



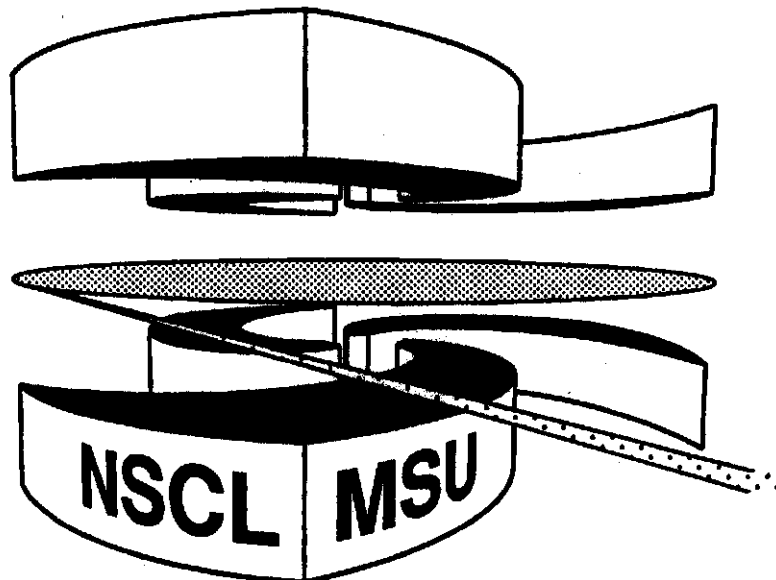
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SHELL-MODEL CALCULATIONS FOR EXOTIC NUCLEI

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**Invited Talk Presented at the International Conference on
Exotic Nuclei and Atomic Masses
Arles, France, June 19-23, 1995**



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Recent shell-model calculations for light nuclei are discussed. The properties of exotic nuclei near the drip lines are emphasized because they provide new and interesting tests of the shell-model predictions and because they are becoming experimentally more accessible. The **full** Op. and Odls spaces are considered and up to $4\hbar\omega$ mixing between them is allowed. The special role of low-lying Odls states in the nuclei ${}^8\text{Be}$, ${}^{12}\text{Be}$, ${}^{10}\text{Li}$ and ${}^{11}\text{Li}$ is examined. In addition, "Super" Gamow-Teller transitions in nuclei up to ${}^{100}\text{Sn}$ are discussed.

1. INTRODUCTION

One of the main advances in the theory of nuclear structure in the last few decades has come from the extension of the shell-model to more complete bases with more accurate effective interactions. ¹⁾ The completeness has been due to our ability to construct and diagonalize Hamiltonian matrices with m-scheme dimensions up to the order of 10^6 and J-scheme dimensions up to the order of 10^5 . This advance has come' from new techniques and computer codes for calculating the matrix elements together with the Lanczos method for efficiently obtaining the low-lying eigenstates for such large dimensions. The accuracy has improved due to a better understanding of the effective one-body and two-body interactions, both at the G matrix level and at the phenomenological level, for specific model spaces and mass regions. More recently, the Monte-Carlo shell-model method has been developed

and applied to the calculation of observables for the ground states of even-even nuclei which effectively incorporate bases with dimensions of up to 10^{10} or more. ²⁾ Future progress with the conventional and Monte-Carlo methods will rely mainly on the continued improvement of the Hamiltonians, and one may eventually hope to obtain a "universal" nuclear Hamiltonian.

In this talk I will focus on some recent work on the shell-model structure of nuclei in the mass region $A=10-20$. This work was initiated in collaboration with Ernie Warburton. At the end I will discuss the role of "super" Gamow-Teller transitions in nuclei up to ^{100}Sn .

2. THE $0p-0d1s$ BASIS FOR THE MASS REGION $A=10-20$

It is well known that the shell-model configurations are a mixture of intrinsic excitations and center-of-mass (spurious) excitations. The lowest $0\hbar\omega$ configurations are particularly simple, since the center of mass is in a $0s$ state. ³⁾ In order to completely separate out the spurious states, the basis must be constructed in a full $N\hbar\omega$ harmonic-oscillator configuration space. For example, the $[0p_{1/2}^{-1}-1s_{1/2}](J=1^-, T=0)$ configuration is partly intrinsic and partly spurious. To remove the spurious state component, the full $0p^{-1}-0d1s$ configuration space must be used. A particularly simple method for removing the spurious component is to add a fictitious Hamiltonian which acts only upon the center-of-mass and whose function is to push up the spurious states to a high excitation energy. ⁴⁾

The investigation of the $0p-0d1s$ model space has proceeded in several steps. In the $(0p)^n$ basis all states which can be constructed from n -particles in the $0p_{3/2}-p_{1/2}$ basis are considered. There are many states in the mass region $A=4-16$ whose structure is known experimentally to be $0p^n$ from which the properties of the $0p$ Hamiltonian can be obtained. In the $(0d1s)^m$ basis all states which can be constructed from m particles in the $0d_{5/2}-0d_{3/2}-1s_{1/2}$ basis are considered. There are many states in the mass region $A=16-40$ whose structure is known experimentally to be $(0d1s)^m$ from which the properties of the $0d1s$ Hamiltonian can be obtained. One can also construct configurations of the form $(0p)^n-(0d1s)^m$ in which both $0p$ and $0d1s$ are active (this means that both are partially full).

By " $0\hbar\omega$ " I will mean the specific $(0p)^n-(0d1s)^m$ configuration which is lowest in energy for a given N and Z value. For example, the ^{16}O $0\hbar\omega$ configuration is $(0p)^{12}$, the ^{11}Be $0\hbar\omega$ configuration is $(0p)^7$, and the ^{16}C $0\hbar\omega$ configuration is $(0p)^{10}-(0d1s)^2$. By " $1\hbar\omega$ " I will mean the configuration in which is higher in energy by a unit of

$1\hbar\omega$ in the harmonic-oscillator model. For example, the ^{16}O $1\hbar\omega$ configuration is $(0p)^{11}-(0d1s)^1$ [or $(0p)^{-1}-(0d1s)^1$], the ^{11}Be $1\hbar\omega$ configuration is $(0p)^6-(0d1s)^1$ and the ^{16}C $1\hbar\omega$ configuration is $(0p)^9-(0d1s)^1$. $N\hbar\omega$ excitations are defined in an analogous manner. For example, the ^{16}O $4\hbar\omega$ configuration is $(0p)^8-(0d1s)^4$. These notations indicate the predominant $0p$ and $0d1s$ parts of the wave functions, but as noted above, other configurations involving the $0s$, $0f1p$ and $0g1d2s$ major shells must also be included in order to eliminate the spurious state. The states with $N\hbar\omega$, $(N+2)\hbar\omega$, etc all have the same parity and in general can mix. I will initially consider only those states which are known experimentally to have rather pure $0\hbar\omega$ and $1\hbar\omega$ configurations in a model which includes only these pure configurations.

Our shell-model wave functions are always described by discrete states as if they were bound in an infinite harmonic-oscillator well. The effects of the finite-potential well become important for higher-excited states in normal nuclei and even for the ground states of nuclei near the drip lines. In the application to these situations one can consider the shell-model wave functions as being applicable in the interior. The interior wave function must be matched onto an exterior finite well or scattering wave function. This is accomplished by considering the one-nucleon parentage of a given state to all core states (in the nucleus with $A-1$). Each of the core states leads to a different part of the extended single-particle wave function. This model is most appropriate when the parentage leads to a unique core state. Its application to the situation where many-core states are important or to those cases where the two-nucleon (or cluster) separation energy is smaller than the single-nucleon separation energy is questionable.

The separation-energy method has been successfully applied to the $E1$ transition between the loosely bound neutron states in ^{11}Be , 5) first-forbidden beta decay, 6) $E2$ transitions and Q moments for loosely bound proton and neutron states $^7, 8, 9$) and to the total-reaction cross sections of nuclei out to the drip lines. $^{10, 11, 12}$)

The effective interaction which is obtained from experimental energies implicitly takes into account the model-space truncation and finite potential well effects. The main assumption is that these effects are uniformly spread over the entire mass region under consideration. At some level this must be an inadequate description, and a binding-energy dependent interaction should be considered. As the wave function becomes more loosely bound, the residual two-body interaction decreases. However, the density-dependence of the effective interaction 13) tends to increase the interaction for loosely bound states (which overlap less with the core density)

and hence to cancel part of the decrease from the geometrical effect.

3. CROSS-SHELL INTERACTIONS FOR PURE $N\hbar\omega$ CONFIGURATIONS

Ernie Warburton and I determined the $0p$ - $0d1s$ cross-shell interaction using both the model-independent (MI) and one-boson exchange potential (OBEP) methods.¹⁴⁾ The interactions derived from a fit to 165 cross-shell energy-level data are denoted by WBT and WBP, respectively.¹⁵⁾ The rms deviation between the calculated and experimental energy level data in the mass region $A=10$ - 22 was about 330 keV. With the OBEP method, harmonic-oscillator radial wave functions were used to calculate the two-body matrix elements. In both cases the states are assumed to be described by pure $N\hbar\omega$ configurations, and only those data where the level is known to be rather pure were considered. The MI method was developed by Wildenthal for the $0d1s$ shell.¹⁶⁾ The OBEP method has also been successfully used for the $0d1s$ shell¹⁶⁾ and our application of it for the $0p$ - $0d1s$ interaction is similar in spirit to the work of Millener and Kurath¹⁷⁾ for this mass region.

When the WBT or WBP interactions are used to calculate the energies of the pure 0^+ , $T=0$ configurations in ^{16}O , the excitation energies of the $2\hbar\omega$ and $4\hbar\omega$ configurations come out at about 9 MeV and 6 MeV, respectively, in reasonable agreement with where these states are experimentally observed in cluster transfer reactions. This is also the case for other nuclei in this mass region. This agreement is perhaps surprising, but can be understood in terms of the weak-coupling models developed many years ago by Arima and others.^{18, 19)}

4. INTRUDER STATES IN ^{11}Be and ^{10}Li

The ground state of ^{11}Be with $J^+ = 1/2^+$ is one of the outstanding exceptions to the simplest shell-model picture. It is an example of an "intruder" $1\hbar\omega$ configuration which lies below the normal $0\hbar\omega$ configuration which with $J^+ = 1/2^-$. Our new $1\hbar\omega$ calculations which by construction are able to reproduce this inversion¹⁵⁾ provide an opportunity²⁰⁾ for a new quantitative understanding of this feature which has long been of theoretical interest.²¹⁾ One can account for the energy gap between the $1/2^+$ ground state and $1/2^-$ excited state at 0.32 MeV in terms of three distinct physical contributions: (i) the monopole energy gap between the $0p$ and $0d1s$ major shells, (ii) the pairing interaction, and (iii) the proton-neutron interaction. In ^{16}O the energy gap between the $0p_{1/2}$ and $1s_{1/2}$ single-particle states is about 6.0 MeV.

When we examine the change in this gap with our interaction as a function of proton number between ^{16}O and ^{10}He , we find a monotonic decrease down to about 2.4 MeV in ^{10}He . The effective single-particle energy gap for ^{11}Be is about 3.6 MeV. This change comes out automatically from our fitted interaction, but it can also be understood as a consequence of the decrease in spacing between levels as they become more loosely bound in a finite well.²⁰⁾

There are two configuration mixing (correlation) effects which lower the energy of the $1/2^+$ configuration in ^{11}Be . There is an extra pairing energy in the $1/2^+$ configuration due to the two neutron holes in the $0p$ shell, which lowers the $1/2^+$ configuration (relative to the $1/2^-$ configuration) by about 2.2 MeV. Also, there is mixing with the $[2^+ \otimes d_{5/2}](1/2^+)$ configuration via the deforming Q-Q interaction, which lowers the energy by about 1.5 MeV. Adding up (i), (ii) and (iii), the $1/2^+$ and $1/2^-$ configurations become essentially degenerate. This degeneracy appears to be due to an accidental cancellation of several effects and not fundamental – but this aspect would be interesting to explore further.

The interplay of the three mechanisms discussed above for the parity-inversion in ^{11}Be are responsible for similar phenomena in other nuclei. In particular, recent investigations of resonances in ^{10}Li have suggested the presence of a narrow low-lying s-wave.²²⁾ Barker and Hickey have suggested that there may be a parity inversion in ^{10}Li , and our WBT and WBP interactions also predict a parity inversion in ^{10}Li . In detail, the calculated $(0+1)\hbar\omega$ spectrum of ^{10}Li with the WBT interaction is $2^-(\text{gs})$, $1^+(0.10 \text{ MeV})$, $2^+(0.32 \text{ MeV})$, $1^-(1.30 \text{ MeV})$, $0^-(1.93 \text{ MeV})$ with eleven more levels up to 4 MeV. Experimental confirmation is important not only from the point of view of the unusual narrowness of s-wave cross section near threshold, but also for the structure of ^{11}Li .

5. MIXED CROSS-SHELL CONFIGURATIONS

It is important to study the mixing of $N\hbar\omega$ configurations. As a prototype, we can consider the mixing of the 0, 2 and $4\hbar\omega$ configurations for ^{16}O . When our WBT interaction¹⁵⁾ is used, the mixing between the $N\hbar\omega$ configurations is very large.²³⁾ The state which is predominantly $2\hbar\omega$ comes out at about the right excitation energy, however, the state which is predominantly $4\hbar\omega$ comes out about 11 MeV too high compared to the well-known state at 6 MeV. This is understood from the fact that the mixed ground state is pushed down by about 11 MeV relative to the pure $0\hbar\omega$ energy by the admixture of the 2 and $4\hbar\omega$ configurations. However, the comparable

6 and 8 $\hbar\omega$ configurations which are needed to push down predominantly $4\hbar\omega$ states are not present in the model space. It is the SU3(20) component of the off-diagonal $\Delta\hbar\omega=2\hbar\omega$ interaction which is responsible for this shift. We have proposed²³⁾ a simple “shift” method to take into account this truncation. This shift can also be regarded in some approximation as giving rise to an effective gap reduction between the 0p and 0d1s shells as used by Haxton and Johnson.²⁴⁾

Another problem in the mixed $N\hbar\omega$ model space is the effective interaction. It is really not appropriate to use our WBT or WBP interactions in the mixed space because they were derived from wave functions in the pure $N\hbar\omega$ model space. Its use in the mixed space results in some double counting. Others have used the bare G matrix in the mixed space.²⁴⁾ An effective interaction has been obtained from a fit to data in an $(0+2)\hbar\omega$ model space,²⁵⁾ however, inconsistencies in the methodology have been pointed out.²⁶⁾ The derivation of a good effective interaction for the mixed $N\hbar\omega$ model space in light nuclei will be an important future project.

When the WBT interaction is applied to the pure $N\hbar\omega$ configurations of the $N=8$ isotones, the pure $1\hbar\omega$ excitations always lie several MeV above the ground state – there is no parity inversion. However, it is remarkable to find that the $2\hbar\omega$ excitations which start out at 9 MeV in ^{16}O quickly come down in energy (8 MeV in ^{15}N , 5 MeV in ^{14}C and 3 MeV in ^{13}B) until they become essentially degenerate with the $0\hbar\omega$ configuration in ^{12}Be , ^{11}Li and ^{10}He . When the $0\hbar\omega$ and $2\hbar\omega$ configurations are allowed to mix in these three nuclei, there will be two states each with about 50-50 admixture of $0\hbar\omega$ and $2\hbar\omega$. There are, in contrast, shell-model calculations for ^{11}Li in which the $2\hbar\omega$ component is small²⁷⁾ or absent,²⁸⁾ due to the fact that the “shift” problem described above was not taken into account or the 0d1s shell was ignored.

Barker²⁹⁾ has pointed out the possibility for a mixed $(0+2)\hbar\omega$ configuration for the ^{12}Be ground state. One of the most direct radioactive beam experiments which would test the structure of the ^{12}Be ground state is the pick-up of one neutron leading to the parity doublet in ^{11}Be (with a resolution capable of separating the 0.32 MeV doublet). A pure $0\hbar\omega$ configuration would lead only to the $1/2^-$ final state, whereas the mixed configuration will also lead to the $1/2^+$ final state (the calculated spectroscopic factor is about 0.4). A better experimental and theoretical understanding of the excited states of ^{12}Be is needed.

The same mechanisms which produce the parity inversion and intruder states in the ^{11}Li region are especially important for the region of ^{32}Mg where the $2\hbar\omega$ intruder states lie several MeV below the $0\hbar\omega$ energies, giving rise to an “island of

inversion." The calculations are well documented in the literature^{30, 31, 32} – perhaps theory is slightly ahead of experiment in this region. This region will be interesting to explore further with new radioactive beam experiments.

It would also be interesting to look for drip-line nuclei which are loosely bound to one- or two-neutron decay, but whose shell-model structure is simpler – such as the neutron-rich C isotopes.³³ ²⁶O is an interesting case where the 0d_{1s} shell calculations predict a very small (but not clearly bound or unbound) two-neutron separation energy.^{34, 35}

6. SUPER GAMOW-TELLER TRANSITIONS

The decay rate for allowed beta decay is given by $ft_{1/2} = 6170/[(g_A/g_V)^2 B(GT) + B(F)]$ where f is the phase-space factor, $t_{1/2}$ is the partial half-life for the decay to a specific final state, $B(GT)$ is the reduced Gamow-Teller transition rate between a specific initial state and a specific final state, $B(GT_{-/+}) = |\langle f | \sum_k \sigma^k t_{+/-}^k | i \rangle|^2 / (2J_i + 1)$, and $B(F)$ is the Fermi decay rate to the isobaric-analogue state. The g_A/g_V is the ratio of the axial-vector to vector coupling constants for the nucleon as obtained from the neutron beta decay. $GT_{-/+}$ corresponds to β^-/β^+ decay with the nucleon isospin raising and lowering operators t_+ and t_- , respectively. The sum rule for the Gamow-Teller transition probabilities is $\sum_f B(GT_-) - \sum_f B(GT_+) = 3(N - Z)$. There are many cases where for nuclear structure reasons either $B(GT_-)$ or $B(GT_+)$ is small (or zero). For example, for the neutron decay $B(GT_+) = 0$ and $B(GT_-) = 3$. For the GT_- transition from ²⁰⁸Pb, which has been studied with the intermediate energy (p,n) reaction, the GT_+ should be small (because of the neutron excess) and hence $\sum_f B(GT_-) \approx 132$. However, for ²⁰⁸Pb as well as most other cases observed in nuclei, the total Gamow-Teller strength is fragmented over many final states, hence the GT strength to any specific final state is small.

In fact, there are only two transitions to specific final states observed so far which are larger than the neutron value of 3. They are 0^+ , $T=1$, ⁶He to 1^+ , $T=0$, ⁶Li decay with $B(GT_-) = 4.72$ ³⁶ and the 0^+ , $T=1$, ¹⁸Ne to 1^+ , $T=0$, ¹⁸F decay with $B(GT_+) = 3.15$.³⁷ In both cases the sum-rule values in the $0\hbar\omega$ shell-model are six and the reduction from this value is due to the higher-order correlations beyond $0\hbar\omega$ which give rise to the well-known quenching of GT strength.³⁷ Also, in both cases, the final states are ground states and they come low in energy because of the attractive particle-particle interaction. As one moves away from the two-particle valence case and adds more valence particles, the strong GT strength moves up in

energy and eventually becomes a “particle-hole” state in the middle of the shell which is pushed up by the residual particle-hole interaction. The high energy usually results in a fragmentation of strength due to mixing with 2p-2h configurations. Borge et al.³⁸⁾ have presented the case for “Super” GT strength in the decays of ^8He , ^9Li and ^{11}Li . However, since the final states lie at a high excitation energy, the strength is probably fragmented over many final states. Op shell calculations for the ^9Li and ^{11}Li decays show this fragmentation.³⁹⁾ Op shell calculations³⁶⁾ for the ^8He decay predict a strength of 7.7 for a state at about 9 MeV in excitation, but the experiment is difficult to interpret because of the large width of the final state.

Where should we look for other Super Gamow-Teller strength? I believe there are two candidates – ^{56}Ni and ^{100}Sn . ^{56}Ni is known to decay to a low-lying final state in ^{56}Co , but the Q value is very small and this particular final state has a very small B(GT). We have predicted⁴⁰⁾ a Super GT to a level just above the Q value with $B(\text{GT}) \approx 5.5$ which may be studied via the inverse reaction $p(^{56}\text{Ni}, ^{56}\text{Co})n$. Also, we have predicted⁴¹⁾ that ^{100}Sn should beta decay by a Super GT to a low-lying state in ^{100}In with $B(\text{GT}) \approx 8.5$.

The GT strength of ^{56}Ni and ^{100}Sn are both examples of a general class of transitions in nuclei with $N=Z$.⁴²⁾ The $N=Z$ nuclei are interesting because the GT sum rule only gives $\sum_j B(\text{GT}_-) = \sum_j B(\text{GT}_+)$; a result of isospin symmetry. There are thus several equivalent ways to obtain the B(GT) values in $N=Z$ nuclei; β^+ decay (where energetically allowed) or (n,p) reactions, β^- decay or (p,n) reactions, or (p,p') reactions. In many cases more than one of these has been observed and compared. As one approaches ^{100}Sn , the beta decay Q values become larger due to the larger Coulomb displacement energy, hence much more of the GT strength can be observed in β decay. Data for the interesting region between ^{56}Ni and ^{100}Sn will rely upon future radioactive beam experiments.

The $\sum_j B(\text{GT})$ are thus completely model dependent and turn out to be extremely sensitive to nuclear correlations. One observes in the middle of the Op and Od1s shells, that the experimental strength is very small compared to the extreme single-particle model (ESP) – for example in ^{28}Si , the experimental GT strength is only about 1/5 of the ESP value. That is, the ^{28}Si experiment is hindered by a factor of 5 compared to ESP. The largest part of this hindrance can be found in the full $0\hbar\omega$ shell-model calculations. But experiment is hindered even with regard to the full $0\hbar\omega$ calculations³⁷⁾ – by about a factor of $1/0.6 = 1.67$. From comparison of M1 and GT matrix elements one can deduce that about two-thirds (in the amplitude) of this comes from higher-order ($>0\hbar\omega$) configuration mixing while one-third comes

from the delta-particle nucleon-hole admixture.⁴³⁾

Calculations for ^{56}Ni ⁴⁰⁾ and ^{100}Sn ⁴¹⁾ at the level of 2p-2h correlations indicate that GT strength in these nuclei should be strong and concentrated into single low-lying final states. The strength is due to the fact that the orbital with $j=\ell+1/2$ is completely filled and the orbital for $j=\ell-1/2$ is completely empty (in the extreme single-particle model). The results for ^{100}Sn are particularly interesting. We predict⁴¹⁾ a Q value of 7.0 MeV for the decay to a final state at an excitation energy of 1.8 MeV with $B(\text{GT})\approx 8.5$ and $T_{1/2}\approx 0.5$ s. The final state is low enough above the proton decay threshold that it should decay entirely by gamma emission. The first experimental indications⁴⁴⁾ are in agreement with the prediction, but a much more accurate experiment with gamma coincidence will be required to determine whether or not this is indeed an example of a Super GT transition. It is important to understand the origin of quenching in heavier nuclei. Only recently²⁾ has it become possible to calculate the total GT strength in a full $0f1p$ shell basis for nuclei such as ^{56}Ni , and we will soon have a better understanding of the role of correlations beyond the 2p-2h (RPA) level.

This work was supported by the US National Science Foundation under grant number PHY-94-03666.

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