bar{d}d\rangle + \frac{\pi^2}{3}\langle\frac{\alpha_s}{\pi}G^2\rangle, \\
c &= \frac{448\pi^3\alpha_s}{81}\langle\bar{u}u\rangle^2,
\end{aligned}$$

where $\langle\cdot\rangle$ denotes the vacuum condensate and S_0 is the continuum threshold in the vacuum sum rule. The factors α, β and γ appeared through the elimination of the simple pole term, and they are defined as $\alpha = 1 - e^{-(S_0'-m^2)/M^2}(1 + \frac{S_0'-m^2}{M^2})$, $\beta = \frac{m^2}{M^2} + \frac{S_0'-m^2}{M^2}e^{-S_0'/M^2} - \frac{S_0'}{M^2}e^{-(S_0'-m^2)/M^2}$, $\gamma = 1 + \frac{m^2}{M^2} - (1 + \frac{S_0'}{M^2})e^{-(S_0'-m^2)/M^2}$ with $m = m_{\rho,\omega}$. If we ignore the S.P. term from the beginning, the corresponding formula is obtained by the replacement; $\beta/\alpha \to 1 - e^{-S_0'/M^2}$, $\gamma/\alpha \to 1$. In eq. (7), we have used the following relations for the matrix elements: (i) πN σ-term $\Sigma_{\pi N}$ is introduced through the relation $m_q\langle\bar{u}u + \bar{d}d\rangle_N = 2m_N\Sigma_{\pi N}$. (ii) The nucleon mass in the chiral limit, m_0, is introduced in favor of $\langle\frac{\alpha_s}{\pi}G^2\rangle_N$ through the QCD trace anomaly: $\langle\frac{\alpha_s}{\pi}G^2\rangle_N = -(16/9)m_0^2$. (iii) Factorization is assumed for the vacuum four-quark condensates, $\langle\mathcal{Q}_5^{\mp}\rangle$ and $\langle\mathcal{Q}^+\rangle$, as is usually adopted in the literature [1, 2]. (iv) Factorization is also employed to estimate the nucleon matrix elements of the scalar four-quark operators $\langle\mathcal{Q}_5^{\mp}\rangle_N$ and $\langle\mathcal{Q}^+\rangle_N$ after making the Fierz transform [7], i.e. $\langle(\bar{q}\Gamma\lambda q)^2\rangle_N \to \langle(\bar{q}q)^2\rangle_N \simeq 2\langle\bar{q}q\rangle\langle\bar{q}q\rangle_N$.

By repeating the same steps as above for J_μ^ϕ, one gets the spin-averaged $\phi - N$ scattering length as

$$a_\phi = \frac{\pi M^2}{3m_\phi^2(m_N + m_\phi)} \frac{r_s\beta/(\alpha M^2) + t_s\gamma/(\alpha M^4)}{(1+\frac{\alpha_s}{\pi})(1-e^{-S_0/M^2}) - 6m_s^2/M^2 + b_s/M^4 - c_s/M^6}, \tag{8}$$

with

$$\begin{aligned}
r_s &= m_s\langle\bar{s}s\rangle_N - \frac{2}{27}m_0^2 + m_N^2 A_2^s, \\
t_s &= -\frac{224\pi\alpha_s}{81}\langle\bar{s}s\rangle\langle\bar{s}s\rangle_N - \frac{5}{3}m_N^4 A_4^s + m_N^2\left(B_1^s + \frac{1}{4}B_2^s + \frac{5}{8}B_3^s\right), \\
b_s &= 8\pi^2 m_s\langle\bar{s}s\rangle + \frac{\pi^2}{3}\langle\frac{\alpha_s}{\pi}G^2\rangle, \\
c_s &= \frac{448\pi^3\alpha_s}{81}\langle\bar{s}s\rangle^2,
\end{aligned}$$

where the strange twist-4 matrix elements B_i^s ($i = 1-3$) are defined similarly to the case of the ρ and ω mesons. For the vacuum condensates and the quark masses in eqs. (7) and (8), we use the standard values at the renormalization scale $\mu = 1$ GeV [2]: $\alpha_s = 0.36$, $m_q = 7$ MeV, $m_s = 110$ MeV, $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = (-0.28\,\mathrm{GeV})^3$, $\langle\bar{s}s\rangle = 0.8\langle\bar{u}u\rangle$. With these vacuum condensates and the continuum threshold $S_0 = 1.75$ GeV^2 for ρ, ω and $S_0 = 2.0$ GeV^2 for ϕ, the experimental values for $m_{\rho,\omega,\phi}$ are well reproduced. We thus fixed S_0 at these values and use $m_{\rho,\omega} = 770$ MeV and $m_\phi = 1020$ MeV in eqs. (7) and (8). As a measure of the strangeness content of the nucleon, we introduce the parameter $y = 2\langle\bar{s}s\rangle_N/(\langle\bar{u}u\rangle_N + \langle\bar{d}d\rangle_N)$ and write $\langle\bar{s}s\rangle_N = y m_N \Sigma_{\pi N}/m_q$. For the nucleon matrix elements we use $\Sigma_{\pi N} = 45 \pm 7$ MeV, $y = 0.2$ and $m_0 = 830$ MeV obtained by the chiral perturbation theory [8]. Since we ignored the twist-2 gluon operators in eqs. (7) and (8), we consistently use the leading order (LO) parton distribution functions of Glück, Reya and Vogt [9] to determine A_i^{u+d} and A_i^s ($i = 2, 4$). It gives $A_2^{u+d} = 0.90$, $A_4^{u+d} = 0.12$, $A_2^s = 0.05$ and $A_4^s = 0.002$ at $\mu^2 = 1$ GeV^2. For the twist-4 matrix elements B_i and B_i^s, we use our recent result [10] extracted from the newest DIS data at CERN and SLAC. It is based on the SU(2) flavor symmetry (i.e. $B_i^s = 0$ ($i = 1-3$)) and a mild assumption on the matrix elements invoked by the flavor structure of the twist-4 operators. Both for the proton and the neutron, it gives $B_{1\mp} + B_2/4 + 5B_3/8 = -0.24 \pm 0.15$ (-0.41 ± 0.23) GeV^2 for the ρ (ω) meson at $\mu^2 = 5$ GeV^2 [11].

Using these numbers for the matrix elements, the Borel curves for the ρ, ω, ϕ-nucleon scattering lengths $a_{\rho,\omega,\phi}$ (eqs. (7) and (8)) are shown in Fig. 1. We determined the values of S_0' in order to minimize the slope of the curves at $0.8 < M^2 < 1.3$ GeV^2. They are 3.32 GeV^2 for ρ, 3.29 GeV^2 for ω and 4.40 GeV^2 for ϕ. With the above parameters, r in eq. (7) reads $r = 0.04 - 0.05 + 0.40 = 0.39$ GeV^2 from the first to the third terms. Thus r is totally dominated by the twist-2 nucleon matrix element A_2^{u+d} and the cancelling contribution from the first and the second terms makes the ambiguity in $\Sigma_{\pi N}$ and m_0 less important. The t-term in eq. (7) reads $t = 0.42 - 0.08 - 0.11 \pm 0.07\,(-0.18 \pm 0.10) = 0.23 \pm 0.07\,(0.16 \pm 0.10)$ GeV^4 for ρ (ω), which shows the contribution from the twist-4 matrix elements is sizable. To get an insight on the sensitivity of the results to the variation of t, we also showed $a_{\rho,\omega}$ without the twist-4 matrix elements in t with $S_0' = 3.35$ GeV^2. One sees that the inclusion of B_i reduces the $a_{\rho,\omega}$ by about 20 % (30 %) for ρ (ω). With the uncertainty in B_i in mind, we assign error bars as $a_\rho = 0.14 \pm 0.05$ fm and $a_\omega = 0.11 \pm 0.05$ fm, taking the values for $a_{\rho,\omega,\phi}$ around $M^2 = 1$ GeV^2. For the case of a_ϕ, the value of $m_s\langle\bar{s}s\rangle_N$ governs the whole result because of large m_s, i.e. $r_s = 0.13 - 0.05 + 0.04 = 0.12$ GeV^2 and $t_s = 0.066 - 0.003 + (\mathrm{twist} - 4 \equiv 0) = 0.063$ GeV^4 from the first to the third terms in r_s and t_s. Due to the uncertainty in $m_s\langle\bar{s}s\rangle_N$, we read from Fig. 1 $a_\phi = 0.035 \pm 0.015$ fm. Some phenomenological analyses on the nucleon form factor [12] and the nuclear force [13] suggest quite a large OZI violating ϕNN coupling constant $g_{\phi NN}/g_{\omega NN} \sim 0.4$. Equation (8) supplies a neat expression for the $\phi N \to \phi N$ interaction strength in terms of the strangeness content of the nucleon, showing the importance of $m_s\langle\bar{s}s\rangle_N$ rather than $\langle\bar{s}\gamma_\mu D_\nu s\rangle_N$. We quote in passing that the scattering lengths obtained without the S.P. term also took the same values within these error bars.

Let us finally discuss the mass shift of the vector mesons in the nuclear medium using the result for the scattering lengths here. In the *dilute* nucleon gas approximation, the V-current correlator in the nuclear medium can be written as

$$\Pi^{N.M.}_{\mu\nu}(\omega,\boldsymbol{q}) = i\int d^4x\, e^{iq\cdot x}\langle T\left(J^V_\mu(x)J^V_\nu(0)\right)\rangle + \sum_{\text{pol.}}\int^{p_f}\frac{d^3p}{(2\pi)^3 2p^0}T_{\mu\nu}(\omega,\boldsymbol{q}). \tag{9}$$

By ignoring the Fermi motion of the nucleon, $\Pi^{N.M.}_{\mu\nu}(\omega,\boldsymbol{q}=\boldsymbol{0})$ can be approximated near the pole position as

$$\begin{aligned}\Pi^{N.M.}_{\mu\nu}(\omega,\boldsymbol{0}) &\simeq f_V^2 m_V^4\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{\omega^2}\right)\left(\frac{1+O(\rho_N)}{\omega^2-m_V^2}+\Delta m_V^2\frac{1}{(\omega^2-m_V^2)^2}\right)\\ &\sim \frac{1+O(\rho_N)}{\omega^2-m_V^2-\Delta m_V^2}+O(\rho_N^2),\end{aligned} \tag{10}$$

where $\Delta m_V^2 = 12\pi a_V\rho_N(m_N+m_V)/m_N$ with the nucleon density ρ_N. From this relation, Δm_V^2 can be viewed as a shift of m_V^2 in the nuclear medium [14]. Our values for $a_{\rho,\omega,\phi}$ suggest that the effective mass for the vector mesons increases by about $27-57$ MeV for ρ, $20-48$ MeV for ω and $5-13$ MeV for ϕ at the nuclear matter density $\rho_N = 0.17\ \text{fm}^{-3}$ [15].

This direction of the mass shift is opposite to the one reported in [7]. The authors of [7] carried out the QSR analysis for the ρ, ω, ϕ-mesons in the nuclear medium with a linear density approximation, eq. (9). They analyzed $\Pi^{N.M.}(\omega,\boldsymbol{0})$ ($\Pi^{N.M.}_{\mu\nu}(\omega,\boldsymbol{0}) = (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi^{N.M.}(\omega,\boldsymbol{0})$) using a simple pole ansatz for the vector meson together with a scattering term ($\sim 1/Q^2$) in the phenomenological spectral function, and found a dropping "effective mass" as the density increases. However, $\Pi^{N.M.}(\omega,\boldsymbol{0})$ contains the density dependent part as a form of $(\rho_N/2m_N)T(\omega^2)/\omega^2$. If one uses unsubtracted dispersion relation for $T(\omega^2)$, $T(\omega^2)/\omega^2$ brings a factor $-\Delta m_V^2(m_V^2/Q^2)$ ($Q^2=-\omega^2>0$) instead of Δm_V^2 in eq. (10). In this case, due to the additional factor $1/Q^2$, the double pole term can not be incorporated into a mass shift. If one uses subtracted dispersion relation, the incalculable subtraction term $T(0)/\omega^2$ remains until the end of the analysis. (Note the subtraction does not affect the calculation of the scattering length at all.) In either interpretation, an inadequate form of the spectral function led to a fictitious "negative" mass shift in [7]. As is shown in eq. (10), in order to see the "mass shift" by a simple pole fitting, $\omega^2\Pi^{N.M.}(\omega,\boldsymbol{0})$ has to be used in the medium QSR, which should lead to a positive mass shift. Since the second term in eq. (9) has an unique relation with the $V-N$ T-matrix around the pole position as is shown in eq. (2), we believe the mass shift of the vector mesons in the nuclear medium in the context of the QSR should be understood as presented in this work.

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Figure Captions

Fig. 1 The Borel curves for the ρ, ω, ϕ-nucleon scattering lengths. The dashed line denotes the one for ρ and ω without the twist-4 matrix elements in eq. (7).

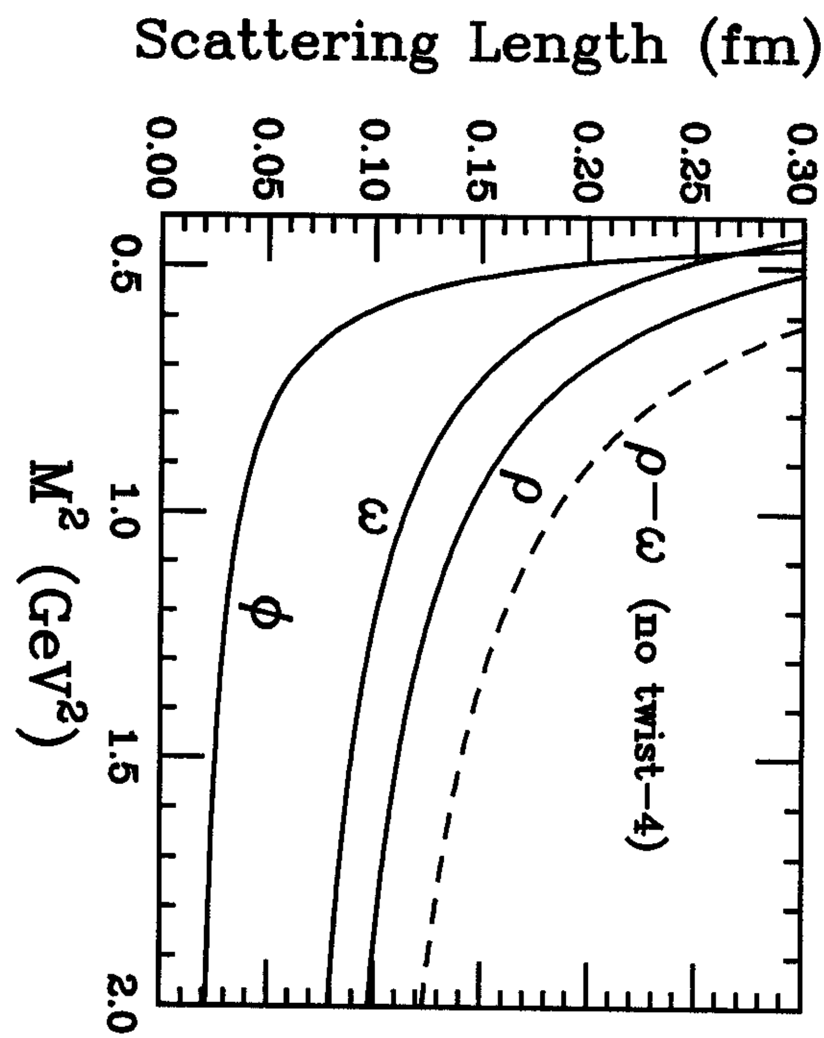
Scattering Length (fm)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
M² (GeV²)
0.5
1.0
1.5
2.0
φ
ω
ρ
ρ−ω (no twist−4)

Fig. 1