

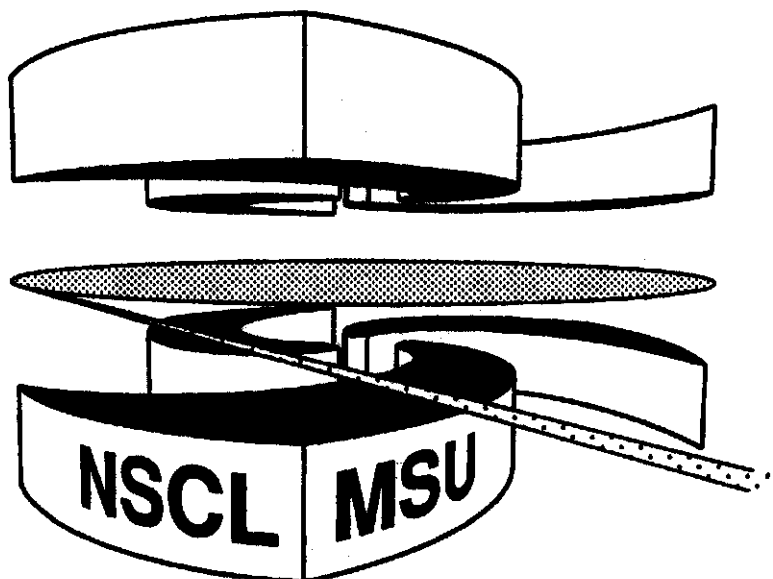


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**CHAOS AND ORDER IN THE SHELL MODEL EIGENVECTORS**

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# CHAOS AND ORDER IN THE SHELL MODEL EIGENVECTORS

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## Abstract

The *energies* and wave functions of stationary many-body states are analyzed to look for the signatures of quantum chaos. Shell model calculations with the **Wildenthal** interaction are performed in the  $J - T$  scheme for 12 particles in the sd-shell. The local level statistics are in perfect agreement with the GOE predictions. The analysis of the amplitudes of the **eigenvectors** in the shell model basis with the aid of the informational entropy and moments of the distribution function shows evidence for local chaos with a localization length reaching 90% of the total dimension in the middle of the spectrum. The degree of **chaoticity** is sensitive to the strength of the residual interaction **as** compared to the single particle energy spacing.

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Quantum chaos in many-body systems was studied mostly from the viewpoint of level statistics which displays a clear relation to the notion of classical chaos [1]. Presumably much more information could be obtained from an analysis of the wave functions and transition amplitudes. Here one expects to encounter the transition from the simple picture of almost independent elementary excitations to extremely mixed compound states which would display new specific features as, for example, so called dynamic enhancement of weak interactions [2].

To perform such an analysis and to check various hypotheses concerning complicated quantum dynamics, one needs a rich set of data which would allow one to make statistically reliable conclusions. Realistic nuclear shell model calculations are one of the most promising candidates for studying this largely unknown structure of quantum chaotic states.

We studied the behavior of the basis-state amplitudes of the shell model eigenvectors produced in the  $J - T$  scheme for 12 particles in the  $sd$  shell. Our model hamiltonian describing a many-body system of valence particles within a major shell contains a one-body part, which is due to an existing core (e.g.  $^{16}\text{O}$  for the  $sd$  shell) and a two-body antisymmetrized interaction of the valence particles

$$H = \sum \epsilon_\mu a_\mu^\dagger a_\mu + \frac{1}{4} \sum V_{\mu\nu\lambda\rho} a_\mu^\dagger a_\nu^\dagger a_\lambda a_\rho . \quad (1)$$

In our calculations the Wildenthal interaction along with the well known procedure to project out of the  $m$ -scheme the states with correct values of the total angular momentum  $J$  and isospin  $T$  were utilized [3,4]. The  $J - T$  projected states  $|k\rangle$  are used to build the matrix of the many-body hamiltonian,  $H_{kk'} = \langle JT; k | H | JT; k' \rangle$ , which is eventually diagonalized producing the eigenvalues  $E_\alpha$  and the eigenvectors

$$|JT; \alpha\rangle = \sum_k C_k^\alpha |JT; k\rangle. \quad (2)$$

They represent the object of our investigation.

The matrix dimension for the  $J^\pi T = 2^+0$  states is 3273. The density of states steeply increases along with excitation energy, reaches its maximum and then decreases again for

the highest energy. This high-energy behavior, as well as the approximate symmetry with respect to the middle of the spectrum, are artificial features of models with finite Hilbert space in contrast to actual many-body systems. For the analysis of the level statistics we used levels 200 - 3000.

Fig. 1 shows the standard quantities which define the chaoticity of a quantum system [1,5], the unfolded distribution of the nearest neighbor spacings  $P(s)$  and the spectral rigidity  $\Delta_3$ , for this class of states. The solid lines in both parts of the figure describe the random matrix results for the Gaussian Orthogonal Ensemble (GOE). The dashed line on the right corresponds to the Poisson level distribution which is characteristic of an ordered system. The closeness of  $\Delta_3$  to the random matrix results even for very large values of  $L$  is remarkable. Previous to this study the largest value of  $L$  considered was less than 100 [6]. Thus, the level statistics manifest generic chaotic behavior.

We next look to the structure of the wave functions which could reveal in more detail how close to chaoticity we are. The appropriate quantities to measure the degree of complexity of a given eigenstate  $|\alpha\rangle$ , eq.(2), with respect to the original shell model basis are, for instance, the informational entropy [7,8],

$$S^\alpha = - \sum_k |C_k^\alpha|^2 \ln |C_k^\alpha|^2, \quad (3)$$

or the moments of the distribution of amplitudes  $|C_k^\alpha|^2$ . The second moment determines the number of principal components (*NPC*) of an eigenvector  $|\alpha\rangle$ ,

$$(NPC)^\alpha = \left( \sum_k |C_k^\alpha|^4 \right)^{-1}. \quad (4)$$

In the GOE all basis states are completely mixed so that the resulting eigenvectors are totally delocalized and cover uniformly the  $N$ -dimensional sphere of radius 1 [5,9,10]. Gaussian fluctuations with zero mean,  $\overline{C_k^\alpha} = 0$ , and width  $\overline{|C_k^\alpha|^2} = 1/N$  lead to the values  $\ln(0.48N)$  and  $N/3$  for the quantities (3) and (4) respectively. Here  $N$  is the total dimension of the model space. In reality, the incomplete mixing of basis states determined by specific properties of the hamiltonian can coexist with the GOE-type level correlations.

The left upper part of Fig. 2 presents the  $\exp(S^\alpha)$  quantity for the  $2^+0$  states. On the  $x$ -axis are the eigenstates numbered in order of their energies. This simple "numbered" scale is equivalent to the "unfolding" procedure described for example by Brody *et al* [5]. "Unfolding" is introduced to separate local correlations and fluctuations from the global spectral properties. The solid line represents the GOE result ( $0.48N$ ). One observes a semicircle-type behavior and a 12% deviation from the GOE even for the maximum entropy in the middle of the spectrum.

It is interesting to study the role of single-particle energies (see eq. (1)) for the chaotic behavior of the amplitudes. The upper right part of Fig. 2 shows the  $\exp(S^\alpha)$  quantity for the  $2^+0$  states for the same hamiltonian but with all single particle energies,  $\epsilon_\mu$  in eq. (1), set to zero. In this degenerate case the GOE limit (solid line) is attained and the chaotic regime extends over a larger part of the spectrum.

To further quantify these effects, we look to the distribution  $P(l_S)$  of  $l_S^\alpha = \exp(S^\alpha)/0.48N$ . One can interpret  $l_S^\alpha$  as a delocalization length  $N^\alpha/N$ . It is expected to be shaped around  $l_S = 1$  in the chaotic limit. The lower left panel of Fig. 2 presents the results for the values of  $l_S$  calculated for the normal hamiltonian and shown on the upper left panel. Here the limit of  $l_S = 1$  is not reached. On the other hand, for degenerate single-particle orbitals (upper and lower panels on the right side), the distribution of localization lengths is more narrow and the full chaotic limit is reached. This is related to the fact that the mean field in general tends to smooth out the chaotic aspects of many-body dynamics [11].

The number of principal components (4) behaves in a very similar way gradually increasing from the edges of the spectrum to the middle, Fig.3 (left). Even the most complicated states are shifted down from the GOE limit of complete mixing. However, for the ratio  $\exp S^\alpha/(NPC)^\alpha$  one obtains the results in the right part of Fig. 3. For a Gaussian distribution of amplitudes  $C_k^\alpha$  of a given eigenvector  $|\alpha\rangle$ , this ratio would be given by the universal ( $N$ -independent) random matrix result equal to 1.44 (solid line). The flattened region indicates that the chaotic dynamics, even if not complete, extends far beyond the region nearby

the maximum of the informational entropy. Again we use the "unfolded" numbered scale rather than the energy scale. The "unfolding" reveals the presence of "local" chaos: in a given small energy range, the eigenstates are characterized by a typical delocalization length  $N^\alpha < N$  and by a Gaussian distribution of the amplitudes  $C_k^\alpha$  with zero mean value and variance  $(N^\alpha)^{-1}$ . This length cancels in the ratio  $\exp(S^\alpha)/(NPC)^\alpha$  for the majority of states in the middle of the spectrum. The flatness of this ratio, as compared to strong  $\alpha$ -dependence of  $\exp(S^\alpha)$  and  $(NPC)^\alpha$  separately, indicates the existence of the local chaotic properties scaling with  $N^\alpha$ . The edge regions with this ratio larger than 1 clearly correspond to relatively weakly mixed states with a reduced  $NPC$ . We note the very narrow dispersion of points in Figs. 2 and 3 for our measures of complexity.

Using the same tools we also analyzed the  $J^\pi T = 0^+0$  states of the model (the dimension of this subspace is 839) and obtained very similar results.

In conclusion, we have studied the chaotic properties of a many-body quantum system which consists of 12 valence particles interacting in the  $sd$ -shell. We have shown that standard signatures of chaos like nearest neighbor spacing distribution or spectral rigidity are not sensitive enough to show deviations from the random matrix results. The informational entropy or moments of the  $|C_k^\alpha|^2$  distribution like  $(NPC)$  are much more suited to reveal these details. Arguments are given in favor of local chaos characterized by a Gaussian distribution of the components of the wave functions with the variance related to the localization length. This length grows as the level density increases but does not reach the GOE limit. Finally, we have shown that the effect of the core (given by the single-particle energies) diminishes the maximal degree of chaoticity which can be obtained when the system consists of interacting valence particles only.

We point out that our signatures of complexity,  $l_S$  and  $(NPC)$ , are basis-dependent, they reflect mutual properties of the eigenbasis of the problem and the original "simple" basis  $|k\rangle$ . In the basis of eigenvectors of a random matrix, unrelated to the mean field of the problem, the results have been checked to coincide with those for the GOE as it should be due to the orthogonal invariance. The basis dependence can give additional physical information

and it should be studied separately. The mean field basis is in some sense exceptional since it can be shown that the mean field itself is generated by averaging out the most chaotic components of many-body dynamics [11]. Therefore it can be considered as a preferential representation for our purpose. It is remarkable that the "natural" choice of the shell model basis sheds a detailed light on the global and local chaotic properties of the wave functions in the many-body system with strong interaction.

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## Figure captions

**Figure 1** Unfolded distribution of the nearest neighbor spacings,  $P(s)$ , left, and the rigidity of the spectrum,  $\Delta_3$ , right, for  $2^+0$  states.

**Figure 2** Left panel: exponential of entropy (upper part), and the distribution of  $l_S = \exp S/0.48N$  for  $2^+0$  states calculated with the full hamiltonian of the model (lower part); right panel: the same quantities for the degenerate model with  $\epsilon_\mu = 0$ .

**Figure 3** The Number of Principal Components, eq.(4), of  $2^+0$  states, left, and the ratio  $\exp S/(NPC)$  for the same states, right.

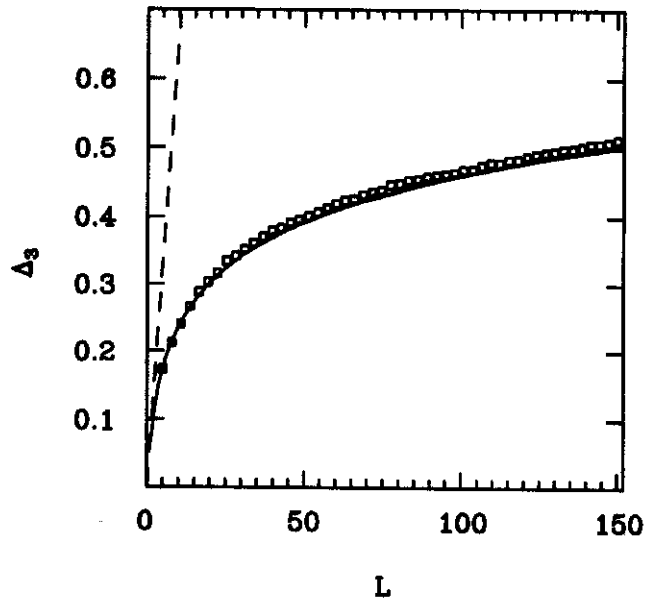
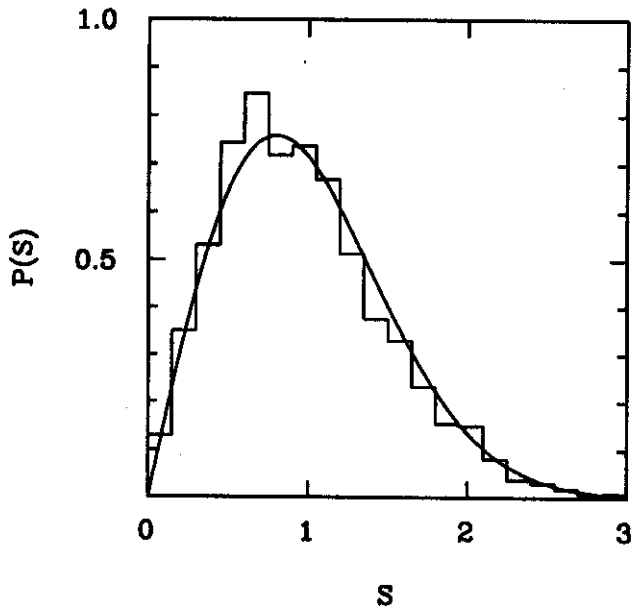


Figure 1:

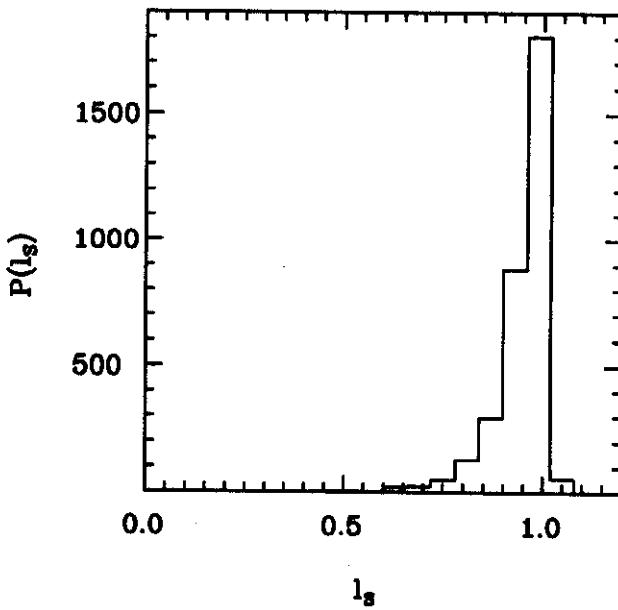
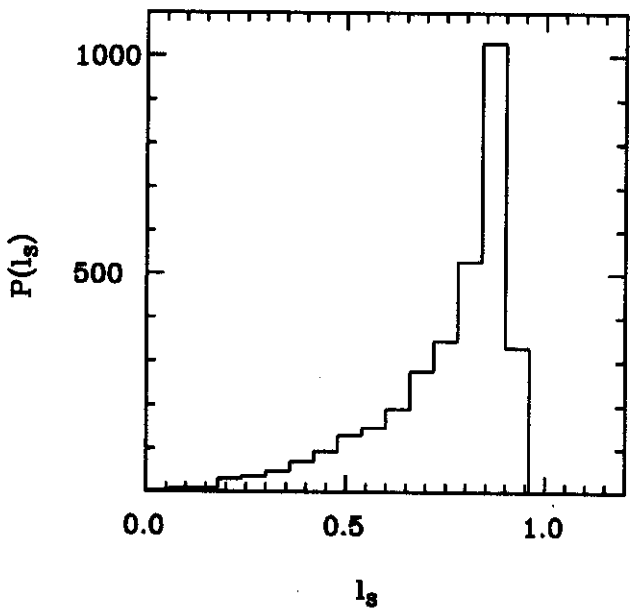
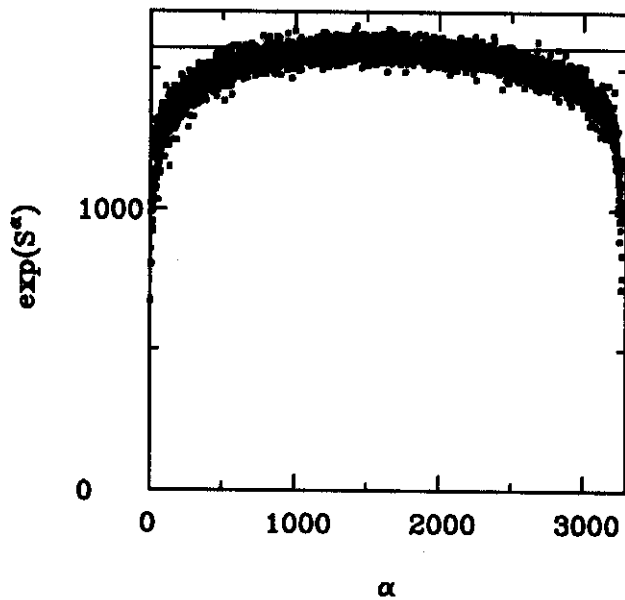
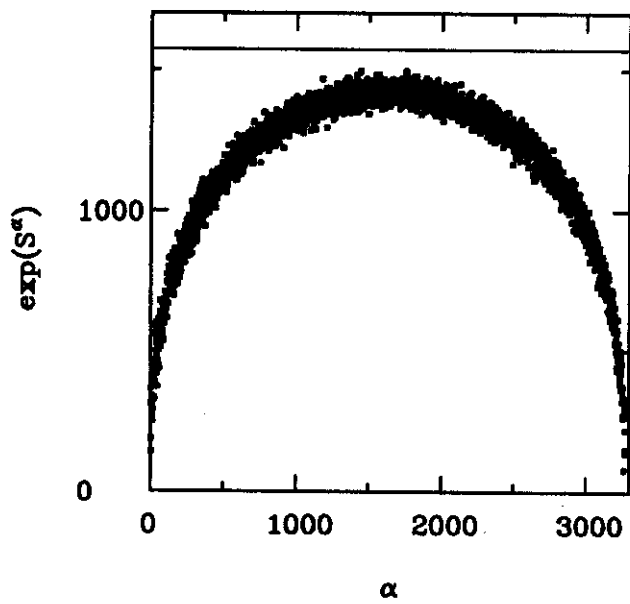


Figure 2:

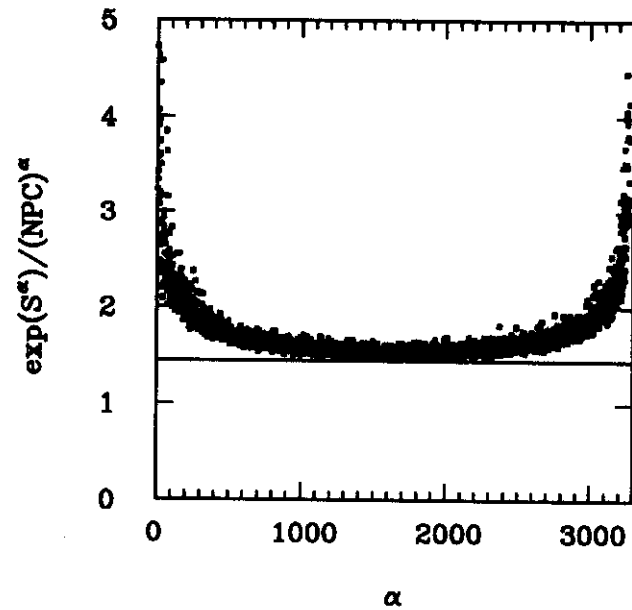
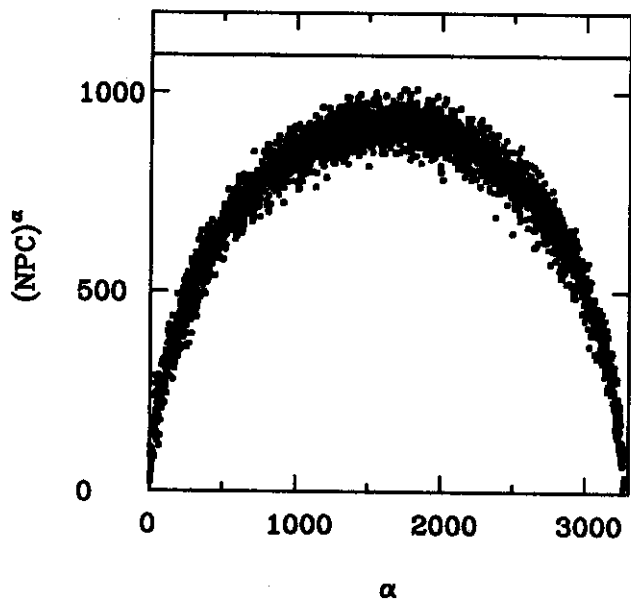


Figure 3: