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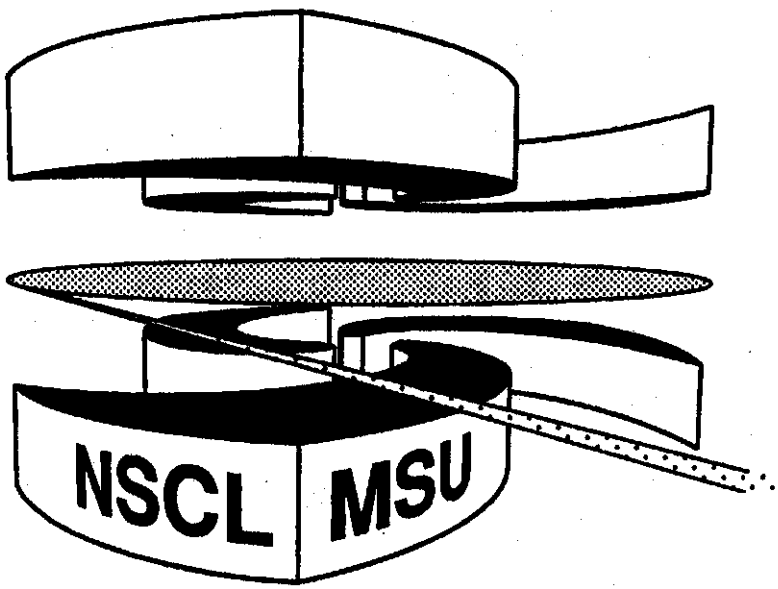

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SCALING OF LARGE FLUCTUATIONS IN FINITE SYSTEMS

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Abstract.

The scaling of critical behavior in molecular and nuclear collisions is discussed employing results from classical and quantum Molecular Dynamics simulations. We have studied collisions of type Xm + Xm (X=nuclei or C cluster, and m=60) at collision energies that lead to fragmentation processes in the MeV and eV range, respectively. The nuclear and molecular systems are scaled according to their radii and binding energies. The critical behavior of the colliding systems is shown in terms of the scaled factorial moments of the cluster mass distribution and the corresponding Campi scatter plot.

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The possibility of the occurrence of a critical behavior related to a liquid-gas phase transition in the fragmentation of hot nuclear systems has been the subject of numerous investigations that study the transition from the liquid-like phase of ordinary nuclear matter (at low excitation energies) to the gaseous phase [1]. Theoretical studies indicate that infinite nuclear matter has an equation of state very similar to that of Van der Waals gas which is characterized by the existence of a liquid-gas phase transition [2]. Experiments on heavy-ion reactions at energies around the Fermi energy have revealed the creation of many fragments in the final stages of the reaction exhibiting a power law in the fragment mass distribution [3]. Such power law is expected for droplet condensation near the critical temperature, indicating a liquid-gas phase transition [4]. A power law behavior in the fragment mass distribution of molecular collisions of fullerenes has been found recently in statistical ensemble quantum Molecular Dynamics (QMD) simulation of $C_{60}^{+} + C_{60}$ collisions at 500 eV total energy [5]. The power law behavior in this investigation has been calculated from an average over all collision impact parameters in the $C_{60}^{+} + C_{60}$ collision simulation. In order to show the scaling of critical behavior we have analyzed the mass fragmentation data of the $C_{60}^{+} + C_{60}$ collisions at fixed impact parameters, and performed collisions of ${}^{60}X + {}^{60}X$ nuclear clusters. The collision energy and impact parameters have been scaled accordingly to ratios of the clusters' collision energy and binding energy, and impact parameter and cluster radius, respectively.

For the interaction of carbon atoms a hybrid of an empirical potential and the density functional linear combination of atomic orbitals in the local density approximation regime by Seifert and Jones [6] has been employed. The potential has been proven to describe the collisions of carbon molecules qualitatively as well as quantitatively [7]. The total kinetic energy per atom in the molecular collision simulation is still low enough such that electronic excitation is not expected to occur [5]. As to the details of the interaction potential and the C_{60} collision simulation the reader is referred to [5] and references therein.

The nuclear cluster collision has been modeled by classical Molecular Dynamics (CMD) [8]. In such an approach the constituent particles ("neutrons and protons") interact through two body Yukawa potentials. Details of the model interaction may be found in Ref. [9]. It is important to stress that for the nuclear cluster collisions the parameters of the force have been chosen such as to give an equation of state for the infinite classical system resembling Van der Waals, and in particular, the predicted nuclear equation of state (of course neglecting the solid phase unavoidable in a classical system).

The hypothesis of this work is that if a finite system retains some features of a phase transition, then the properties of the system near the critical point are independent of the details of the force, and thus different systems may be compared when being scaled appropriately. Thus, we compare our nuclear and molecular models by scaling energies and impact parameters by their binding energies and radii, respectively: $E_r=E_{c.m.}/$ B.E.; $b_r=b/(2R)$. Here, $E_{c.m.}$ is the center of mass energy per particle, B.E. is the (absolute value) of the binding energy of the nuclei (9.5 MeV, without Coulomb) and the respective carbons (9.5eV). R and b are the impact parameter and the radii of projectile and target,

respectively. This is a reasonable ansatz in case the critical temperatures of the two systems are proportional to their respective binding energies. The reduced energy E_r and length scale b_r are defined by the given data set of the molecular cluster collision [5]. Thus, we expect similar collision properties for the nuclear cluster collision at $E_r = 0.45$. In Fig. 1 we plot the normalized fragmentation mass yield at the scaled impact parameter $b_r = 0.32$. Note the fairly good agreement in the probability yield of the two different model systems. An equally good agreement of the fragmentation mass distribution can be found for the larger impact parameters as well.

The results of collision simulations may be discussed in terms of the Fisher's droplet model. In such approach the mass yield is given by

$$Y(A) = Y_o \exp\left[\frac{\mu_g - \mu_l}{T} A - \frac{4\pi r_o \sigma}{T} A^{2/3} - \tau \ln A\right] = Y_o A^{-\tau} Z^A X^{A^{2/3}} \quad (1)$$

where μ_g and μ_l are the gas and liquid chemical potential, respectively, σ is the surface tension of the liquid (σ=0 for $T>T_c$), and τ=2.23 is the curvature term. At $T=T_c$ the surface (X) and volume (Z) term are equal to one, and only the curvature term contributes. The least square fit of Eq. (1) to both model results shows that both systems are very close to the critical temperature ($X_{mol} \approx Z_{mol} \approx X_{nucl} \approx Z_{nucl} \approx 1$) for the central collisions. In order to verify the scaling assumption on an event by event basis we performed a Campi analysis of our data [10]. In the Campi analysis we have plotted the logarithm of the largest fragment (P) versus the logarithm of S_k with k=2, and

$$S_k = M_k / M_1, \quad (2)$$

$$M_k = \sum_A A^k N(A, T - T_c), \quad (3)$$

M_k being the moments of the cluster size distribution, and A and $N(A,T-T_C)$ the cluster mass (size) and multiplicity of cluster size A at fixed distance $T-T_C$ from the critical point. The sum in Eq. (2) is performed over all the fragments but the largest one (P). The Campi plot gives us information on the size fluctuations of the largest fragments on an event by event basis. Near the critical point such fluctuations should be at maximum, i.e., resulting in a large range of S_2 values. In Fig. 2 the Campi plot for the molecular fragmentation is shown at the reduced impact parameter b_r =0.11. We have performed classical nuclear MD calculations at many incident energies in order to show some of the salient features of the Campi plots. In the Fig. we have plotted the results at b_r=0.11 and at c.m. reduced energies of 0.24, 0.45 and 1.05, respectively. For the smallest collision energy we obtain the events concentrated in the left upper corner of the Campi plot. This implies the formation of large fragments with little fluctuation in the fragment size (represented by the small range of S_2). At the medium collision energy, which corresponds to the collision energy of the molecular system, the events result in the largest range of S_2, while at the highest energy small fragments with little fluctuation (small range in S_2) are formed. We like to stress that the 0.45 energy case is the one exhibiting the largest fluctuations in the size distribution among all the energies we have studied in the CMD simulation. The molecular collision events (at the same b_r=0.11) are concentrated at the right corner of the Campi plot, i.e., with large fluctuations. In Fig. 3 we have plotted the fragmentation events

of both collision systems for all studied impact parameters with fixed energy E_r= 0.45. The overlap of fragmentation events and respective fluctuations is astonishing well.

The analysis of the region of maximum fluctuations in Fig. 3 shows that these events are originated in central impact parameters collisions while the fluctuations decrease when going from central to peripheral collisions. Note that the CMD events fill the Campi plot much more for in the case of CMD it has been possible to calculate much more collision events.

In conclusion, we have shown that two completely different finite size systems, a classical one (which has some features of nuclei) in the MeV energy and fm range can be compared to a quantum system (with some molecular features) in the eV and Å range. The comparison has done by appropriately scaling the collision energy and distances by the binding energies and radii of the two systems. This procedure is similar to what is obtained in infinite systems where universal curves can be obtained in the vicinity of the critical point for a liquid gas phase transition. Very large fluctuations are seen for both systems at the same reduced energy, E_r =0.45. If the features discussed in this work will be confirmed by experimental observations both in heavy ion collisions and fullerenes collisions, one may hope to be able to obtain universal curves also for microscopic systems containing as little as 100 particles.

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Figure Caption.

Fig. 1: Mass yield for CMD calculations (circles) and QMD (filled squares) at the reduced energy E_Γ=0.45, and reduced impact parameter b_Γ=0.32.

Fig. 2: Campi plots for QMD (filled squares) at E_Γ=0.45 and CMD at three reduced energies 0.24 (triangles), 0.45 (diamonds) and 1.05 (circles), respectively, calculated at b_Γ=0.11.

Fig. 3: Campi plots at fixed reduced energy E_Γ=0.45 including all impact parameters considered in the QMD (triangles) and CMD (dots) simulations. Note that there is a dense population of CMD events for S_2>2.5 that are not visible in this plot due to overlapping symbols of the QMD events.

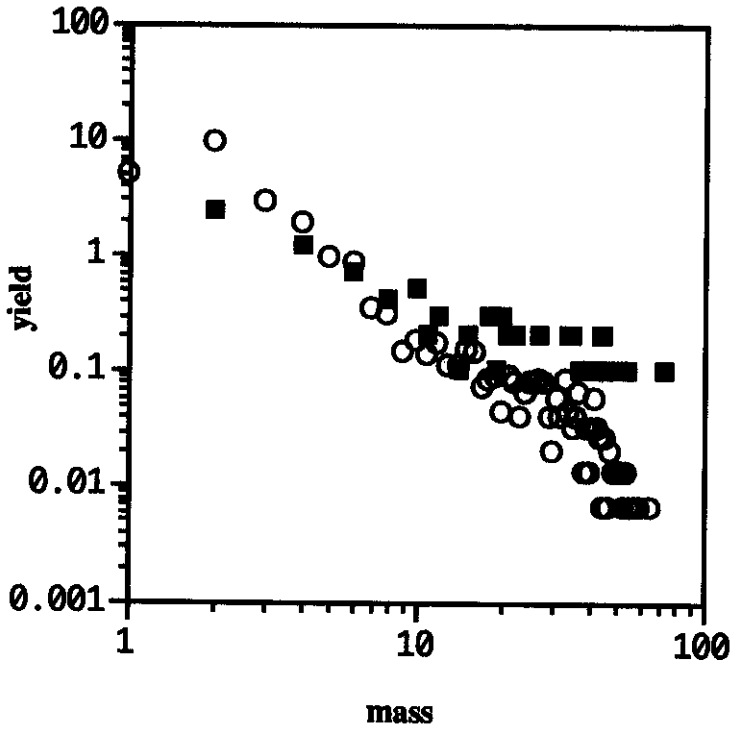


Fig. 1; A. Bonasera, J. Schulte

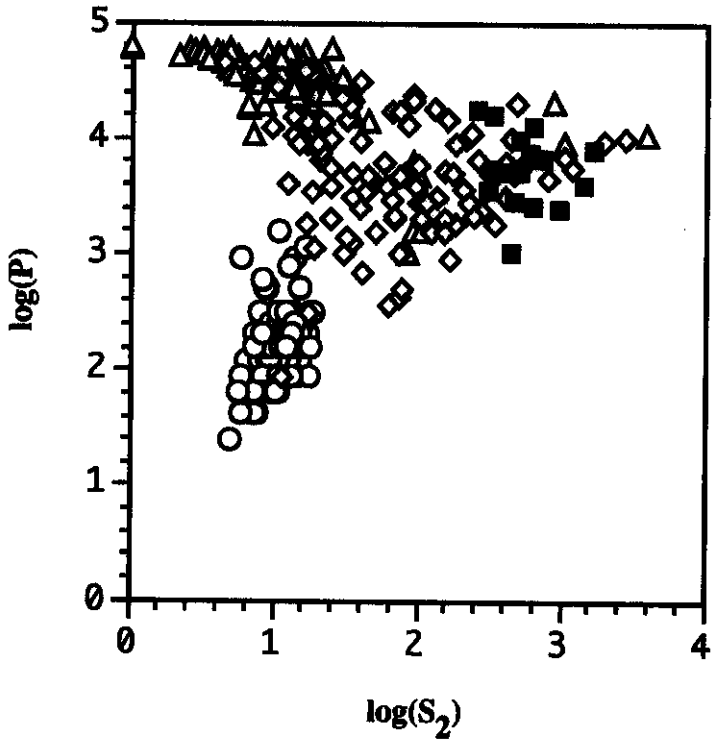
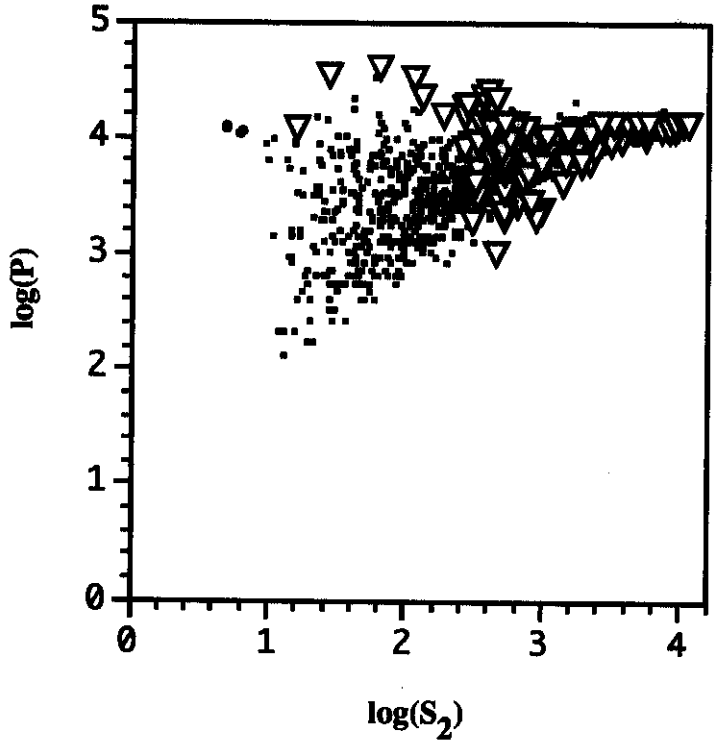


Fig. 2; A. Bonasera, J. Schulte



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