MICHIGAN STATE UNIVERSITY

CYCLOTRON PROJECT

Ion Injection in a Cyclotron with Double-Mode Dee System

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February 1963

Department of Physics

East Lansing, Michigan

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ABSTRACT

For the MSU cyclotron a double-mode dee system is being planned with two 144° dees and two dummy dees operating for first- and third-harmonic acceleration in the push-pull mode and for second-harmonic acceleration in the push-push mode. The characteristics and possibilities of such a system with respect to ion injection and central-region layout are investigated. In particular two injection schemes are studied: (a) a normal-injection scheme in which ion source and puller are readjusted radially and azimuthally when the mode of operation is being changed to optimize starting conditions and beam quality; (b) a d.c.-injection scheme where the ion source is on a high d.c. potential and the ions are injected at a fixed point in all 3 modes of operation. The computations show that in the normal-injection scheme very favorable conditions with respect to beam quality and ion-capture efficiency can be achieved for all modes of operation. In the d.c.-injection case only the results for the first-harmonic mode are favorable while the situation in the cases of second- and especially third-harmonic acceleration is extremely unsatisfactory (poor beam quality and low ion-capture efficiency).

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1. Introduction.

The initial proposal for the MSU cyclotron included a r.f. system with two dees operating in the push-pull mode for first- and third-harmonic acceleration with a frequency range of 15 to 21 Mc/s. Since such a system does not accelerate particles with orbital frequencies between 7 and 15 Mc/s, it was decided later on to investigate a double-mode system which, in the push-push mode, allows second-harmonic acceleration of ions with orbital frequencies between 7.5 and 10.5 Mc/s and so covers most of this undesirable gap in the frequency range.

In a double-mode system the dee angle must be smaller than 180° and dummy dees in between are necessary to prevent overlapping and neutralization of the electric fields in the push-push mode. While the great advantages of additional second-harmonic acceleration are obvious, ion injection and design of central geometry are causing some problems. To program the useful part of the beam, ion source and puller positions have to be changed when turning from one mode of operation to another and the presence of dummy dees, which must extend into the very center of the machine, imposes geometric limitations for these readjustments. Therefore a careful analysis of injection possibilities and central orbits was made using various computer programs worked out for the MISTIC, MSU's present computer facility. In this report the basic considerations for the layout of central geometry and approximate calculations of motion in the median plane with the "Uniform Field Code" are discussed. On the

basis of these results a mock-up of the center was built for electrolytic-tank measurements, and more accurate numeric calculations with experimental field data are to give the final optimum arrangement. These investigations as well as computations of vertical motion will be presented and discussed in other reports.

In connection with the problem of getting larger duty cycles, some people suggested that injection of preaccelerated ions into the cyclotron might give a remarkable improvement. Since this question is of great interest it was decided to study also the possibility of d.c. injection and compare the results with a normal injection scheme.

2. Characteristics of the Double-Mode Dee System.

The possibilities of accelerating particles on different harmonics in a cyclotron with a double-mode dee system were investigated theoretically by Jacobson and Smith.[1] Dummy dees are necessary in such a system to separate the electric fields which, in the push-push mode, would otherwise overlap and mutually cancel. The maximum energy gain per turn ΔE_{kmax}, assuming infinitesimal gap width, is given by the formula

$$(1) \qquad \Delta E_{kmax} = 4eU_0 \left| \sin\left(N \frac{\beta}{2}\right) \right|,$$

where e is the electric charge of the ions, U_0 the peak dee voltage, $N = \frac{\omega_e}{\omega}$ the ratio of electric frequency to orbital

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1) Phys. Rev. 93 (1954), 303.

frequency and β the dee angle ($\pi-\beta$ = angle of dummy dee). Fig. 1 shows the relative maximum energy gain per turn $\Delta E_{kmax}/4eU_o$ as a function of dee angle β for different harmonic numbers N. It is seen that, for every number N, there are dee angles where no acceleration occurs. The most important cases are $\beta=180^o$ for N=2 and $\beta=120^o$ for N=3, and since second- and third-harmonic acceleration are essential, the dee angle must be smaller than 180^o and either smaller or larger than 120^o.

As mentioned, in the central region of a double-mode dee system there are some difficulties which are mainly caused by geometric limitations. The ideal r.f. phase θ_m where the particles must cross the acceleration gap to get the maximum energy gain per turn is for first- and third-harmonic acceleration $-N\frac{\pi-\beta}{2}$ when the ions enter the dummy dee and $+N\frac{\pi-\beta}{2}$ when they leave the dummy dee (i.e., the ions cross the center line of the dummy dees at the peak-voltage phase $\theta=0^o$).

In the second-harmonic acceleration mode (push-push) the ions enter the dummy dees at the phase $\theta_m = \beta-\frac{\pi}{2}$ and leave them at $\theta_m = -\left(\beta-\frac{\pi}{2}\right)$; i.e., they cross the center lines when the voltage goes through zero. The starting phase θ_o, on the other side, where the useful group of ions leave the ion source to enter the dee depends on the operating conditions, especially the peak voltage U_o, and is in all cases negative. As a result of this, to obtain the proper phase for the gap crossings, the ion source must be positioned at different azimuths for the three modes of operation, the puller forming a long protrusion of the dee which can be pulled back and forth.

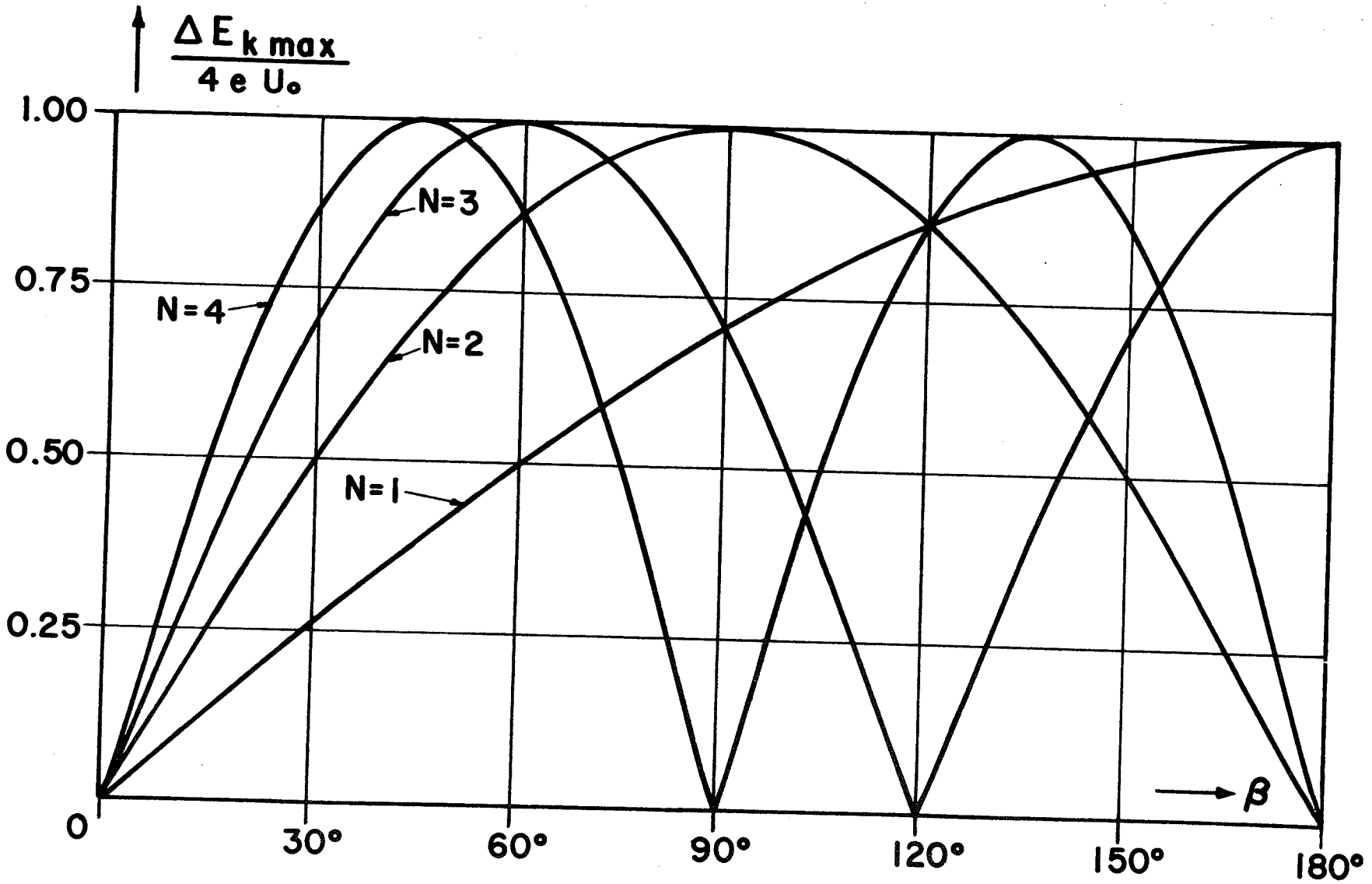

Fig. 1: Maximum energy gain per turn in a double-mode dee system as a function of dee angle β for different modes of operation (N odd: push-pull mode, N even: push-push mode).

This situation is illustrated schematically in Fig. 2, where in the 3 modes of operation the phase intervals are plotted, at which the ion beam crosses the acceleration gaps. These intervals are centered at the favorable starting phase in the first gap and at the phase of maximum energy gain θ_m in the subsequent gap crossings. Taking the N=1 case, one can see that the favorable starting phase 1 is negative, i.e., displaced by a large interval from the position 1 where it should be. Therefore a phase shift is required to bring the beam to the proper phase position in the following gap crossings. A similar situation exists in the N=3 case, while in the push-push mode (N=2) the favorable starting phase is very close to the point where it should be.

This necessity for large readjustments, together with the necessary presence of dummy dees at the first turn already, forbid any large radial displacements of source and puller, and, consequently, only a constant-orbit program[2] can be realized. Some difficulties for a constant-orbit program, arising from the fact that the voltage (and energy gain) in the first gap is different from the effective voltage accelerating the ions in the following gap crossings, will be discussed in the next section.

Another important characteristic of the double-mode dee system, in contrast to the conventional 180° system, is that the vertical electric field is alternatingly focusing and defocusing since the phase of gap crossing changes alternately

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2) M. Reiser, Nucl. Instr. and Meth. 18, 19 (1962), 370.

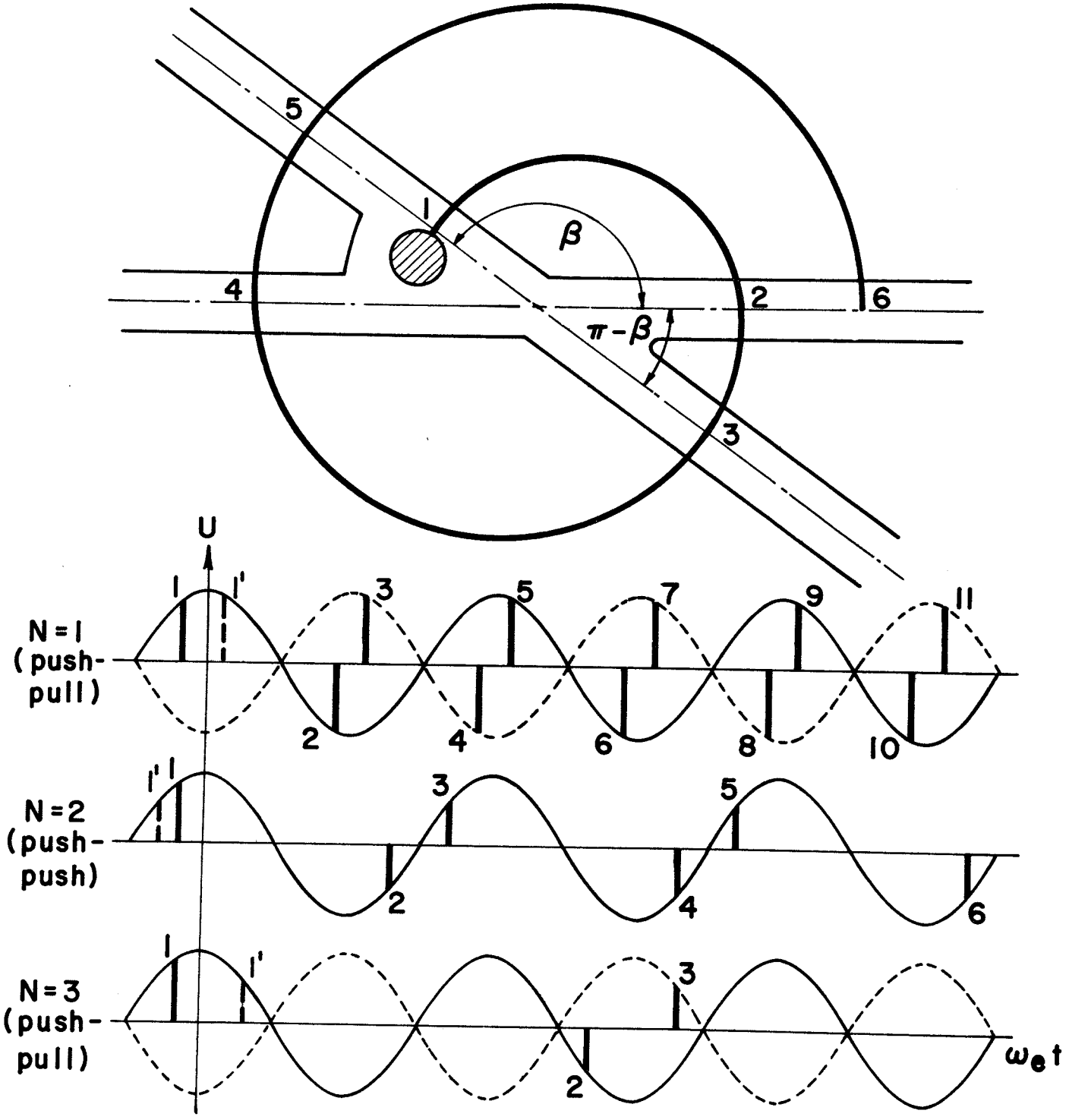

Fig. 2: Favorable starting phase (1) and gap-crossing phases for maximum energy gain per turn (1', 2, 3, 4, etc.) in a double-mode dee system for different modes of operation.

between positive and negative values. This fact may eventually cause overfocusing if the particles, in the first focusing gap, experience a very large focusing momentum (strong crossover) which cannot be fully counterbalanced in subsequent gaps since the electric-lens effect decreases rapidly with energy.

3. Conditions for Constant Orbit Geometry and Determination of Dee Angle.

The condition for constant orbit geometry, neglecting transit time effects in the accelerating gaps and relativistic effects, is[2)]

$$\frac{B^2 e/m}{U} = \text{const} \quad , \tag{2}$$

where B is the central magnetic field in the cyclotron, e/m the specific ion charge and U the voltage representing the effective energy gain across the dee gap. From (2) it follows, that to vary the final energy of a particular group of ions (e/m = const) without changing the orbit geometry, the voltage U must be changed with the square of the magnetic field B. Likewise, changing the mode of operation, i.e., turning from one group of particles to another with different e/m at the same magnetic field, requires that the voltage must be varied as the ratio of the specific charges if the orbit geometry is to be maintained. If the particles with first-harmonic acceleration are taken as the reference case, the ratio of the

specific charges is simply the harmonic number N (since B was assumed constant) and if U_1 is the voltage for the first-harmonic mode, U_N the voltage for the N-th harmonic mode of operation, the condition for constant orbit geometry can be written in the form

$$(3) \qquad \frac{U_1}{U_N} = N \qquad \text{or} \qquad U_N = \frac{U_1}{N} .$$

Thus, turning from first- to third-harmonic acceleration, means that the voltage has to be decreased by a factor of 3.

In a double-mode dee system, as is considered here, the voltage corresponding with the effective energy gain rather than the peak value must be taken. From equation (1) this effective voltage, introducing the index N, is

$$(4) \qquad U_N = U_{ON} \left| \sin \left(N \frac{\beta}{2} \right) \right| ,$$

where U_{ON} means the peak voltage for the N-th mode of operation. The condition for constant orbit geometry in a double-mode dee system, regarding the peak voltage, is then

$$(5) \qquad U_{ON} = \frac{U_{O1} \left| \sin \frac{\beta}{2} \right|}{N \left| \sin \left(N \frac{\beta}{2} \right) \right|}$$

with U_{O1} being the peak value for first-harmonic acceleration. Condition (5) would secure uniform radius gain per turn in all modes of operation, except for the first gap (between ion source and puller), where, as mentioned in the previous section, the peak voltage U_{ON} rather than the effective voltage U_N determines the energy gain and the corresponding first radius of curvature r_{1N}. In rough approximation one can use

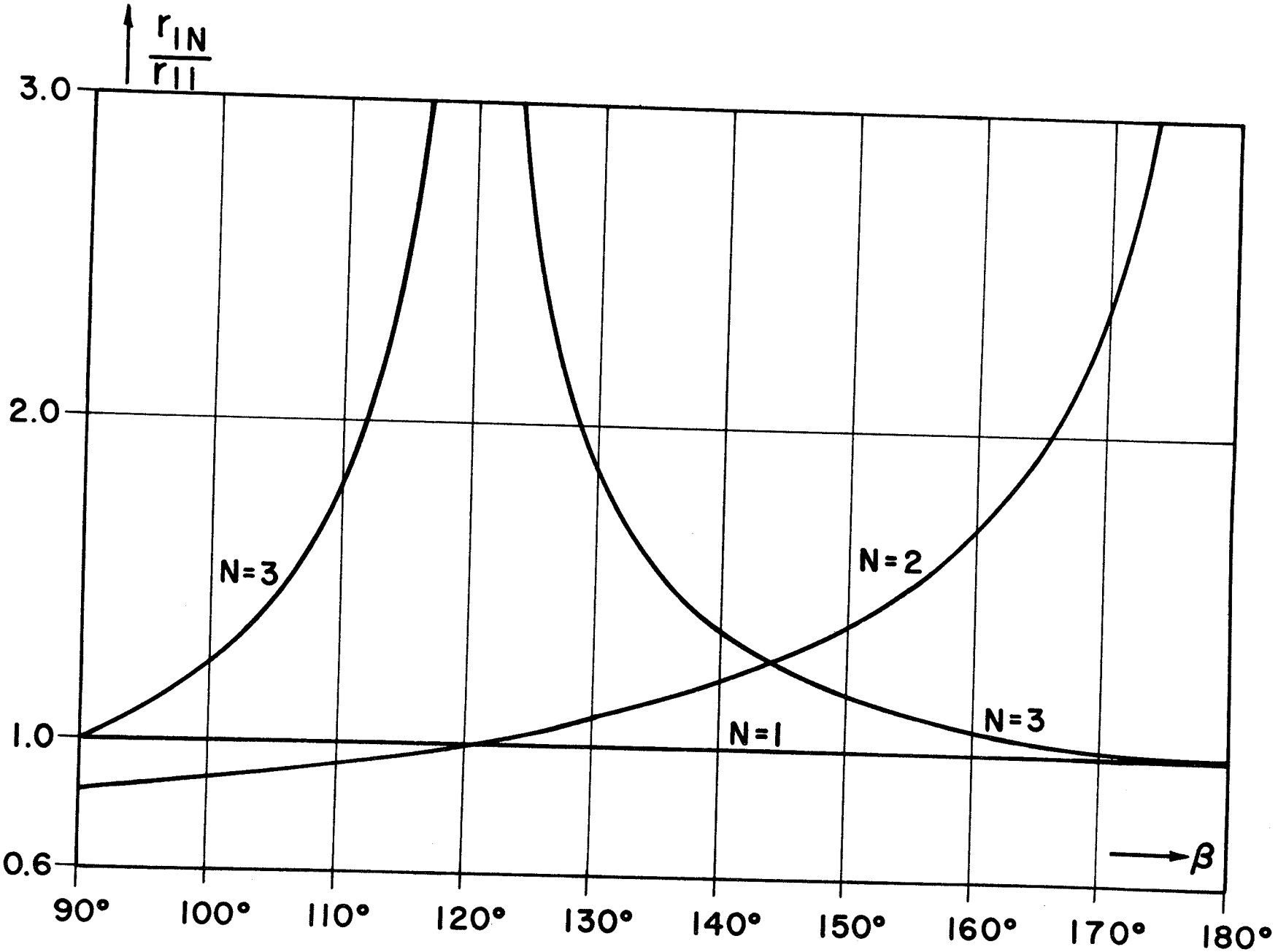

Fig. 3: Radius of curvature after first gap traversal as a function of dee angle β in a "constant-orbit-geometry" program.

the formula

$$(6) \qquad r_{1N} = \frac{C}{B}\sqrt{2\left(\frac{m}{e}\right)_N U_{ON}} \,,$$

where C is a factor smaller than 1 representing a correction due to starting phase and transit time of the useful particles. Remembering that $\left(\frac{m}{e}\right)_N = N\left(\frac{m}{e}\right)_{N=1}$ and using formula (5) for the peak voltage U_{ON} we obtain

$$(7) \qquad r_{1N} = \frac{C}{B}\sqrt{2\left(\frac{m}{e}\right)_1 U_{01}\,\frac{\left|\sin\frac{\beta}{2}\right|}{\left|\sin\left(N\frac{\beta}{2}\right)\right|}} \,.$$

Assuming that, within this rough estimate, the factor C is constant for all modes of operation*), one can introduce the radius r_{11} for first-harmonic acceleration and finally obtains

$$(8) \qquad r_{1N} = r_{11}\sqrt{\frac{\left|\sin\frac{\beta}{2}\right|}{\left|\sin\left(N\frac{\beta}{2}\right)\right|}} \,.$$

Thus the first orbit radius is seen to depend on the harmonic number N and the dee angle β. Fig. 3 gives an idea of how the radii for second- and third-harmonic operation differ from the radius of first-harmonic acceleration if the dee angle β takes values between 90° and 180°. This graph indicates clearly that, despite the fact that condition (5) is satisfied, an exact constant orbit geometry cannot be achieved. Nevertheless, as the figure shows, it is possible to minimize the differences in radii and consequently the radial displacements of source and puller, if one chooses a dee angle where the radius for

*) This can be achieved to a certain degree by changing the width of the first gap.

N=2 is the same as for N=3. If r_{12} and r_{13} are to be equal one obtains from (8) the equation

$$(9) \qquad \left|\sin \beta\right| = \left|\sin\left(\frac{3}{2}\beta\right)\right| ,$$

which condition is satisfied by $\beta = 144^{\circ}$. Therefore a value of 144° was established for the dee angle, and all further calculations are based upon this special angle.

4. Normal Injection

4.1 Approximate Determination of Favorable Starting Conditions.

A method to determine approximately the favorable starting conditions has been discussed in reference 2 for a 180° dee system. It is quite valuable for either designing the experimental layout of the central geometry without detailed theoretical analysis or for quickly finding a first set of initial conditions for more exhaustive computer runs. The application of this method to a dee system of different geometry is straightforward, and in the following the formulas for the double-mode system shall be derived. The main difference to the 180°-dee case is that in a 4-gap configuration the instantaneous orbit centers (after each gap traversal) no longer move along a straight line but in a rhomboidal way, the individual center displacements occurring parallel to the gap symmetry lines.

A possible geometric arrangement in the center of a double-mode cyclotron is outlined in Fig. 4, showing a

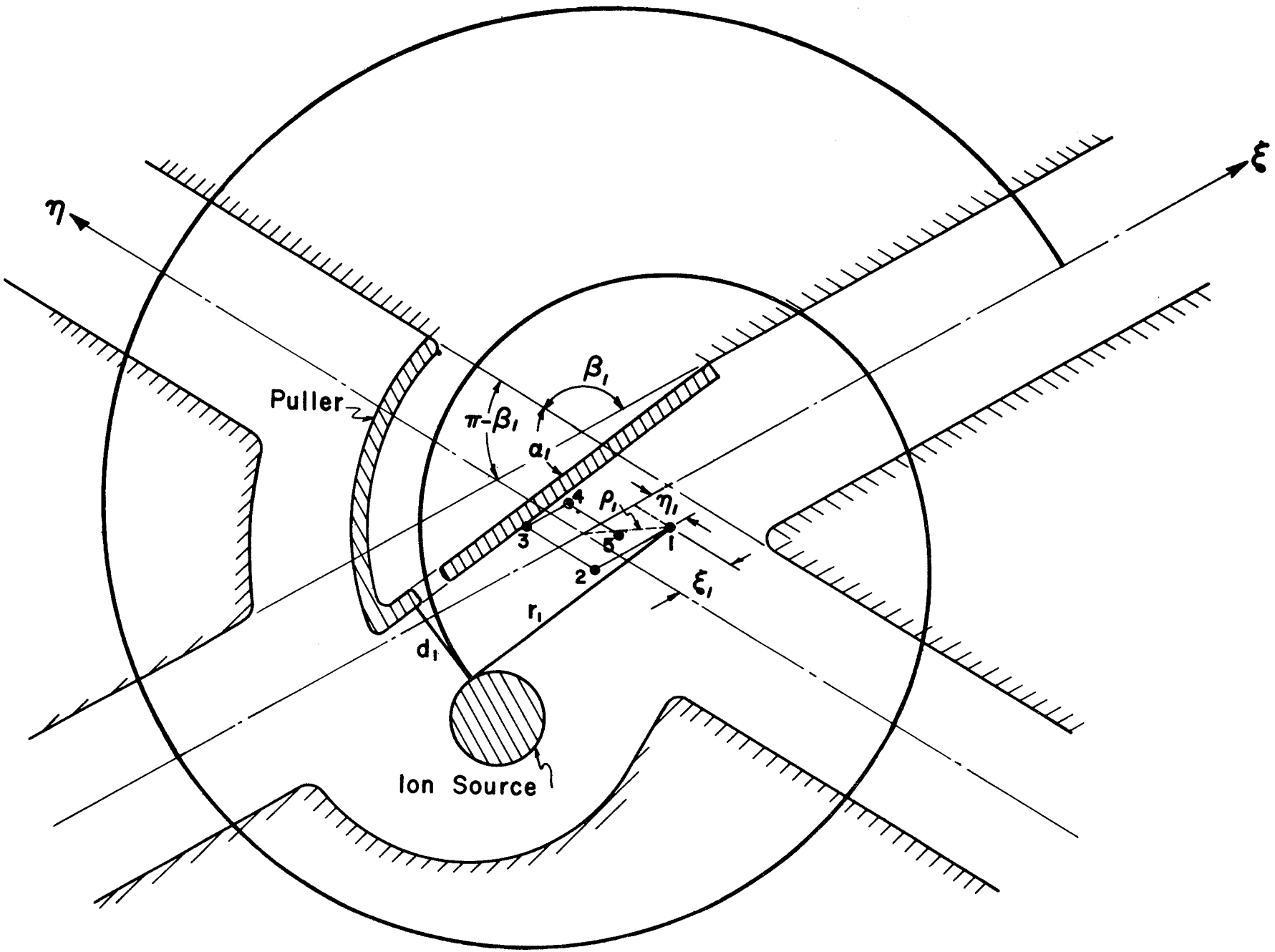

Fig. 4: Central arrangement and geometric parameters in a double-mode dee system.

particle trajectory with corresponding orbit centers. In order to center the orbits after many revolutions, one must displace the first orbit center (after traversal of the source-to-puller gap) from the cyclotron center by a distance ρ_1; the coordinates of this point in the oblique frame formed by the gap symmetry lines are denoted by ξ_1 and η_1.

The puller-to-dee angle α_1 necessary to shift the phase of the particles to a desired value θ_{m2} at the following gap center line can be calculated in similar manner as in reference 2. Let θ_o denote the starting phase at the source, τ_d the transit angle in the first gap and d_1 the gap width. Using some trigonometric relations which can be established from Fig. 4, the following formulas for α_1 can be derived:

$$\alpha_1 = \pi - \beta_1 + \frac{d_1}{r_1} + \frac{\theta_{m2} - \theta_o - \tau_d}{N} + \frac{|\eta_1| \cos\left(\beta - \frac{\pi}{2}\right)}{r_1} \tag{10}$$

for the push-pull mode (N=1, 3, 5, etc.), and

$$\alpha_1 = \frac{\pi}{N} - \beta_1 + \frac{d_1}{r_1} + \frac{\theta_{m2} - \theta_o - \tau_d}{N} + \frac{|\eta_1| \cos\left(\beta - \frac{\pi}{2}\right)}{r_1} \tag{11}$$

for the push-push mode (N = 2, 4, etc.).

The coordinates ξ_1 and η_1 of the first orbit center can be calculated approximately by using a simplified mechanism for the energy gain per gap and the corresponding changes of orbit radius throughout the acceleration process. Assuming that the ion motion is strictly isochronous and considering

only the average magnetic field and the average orbit radius, let θ_{m2} denote the phase of the particles when crossing the gap center line $\eta=0$, i.e., the number of gap crossing η is even ($\eta = 2, 4, 6$, etc.), and θ_{m3} the phase at the $\xi=0$ line ($\eta = 1, 3, 5$, etc.). Then the relative energy gain per gap is

(12) $\Delta\epsilon_2 = \cos\theta_{m2}$ for even gap crossings and

$\Delta\epsilon_3 = \cos\theta_{m3}$ for odd gap crossings.

The total energy after the n-th gap traversal is given by the formulas

(13) $\epsilon_n = \epsilon_1 + \frac{n}{2}\cos\theta_{m2} + \frac{n-2}{2}\cos\theta_{m3}$ for $n = 2, 4, 6$, etc., and

$\epsilon_n = \epsilon_1 + \frac{n-1}{2}\left(\cos\theta_{m2} + \cos\theta_{m3}\right)$ for $n = 3, 5, 7$, etc.

The corresponding average orbit radii are

(14) $r_n = r_o\sqrt{\epsilon_n}$ with $r_o = \frac{1}{B_o}\sqrt{\frac{2m_oU_o}{e}}$

(where B_o is the central magnetic field).

The coordinates ξ_n, η_n of the instantaneous orbit center after the n-th gap traversal (in the oblique frame of the gap symmetry lines) are approximately given by the recurrence formulas:

(15a) $$|\xi_n| = r_n - r_{n-1} - |\xi_{n-1}| \quad \text{for} \quad n = 2, 4, 6, \text{etc.},$$

$$\xi_n = \xi_{n-1} \quad \text{for} \quad n = 3, 5, 7, \text{etc.},$$

(15b) $$|\eta_n| = r_n - r_{n-1} - |\eta_{n-1}| \quad \text{for} \quad n = 3, 5, 7, \text{etc.},$$

$$\eta_n = \eta_{n-1} \quad \text{for} \quad n = 2, 4, 6, \text{etc.}$$

With the geometry of Fig. 4 the first orbit center would have to be located in the fourth quadrant of the ξ,η-system which means that ξ_1 would be positive and η_1 would have a negative sign. Then the sign changes alternately at each even gap crossing for the ξ_i-coordinates and each odd gap crossing for the η_i-coordinates.

The condition for centered orbits is that the instantaneous orbit centers asymptotically approach the origin of the coordinate system as n increases to very large numbers. Since in any case the total number of gap crossings has a finite value, the orbit centers should always remain some small distance away from the origin, and as a good condition for centering one may postulate that the displacement of the center point from the origin after the n-th gap crossing must equal that after the (n-2)-th crossing, or, equivalently,

(16a) $$|\xi_n| = |\xi_{n-2}| \quad \text{for n even, and}$$

(16b) $$|\eta_n| = |\eta_{n-2}| \quad \text{for n odd.}$$

Since $\xi_{n-1} = \xi_{n-2}$ (for n even), introducing (16a) in (15a) leads to the equations

(17a) $$r_n - r_{n-1} - 2\left|\xi_{n-2}\right| = 0 \quad \text{(n even), and likewise}$$

(17b) $$r_n - r_{n-1} - 2\left|\eta_{n-1}\right| = 0 \quad \text{(n odd).}$$

From these equations one can find ξ_1 and η_1 by choosing a sufficient large number of n and calculating $\left|\xi_n\right|$ and $\left|\eta_n\right|$ recursively from (15) in terms of the r_i (i = 1, 2, ..., n) and ξ_1 and η_1, respectively.

Since this is only a rough approximation and the geometric adjustments in the cyclotron center, on the other hand, are not very accurate, n must not be very large and values between 10 and 15 have turned out to be quite sufficient. So, for example, one gets

$$\left|\xi_2\right| = r_2 - r_1 - \xi_1 \ ,$$

$$\left|\xi_3\right| = \left|\xi_2\right| \ ,$$

$$\left|\xi_4\right| = r_4 - r_3 - \left|\xi_3\right| = r_4 - r_3 - r_2 - r_1 + \left|\xi_1\right| \ , \text{ and}$$

$$\left|\xi_6\right| = r_6 - r_5 - r_4 - r_3 - r_2 - r_1 - \xi_1 \ ,$$

which introduced in (17a) yields

$$2\left|\xi_1\right| = -r_8 + r_7 + 2(r_6 - r_5 - r_4 + r_3 + r_2 - r_1)$$

and likewise for larger n or for $\left|\eta_1\right|$. The radii are to be

calculated from (14), using (13) for the energy ϵ_n. The energy gain in the first gap ϵ_1, as well as the favorable starting phase θ_o and transit angle τ_d can be determined as in reference 2.

Table I shows the parameters and starting conditions as calculated by the method described for the high-energy and the low-energy modes of operation (21 and 15 Mc/s respectively). The peak voltage U_o was determined by choosing the value of 70 kV for the high-energy operation with protons (N=1) and calculating the values for the second- and third-harmonic modes of operation from equation (5). The voltages for the low-energy mode follow from the constant-orbit condition (2), which, in the case of the protons, leads to

$$U_2 = U_1 \left(\frac{B_2}{B_1}\right) = U_1 \left(\frac{f_{e2}}{f_{e1}}\right)^2 = 70 \times \left(\frac{15}{21}\right)^2 = 35.7 \text{ kV}.$$

The width d_1 of the first gap, i.e., the extension of the electric field (which is larger than the source-to-puller spacing) was chosen to be 1.0 cm for the N=1 mode at high energies. It was assumed, then, that this distance is being changed at different modes proportional to the voltage U_o so as to keep the field strength $\frac{U_o}{d_1}$ always constant. The parameter $\chi = \frac{d_1^2 B^2 e/m}{U_o}$ determines the favorable starting phase θ_o, the maximum energy gain ϵ_1 and the transit angle τ_d in the first gap.

It should be noted that these starting conditions refer only to the favorable starting phase θ_o where the maximum energy gain in the first gap occurs. The values for the phases θ_{m2} at even and θ_{m3} at odd gap crossings are chosen more or less arbitrary by qualitative reasoning: In the two push-pull cases (N=1 and N=3), the phases were shifted by about 10^o beyond the maximum-energy-gain position to make sure that there was an overall focusing effect. In the push-push case (N=2), where the particles cross the second gap at a positive phase with strong electric focusing, such a shift was considered unnecessary, and it was assumed that both phases are symmetric with respect to the peak voltage.

	High-Energy Mode f_e = 21 Mc/s			Low-Energy Mode f_e = 15 Mc/s		
	N=1	N=2	N=3	N=1	N=2	N=3
U_o [kV]	70	56.6	37.7	35.7	28.9	19.2
d_1 [cm]	1.0	0.8	0.54	0.51	0.41	0.28
χ	0.257	0.105	0.046	0.067	0.027	0.012
θ_o	-29°	-38°	-37°	-15°	-18.5°	-18°
τ_d	43.5°	52°	54°	22°	25.8°	25°
ϵ_1	0.984	0.977	0.977	0.997	0.995	0.990
r_o [cm]	2.76	3.52	3.52	2.76	3.52	3.52
r_1 [cm]	2.74	3.48	3.48	2.76	3.51	3.50
θ_{m2}	-11°	54°	63°	-11°	54°	-45°
θ_{m3}	25°	-54°	-45°	25°	-54°	63°
α_1	36.7°	-16.3°	27.4°	34.8°	-20.0°	26.3°
ξ_1 [cm]	0.69	0.56	0.66	0.72	0.56	0.66
η_1 [cm]	0.42	0.45	0.33	0.48	0.42	0.33

Table I: Parameters and starting conditions for the high-energy and low-energy modes of operation in a normal-injection scheme.

4.2 Computer Results with the Uniform-Field Code.

To obtain some better information about the character of the ion trajectories on the first turns and to check the calculated starting conditions a computer program, called the "Uniform-Field Code", has been written by T. I. Arnette and D. Johnson. This program is basically the same as that described in a former article about the Karlsruhe cyclotron[3]; a couple of modifications and improvements were made to account for the special features of the MSU machine.

The Uniform-Field Code is based on the assumption that the magnetic field in the center of the cyclotron is uniform and that the electric field is concentrated in the acceleration gaps and is also uniform. In this way the central region is subdivided into electric gaps and sectors with no electric field, and the analytical solution of the equations of motion is used for the computations. The code calculates the r.f. phase, the coordinates and velocity components of a particle with given starting phase when it enters the electric-field gaps and the phase, coordinates, velocity components, kinetic energy, and radius of curvature at the end of each gap traversal. The great advantage of this code is that of any simplified model: With a minimum of theoretical effort and computer time one obtains a good survey of the general features,

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3) M. Reiser, Nucl. Instr. and Methods 13 (1961), 55.

geometric limitations, and physical problems of the initial acceleration process by idealizing the field configurations and neglecting the finer details of the dynamic behaviour of the particles. The disadvantage of such an approach is that it does not describe the effects produced by the nonuniformities of the magnetic and electric fields, gives only qualitative informations about the vertical focusing (from the phase of gap crossing) and so far is not very accurate. However, the accuracy can be quite good if the central magnetic field is essentially uniform and the electric field defined by attaching electrodes with small beam slits to the dee edges, as was demonstrated by the experiments with the Karlsruhe cyclotron analogue.

The Uniform-Field Code uses a righthanded system of cartesian coordinates (origin at the cyclotron center) which rotate from gap to gap when the ion trajectories are traced.

The x-axis is always parallel to the electric field lines while the y-axis is located at the gap center lines, except for the first gap where different conditions may exist. The phase is always related to the accelerating peak voltage (phase 0) in each individual gap. The coordinates of the starting point x_{1a}, y_{1a} (in the first coordinate system) were determined graphically from a figure similar to Fig. 4, using the calculated parameters of Table I.

It turned out that these values were quite satisfactory and that only one run for each mode of operation was necessary, except for some special cases.

A somewhat critical parameter is the gap width. Since in the push-push mode the tip of the dummy dee must be close enough to the center so that the first turn passes through it, one must have a rather small spacing between dee and dummy dee — the smaller the better. On the other hand the danger of sparking puts a lower limit to the possible values. Which spacing for a given voltage can be considered safe is a very delicate question. Communications with various cyclotron groups revealed that the experience differs widely from machine to machine as reflected by a factor of 10 between the extreme values obtained. The geometric restrictions dictated that we go closer to the optimistic limit, and therefore we chose a spacing of 0.5 in. hoping that it will hold the maximum voltage of 70 kV.

As for the computer runs the next question was to choose a reasonable value for the extension of the electric field, i.e., for the width d of the electric field region. Actually the electric field extends beyond the edges of the dee gap and in the center overlaps with the field of the following gap. However, since the philosophy is to attach electrodes to the dee edges along the first turn, the main field would be confined to the gap region, at least on the first revolution. The code, on the other hand, works only if the electric field gaps are separated by the dummy-dee sectors. Therefore the width of the electric field gaps was chosen to correspond with the dee spacing of 0.5 in. (1.27 cm), after

a test run with $d = 2.00$ cm clearly indicated that there was no significant difference between the trajectories of the two cases.

In the following the results of the computer runs for the high-energy operation shall be discussed. In all 3 modes the r.f. frequency was assumed to be 21 Mc/s which leads to a central field of 13,774 gauss for the protons.

1) N=1 (Protons). The coordinates of the starting point were $x_{1a}=-0.3$ cm, $y_{1a}=1.7$ cm. The puller-to-dee angle was chosen to be 36° so that the total angle between first and second gap is 180°. (In all these runs the ion source was considered a point source, and in the normal-injection scheme the particles are started with zero initial momentum.) Fig. 5 shows the trajectories of several particles with different starting phases θ_o over two full revolutions. The figure demonstrates that a relatively large group of starting phases — between -60° and 0° — is well centered and bunched radially, while beyond 0° the eccentricity and divergence in radial motion increase quickly as seen for the $+20^{\circ}$ starting phase (In all these figures the useful group of trajectories is shaded and for comparison one undesired trajectory is shown). The explanation for these effects is essentially the smaller energy gain of particles starting while the voltage is decreasing. In Fig. 6 the kinetic energy after consecutive gap crossings is plotted versus the starting phase.

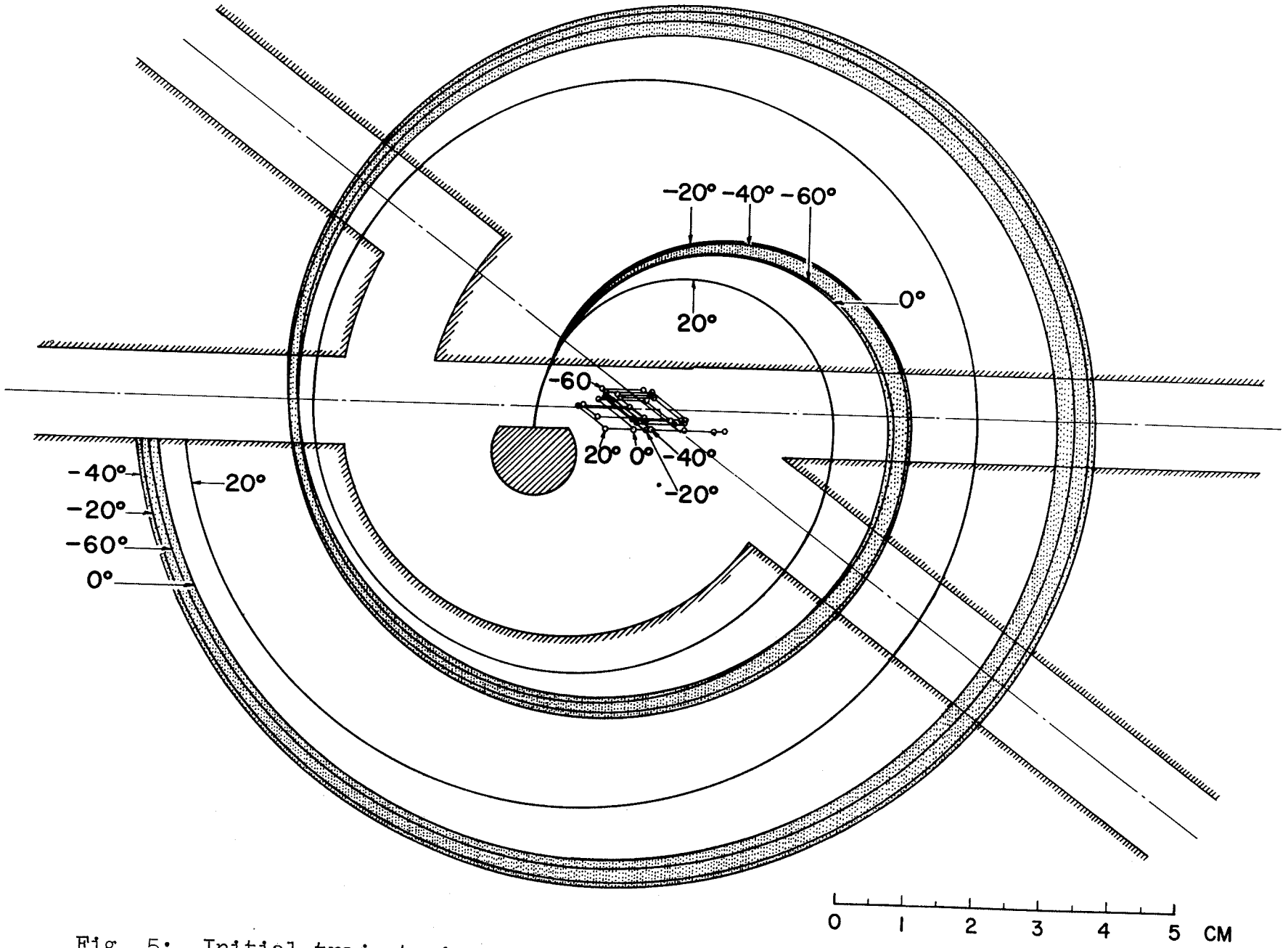

Fig. 5: Initial trajectories and instantaneous centers of curvature in the N=1 mode (Protons) of operation for different starting phases; puller-to-dee angle $\alpha = 36^{\circ}$.

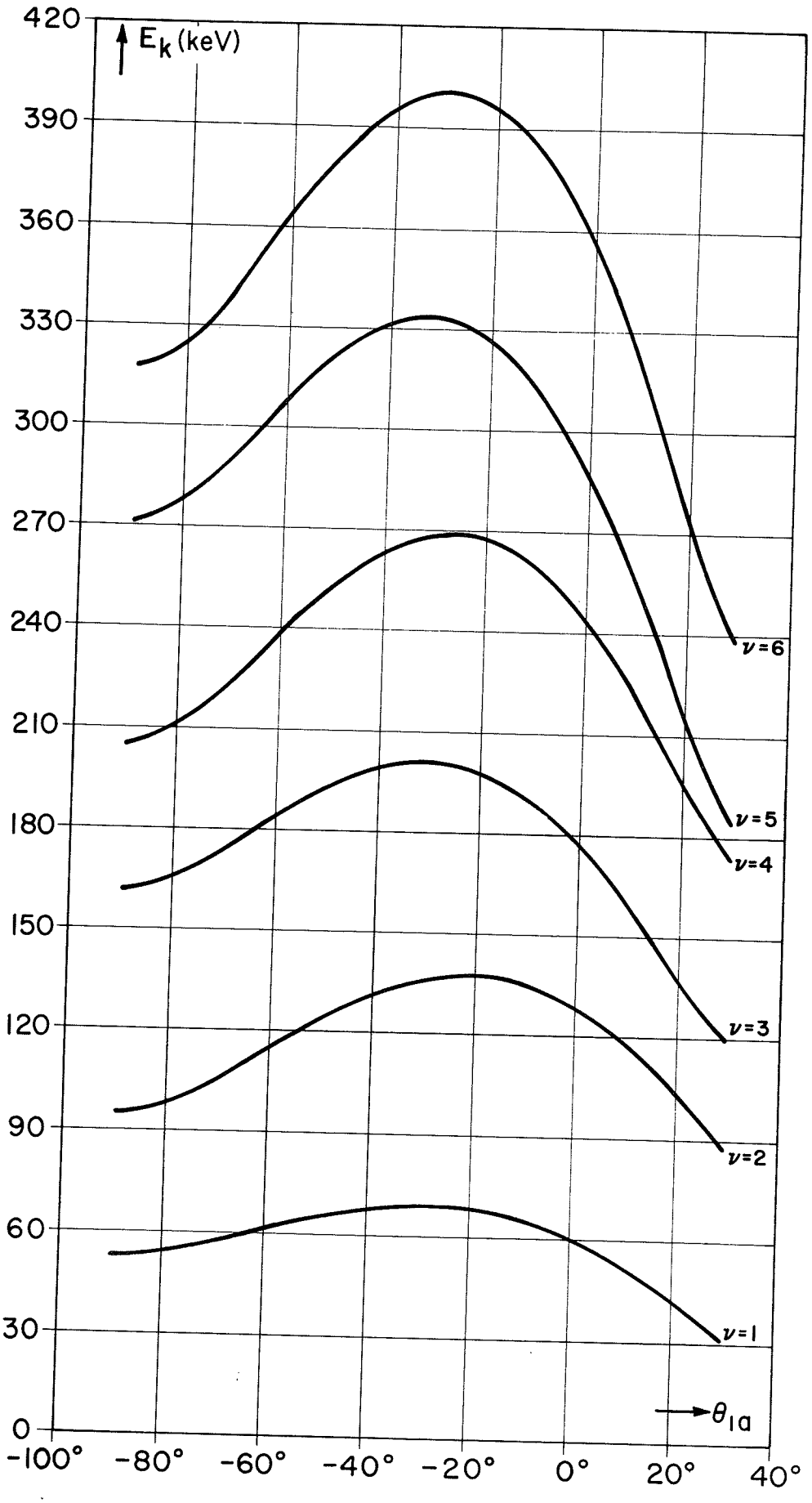

Fig. 6: Kinetic energy after consecutive gap crossings as a function of starting phase in the N=1 case.

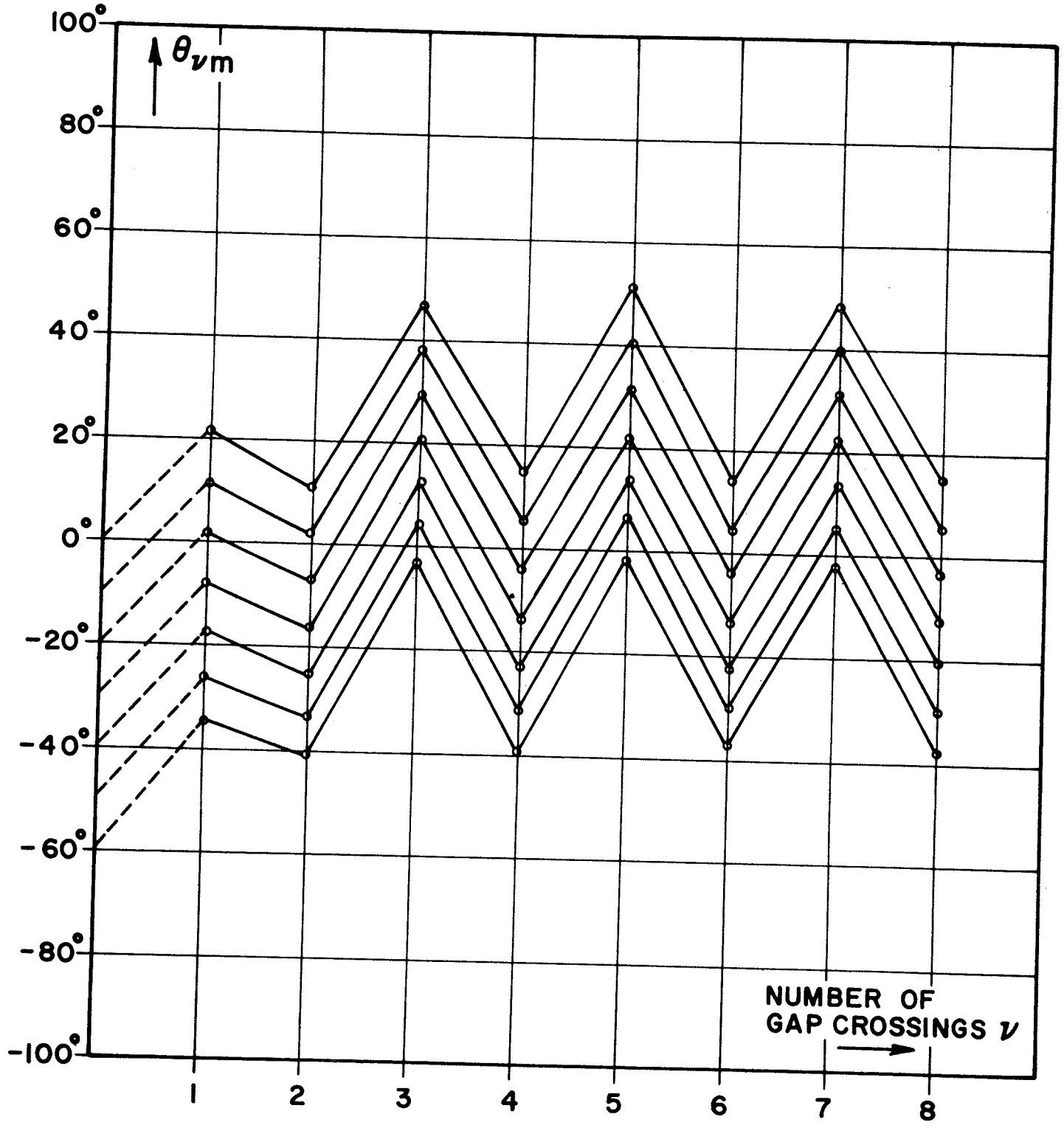

Fig. 7: Average phase of gap crossing for particles with different starting phase in the case N=1.

There is a flat maximum at about -30°, the most favorable starting phase and a strong fall-off beyond 0°. The trajectories of particles close to the favorable starting phase do not differ significantly from one another. For example, the trajectories for the -20° and -40° starting phase are almost identical along the 2 calculated turns.

The phase history through the first eight gap crossings is plotted in Fig. 7. Since it would have complicated the code to calculate the true phase $\theta_{\nu m}$ at the center line of each gap, the arithmetic average $\frac{1}{2}(\theta_{\nu a}+\theta_{\nu e})$ between the phases at gap entrance $\theta_{\nu a}$ and at the end of the gap $\theta_{\nu e}$ was taken as a good approximation for $\theta_{\nu m}$ (actually $\theta_{\nu m}$ is a little bit larger than $\frac{1}{2}(\theta_{\nu a}+\theta_{\nu e})$ since the particle travels longer in the first half than in the second half of the gap because of its increasing velocity). The most favorable interval of starting phases in terms of energy gain and radial bunching is -20° to -30° as seen in the previous figures. Following now the phase history of this group of particles one sees that, for example, the -30°-starting-phase particle has an average phase of -8.5° in the first gap, -16.5° in the second, 21.5° in the third, -14°in the fourth, 22° in the fifth gap, etc. This sequence means that, qualitatively, there should be quite sufficient electric focusing for this group of ions. In fact, since the focusing effect extends somewhat beyond the 0° to negative phases, one would expect an overall focusing momentum also for the -40° starting phase, while at more

negative values an increasing number of particles would be defocused and eventually hit the dee structure. Another interesting point in Fig. 7 is that there is only a slight phase-bunching effect. The phase interval from -60° to 0° at the start is bunched to a width of only 50° in the next gaps. This is important with respect to duty-cycle considerations which will be discussed later.

2) N=2 (Deuterons). The previous calculations of favorable starting conditions resulted in a negative puller-to-dee angle of -16° (Table I) which means that the dee-angle has to be reduced from 144° to 128° at the first gap. In the coordinate system x_1, y_1 (which is displaced from the second gap by an angle of 128°) the starting point then was found as $x_{1a}=0.5$ cm, $y_{1a}=2.7$ cm. The width of the first gap was assumed to be 0.8 cm and the peak voltage $U_o = 56.6$ kV . In Figures 8 to 10 the trajectories, kinetic energy and phase history curves obtained from the computer runs are plotted. The contrast to the N=1 mode can best be seen by looking first at the energy curves in Fig. 9. In the first gap the kinetic energy gain versus starting phase is similar to the N=1 case except that the maximum occurs now at about -40°, i.e., about 10° earlier. In the second gap the ions travel through the electric field at a time where the instantaneous voltage is rapidly decreasing (large positive phases) and then changing its polarity, as is clear from Fig. 3. The result is that the early starting

particles (i.e., starting phases close to -90°) experience the highest energy gain, those coming later gain less and less energy and, finally, if they started later than -5° they are even decelerated and lose part of the energy they gained in the first gap crossing. Thus, if one compares a certain phase interval centered at the favorable starting phase one finds a larger energy spread after the second gap traversal in the N=2 mode than in the N=1 mode. In the third gap, where the ions travel through the electric field at a time where the voltage is rising, the situation is just reversed. Now the particles that had started at the favorable phase of -40° and a little later have the highest energy gain. This pattern, where the maximum energy gain shifts alternately from one range of starting phases to another, continues now through the whole acceleration process.

The larger nonuniformity in energy gain compared with the N=1 mode is reflected by the trajectories in Fig. 8. Considering again the particles with starting phases between -60° and 0°, there is less radial bunching than in Fig. 5. In fact, the 0° particle is already useless for further acceleration. There is also a larger spread of orbit centers; only the particles close to the favorable starting phase of -40° are sufficiently centered.

The phase history in Fig. 10 indicates that the particles with starting phases close to -40° cross the following gaps at about the maximum-energy-gain phases of $+54^{\circ}$ and -54°. There

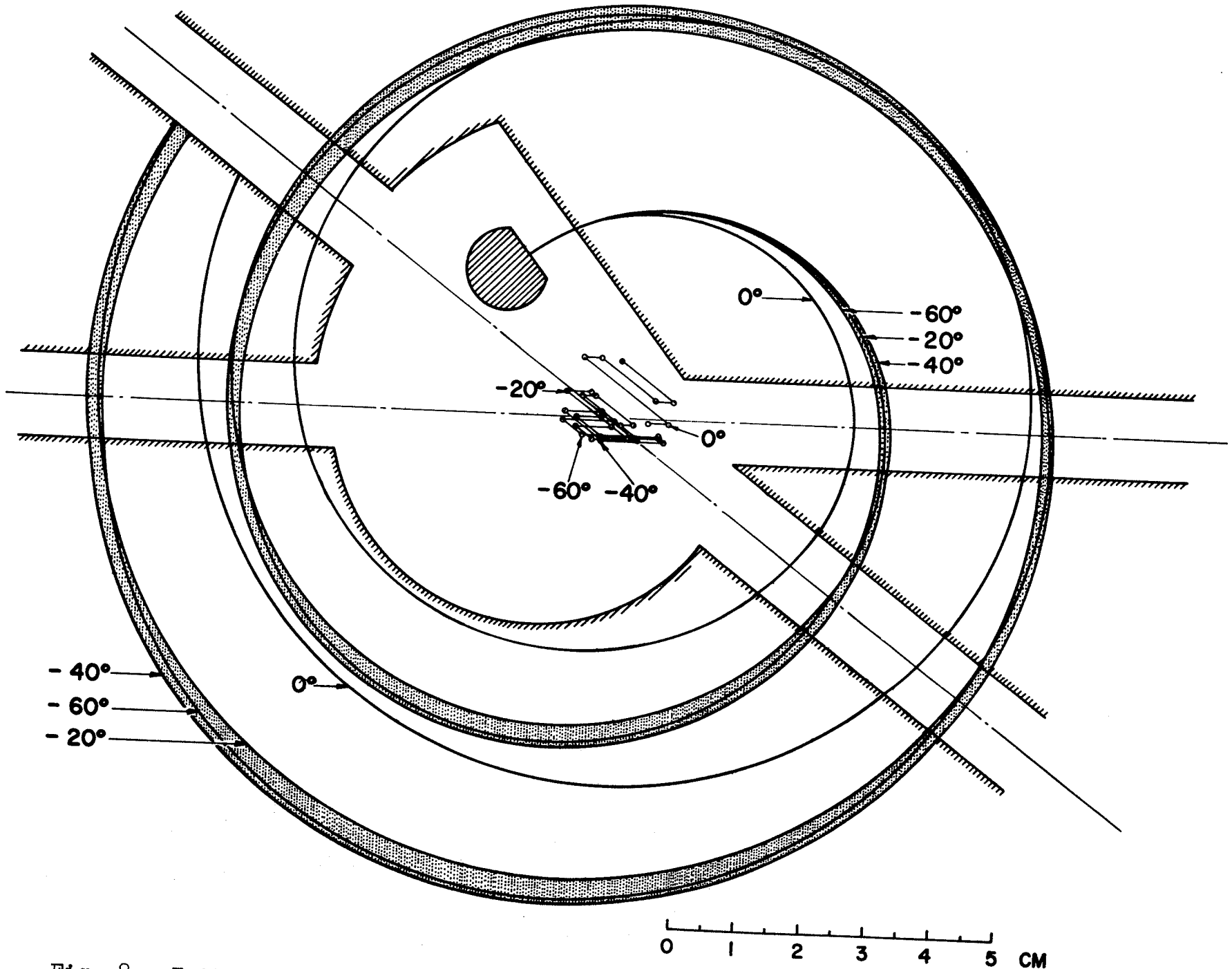

Fig. 8: Initial trajectories and centers of curvature in the N=2 case; puller-to-dee angle $\alpha = -16^{\circ}$.

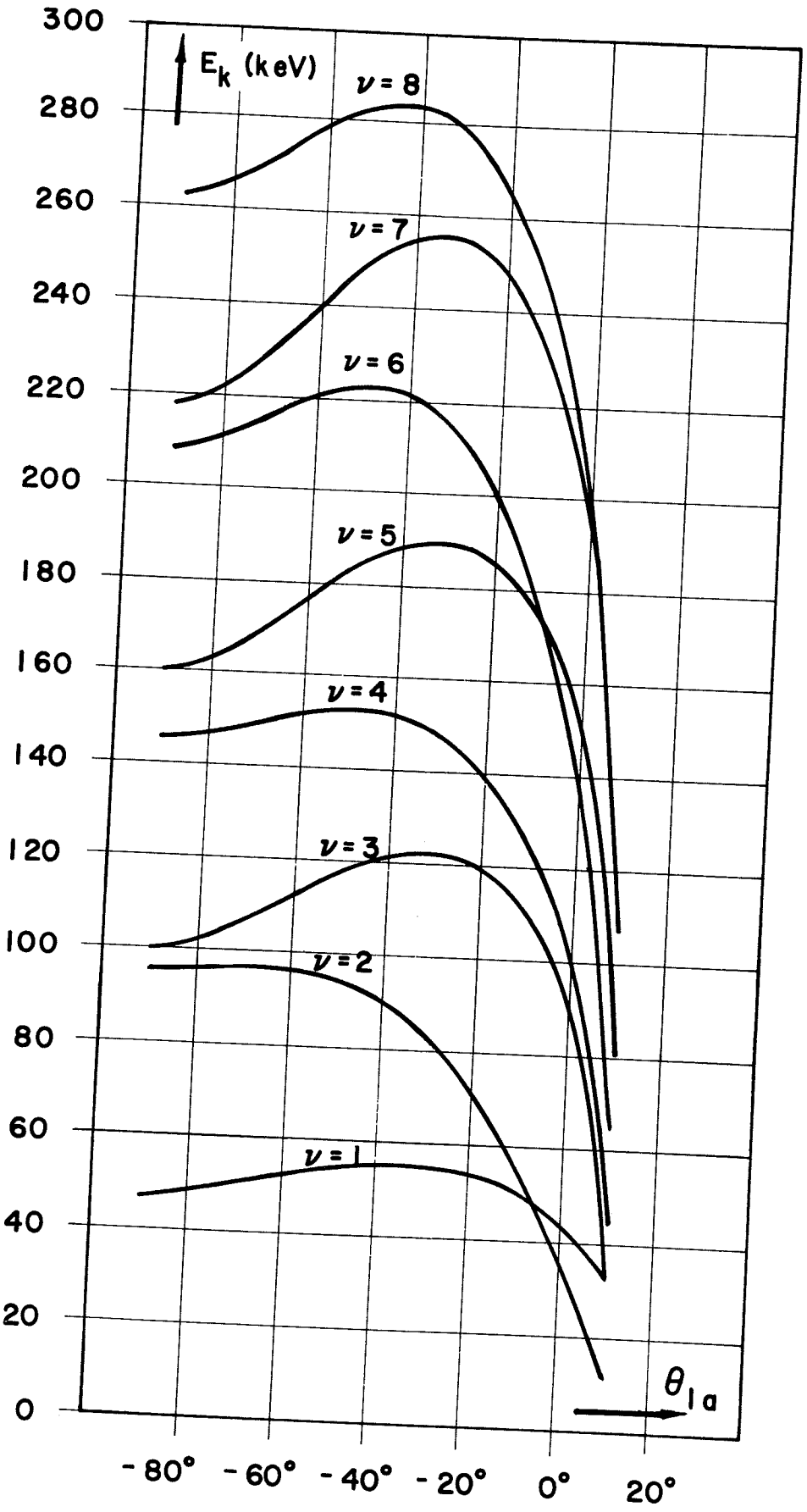

Fig. 9: Kinetic energy versus starting phase after consecutive gap crossings (N=2).

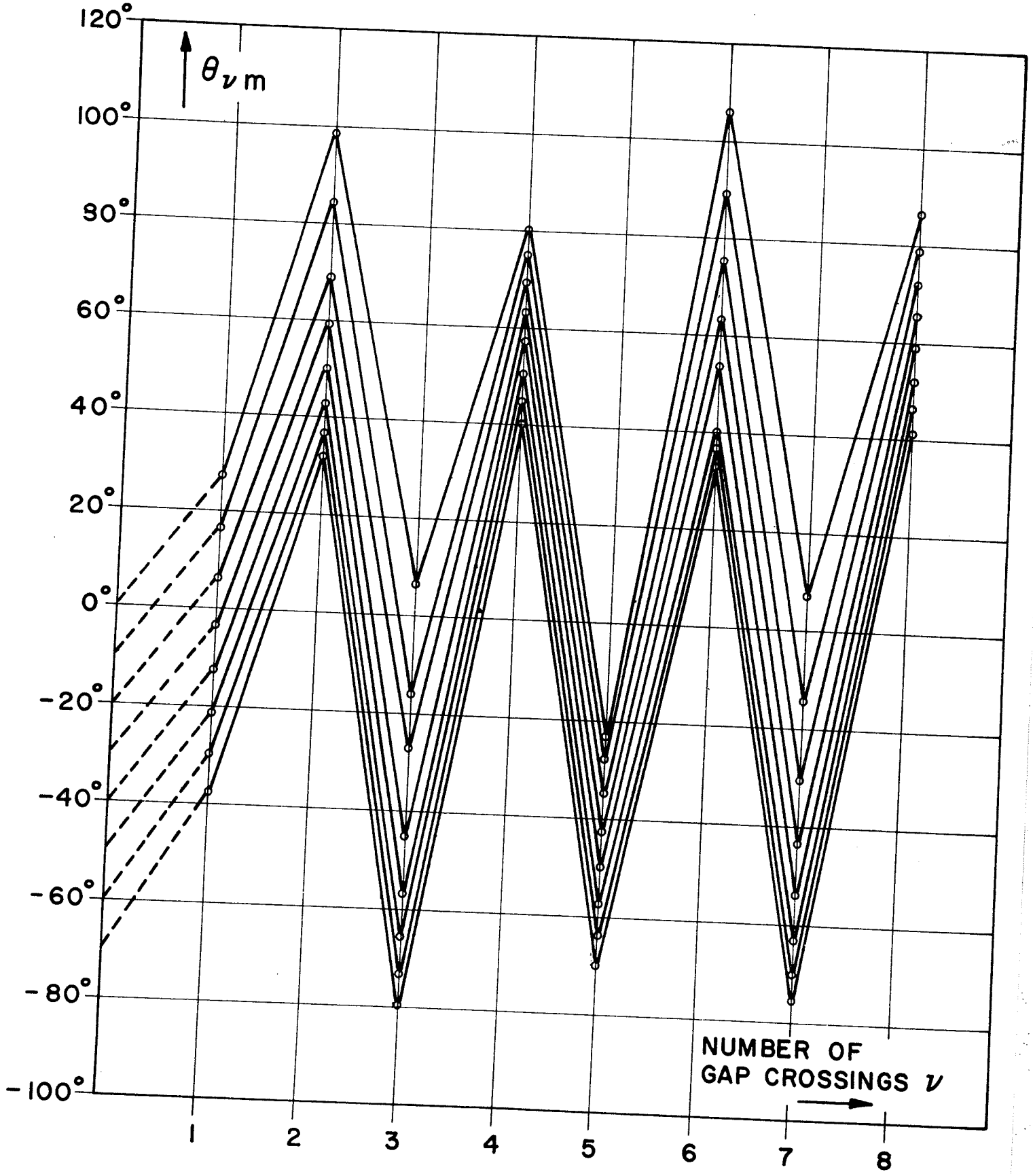

Fig. 10: Average phase of gap crossing for particles with different starting phase in the case N=2.

is a significant difference with respect to the N=1 case: The sequence of alternating focusing starts in the second gap with a strong focusing effect while in the N=1 case the second gap was defocusing. This strong momentum can eventually result in overfocusing, i.e., in a crossover which in subsequent gaps cannot be fully corrected since the electric focusing effect decreases rapidly with the energy of the particles. As a counter-measure against overfocusing one could think of changing the phase history by further reduction of the dee angle at the first gap so that in the average the phase of gap crossing is more negative than positive. The focusing force in the second gap would still be strong enough to predominate over subsequent defocusing gap crossings except that the overfocusing action would be more or less compensated. Another feature in Fig. 10 contrasting with the N=1 case is that there is a different phase-bunching effect. If one takes the 50° phase interval from -60° to -10° at the start, there is practically no bunching in the second and third gaps, strong bunching to 30° in the fourth and 37° in the fifth gaps; and then this cycle seems to repeat itself again in the following gap crossings. This alternating bunching effect is obviously related to the movement of the center points. In the N=1 case the energy gain of the favorable group of particles was quite uniform and, as a result, there was only little spread in instantaneous orbit centers, whereas in the N=2 mode this spread in the center points of particles with different starting phase

is quite significant. It is obvious, of course, that for two given ions the path length in the field-free sectors between gaps is different if their orbit centers are not identical, and therefore their time of flight through the sectors is different.

3) N=3 ($_{12}C^{4+}$). The puller-to-dee angle was determined to be 27°, which value results in a dee angle β_1 of 171° instead of 144°. For the starting point in the first coordinate system, determined by this angle, the calculations yielded x_{1a}=-0.1 cm , y_{1a}=2.5 cm , while the first gap width was assumed to be 0.54 cm and the peak voltage as 37.7 kV. In general this push-pull mode of operation is much similar to the N=1 case; the main difference is that the phase of gap traversal is, as in the N=2 case, farther away from the peak voltage and, of course, the ions spend a larger part of the r.f. period in the electric field. The trajectories in Fig. 11 look better than in the previous N=2 case. Both radial bunching and centering are quite effective for a large group of starting phases. However, this effect is not as uniform as in the N=1 mode, the nodes and amplitudes of radial motion are more accentuated than in that previous case. This, again, is mainly due to the fact that the particles cross the second gap at rapidly increasing voltage which leads to a larger spread in energy gain and orbit centers than for the first-harmonic mode of acceleration. From Fig. 12 one can see that

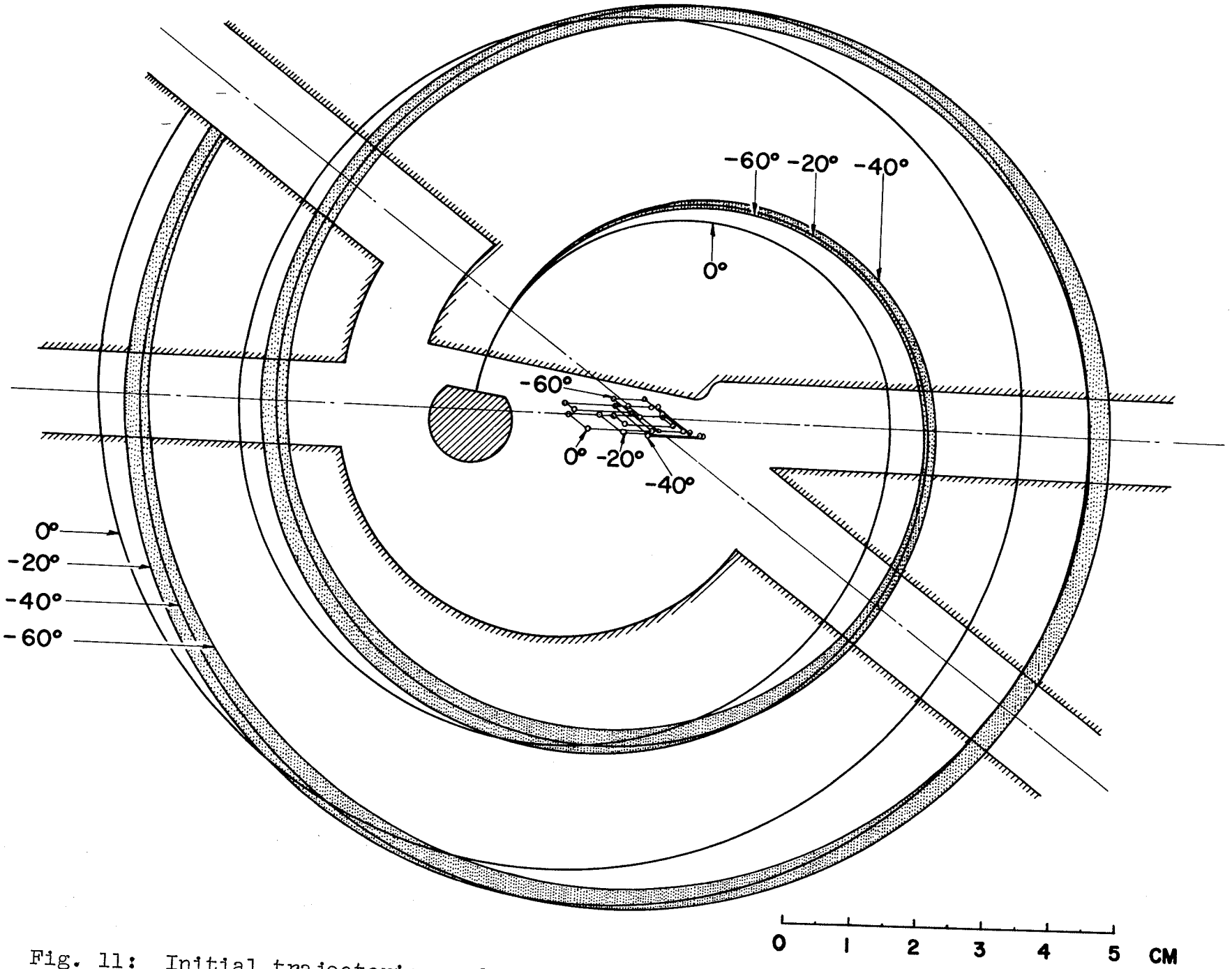

Fig. 11: Initial trajectories and centers of curvature in the N=3 case; puller-to-dee angle $\alpha = 27^{o}$.

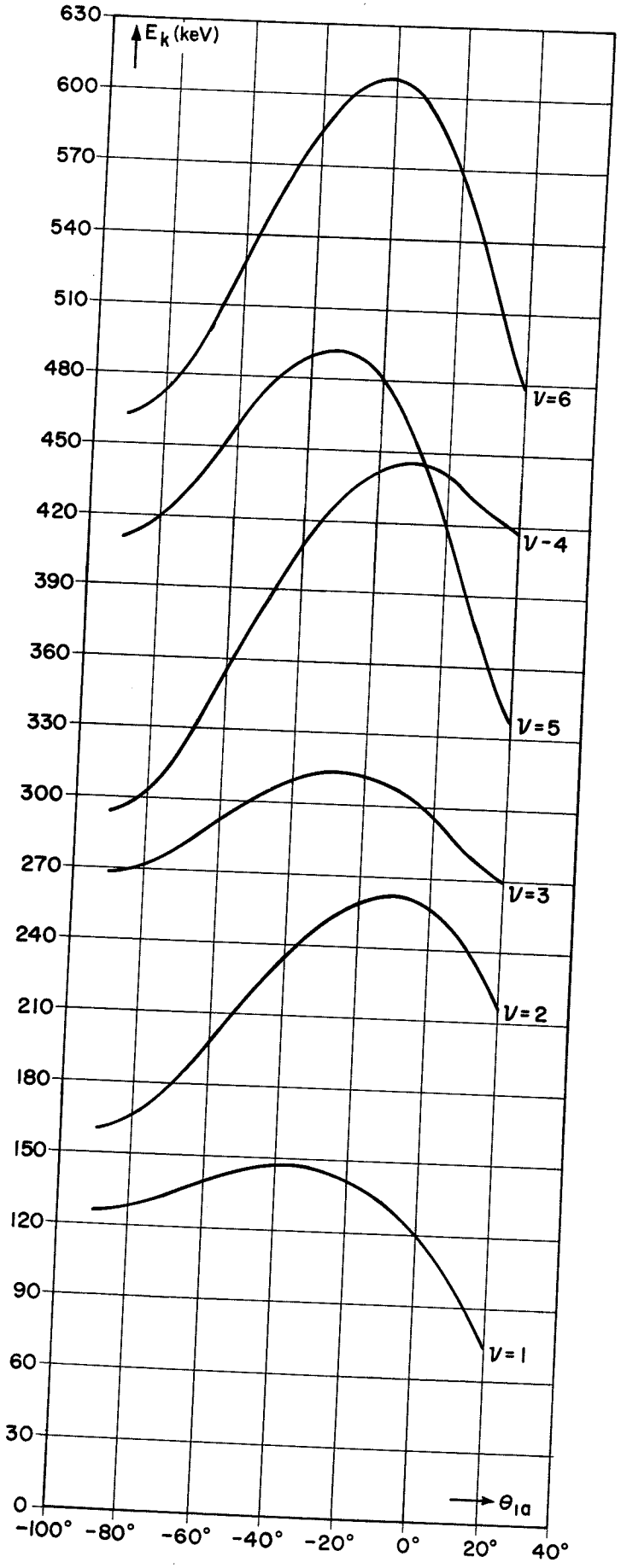

Fig. 12: Kinetic energy versus starting phase after consecutive gap crossings (N=3).

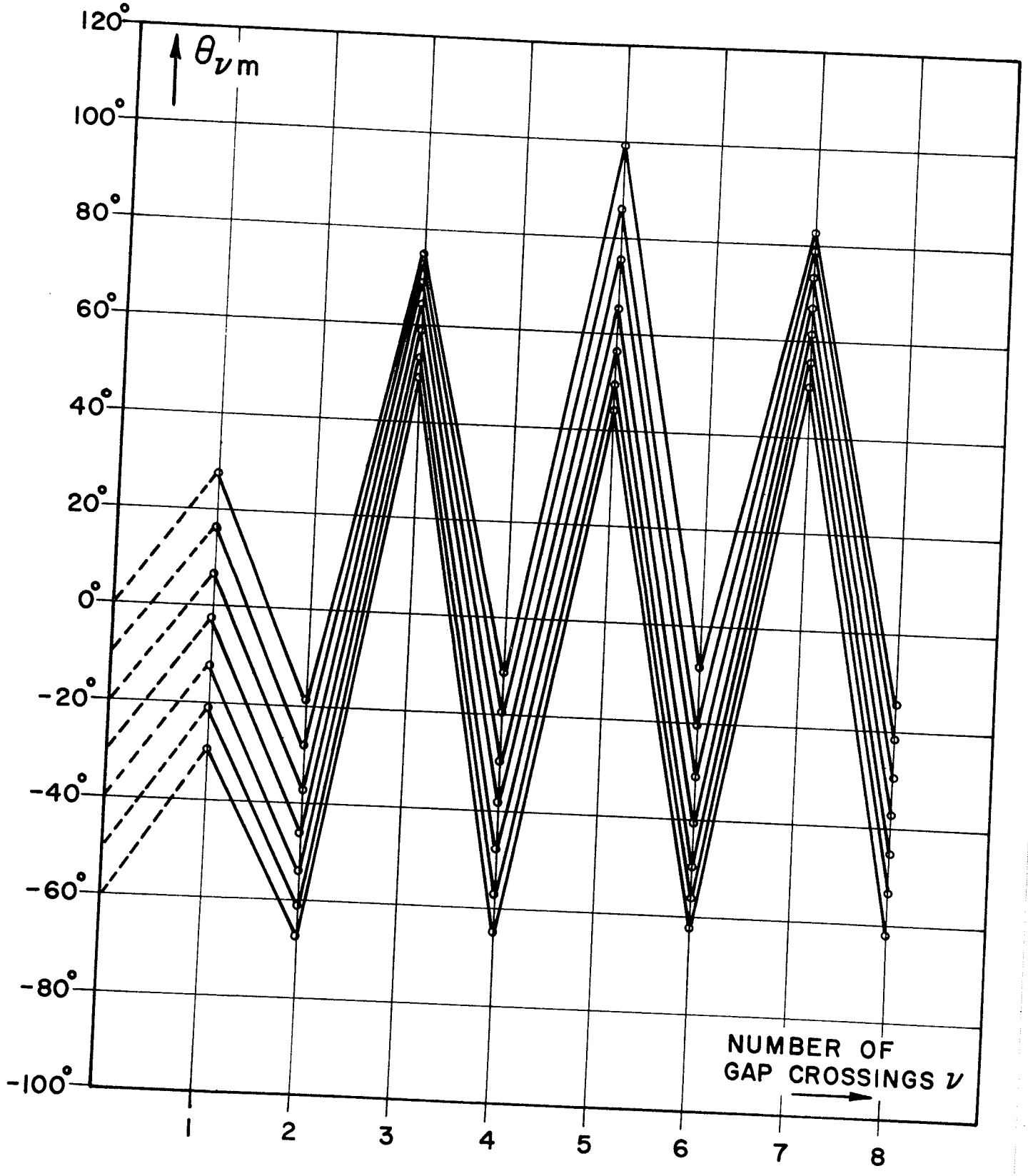

Fig. 13: Average phase of gap crossing for particles with different starting phase in the case N=3.

the favorable starting phase in terms of maximum energy is between -30° and -40° regarding the first gap. In the following gaps this maximum shifts alternately back and forth between about -35° and -15° as one expects from the phase history which is plotted in Fig. 13. As in the N=1 mode, and contrary to the push-push situation, the phase in the second gap starts with large negative values which means that the very early particles could be defocused (the focusing action is not symmetric, i.e., the range of focusing extends to negative values while only the large negative phases are defocusing). Again, however, the useful group of particles should experience an overall focusing effect. As in the N=2 case there is an alternating but stronger phase bunching effect which decreases the 60°-phase interval from -60° to 0° at the start to a width of 48° in the second, 27° in the third, about 55° in the next three gaps, 34° in the seventh gap, etc. As in the other two cases the approximate calculations of starting conditions turned out very satisfactory, and only one computer run was necessary.

These were the results obtained for the high-energy mode of operation. At low energies the situation is basically all the same. The only difference is that one reduces the source-to-puller spacing which results in minor changes of the puller-to-dee angle (see Table I) and the starting points. Three computer runs were made for the three modes of operation at the lowest energies (corresponding to the 15 Mc/s of

the r.f. system), but they will not be discussed here since the results are practically a complete analogue to the high-energy situation.

4.3 Central Geometry for the Normal-Injection Scheme.

The next important question that remains to be discussed after the theoretical results have turned out quite satisfactory is: How would one design the central geometry or, in other words, how could the adjustments that are necessary in such an injection scheme be achieved in a practical layout of the cyclotron center? Obviously, both the ion source as well as the puller structure must be adjustable within a certain area of the center. The following adjustments must be provided for the ion source: (a) radial displacement with respect to the cyclotron center of about $\Delta r=0.5"$; (b) azimuthal rotation over an angle of about 45°; (c) rotation of the source output slit about the vertical axis of the source. If the ion source in the MSU cyclotron will be inserted vertically through a hole in the upper central yoke and pole tip, one could use a mechanism similar to that of the Berkeley 88" Cyclotron.[4] The puller should have the same degree of adjustment as the source, i.e., (a) rotation about a given axis, and (b) displacement of this axis of rotation in 2 directions.

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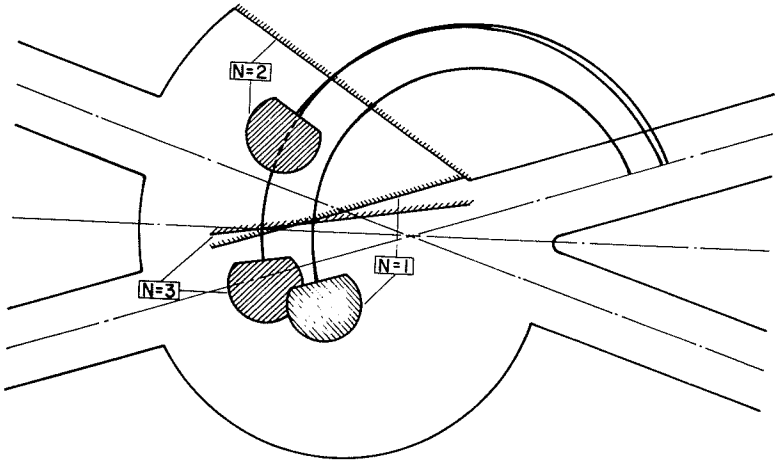
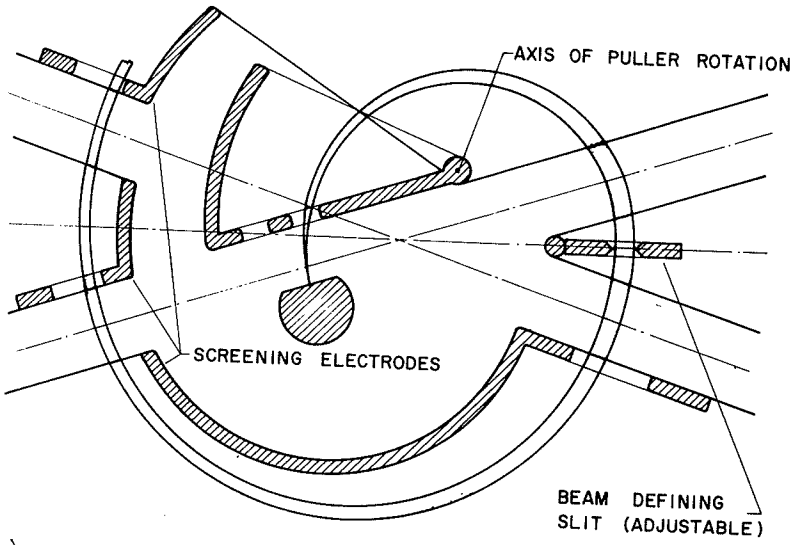
4) J. M. Haughian and R. J. Burleigh, Nucl. Instr. and Meth. 18, 19 (1962), 559.

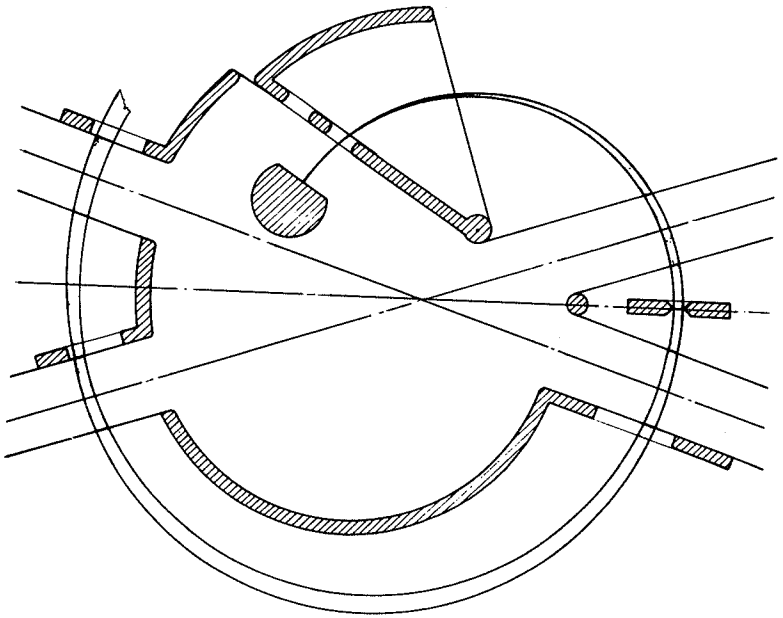
The accomplishment of these motions (especially the displacement of the axis) is somewhat complicated because it has to be done through the dee system. One could reduce the difficulties by designing the puller such that only a rotation is necessary. This situation, of course, is a flexibility limitation which means that the theoretical requirements cannot be realized exactly but have to be compromised in the practical design. The way that such a compromise can be accomplished is shown schematically in Fig. 14. In the first of the four pictures, the three main positions of source and puller are indicated, the puller being represented by its front line which defines the angle to the dee. If one now designs the puller with two slits and locates the axis of rotation properly within the dee, all three conditions can be approached quite good by merely turning the puller about its axis as demonstrated in the three following pictures of Fig. 14. The outer slit is used for the N=3 mode of operation, while the inner slit fits to the N=1 and N=2 cases. It is to be noted that the theoretical puller angles in the N=1 and N=2 modes are matched exactly while there is a slight discrepancy in the N=3 position. However, the whole situation becomes less ideal if one considers the slight readjustments in each main position when the energy of the particles is to be changed. As said previously, one would like to reduce the source-to-puller spacing if the voltage is lowered to increase the output current.



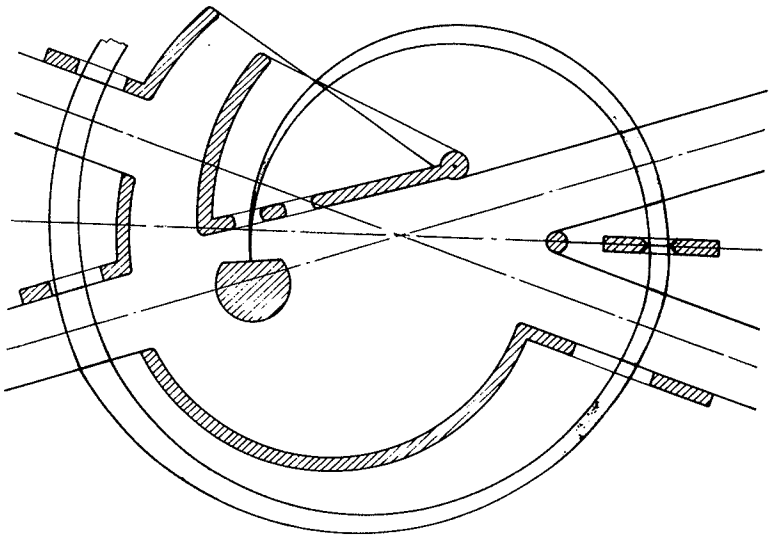
(a) Calculated optimum positions of source and puller.



(b) Central geometric arrangement with ion source, puller and defining slit in the N=1 position (push-pull mode).



(c) Source, puller and defining slit in the N=2 position (push-push mode).



(d) Source, puller and defining slit in the N=3 position (push-pull mode).

Fig. 14: Schematic layout of central geometry with main positions of source and puller in the 3 modes of operation.

The first orbit center is determined by the direction of the electric field between source and puller and, if the field is approximately uniform close to the source, is located at a line which extends through the output slit and is perpendicular to the field direction. Therefore, if one wants to keep the orbits centered, it is better to reduce the gap width by moving the puller rather than the source. However, moving the puller towards the source by means of rotation through a fixed axis necessarily changes the puller-to-dee angle and hence the direction of the electric field and the position of the first orbit center. To reduce this effect one could (a) locate the puller axis in the middle between the two optimum positions at the high- and low-energy operation, and (b) build a source which has a flat surface at the front, and slightly rotate this surface opposite to the change in the puller-to-dee angle, as is indicated in Fig. 14d. By measure (b) the instantaneous orbit center would be off the desired position at the beginning of motion, but, as the particle approaches the puller side of the gap, would move towards the point which is necessary to achieve subsequent centering of the orbits.

Summarizing this discussion about the aspects of a fixed axis of rotation for the puller one can say that, despite certain difficulties in matching the theoretical requirements, a satisfactory solution should be possible.

Another serious problem is how one can achieve, at least with some approximation, a field distribution similar to that used in the Uniform-Field Code. It is clear that the uniformity of the field is an idealization which cannot be matched perfectly since one would always need slits through which the beam passes; in fact, uniformity is even undesirable because a uniform field would provide no vertical focusing. The Uniform-Field Code implies two assumptions at which one should aim in the actual design (and the better they are realized the more accurate are the theoretical results for optimum source and puller location): (a) The main field should be concentrated within the gap region (the width of the acceleration gap may be somewhat larger, of course, than the dee spacing), and (b) the field direction should essentially be perpendicular to the gap center line. These requirements mean that no strong electric fields should be present along the particles' trajectories outside the immediate gaps, in particular, any strong field components normal to the trajectories should be avoided.

It is clear that these desirable conditions are very well satisfied outside the center of the cyclotron by the geometry of the dee system itself. In the very center, however, the structures of ion source and puller introduce distortions with strong radial field components, which unfortunately, change if one turns to another mode of operation, i.e., when readjustments are made. A look at Fig. 14 may help to understand this situation in more detail. In order to provide enough space for the

insertion of source and puller and to make the necessary adjustments, one has to cut out certain areas of the dee structures. In addition, these modifications of the dees and dummy dees must be large enough that in every mode of operation the spacings between neighboring surfaces of different potentials are always above the breakdown safety distance. It can be seen that strong electric fields normal to the expected ion trajectories will exist especially at the dee edge behind the source and at the tip of the dummy dee on the left. To overcome this difficulty one has to shield the beam path from the influence of these undesired stray fields by introducing screening electrodes along the edges of the dee and dummy dee as indicated in Fig. 14. One could also think of putting a beam-defining slit in the tip of the right dummy dee, as outlined in the figure, and this operation would then perform several functions simultaneously: (a) stop unwanted beam, (b) prevent overlapping of the fields from both gaps (otherwise this overlapping severely reduces the voltage drop experienced by the particles in the push-push mode), (c) define and concentrate the electric field in a favorable way.

The insertion of electrodes, though most useful for correcting the electric field configuration, has one great disadvantage: It increases the impedance for the gas flow and thus reduces the pumping speed at a point where it is of vital importance with respect to breakdown safety. Clearly, if the neutral gas streaming out of the source cannot pass

freely but is repelled by walls, one may have a dangerous pressure build-up. The ideal solution to this problem would be to use a wire net or a grid instead of solid metal walls. A grid would still efficiently screen the electric field but, at the same time, represent no serious barricade to the vacuum pump. The difficulty with a grid structure is that it is likely to be burned or sputtered if it is hit by the ion beam, unless one takes thick wires of a heat-resistive metal. Another possibility to reduce the pumping impedance would be to provide for adequate openings in the solid walls.

Whatever measures seem advisable, the best way of studying these problems is the electrolytic-tank method. In addition, the shortcomings of the Uniform-Field Code in neglecting the effects of nonuniformities demand a more realistic model for the electric field configuration. Therefore an electric-tank mock-up of the cyclotron center was built (in a 3:1 scale) using the Uniform-Field-Cde computations for the first set-up. A photograph of this model is shown in Fig. 15. The axis of puller rotation can be adjusted to the optimum positions. We plan to measure the field configuration and, with the data obtained, calculate numerically with the so-called "Cartwheel Code"[5] the trajectories of the ions along the first two turns. Various shapes and arrangements for the shielding electrodes will be investigated for determination of the final layout of central geometry.

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5) T. I. Arnette, H. G. Blosser, M. M. Gordon, and D. A. Johnson, Nucl. Inst. and Meth. 18, 19 (1962) 343.

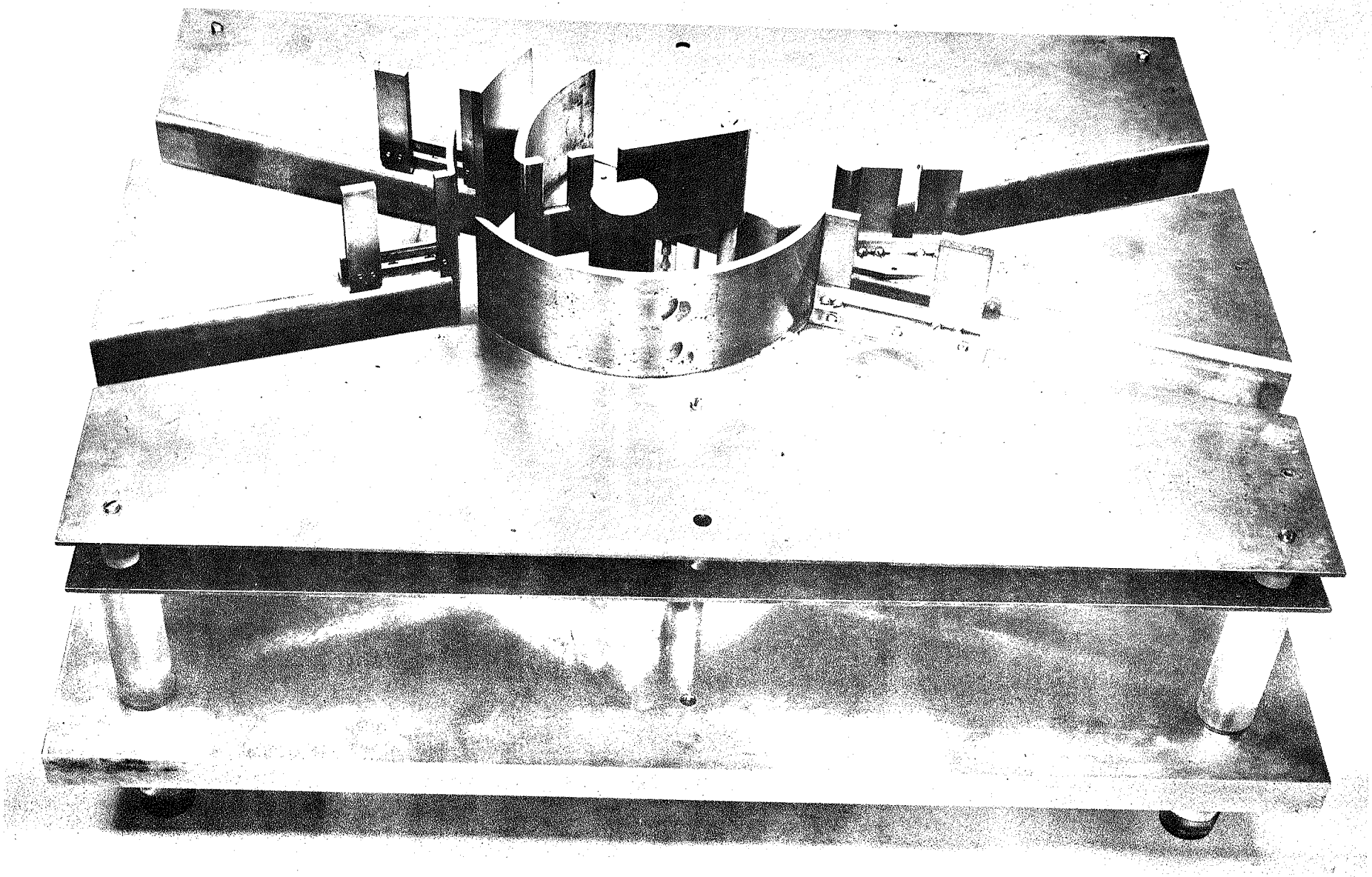

Fig. 15: Electrolytic-tank model of the central part of the dee system.

5. D.C. Injection.

5.1 General Considerations.

Preacceleration and injection of the ions into the cyclotron with a high initial velocity has been suggested several times for various reasons. In the first design scheme for the Karlsruhe cyclotron with its cloverleaf type of dee system, d.c. injection was envisaged to get the ions out of the center where the electric field, and hence the energy gain, is very small due to the strong overlapping inherent in such a system. After the experimental results with the central-region analogue had shown that the electric field could be effectively increased and defined by insertion of slit diaphragms along the first turn, the idea of d.c. injection was abandoned.[6] The Birmingham group developed a scheme for injection of preaccelerated polarized ions through an axial hole in the center of the magnet. The 10-keV ion beam is bent from its vertical direction into the median plane by a deflector electrode located between dee and dummy dee.[7] At the cyclotron conference in Los Angeles in April, 1962, d.c. injection was proposed as a possibility of

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6) M. Reiser, Dissertation, Mainz 1960.

7) A. J. Cox, D. E. Kidd, W. B. Powell, B. L. Reece, P. J. Waterton, Nucl. Instr. and Meth. 18, 19 (1962), 25.

improving the duty cycle.[8] Clearly, the continuous emission of a constant beam of particles during the whole r.f. period appears to be an improvement although, in the final outcome, it is the r.f. structure of the dee voltage and the extraction process which mainly determine the duty cycle. Another point in favor of a d.c. injection is that the ion source emittance is better understood theoretically (Child's Law) and can more easily be determined experimentally than in the case of r.f. voltage.

Nevertheless, there are a great many of problems and technological difficulties associated with such an injection scheme, and a careful investigation should be made in each particular case whether the expected advantages can be realized without substantial increase in efforts, costs and difficulties.

In the case of the MSU cyclotron practically all the points of view that were mentioned enter into the discussion of a d.c. injection: (a) the low electric field due to overlapping in the push-push mode, (b) the possibility of accelerating polarized ions, (c) the possible improvement of duty cycle, and (d) the better knowledge of emittance properties in a d.c. case. In addition, however, other arguments have to be considered: The normal-injection scheme discussed in the previous sections has the disadvantage that it requires a large range of adjustment for both source and puller; the mechanical devices to

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8) A. I. Yavin, Nucl. Instr. and Meth. 18, 19 (1962), 610.

perform the individual motions required are costly and complicated. If the source were at a d.c. potential, one would have the d.c. voltage as an additional parameter which could possibly eliminate the necessity of readjustments. Instead of moving the source to a different radial position, one would merely change the d.c. potential, and as for the azimuthal position one would abandon the demand of variability. In fact, the d.c.-injection scheme can only compete successfully with normal injection, it is felt, if it not only increases the duty cycle substantially but if it works without the necessity of large readjustments. On the other hand, if neither duty cycle nor ion capture efficiency and beam quality are substantially better, the mere fact that no major readjustments are necessary alone would not justify the d.c.-injection scheme. Even if the costs for putting the source and its electrical supply units on a d.c. potential were not higher than those for the adjustment devices, there is the problem that by far the largest part of the beam, continuously injected into the machine, is not useful for further acceleration and is somehow dissipated in the center. This problem, together with the additional presence of a structure with high d.c. potential, may seriously increase the electric-breakdown troubles which are so common with many accelerators. A normal-injection scheme would then clearly be the safer and better solution.

Since a final decision between the two injection schemes can be made only on the basis of a quantitative comparison, a

theoretical investigation of the d.c. injection as an alternative to the normal scheme with a grounded ion source was carried out with the Uniform-Field Code. There are several possibilities for the arrangement of the source with respect to the dee system, and this question had to be cleared first. The ion source can be outside the vacuum chamber, e.g., on top of the magnet as in the Birmingham case. Or it may be the usual cyclotron-type source facing the dee but carrying a high d.c. potential; in the first gap, the accelerating field which extracts the ions from the source would then consist of a d.c. component with a superimposed r.f. component. Another possibility is that a grounded slit diaphragm is inserted between the source and the dee separating the d.c. and r.f. fields. The special geometry of the MSU cyclotron favored the last case since it is equivalent to injecting the beam into a dummy dee. The tip of the dummy dee could be shaped adequately to fulfill the function of a grounded extractor slit. H. G. Blosser suggested another possibility in which one could connect the extractor electrode to the source mechanically by means of a grounded tube surrounding the source structure. This outer tube would simply have a slit opposite the output opening of the arc chamber. The advantage of such an arrangement is that one could perform minor readjustments of both puller and source simultaneously and thus better optimize the starting conditions.

Whatever type of arrangement would be used is not important for this investigation. It is simply assumed that the ions are injected into the acceleration gap from a dummy dee with a certain initial energy and under a given angle with respect to the dee edge which condition holds in either case. The injection voltage was another parameter that had to be chosen. It should be as high as possible, of course, but the geometry and insulation problems impose an upper limit, and it was felt that 50 kV was about the maximum one could risk. So this value was assumed for the high-energy mode of the protons (N=1), while for the other modes the d.c. voltage was changed according to the constant-orbit condition.

5.2 Computer Results.

For the computer runs with the Uniform-Field Code the high-energy situation was chosen with the general parameters being the same as in the case of normal injection: electric frequency f_e = 21 Mc/s, central magnetic field B_o = 13,774 gauss, gap width d = 1.27 cm (0.5 in.). For the dee voltages the same values were used as in the previous calculations, i.e., 70 kV for N=1, 56.6 kV for N=2, and 37.7 kV for N=3.

N=1 (Protons): After a first test run which was not quite satisfactory a second run yielded good results with the following starting conditions: x_{1a} = -0.635 cm, x_{1e} = 0.635 cm,

$y_{1a} = 2.350$ cm, $\dot{x}_{1a} = 2.8338 \times 10^8$ cm/s, $\dot{y}_{1a} = 1.2442 \times 10^8$ cm/s. The initial velocity components correspond to an injection voltage of 50 kV and an injection angle $\gamma = \arctan \frac{\dot{y}_{1a}}{\dot{x}_{1a}}$ of 23.7° ($\gamma = 0^\circ$ would mean that the particles are injected perpendicular to the gap). Fig. 16 shows the trajectories for starting phases, i.e., the r.f. phases where the particles enter the first gap, between -40° and $+60^\circ$. It is seen that all these orbits are well centered, but only a group of particles with starting phases between -40° and $+20^\circ$ is bunched radially and as a whole has good turn separation. The $+60^\circ$ starting phase is still well centered but has considerably less energy and radius gain than the others. Again, only the best trajectories are shown; particles that start earlier or later are practically useless and their trajectories would only confuse the picture. In Fig. 16 a possible geometric arrangement is outlined, where the ions are injected from the source into the specially designed tip of the dummy dee. A better insight into the acceleration process can be obtained from Fig. 17 where the kinetic energy after consecutive gap traversals is plotted versus the starting phase. Obviously, the particles starting around 0° have the highest energy gain and as one comes to large negative or positive phases the energy gain sharply decreases. The excellent behavior of the particles in the previous picture can now be understood from the fact that the maximum of energy gain with respect to starting phase in the first gap is very close to the maximum

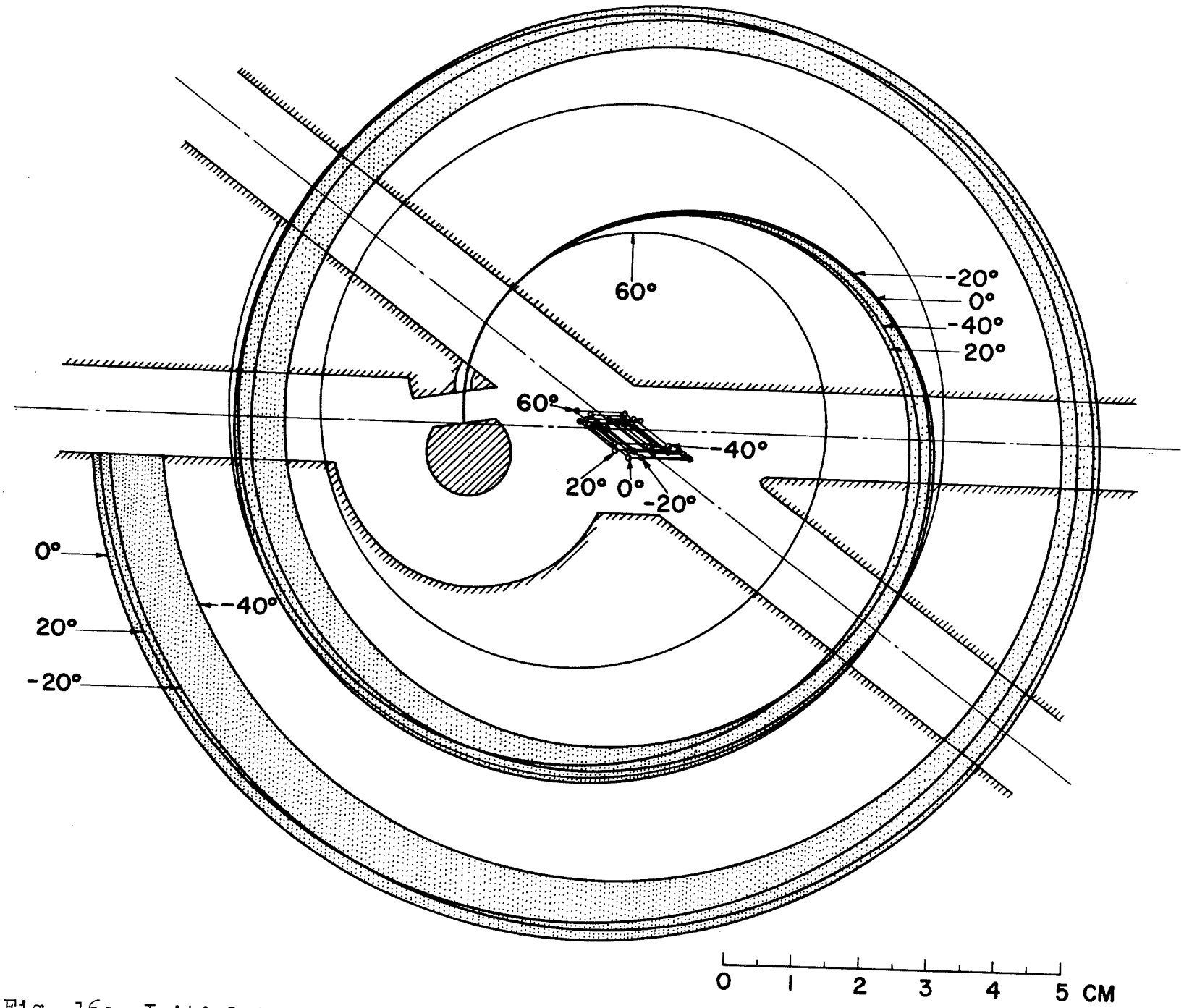

Fig. 16: Initial trajectories and orbit centers in the N=1 mode of the d.c.-injection scheme for different injection phases.

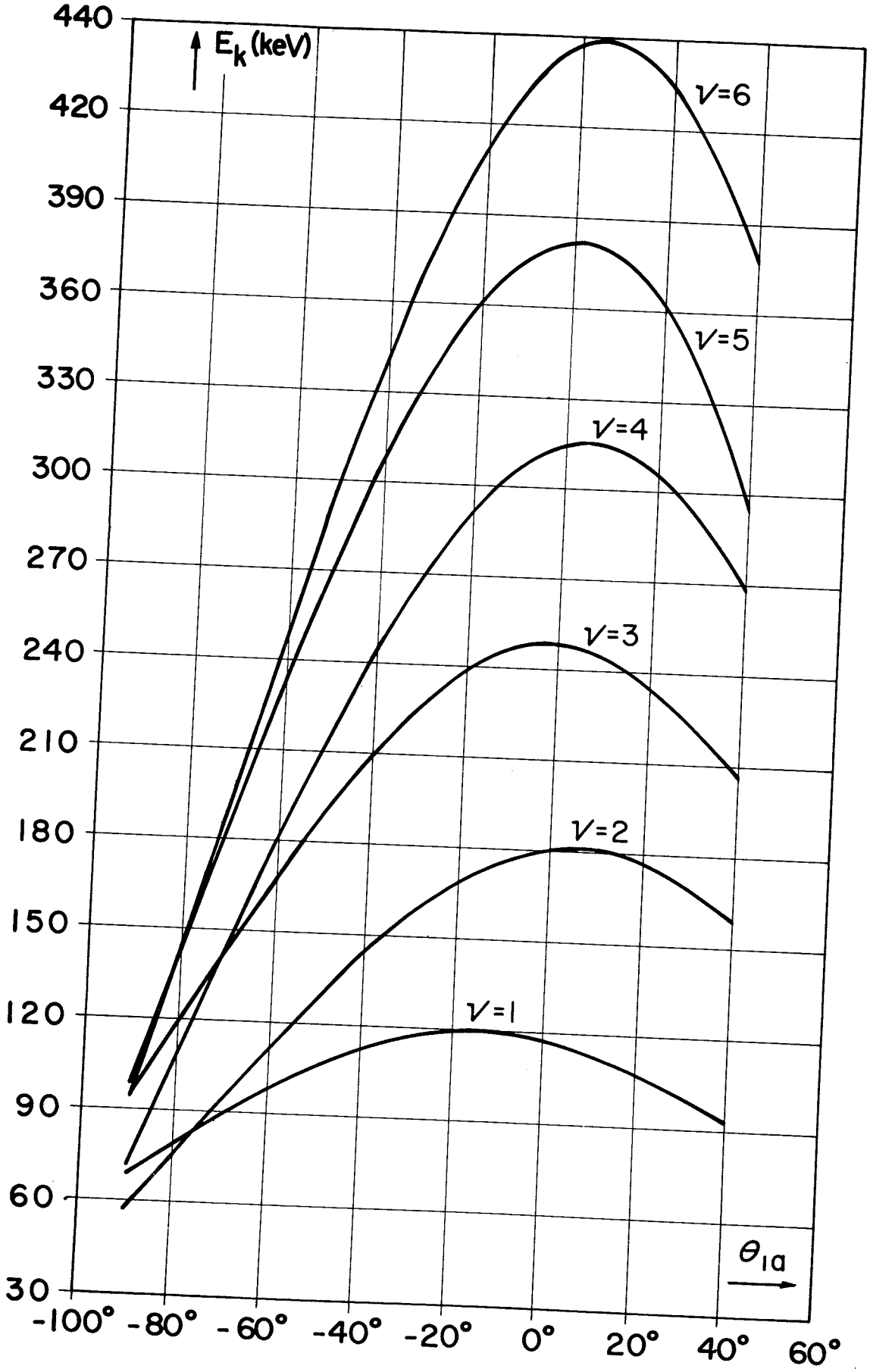

Fig. 17: Kinetic energy of the protons after consecutive gap crossings as a function of injection phase (N=1; d.c.-injection).

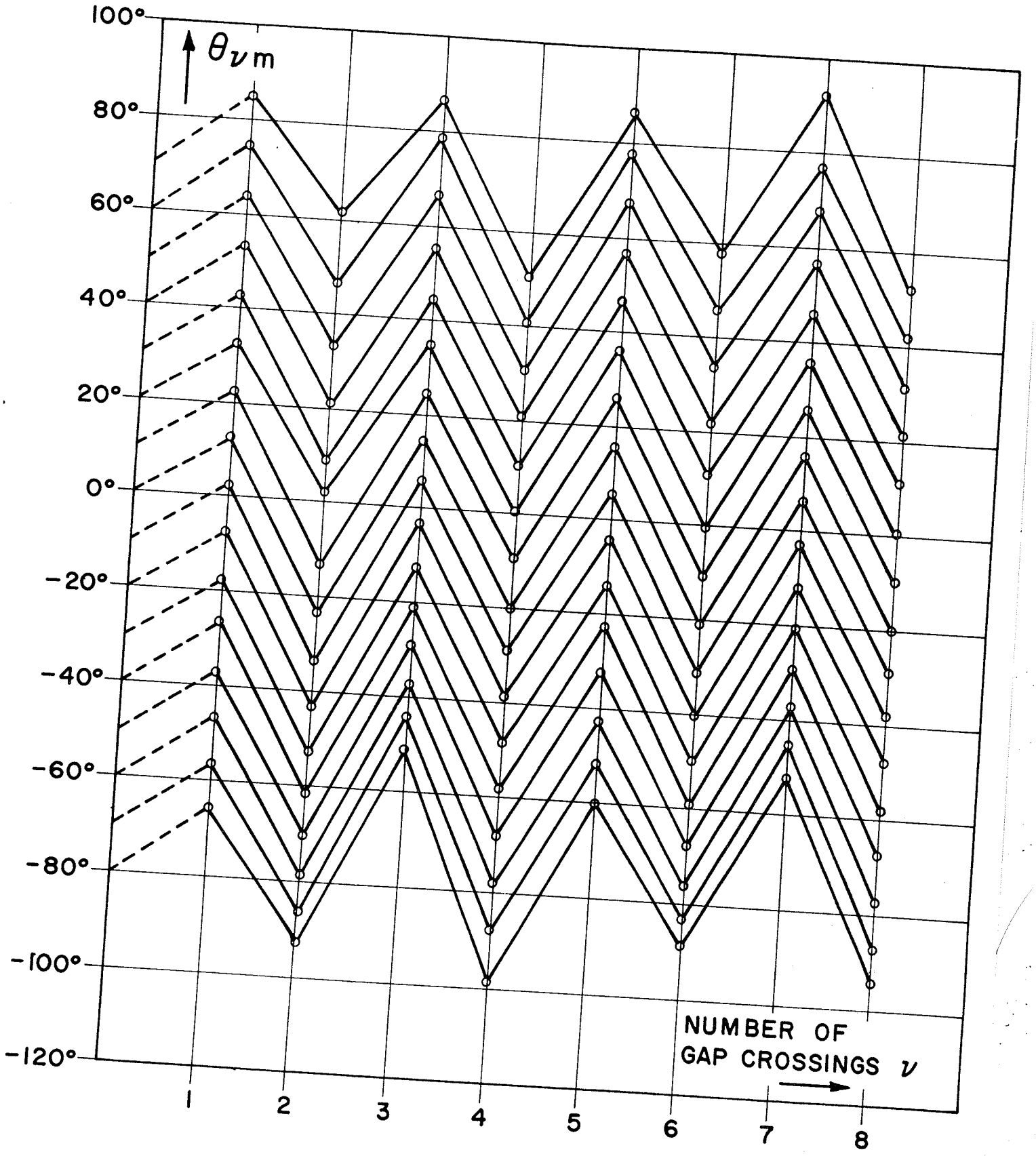

Fig. 18: Average phase of gap crossing for particles with different injection phase (N=1, d.c.-injection).

in subsequent gaps. A look at the phase history in Fig. 18 shows that the average phase of gap crossing $\theta_{\nu m}$, which determines the electric focusing effect, is predominantly negative for particles starting earlier than 0° and positive for the ions with positive starting phases. As in the equivalent normal-injection case there is practically no phase bunching. A phase shift of about 15° between the first and second gaps would certainly improve the situation still further (it would bring the energy maximum in all gap crossings to the most favorable starting phase of -15° and improve the electric focusing effect for this phase), but even without this shift, this particular case of d.c. injection can quite successfully compete with the N=1 acceleration mode in the normal-injection scheme.

N=2 (Deuterons): The injection voltage was 25 kV, the starting point was the same as for the protons (y_{1a} = 2.350 cm) and again the best results were obtained with an injection angle of 23.7° corresponding to $\dot{x}_{1a} = 1.4169 \times 10^{8}$ cm/s, and $\dot{y}_{1a} = 0.6221 \times 10^{8}$ cm/s; the r.f. peak voltage was 56.6 kV.

The computer results obtained were not very satisfactory compared with the previous case. It turned out that only the particles entering the first gap during the phase interval from -100° to -40° behaved reasonably to some degree, but even they, as Fig. 19 demonstrates, are diverging in their trajectories and their orbit centers are spread over an area with a 3-cm longitudinal diameter. This unfavorable situation can be understood best if one examines the relationship between

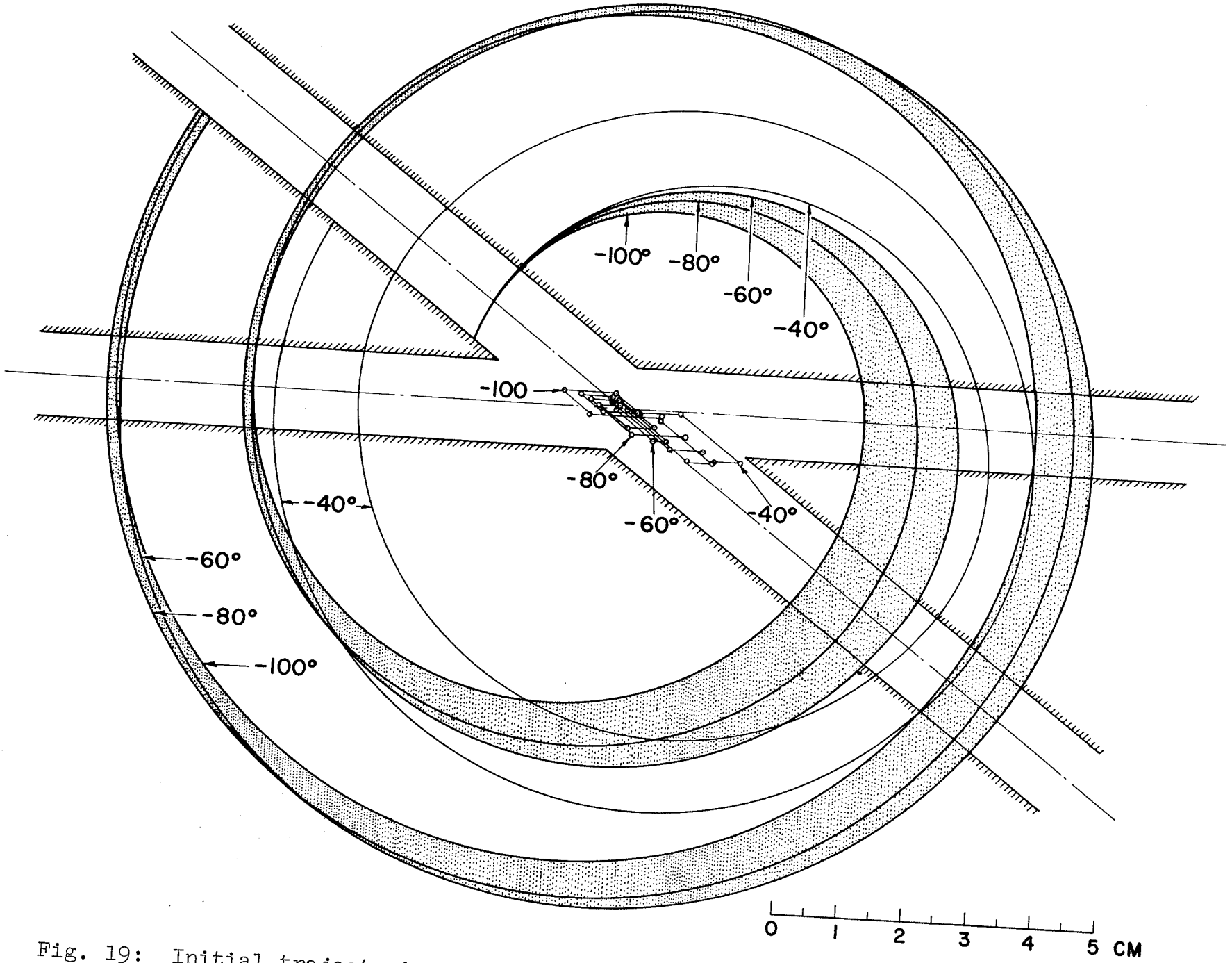

Fig. 19: Initial trajectories and orbit centers in the N=2 case of d.c.-injection.

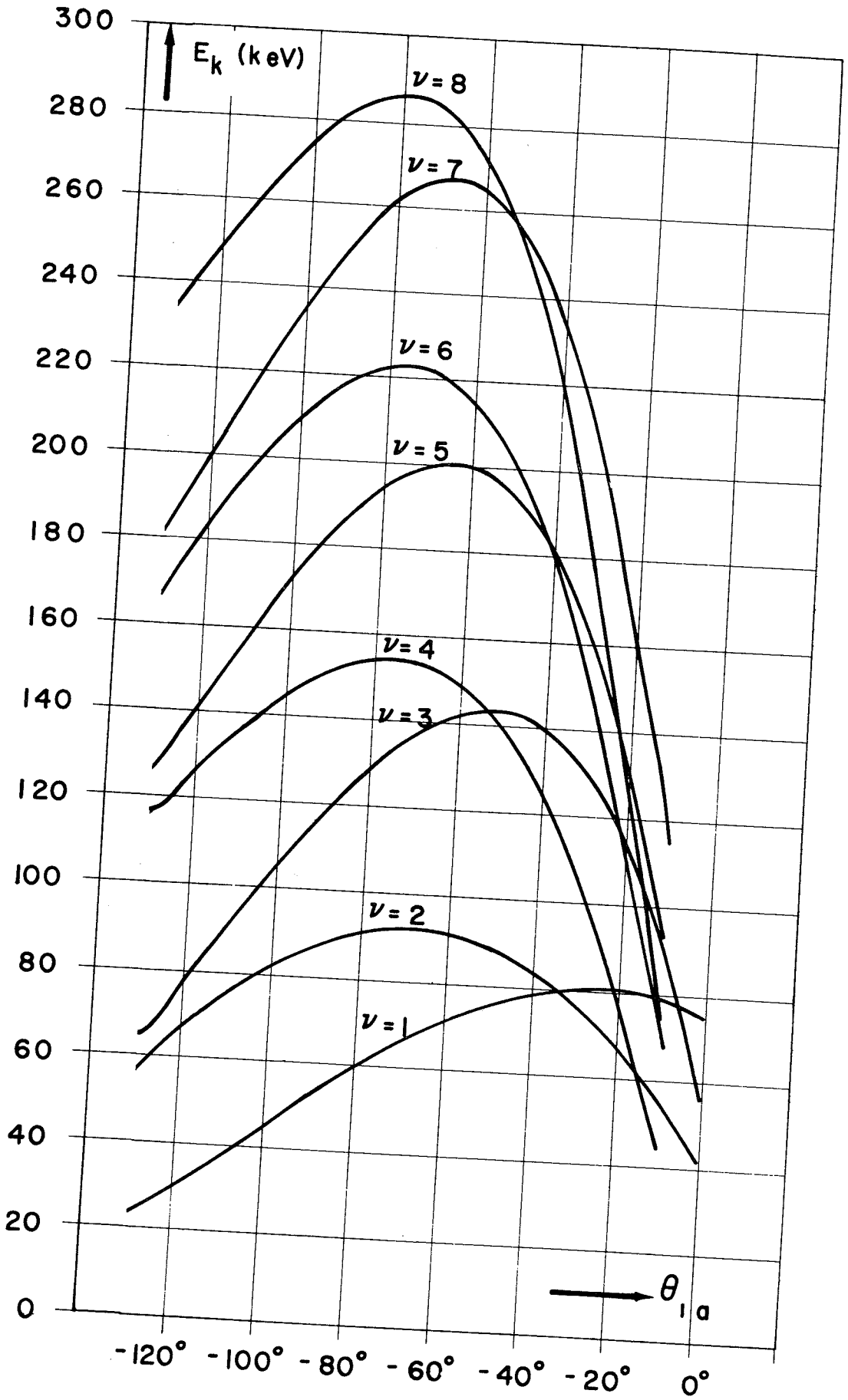

Fig. 20: Kinetic energy versus injection phase after consecutive gap crossings. (N=2, d.c. injection)

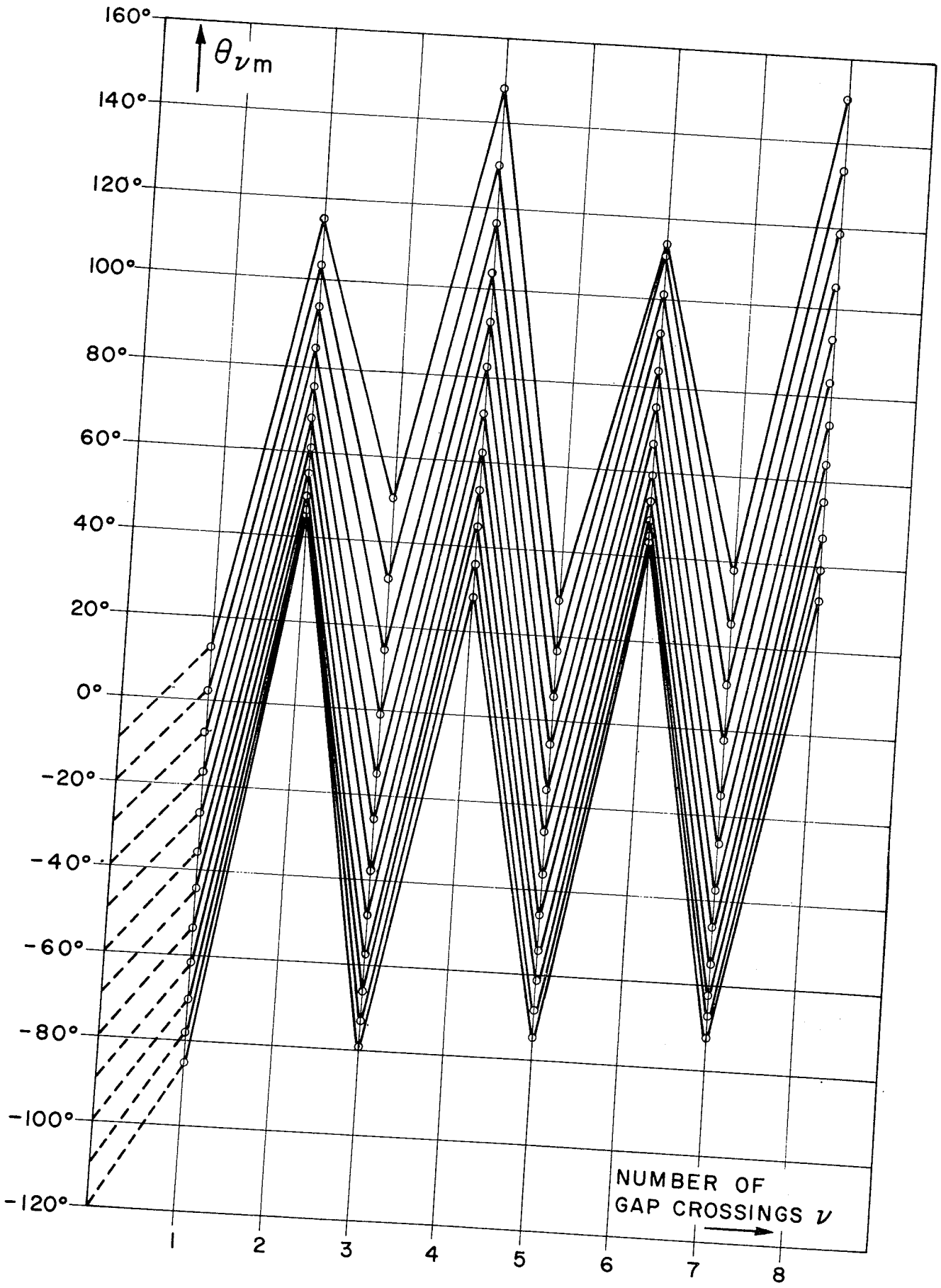

Fig. 21: Average phase of gap crossing. (N=2, d.c. injection)

energy gain and phase of gap traversal (i.e., the r.f. phase θ_{vm} where the ions cross the gap center lines). In the N=2 case the phase for maximum energy gain per turn is -56° in the gap where the ions enter the dees and $+56^{\circ}$ where they enter the dummy dees. In the first gap, where the transit time is relatively long, an average phase $\theta_{1m} = -56^{\circ}$ corresponds to a starting phase θ_{1a} of about -80°; the favorable phase interval in terms of maximum and uniform energy gain, however, is centered at the starting phase of -25°, as is shown in Fig. 20. The comparison of a 30° interval centered at -25° with one centered at -80° accentuates the great difference: In the first case the spread in kinetic energy at the end of the first gap is about 2%, and the corresponding spread in radius of curvature 1%, while in the second case the energy spread is more than 30%. In the normal-injection scheme, where source and puller were readjusted in each mode of operation, the favorable interval of starting phases was made to coincide with the maximum-energy-gain phase at the subsequent gap crossings, and therefore this difficulty did not occur.

From these considerations it is clear that the large difference in energy gain during the first gap traversal of particles starting at different phases in the "useful" interval about -80° is the direct cause for the large spread of initial orbit centers and the correlating divergence

of the orbits in Fig. 19. Only the trajectory belonging to the "best" starting phase of -80° is well centered, while the other orbits shown are more or less off-center. The ions injected at -20°, for example, start out with a large radius of curvature after the first gap and a correspondingly large displacement of the initial center point, but then are decelerated in the second gap and gain so little energy in the following gap crossings that their second turn overlaps with the first turn of particles starting closer to -80°. The phase history, plotted in Fig. 21, is quite similar to the equivalent normal-injection case (except that the dee angle between first and second gaps is not reduced) and therefore needs not to be discussed.

N=3 ($_{12}C^{4+}$): For the N=3 case the condition of constant orbit geometry required an injection voltage of 16.667 kV and a dee voltage of 37.7 kV. Again the particles were injected into the first gap at $y_{1a} = 2.350$ cm with an angle $\gamma = 23.7^{\circ}$, $\dot{x} = 0.9446 \times 10^{8}$ cm/s, and $\dot{y}_{1a} = 0.4147 \times 10^{8}$ cm/s.

After the previous discussion about the relationship between phase of gap crossing, maximum energy gain, energy spread, and spread of orbit centers it is clear that the situation in the N=3 mode should be likewise unfavorable and probably even worse than the N=2 case. The phase of maximum energy gain per turn is -56° if the particles enter the dummy

dees and 56° if they enter the dees; this situation means that the useful injection phase in the first gap is somewhere between 0° and 40° (depending on the transit time), i.e., in a region which is most unfavorable because of the large spread of initial energy gains and orbit centers for different starting phases.

The computer results, as plotted in the three following figures, confirm these pessimistic expectations. Looking first at the energy curves after consecutive gap crossings in Fig. 23, one sees that in the first gap the favorable group of ions, injected in the neighborhood of -40°, is considerably decelerated in the second gap and practically useless for the subsequent acceleration process. The particles which have the proper phase of -56° and +56° in the next gaps are starting at about 20°, i.e., in the undesirable fall-off region of the first energy curve. The trajectories in Fig. 22 reflect this extremely unsatisfactory situation very clearly. The particles starting at -40° with the largest energy and orbit radius after the first gap lose energy in the second gap, get off-center, and probably hit the source structure after the first turn. The "useful" particles injected at 20°, on the other hand, have a smaller radius after the first gap, but then, in a well-centered motion, experience the highest energy gain per turn. Nevertheless, the great disadvantages of this phase interval become evident if one looks at starting phases in the neighborhood of 20°. The large difference ...

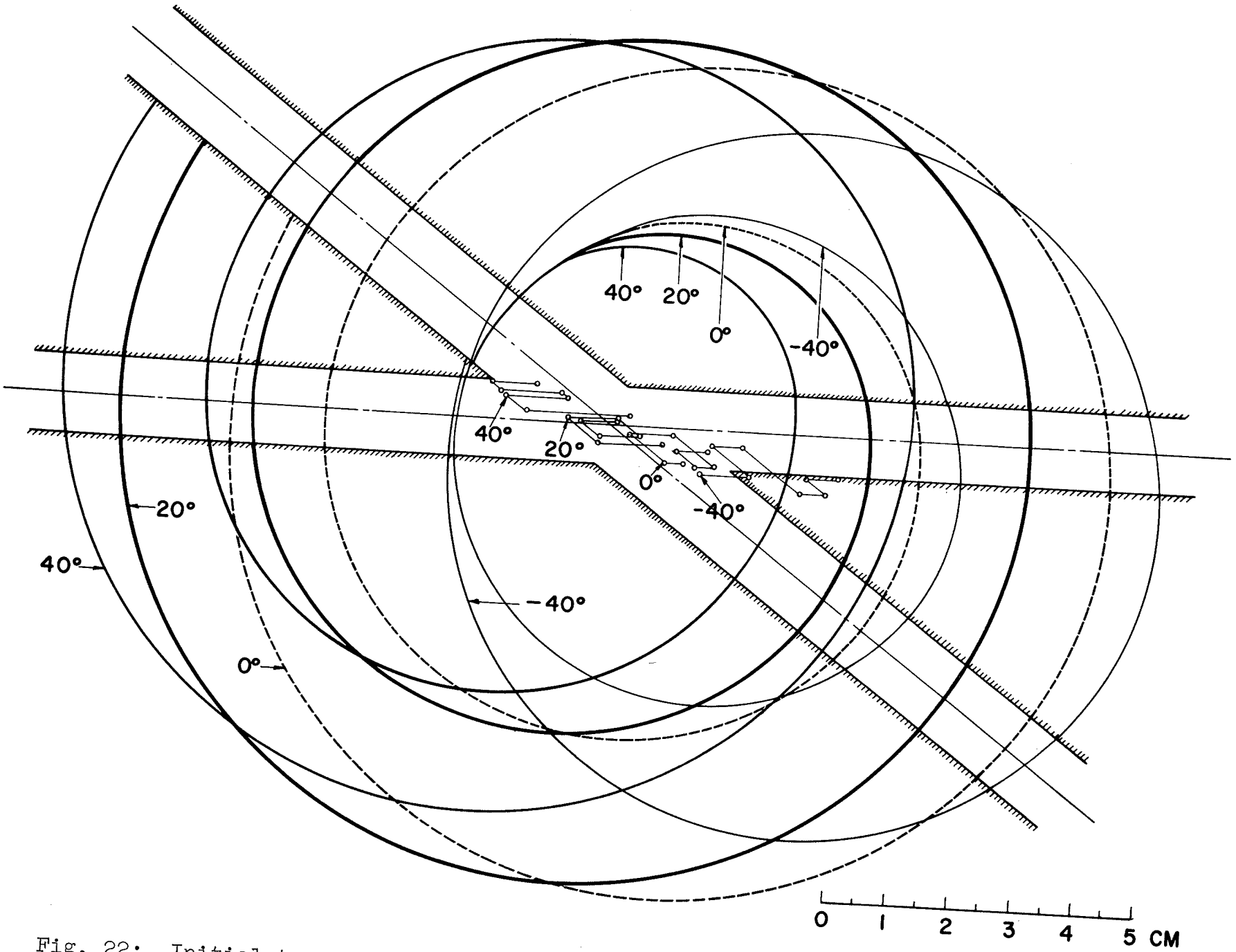

Fig. 22: Initial trajectories and orbit centers in the N=3 case of d.c.-injection.

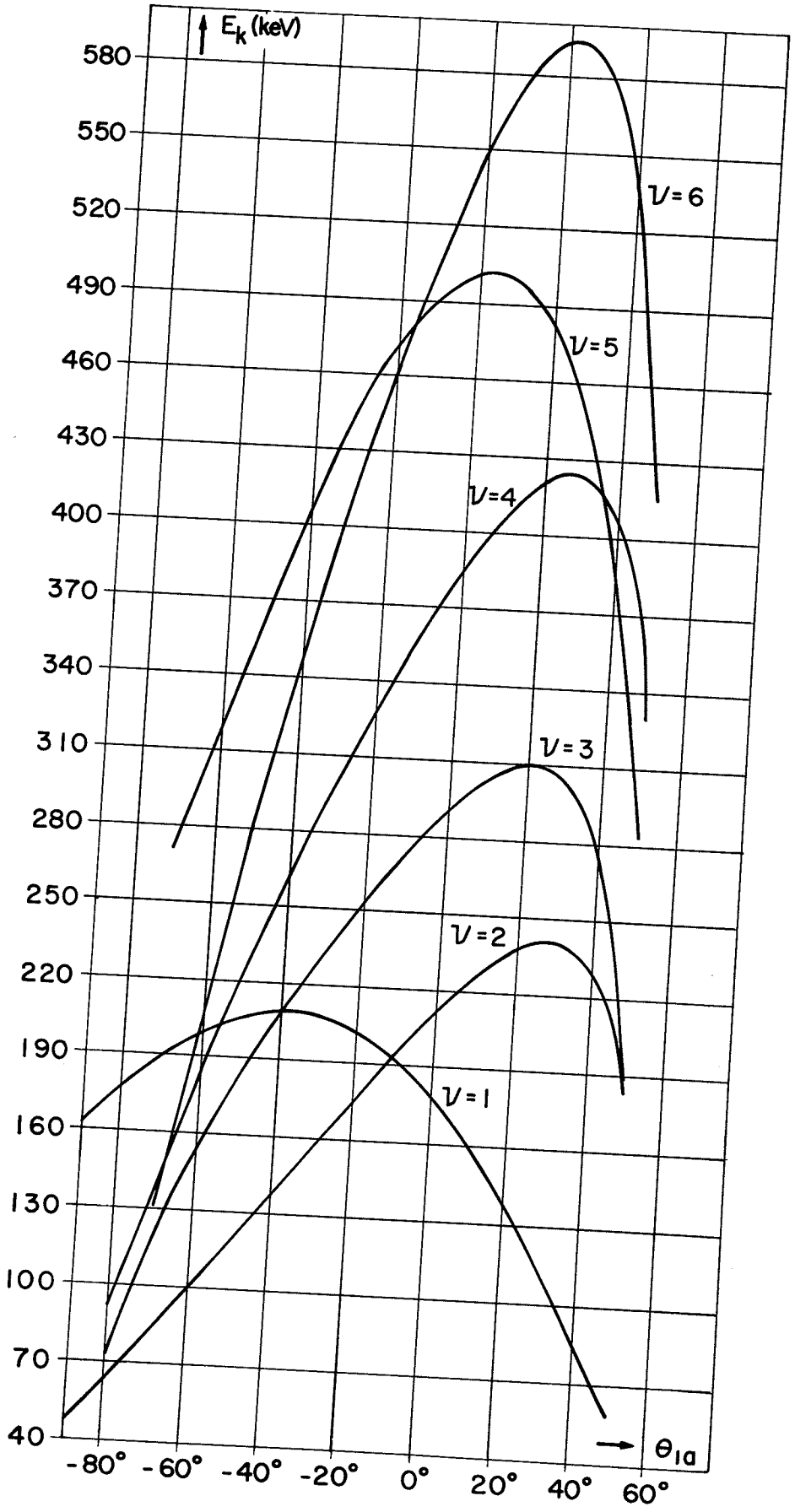

Fig. 23: Kinetic energy versus injection phase after consecutive gap crossings. (N=3, d.c. injection)

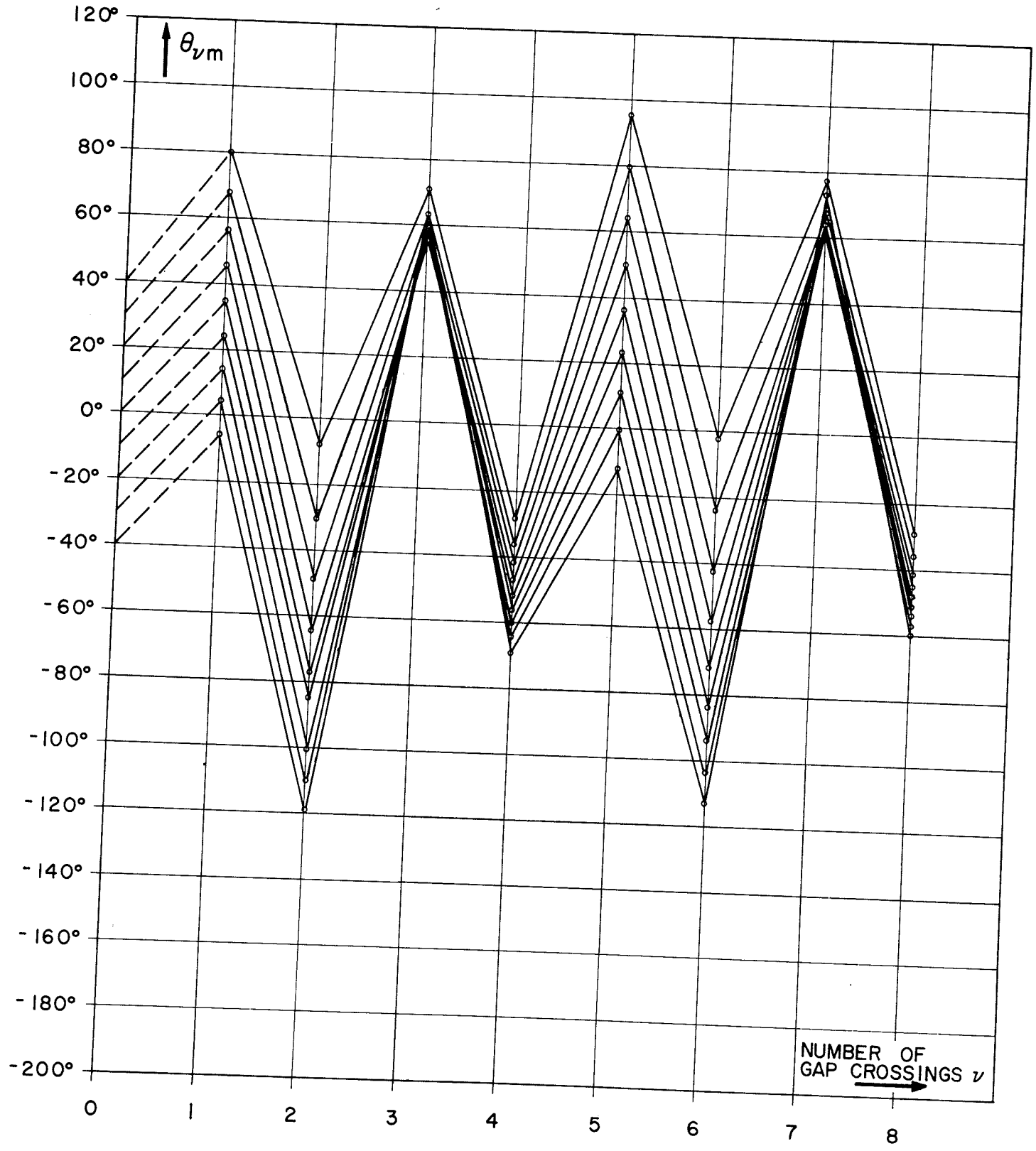

Fig. 24: Average phase of gap crossing (N=3, d.c. injection).

in the initial energy gains of particles starting earlier and those starting later than 20° results in an unusual amount of eccentricity and wide spread of orbit centers (about 4 cm) as demonstrated by the 0°- and 40°-trajectories, and so far the N=3 mode of operation looks even more unsatisfactory than the N=2 case. The phase history plot in Fig. 24 underlines the previous remarks and illustrates the irregular and asymmetrical pattern of the acceleration process.

5.3 Comparison of the Computer Results for the two Injection Schemes.

The computer results indicate that the objectives of the d.c. scheme, namely to get the same or possibly better beam quality than in the normal-injection case, were not achieved as well as one would like. Only the N=1 modes of the two schemes seem comparable, while in the cases of second- and especially third-harmonic acceleration the d.c.-injection results are unsatisfactory. This general conclusion drawn from the figures in the previous section can be substantiated in greater detail by a numeric comparison of the two schemes with respect to beam-quality properties.

The following table compares the most useful interval of starting phases in the d.c. scheme with the best interval of equal width in the normal-injection case. The first line shows the favorable starting phase if one considers the motion in the

	N = 1				N = 2				N = 3			
	Normal Injection		D.c. Injection		Normal Injection		D.c. Injection		Normal Injection		D.c. Injection	
	Gap 1	Gap 6	Gap 1	Gap 6	Gap 1	Gap 6	Gap 1	Gap 6	Gap 1	Gap 6	Gap 1	Gap 6
Favorable starting phase	-30°	——	-15°	——	-40°	——	-25°	——	-37°	——	-40°	——
Useful starting-phase interval	-60° to 0°	——	-30° to 30°	——	-60° to -20°	——	-100° to -60°	——	-50° to -10°	——	0° to 40°	——
Maximum energy E_{kmax} [keV]	68.9	401.2	119.3	447.4	55.1	222.5	70.9	222.1	146.9	607.3	178.7	594.1
Minimum energy E_{kmin} [keV]	60.6	355.4	100.0	385.9	52.2	199.4	44.6	208.7	132.8	561.7	86.8	538.8
Energy spread ΔE_k [keV]	8.3	45.8	19.3	61.5	2.9	23.1	26.3	13.4	14.1	45.6	91.9	55.3
$(\Delta E_k/E_{kmax})$ x 100	12.0%	11.4%	16.2%	13.7%	5.3%	10.4%	37.0%	6.1%	9.6%	7.5%	51.4%	9.3%
Maximum radius r_{max} [cm]	2.75	6.64	3.62	7.02	3.48	7.00	3.95	6.99	3.48	7.08	3.84	7.00
Minimum radius r_{min} [cm]	2.58	6.25	3.32	6.52	3.39	6.62	3.13	6.78	3.31	6.81	2.86	6.67
$\Delta r = r_{max} - r_{min}$ [cm]	0.17	0.39	0.30	0.50	0.09	0.38	0.82	0.21	0.17	0.27	0.98	0.33
$(\Delta r/r_{max})$ x 100	6.2%	5.9%	8.3%	7.1%	2.6%	5.4%	20.4%	3.0%	4.9%	3.8%	25.5%	4.7%
Radial width of beam [cm]	0.03	0.63	0.00	0.70	0.01	0.19	0.02	0.93	0.00	0.75	0.00	3.38
Maximum spread of centers [cm]	0.18	0.24	0.31	0.57	0.09	0.51	0.84	0.92	0.18	0.84	1.18	3.40
Width of phase interval	53.0°	51.8°	60.7°	60.7°	32.3°	36.7°	30.0°	26.6°	35.9°	35.8°	51.2°	71.0°
Ion-capture efficiency	45.6%	——	16.7%	——	26.3%	——	<11.1%	——	31.4%	——	<11.1%	——

Table II: Comparison of beam-quality properties.

first gap only. The next line gives the useful starting-phase interval, i.e., the range of starting phases centered at the phase of maximum-energy-gain per turn in the following gap crossings. (In the normal-injection scheme this phase coincides with the favorable starting phase for first-gap motion.) The following lines show the spread in kinetic energy, radius of curvature, and orbit centers, the beam-width, the width of the phase interval, and the corresponding duty cycle after the first and sixth gap crossings for the useful group of particles; the numbers for the ion-capture efficiency in the last line of the table are explained below.

In the N=1 mode a 60° interval extending from -60° to 0° in the normal-injection scheme and from -30° to 30° in the d.c. scheme was selected. This useful interval is in the normal-injection case centered at the favorable starting phase with respect to first-gap motion, while in the d.c. case the favorable phase is slightly off-center. In the first case the energy spread is 12.0% after the first gap crossing and 11.4% at the end of the sixth gap; the corresponding figures for d.c. injection are 16.2% and 13.7%, respectively. The maximum spread of orbit centers within the useful group of particles is relatively small, only a few millimeters, but again the d.c. case is less favorable than the normal-injection situation.

The figures for the width of the phase interval show that there is a small but insignificant amount of phase bunching in the case of normal injection which results from the fact that

transit-time effects have a greater influence if the particles start with zero energy and spend more time in the first electric field region. Here one has an indication that a d.c.-injection scheme, if properly designed, may improve the duty cycle somewhat.

In the case of second-harmonic acceleration the selected phase interval has only a width of 40°, extending from -60° to -20° (centered at the favorable starting phase of -40°) in the normal-injection scheme, and from -100° to -60° in the d.c.-case. In the d.c. case the useful phase interval does not coincide with the favorable starting phase, and as a result the energy spread after the first gap is very large and amounts to 37%. Although this unfavorable initial nonuniformity in energy gain is considerably reduced in the following gap crossings, it causes a large spread of initial orbit centers (of the order 1 cm), and this spread is not balanced out during the subsequent acceleration process. The phase-bunching effect is more pronounced than in the first-harmonic mode and, contrary to the previous case, is stronger in the d.c. scheme. This can be explained by the fact that the strongest phase bunching in the first gap occurs in the neighborhood of -90° (2), and in the d.c. case the useful interval of starting phases is closer to this region than in the normal-injection case. If one asks for a large duty cycle, then in this case d.c. injection brings no improvement at all, demonstrating how strongly this problem

depends on design details especially on the question of which starting-phase interval one can use for further acceleration.

In the N=3 mode, where again a phase interval of 40° is compared, the useful phase range in the case of d.c. injection is even more displaced from the favorable phase than it was in the second-harmonic mode. The energy spread after the first gap is here 51.4% resulting in a large initial distribution of centers over a range of about 1.2 cm. Again the nonuniformity in energy gain is largely balanced out in subsequent gap crossings, but the spread in orbit centers increases substantially and reaches the amount of 3.4 cm at the end of the sixth gap. The figures for the width of the phase interval after the first and sixth gap are in both cases not very informative since we know from Figs. 13 and 24 that there is a very pronounced alternating bunching and debunching process, and especially in the d.c. case the first and sixth gap are strongly debunching.

Very interesting is the comparison of injected current and ion-acceptance efficiency in both schemes. To simplify this problem, it is assumed that Child's Law can be applied, meaning that the current extracted from the source varies with the 3/2 power of the instantaneous voltage. Another assumption is that all ions starting in the useful phase interval are accepted. In a strict sense this is only justified for the N=1 case, where the beam quality is comparable in both schemes while in the second- and third-harmonic modes of the d.c. scheme the

acceptable phase range would be much smaller than the "useful" interval. Nevertheless, the figures for acceptance efficiency in the table refer to the useful phase interval, and in the N=2 and N=3 cases of the d.c. scheme it is indicated that the real values are smaller. The acceptance efficiency is defined as the ratio of the number of ions leaving the source in the useful phase interval to the total number of injected ions during one r.f. period. This efficiency is naturally smaller in the d.c. scheme than in the normal-injection case because of the continuous source emittance during the whole period in the d.c. case.

The numbers for the ion-capture efficiency alone do not yet give a full picture of the situation. More informative is a comparison of the injected and accepted currents. Taking the N=1 case as an example, let us suppose that the peak proton current in the normal-injection scheme is 100 mA, the peak r.f. voltage being 70 kV. Then the total current emitted from the source during the negative half period is roughly 27.8 mA and the accepted current about 12.7 mA (with the calculated efficiency of 45.6%). In the equivalent d.c. case with a voltage of 50 kV, if we assume that the spacing d between extraction electrode and ion source is decreased proportional to the voltage, we obtain a total current of about 118.3 mA according to Child's Law, and the accepted current would be 19.7 mA. This means that the useful current in the d.c. scheme is about 35% larger than in the normal-injection scheme but at the same time one has in the d.c. case more than six times as much unwanted ion current than in the case of normal injection.

6. Conclusions.

The numeric comparison of the theoretical results for the two injection schemes shows that, in terms of beam quality, the normal-injection scheme is superior to the d.c.-injection system. If one includes the general considerations of Section 5.1, one may summarize the various aspects of this comparison as follows:

(a) Both schemes employ a special "parameter" or mechanism to achieve the desired optimum conditions for efficient ion injection and acceptance. In one case this special parameter is the possibility to readjust the source and puller radially and especially azimuthally; in the other case it is the d.c. potential of the source.

(b) In either case a number of technical problems are associated with this additional feature, but it seems that the difficulties as well as the costs involved are comparable, or at least do not favor one of the schemes significantly.

(c) Because of this fact the decisive criterion is mainly the beam quality (energy spread, centering of the orbits, etc.).

(d) In the particular cyclotron considered here, with its various modes of operation, the possibility of placing the injection point and the first acceleration gap at different azimuthal positions is clearly the better method of optimizing the starting conditions and ion capture

efficiency. The reason is that the spread of orbit centers of particles with different starting phases strongly depends on the energy gain in the first gap and the correlated-initial center distribution. The freedom of readjusting the injection system makes it possible to program the transit angle between first and second gap so that the favorable starting phase for the first gap is shifted to the phase of maximum-energy-gain per turn at the following gap crossings.

(e) With respect to duty cycle the d.c.-injection scheme discussed here did not give a significant improvement compared to the normal-injection scheme. One should note, however, that in both schemes (especially for the N=1 mode) the duty cycle seems to be larger than is normally reported from cyclotrons. (The 60° interval of useful starting phases in the N=1 mode was not the upper limit one can get; with some sacrifice of beam quality it can be extended to about 80° or 90°.) The results indicate that with a conventional grounded source one can considerably improve the duty cycle by proper design of the central geometry, i.e., by taking advantage of the favorable starting-phase interval with uniform energy gain and by reducing the transit-time and phase-bunching effects. D.c.-injection may achieve equally high or even somewhat better duty factors only if the same degree of adjustment is provided to optimize the starting conditions and if

the injection voltage is not too small.

Finally, a few questions which were not specifically discussed in this investigation may be raised. To what extent are the results obtained influenced by the idealized assumptions made in the uniform-field mode? In particular, what effects and changes can be expected from the fact that the real electric field distribution is nonuniform and the "gaps" broader and less defined than assumed? What effects are produced by the finite width and initial momentum distribution, i.e., the full phase-space emittance of the source? How effective is electric focusing?

To the first of these questions one can argue that the basic conclusions regarding beam quality, especially the relationship between favorable starting-phase interval and the subsequent acceleration process, result from the wave shape of the accelerating voltage and the geometry of the dee system. If the gaps are broader than assumed (d.c.-injection scheme with no field defining slits along the first turn) then the transit-time effects are stronger, i.e., there is more phase bunching and smaller energy gain; the favorable phase interval shifts more towards the -90° starting phase and so increases the mentioned difficulties in the N=1 and N=3 modes of the d.c.-injection scheme. In the normal-injection scheme the important first r.f. gap is well defined by the source-to-puller arrangement, only the energy gain and radius of curvature in the following gaps may be somewhat smaller than

calculated due to field overlapping and stronger transit-time effects.

The fact that the ion source has a finite output slit (the radial slit width is usually in the order of 1 mm) means that upon each individual trajectory shown in the figures a "beam pattern" is superimposed with radial oscillations whose amplitudes depend on the source emittance and the field-nonuniformities. The main objective of this investigation was to study the effects of starting phase and phase of gap crossing on ion-acceptance efficiency and beam quality (and the r.f. phase is certainly the primary factor in this regard). The uniform-field model is a very useful tool for this type of investigation, but is not suited for the study of radial oscillations and detailed beam shape. In the latter case numeric integration with the electrolytic-tank field or with the so-called "Silax Code" provides much better information. The "Silax Code" has been developed by H. G. Blosser, M. M. Gordon, and T. I. Arnette; it calculates both radial and axial motion in a simulated electric field distribution derived theoretically from an undistorted, symmetric dee geometry.[5] The results of the computer runs with this code indicate that electric focusing is quite effective for the group of particles which cross the gaps in the most useful phase interval of maximum-energy-gain per turn.