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Abstract

Pion and **kaon** mean free paths within a thermal hadronic background are calculated using relativistic kinetic theory. Free cross sections are used which include contributions from ρ, K^*, A and heavier resonances. Given **pion** to baryon ratios appropriate for the breakup stage of a $200 \cdot A$GeV relativistic-heavy-ion collision, we find for temperatures less than 100 MeV, kaons have a shorter mean free path than pions, while for higher breakup temperatures the reverse is true. Since breakup temperatures should be in the neighborhood of 100 MeV, this suggests that **kaon** interferometry samples the same emission distribution as **pion** interferometry.

By measuring outgoing hadrons in a heavy-ion collision one can reconstruct a reaction's breakup stage. The momenta of outgoing particles can be measured directly and a space-time picture of the last collisions may be constructed using the techniques of two-particle interferometry [I]. Space-time information is especially useful. For instance, if the time a reaction takes to proceed is much longer than 10 fm/c, it would signal a reduction in pressure, inferring a first-order phase transition [2].

Unfortunately, such information regarding the reaction's lifetime can be masked by the presence of long-lived resonances [3], particularly the ω which has a lifetime of 20 fm/c. For this reason Padula and Gyulassy campaigned for two-kaon interferometry. The only long-lived resonance responsible for a significant portion of kaons is the $K^*(892)$ which has a lifetime of 8 fm/c, smaller than characteristic times for crossing the reaction zone.

In this letter we study mean free paths of pions and kaons at temperatures and densities characteristic of hadronic matter from the breakup stage of a relativistic heavy-ion collision at CERN. We find that in this environment mean free paths are remarkably similar and conclude that information from kaon interferometry describes the dissolution of the entire system, as kaons should escape at the same time as pions which comprise the bulk of the matter.

In baryon-rich matter, such as is characteristic of heavy-ion collisions at the AGS, most charged kaons are positive since s quarks are absorbed into baryons, leaving $\bar{s}$ quarks to form hadrons. Positive kaons interact little with baryons as opposed to pions which interact vigorously due to the Δ resonance. Thus positive kaons escape baryonic matter much more easily than pions and the results of kaon interferometry indeed reflect this fact [4]. For sulfur projectiles at 200 A$\cdot$ GeV incident on heavy targets, resulting pion multiplicities are ten times as large as baryon multiplicities at mid-rapidity. Thus a meson's escape probability depends principally on its cross section with pions.

In the context of a thermodynamic model we estimate mean free paths of a given meson of type a given its momentum. Assuming the meson interacts with an assortment of hadrons of type b whose density is given according to a relativistic Boltzmann distribution and assuming we know the cross sections σ_{ab}, the mean free path is:

$$\lambda_a(\boldsymbol{p}_a) = \frac{\boldsymbol{p}_a}{E_a}\frac{1}{R_a^{net}(\boldsymbol{p}_a)}. \tag{1}$$

Here $\boldsymbol{p}_a$ and E_a represent the meson's momentum and energy while R_a is the collision rate. The net collision rate includes contributions from all different species.

$$R_a^{net}(\boldsymbol{p}_a) = \sum_b R_{ab}(\boldsymbol{p}_a) \tag{2}$$

$$R_{ab}(\boldsymbol{p}_a) = \int ds \frac{d^3p_b}{(2\pi)^3} f(\boldsymbol{p}_b)\sigma_{ab}(s)v_{rel}\delta\left(s - (p_a + p_b)^2\right)$$

where

$$v_{rel} = \frac{\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}}{E_a E_b}, \qquad f(\boldsymbol{p}_b) = (2s_b + 1)e^{-(E_b-\mu)/T}.$$

Only baryons are given a chemical potential which is chosen to result in a free π^+ to proton ratio of 10. Relative densities of hadron species are dependent on the temperature. They are illustrated in Fig. 1. Species included in the calculation are π, K, ρ, η, ω and K^*.

Cross-sections are dominated by contributions of resonances. For pions the largest contributor to the collision rate is the resonant reaction through the ρ while for kaons the most common collision is with pions through the K^* resonance. Resonant cross-sections are assumed to have the form:

$$\sigma_{ab}(s) \propto \frac{s - (m_a - m_b)^2}{(s - m_R^2)^2 + m_R^2\Gamma_R^2} \tag{3}$$

The mass and width of the resonance are m_R and Γ_R respectively. Normalizations to this form are chosen to yield maximum cross sections consistent with unitarity limits. The unitarity limits for cross sections through a resonance are proportional to $1/k^2$ where k is the relative momentum of the scattering.

For a narrow width Γ_R the contribution to the rate in Eq. (3) is proportional to $\bar{f}(k_R)\Gamma_R$, where k_R is the magnitude of the reduced relative momentum at the resonance and $\bar{f}$ is the average Boltzmann factor for a particle b having a momentum corresponding to a resonance with a. One expects $\pi\pi$ scattering to be weaker than πK because the K^* has less decay energy than a ρ, hence the Boltzmann factor is larger for the K^* case. However, the width of the K^*, 50 MeV, is one third the width of the ρ. At sufficiently high temperatures pions scatter more readily when travelling through a pion gas than do kaons.

Fig. 2 shows the mean free paths of particles travelling through a hadron gas at a temperature of 120 MeV as a function of their momentum. The dotted lines show the mean free paths of a pion (Fig. 2a) and a kaon (Fig. 2b) in a pion gas where only interactions through the ρ and K^* resonances are considered. These results are consistent with previous calculations which used similar assumptions [5,6]. The mean free paths go to zero with zero momentum due to the velocity term in Eq. (1). The rise of the mean free path for pions at low momentum demonstrates how pions without much energy have difficulty colliding through the ρ resonance since it is difficult to find a second pion with sufficient energy to produce a ρ. For kaons this is not so difficult, since K^*s do not require so much center-of-mass energy; and since the kaon is heavier, less of a colliding particle's energy is lost to center-of-mass motion.

The long-dashed curves result from adding to the resonant ρ and K^* an s-wave contribution. Boltzmann weighting favors near-threshold interactions and therefore enhances the effect of the relatively small s-wave for which a constant cross section of 8.0 mb is taken. Next we add the heavier resonances. For the pion results (Fig. 2a) we add K^*, $a_1(1260)$ and Δ for pion-nucleon interactions. They are presented as short-dashed, dot-dashed and solid lines, respectively. We also studied the effect of even heavier resonances but conclude they are of no importance for these temperatures. The effect of the resonances beyond ρ is to reduce the mean free path of order 30% at low pion momentum and 15% at high momentum.

To the kaon interactions we add ϕ and $K_1(1270)$ for interactions with other kaons and with ρs. We also add $\Lambda(1520)$ and $\Lambda(1800)$ for $K^{-,\bar{0}}$s interacting with nucleons. The results are again shown as long-dashed, short-dashed, dot-dashed and solid lines, respectively. The effect of the ϕ is quite small as expected. The $K_1(1270)$ is near enough to threshold that the unitarity limit is relatively large resulting in a rather strong interaction and a noticeable change in the kaon mean free path. Finally, the presence of nucleons further reduces the kaon mean free path but only slightly. Overall these heavier resonances play a modest role. On the other hand, quantitative comparisons between experimental data and model calculations that neglect these heavier resonances such as refs. [7–9] are only reliable, as we have shown, up to tens of percents. Many authors have studied mean collision times for pions [10–13], but we choose to compute average mean free paths of pions and compare to kaons. The result is shown in Fig. 3 as a function of the temperature. The average mean free path is

$$\bar{\lambda} = \frac{\int d^3p f(\mathbf{p}) \lambda(\mathbf{p})}{\int d^3p f(\mathbf{p})}. \tag{4}$$

All resonances and s-wave contributions are included in the calculation. The reaction should end when the mean free path is near the size of the system. From interferometry the size of the dissolving system appears similar to the size of a lead nucleus which has a radius of seven fm. A mean free path of seven fermi corresponds to a temperature of approximately 110 MeV. One must be cautious of such conclusions both because of the lack of geometric detail involved in the inference and the questionable assumption of zero chemical potential. Rapid expansion can outrun a system's ability to stay in chemical equilibrium, resulting in large chemical potentials [14–16], higher densities and therefore shorter mean free paths. This would allow the system to stay together longer, resulting in lower breakup temperatures.

The most remarkable aspect of Fig. 3 is that the mean free paths of kaons and pions are so similar. This means that kaons can be used to view the final stage of the collision without the qualification that they have escaped prematurely. This does not mean that correlation functions from kaons and pions should have the same apparent source sizes, and indeed at

CERN preliminary measurements point to smaller sizes for kaons than pions [17]. Even if one can account for pions from long-lived resonances, kaon sources can appear smaller due to collective expansion. A heavier particle with a given velocity is more confined to the region with the same collective velocity [18,2]. Given that pions and kaons have such similar escape probabilities, one can then compare and interpret correlation results from kaons and pions. This clarifies the meaning of both measurements. In addition the effect of heavier resonances and s-wave contributions has been cataloged, showing they matter at the level of 10–20 percent. These calculations can provide guidance to those constructing transport models. By using the results of Fig. 2, one can see which channels should be included and which reactions can safely be neglected.

ACKNOWLEDGMENTS

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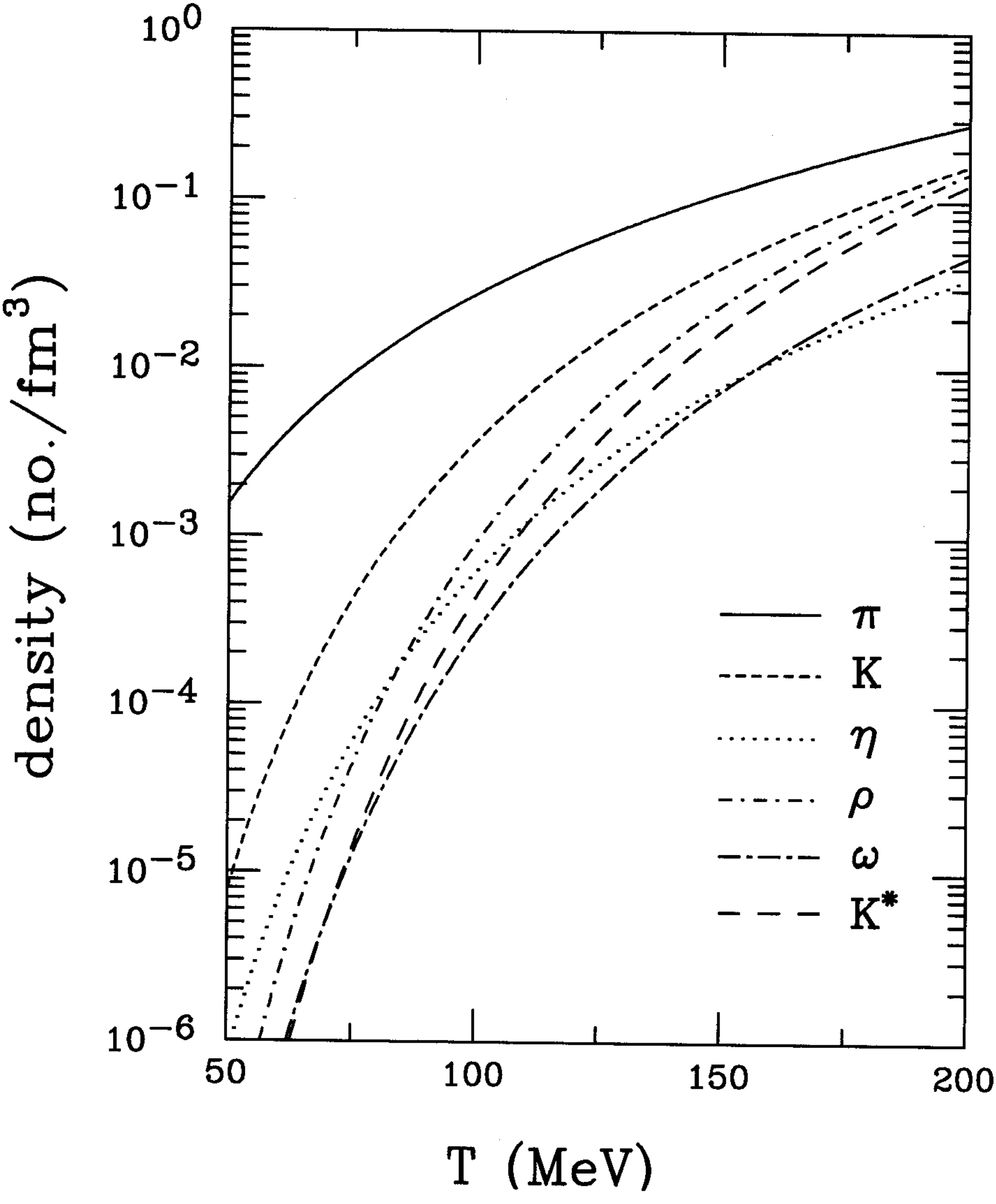
FIGURES

FIG. 1. Thermal ensemble of hadrons and their number densities as a function of temperature.

FIG. 2. Mean free paths of pions (a) and kaons (b) as a function of their momentum. Resonant ρ and K^* cross sections are included in the dotted curves. The the effects of adding to this the s-wave (long-dashed curves) into the cross section are shown. Finally, K^*, $a_1(1260)$ and Δ into the pion results and ϕ, $K_1(1270)$, $\Lambda(1520)$ and $\Lambda(1800)$ are shown as short-dashed, dot-dashed and solid lines, respectively.

FIG. 3. Average mean free paths of pions (solid curve) and kaons (dashed curve) as they depend on the temperature.



density (no./fm^3)
T (MeV)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
50
100
150
200
π
K
η
ρ
ω
K*

Figure 1

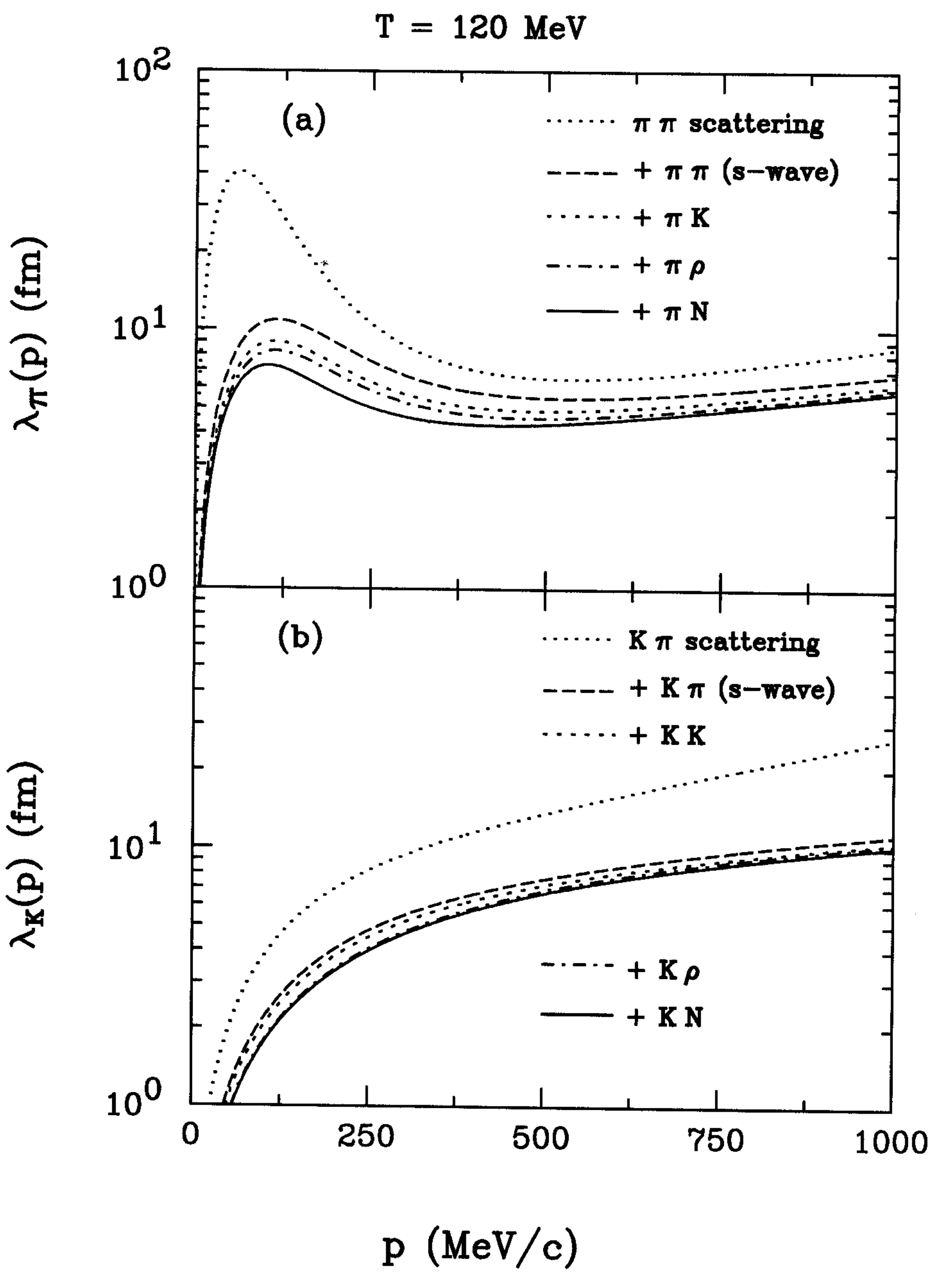
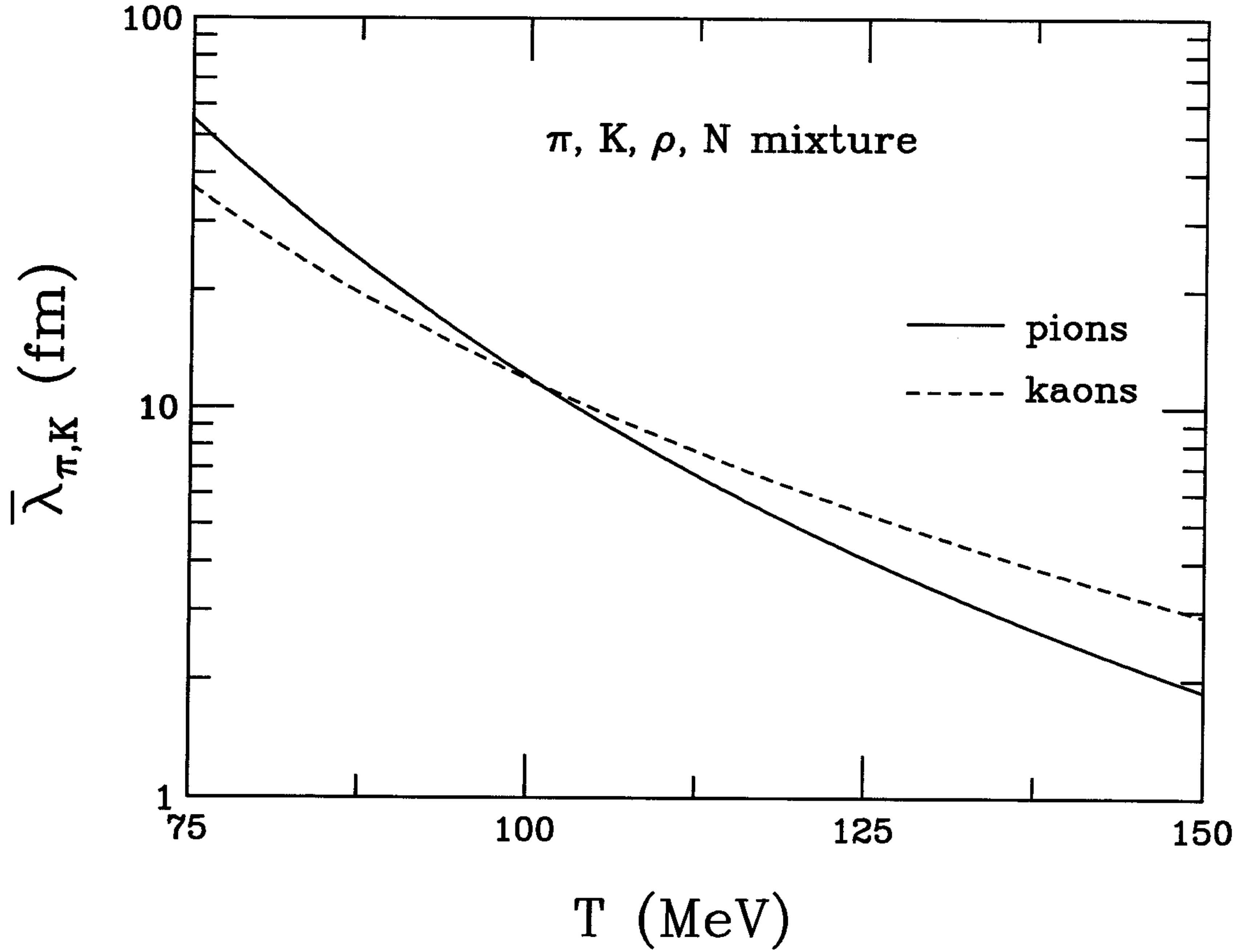

T = 120 MeV
(a)
π π scattering
+ π π (s−wave)
+ π K
+ π ρ
+ π N
λ_π(p) (fm)
(b)
K π scattering
+ K π (s−wave)
+ K K
+ K ρ
+ K N
λ_K(p) (fm)
10^2
10^1
10^0
0
250
500
750
1000
p (MeV/c)

Figure 2

Figure 3



100
10
1
75
100
125
150
π, K, ρ, N mixture
pions
kaons
$\bar{\lambda}_{\pi,K}$ (fm)
T (MeV)