MSUCP-17

MICHIGAN STATE UNIVERSITY

CYCLOTRON PROJECT

Cyclotron Duty-Factor Improvement by Reduction of Phase Bunching in the Central Region

M. Reiser

March 1963

Department of Physics

East Lansing, Michigan

---

*Research Supported in part by the National Science Foundation (Grant NSF-G19978)

Cyclotron Duty-Factor Improvement by Reduction of Phase Bunching in the Central Region.*

M. Reiser

Michigan State University
East Lansing, Michigan
March, 1963

ABSTRACT

In an isochronous cyclotron, if negative ions are accelerated and beam extraction is accomplished by electron stripping, the duty cycle of the transmitted beam is essentially determined by the phase-bunching effect in the central region, the radial stability limit of the magnetic field, and the tolerable maximum spread of kinetic energy at deflection radius. The effect of phase grouping and the associated spread of orbit centers are analyzed for different situations of central geometry in the case of conventional as well as d.c. injection. The results of the calculations show that it is possible to substantially reduce the phase-bunching effect, which is the main cause for the low duty factors in conventional cyclotron beams, and produce a central beam with a pulse length approaching 180° by (a) operating at low magnetic fields and high dee voltages to increase the orbit radius on the first turns, (b) limiting the electric field to a narrow gap by attaching diaphragms to the dees in the center, and (c) if necessary employing a d.c. injection scheme.

*Research supported in part by the National Science Foundation.

1. Introduction.

One of the dominant factors determining the low duty cycle of conventional cyclotron beams is the phase-grouping effect which takes place during the initial revolutions in the center where the ions spend most of their rotation period within the electric-field region. The existence of phase bunching in the central region of the cyclotron was first proved theoretically by Bohm and Foldy[1] and later investigated in more detail by Cohen.[2] In both cases the calculations were based on the assumptions of uniform electric and magnetic fields. Direct phase measurements of deflected[3-5] as well as internal[6-10]

---

1) D. Bohm and L. Foldy, Phys. Rev. 72 (1947), 649.

2) B. L. Cohen, Rev. Sci. Instr. 24 (1953), 589.

3) M. Jakobsen, J. H. Manley, Phys. Rev. 95 (1954), 600.

4) S. D. Bloom, Phys. Rev. 98 (1955), 233.

5) F. Tinta, N. Martalogu, R. Dumitrescu and T. Magda, Nucl. Instr. and Meth. 12 (1961), 138.

6) M. Konrad, Rev. Sci. Instr. 29 (1958), 840.

7) M. Reiser, Nucl. Instr. and Meth. 13 (1961), 55.

8) A. J. Cox, D. E. Kidd, W. B. Powell, B. L. Reece and P. J. Waterton, Nucl. Instr. and Meth. 18-19 (1962), 25.

9) W. H. White, B. Duelli and R. J. Jones, Nucl. Instr. and Meth. 18-19 (1962), 601.

10) C. G. Dols, Nucl. Instr. and Meth. 18-19 (1962), 595.

ion beams confirmed that the length of the ion pulse is only a very small fraction of the r.f. period: typically 10° to 20°, sometimes up to 30° (out of 360°) depending on operating conditions.

For a certain group of experiments, like time-of-flight spectrometry of fast neutrons, the short pulse duration of the cyclotron beam is a desired feature. In other investigations, however, involving coincidence measurements, it is a great disadvantage, and this fact started the present discussions of how to improve the cyclotron duty cycle[11-13]. One possibility of increasing the duty factor is to aim at a reduction of the phase-bunching effect in the center. A first step in this direction is the insertion of "feelers" or an extractor electrode at the dee opposite the ion-source output slit, a technique which is now used in almost every operating cyclotron to improve the source output and the initial conditions. The use of "feelers" is equivalent to injecting the ions with a certain initial velocity and this decreases the phase-bunching effect, as was already pointed out by Cohen[2].

---

11) R. M. Eisberg, NAS-NRC 656 (1959), 289.
12) Tat K. Khoe, Nucl. Instr. and Meth. 18-19 (1962), 606.
13) A. I. Yavin, Nucl. Instr. and Meth. 18-19 (1962), 610.

Another possibility is d.c. injection, which is often discussed and has been employed for the first time in Birmingham[8]. According to the uniform-field theory the pulse length under ideal phase-bunching conditions is somewhat less than 10° for a starting-phase interval of 180°. Pulse durations of 20° to 30° have been measured in Birmingham as well as in other places with a normal injection scheme and illustrate the improvements made so far with either d.c. injection or the normal puller technique.

Phase bunching is of course not the only effect which limits the duty cycle. Especially if one aims at larger pulse durations other factors like electric defocusing, the transmission capability of the magnetic field (radial phase-space stability limits) and most of all the extraction process become more and more important. The resonance-extraction scheme, for example, is highly phase selective and, depending on final energy or number of turns, only a narrow phase interval of a few degrees of the r.f. period can be efficiently extracted,[14][15] and in this case a high duty cycle of the internal beam would be undesirable. No such restrictions exist, however, if negative ions are accelerated and beam deflection is accomplished by electron stripping. In this

---

14) H. G. Blosser and M. M. Gordon, Nucl. Instr. and Meth. 13 (1961), 101.

15) A technique to improve this situation by sacrificing in beam quality has been discussed by H. G. Blosser and M. M. Gordon in Nucl. Instr. and Meth. 18-19 (1962), 488.

investigation it was therefore assumed that extraction is performed by the stripping method (or any other mechanism which is not phase selective) in which case the duty cycle depends mainly on the conditions in the center and the magnetic stability limits. The relative importance of various central-region factors is being discussed and an estimate was made of possible improvements of the pulse duration (beyond the 30° observed so far) by a comparison of several different cases of central geometry.

2. General Considerations.

The amount of phase bunching in an isochronous cyclotron depends very strongly on the operating conditions and the central geometry, both of which determine the energy gain, the radius of curvature, and the fraction of r.f. period the particles spend in the electric field during one revolution. One distinct possibility is the ideal phase-grouping situation in the open-dee geometry where the first turn of the ions is completely within the electric-field region, i.e., the radius of curvature r at the first revolution is smaller than the half width d of the electric-field gap. In this case all particles starting at the source within the accelerating half period of the r.f. voltage are sharply bunched in phase to a narrow pulse of less than 10° length after the first turn.

There is, however, a large spread of kinetic energy and orbit centers and only the group of particles whose orbit centers are within a certain radial tolerance limit Δr_{tol}, which will be explained below, are useful for further acceleration. The duty cycle is thus practically determined by the phase-bunching effect alone, while the interval of useful starting phases, i.e., the acceptance rate, is defined by the tolerance limit. The other distinct situation is the narrow-gap case where the electric field is confined to a small region (e.g., by insertion of diaphragms) and the time of gap crossing is only a very small fraction of the r.f. period ($r \gg d$). In this case no phase bunching occurs at all, and the actual pulse length is defined by the accepted interval of starting phases, i.e., both duty cycle and acceptance rate are directly determined by the radial tolerance limit.

The acceptable spread of initial orbit centers in an isochronous cyclotron with stripping extraction is determined by the radial stability limit of the magnetic field and the tolerable spread of final energy at deflection radius. If, for example, the relative energy spread $\Delta E_k/E_k$ of the negative ions hitting the stripping foil at a given deflection radius r_d is to be smaller than 1% then the initial distribution of orbit centers must stay below the value $\Delta r_e = 5 \times 10^{-3} r_d$ or below the magnetic stability limit Δr_{st}, whichever factor is smaller. The parameters Δr_e and Δr_{st} thus define what was called the radial tolerance limit Δr_{tol}.

The previous description gives still an incomplete and somewhat simplified picture of the real situation. There are some other factors which also influence to some extent the duty cycle and which will be discussed briefly. One important factor is the geometric structure of the injection system (ion source, extractor electrode, etc.) which forms a barricade for all particles with too low energy gain due to an unfavorable starting phase. In the theoretical analysis this obstacle can be represented by a disk stopping all particles whose radius of curvature at the first turn is smaller than the radius D/2 of this disk. Another factor is the electric focusing effect which is strongly phase sensitive and results in vertical defocusing for ions crossing the electric gap far ahead of the peak-voltage phase, i.e., at large negative phases, and in overfocusing for those which are lagging too far behind (large positive phases). The number of particles actually lost to the dee structure depends very strongly on details of the central-region layout like the ratio of the vertical height of the source output slit to the internal dee height, the strength of the vertical electric forces compared to the magnetic focusing effect, etc. There is no sharp limit for the phase of gap-crossing since only the off-median-plane particles are effected, i.e., electric defocusing or overfocusing may cut off a certain fraction of the group of particles which start at too early or too late phases of the r.f. voltage, but the particles close to the median plane will always pass through this region. So far this effect has only a partial and indirect influence on the duty cycle. Besides, electric defocusing can be substantially reduced if one programs the phase history for the particles that are within the tolerance limit; if desired one could almost completely eliminate electric defocusing by inserting diaphragms with long vertical slits (like the focusing grids) on both sides of the accelerating gap and by providing the necessary vertical focusing with the magnetic field alone. A similar argument applies for the radial oscillations and beam width. The finite width of the source-output slit or source emittance means that all particles starting at a particular moment of the r.f. period cover a certain area $\Delta r \Delta p_r$ in radial phase space and only the fraction lying within the stability or tolerance limit is being accepted.

For the duty-cycle calculations these effects, resulting from the finite vertical and radial width of the source-output slit, shall be neglected and only the energy-time relationship will be analyzed. In other words, the ion source is considered a point source and only the radial motion or median-plane trajectories are being investigated for various situations. We are then concerned with the problem of calculating the duty cycle of the acceptable fraction of the beam for different operating and starting conditions for given values of the parameters Δr_{tol} and D. The operating conditions, i.e., the central magnetic field B_o, the dee voltage U_o, the specific charge e/m of the ions, and the half width d of the electric-field region, can be represented by the similarity parameter χ with

$$(1) \qquad \chi = \frac{d^2 B_o^2 e/m}{U_o} \quad .$$

The starting conditions are the position of the source-output slit x_o, y_o, the initial velocity $\dot{x}_o, \dot{y}_o$ or injection voltage U_i, and the starting phase θ_o with respect to the r.f. voltage $U = U_o \cos(\theta_o + \omega_e t)$. In order to obtain a quantitative estimate we consider the particular case of an isochronous cyclotron with two dees of 180^o in first-harmonic, push-pull mode, accelerating negative hydrogen ions (H^-) with $U_o = 70$ kV and $B_o = 13{,}774$ gauss. For the radial tolerance limit a value of $\Delta r_{tol} = 0.5$ cm was assumed and for the "disk" diameter D = 2.0 cm (this value is larger than the actual diameter of the arc chamber, taking into account that the immediate neighborhood of the source is not useful due to the distorted electric field). Two situations for the electric field region were considered, the open-dee or large-gap geometry and the narrow-gap case, and the calculations were carried out for both normal injection ($U_i = 0$) and d.c. injection with an injection energy corresponding to $U_i = 70$ kV.

3. Open-Dee Geometry (Large Gap, $r \leq d$).

3.1 Radial Grouping.

For the calculation of initial trajectories in the large-gap case we neglect the distortions produced by the ion source and consider an idealized two-dee geometry. The electric field configuration in this case depends on the ratio of dee spacing 2k to dee height 2h and has been both measured[16) and calculated[17)18) for various values of the parameter k/h. For our purpose we choose the values $k/h = 1.0$ and $h = 2.0$ cm, for which case the field and potential distributions in the median plane are plotted in Fig. 1. It is seen that the field penetrates deep into the interior of the dee, reaching the potential of 90% at about $x = 4.0$ cm, i.e., 2.0 cm beyond the dee edge. The median-plane trajectories in this field were calculated numerically with the Cartwheel Code[19) for different starting phases $-90^\circ \leq \theta_o \leq 90^\circ$ (with initial conditions $x_o = y_o = \dot{x}_o = \dot{y}_o = 0$ and with the operating conditions

---

16) R. R. Wilson, Phys. Rev. 53 (1938), 408.

17) R. L. Murray and L. T. Ratner, Journ. Appl. Phys. 24 (1953), 67.

18) J. W. Beal, MSU-Cyclotron Report 12 (1961).

19) T. I. Arnette, H. G. Blosser, M. M. Gordon and D. A. Johnson, Nucl. Instr. and Meth. 18-19 (1962), 343.

specified in the previous section) and are plotted in Fig. 1. The picture shows the typical pattern known from uniform-field calculations (the differences with respect to the uniform-field case will be discussed below): the large radial spread after the first half turn, followed by sharp radial grouping of all trajectories at the end of the turn. Notice that there are two very accentuated nodes or "focal points" before and after the particles cross the center line (x=0) at the left side. To illustrate the phase-bunching effect, the positions of the particles at times $\theta = \theta_o + \omega_e t = \pi$ and $\theta = 2\pi$ are marked. The dotted curves show the actual shape of the "sausage" formed by the whole group of ions at these moments; especially the curve at the end of the first turn illustrates how close the particles are together, i.e., bunched in phase, as they cross the center line.

To evaluate the relative influence of the different parameters on phase bunching, radial grouping, etc., in this large-gap situation, one could now carry out a great number of similar computer runs varying the initial and operating conditions as well as the geometry in a systematic way. But, as we shall see, the uniform-field approximation gives a very accurate description of the energy-phase relationship we are interested in, and we shall therefore employ this analytic approach. Following in part the pioneering work of Cohen we shall derive a few formulas which describe the effects of

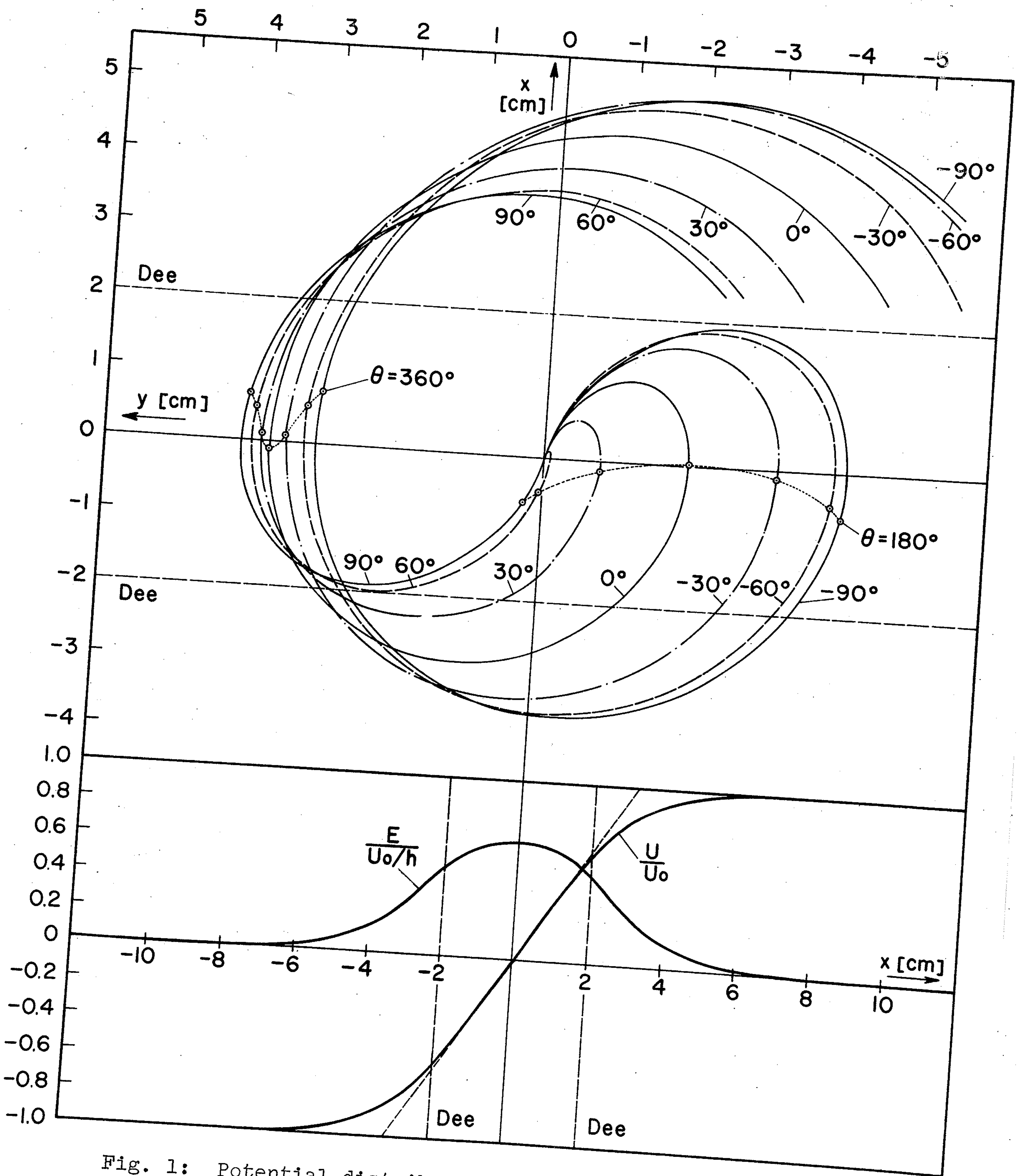

Fig. 1: Potential distribution and initial trajectories of H^- ions with different starting phases θ_o in an open-dee geometry. Dee spacing $2k = 4$ cm, dee height $2h = 4$ cm; dee-to-ground voltage $U_o = 70$ kV, magnetic field $B = 13.774$ kG.

radial grouping, phase bunching, and spread of orbit centers as functions of the operating parameter χ and the starting conditions. Using the coordinate system of Fig. 1 and assuming that the ions are injected at $x_o = 0$, $y_o = 0$ with initial velocity $\dot{x}_o \neq 0$, $\dot{y}_o = 0$ at $t = 0$, integration of equations of motion leads to

(2) $$\frac{x}{d} = \frac{1}{2\chi} [\tau \sin(\theta_o+\tau) - \sin\theta_o \sin\tau] + \frac{\dot{x}_o}{\omega d} \sin\tau,$$

(3) $$\frac{y}{d} = \frac{1}{2\chi} [\tau \cos(\theta_o+\tau) + 2 \sin\theta_o - 2 \sin(\theta_o+\tau) + \cos\theta_o \sin\tau] - \frac{\dot{x}_o}{\omega d} (1-\cos\tau),$$

(4) $$\frac{\dot{x}}{u} = [\sin(\theta_o+\tau) - \sin\theta_o] + \chi \frac{y}{d} + \frac{\dot{x}_o}{u},$$

(5) $$\frac{\dot{y}}{u} = -\chi \frac{x}{d},$$

where $\tau = \omega_e t$, $\omega_e = \omega$, and

(6) $$u = \frac{U_o}{dB_o} = \frac{\omega d}{\chi}.$$

The coordinates after consecutive half turns, i.e., at times where

(7) $$\theta_o + \tau = \nu\pi \qquad (\nu = 1, 2, 3, \text{etc.}),$$

are

(8) $$\frac{x}{d} = \frac{(-1)^\nu}{2\chi} \sin^2\theta_o - \frac{\dot{x}_o}{\omega d}(-1)^\nu \sin\theta_o ,$$

and

(9) $$\frac{y}{d} = \frac{(-1)^\nu}{2\chi} \left[\tau + (-1)^\nu\, 2 \sin\theta_o - \frac{1}{2}\sin 2\theta_o\right] - \frac{\dot{x}_o}{\omega d}\left[1 - (-1)^\nu \cos\theta_o\right] .$$

These formulas are only valid in the region where $x/d \leq 1$, i.e., as long as the trajectories are within the electric-field gap. From equation (9) we can derive some interesting relations. The points where the particle starting at $\theta_o = 0$ crosses the y axis are

(10a) $$\frac{y}{d} = -\frac{\nu\pi}{2\chi} + 2\frac{\dot{x}_o}{\omega d} \quad \text{for } \nu = 1, 3, 5, \text{etc.},$$

and

(10b) $$\frac{y}{d} = \frac{\nu\pi}{2\chi} \quad \text{for } \nu = 2, 4, 6, \text{etc.}$$

It is seen that injection with initial velocity $\dot{x}_o$ moves the points at the negative y axis farther away from the starting point, but leaves the points at the positive side unchanged. Another conclusion is that the spacing Δs between turns at the center line is always constant, i.e.,

$$\Delta s = \frac{y}{d}\Big|_{\nu+2} - \frac{y}{d}\Big|_{\nu} = \frac{\pi}{\chi} \, . \qquad (10c)$$

The total width Δy of the family of trajectories with $-\frac{\pi}{2} \leq \theta_o \leq \frac{\pi}{2}$ at consecutive half turns is constant and independent of the injection velocity $\dot{x}_o$:

$$\Delta y = \frac{1}{2\chi} [4 + \pi], \quad \text{for} \quad \nu = 1, 3, 5, \text{etc.}, \qquad (11a)$$

and

$$\Delta y = \frac{1}{2\chi} [4 - \pi], \quad \text{for} \quad \nu = 2, 4, 6, \text{etc.} \qquad (11b)$$

The trajectories have a large radial spread when they cross the negative y axis, but are strongly bunched after each full turn, the radial width at this moment being a small fraction of the turn separation

$$\frac{\Delta y}{\Delta s} = \frac{4-\pi}{2\pi} = 0.1366 \, . \qquad (12)$$

While the initial velocity $\dot{x}_o$ does not effect the total "beam" width, it causes a redistribution within the group of trajectories.

The trajectories in the phase interval $-\frac{\pi}{2} \leq \theta_o \leq 0$ are radially bunched when they cross the negative y axis and debunched after each full turn according to

(13a) $$\Delta\eta' = \frac{1}{2\chi}\left[2 + \frac{\pi}{2}\right] - \frac{\dot{x}_o}{\omega d}\,, \quad \nu = 1, 3, 5, \text{etc.},$$

and

(13b) $$\Delta\eta' = \frac{1}{2\chi}\left[2 - \frac{\pi}{2}\right] + \frac{\dot{x}_o}{\omega d}\,, \quad \nu = 2, 4, 6, \text{etc.}$$

The "beam" width is the same at all half turns (i.e., $\Delta\eta' = \Delta\eta''$) if $\dot{x}_o/u = \pi/4$.

For the trajectories with positive starting phases $0 \leq \theta_o \leq \frac{\pi}{2}$ it is just the opposite way: they are more strongly debunched or bunched at the respective half turns than they would be in the case $\dot{x}_o = 0$.

3.2 Phase Bunching.

The phase θ at the end of each full turn, i.e., at times $\theta_o + \tau = 2\pi n$ (n = 1, 2, 3, etc.) can be defined by the relation

(14) $$\tan\theta = -\frac{x}{y}\,,$$

and since the trajectories are strongly bunched at these moments we can use in good approximation formula (10b) for y/d and obtain with (8)

$$\tan\theta = -\frac{\sin^2\theta_o}{2\pi n} + \frac{\dot{x}_o}{u}\frac{\sin\theta_o}{\pi n}, \quad (n = 1, 2, 3, \text{etc.}) \tag{15}$$

With this formula we can now analyze the phase-bunching effect in the large-gap situation. First we notice that phase grouping becomes stronger with the number of turns n, but decreases with increasing initial velocity $\dot{x}_o$. If we introduce the injection voltage U_i with the relation $\dot{x}_o = \sqrt{2eU_i/m} = u\sqrt{2\chi\, U_i/U_o}$ we obtain for the phase at the end of the first turn (n=1)

$$\tan\theta = -0.159\sin^2\theta_o + 0.450\sqrt{\chi\, U_i/U_o}\,\sin\theta_o \tag{16}$$

In the normal situation where the ions start with zero initial energy the phase at the end of the first turn is independent of the operating conditions and is approximately

$$\theta \simeq -9^o\sin^2\theta_o\,. \tag{17}$$

This formula means that all particles starting in the interval $-90^o \leq \theta_o \leq 90^o$ are bunched to a narrow pulse of only 9^o width. If the ion trajectories continue to be within the electric-field region, the total pulse width will be further reduced to 4.5^o after the second turn, 3.0^o after the third turn, etc., according to (15). This applies mainly to synchrocyclotrons with their small energy-gains per turn. The normal situation in the fixed-frequency cyclotron is, however, that only the first turn is completely within the electric field and that during the following revolutions the fraction of the rotation period spent within the field region becomes gradually smaller; formula (17) represents therefore a good estimate of what is happening under ideal phase-bunching conditions. In the upper part of Fig. 2 the phase θ for zero injection energy ($U_i/U_o = 0$) is plotted as a function of starting phase θ_o. This curve demonstrates that the pulse width is considerably shorter if one takes a smaller interval of θ_o, e.g., the interval $-20^o \leq \theta_o \leq 20^o$ is bunched to 1^o, the interval $-50^o \leq \theta_o \leq 50^o$ has a width of about 5^o after the first turn.

The use of an extractor electrode or the injection of the ions with initial energy reduces the phase-bunching effect, and the controlling factor is here the product of the parameter χ, which represents the operating conditions, and the ratio of injection voltage to dee voltage U_i/U_o. In Fig. 2 the phase after the first turn is plotted versus starting phase θ_o for several values of this product. Normally χ is a fixed parameter and thus the curves show how the pulse length of a given starting-phase interval increases with the injection energy. In the case of $U_i/U_o = 1.0$, $\chi = 4.0$ (curve 4), for example, the total phase range extends

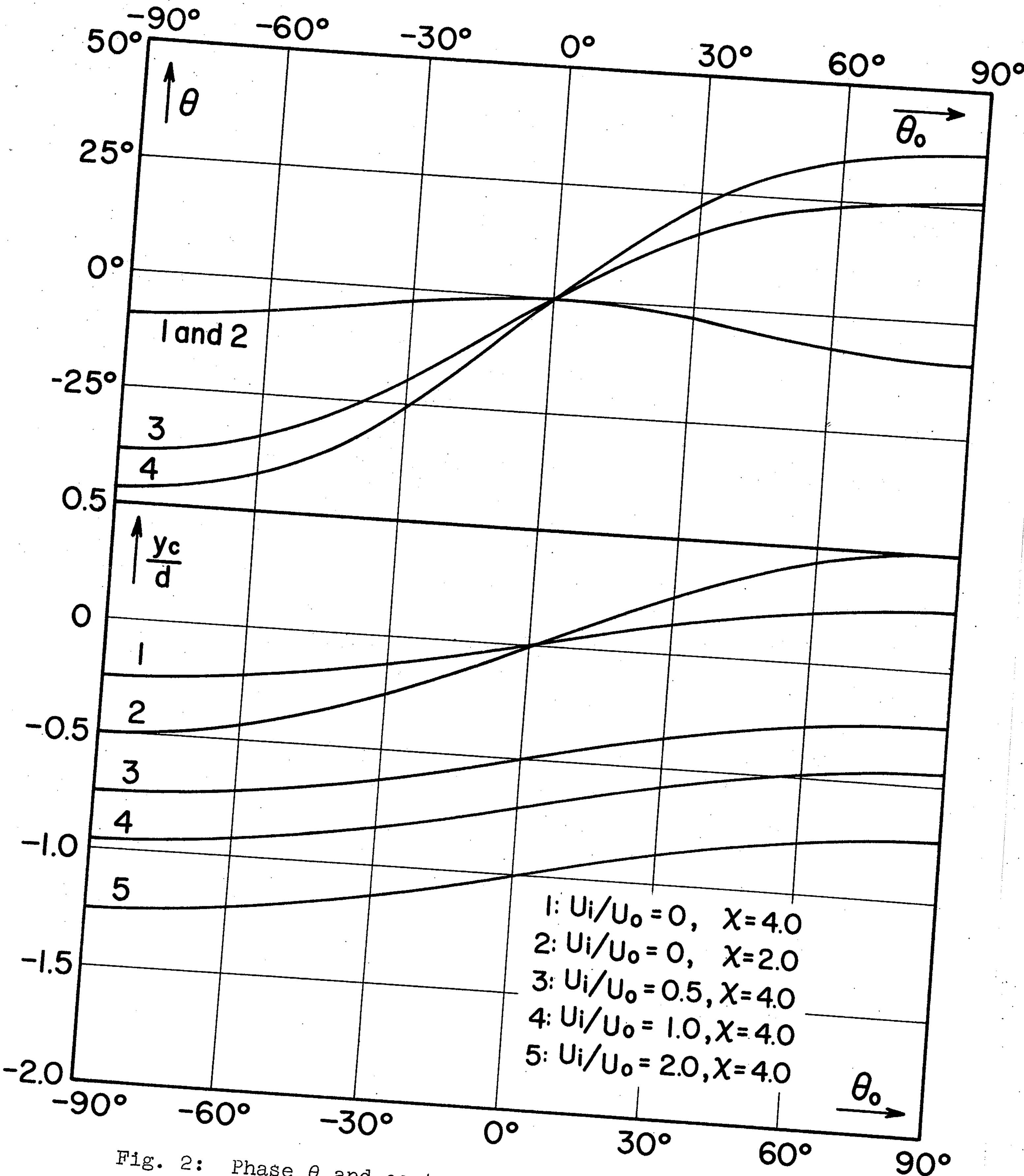

Fig. 2: Phase θ and center-point coordinate y_c/d at the end of the first turn as a function of starting phase θ_0 for various operating conditions.

from -46.6° to 37.2°, i.e., d.c. injection has broadened the pulse length to 83.8° as compared to 9° with zero injection energy. It should be noted however, that according to formula (15) this pulse width shrinks to some extent if large fractions of subsequent turns are still within the electric-field region.

3.3 Spread of Orbit Centers.

The coordinates x_c, y_c of the instantaneous orbit centers are given by the equations

$$x_c = x + \rho \frac{\dot{y}}{v} , \tag{17}$$

and

$$y_c = y - \rho \frac{\dot{x}}{v} , \tag{18}$$

where v is the velocity of the particles and ρ the radius of curvature as defined by

$$\rho = \frac{mv}{eB} [1 - \frac{E_n}{vB}]^{-1} . \tag{19}$$

If the electric field component normal to the trajectory E_n is very small or zero, as is the case near the center line $(x=0)$, one gets, with $eB/m = \omega$,

(20) $$x_c = x + \frac{\dot{y}}{\omega} ,$$

and

(21) $$y_c = y - \frac{\dot{x}}{\omega} .$$

With the initial conditions $x_o = 0$, $\dot{y}_o = 0$, and using (5) and (6) one obtains

(22) $$x_c = 0 .$$

The y coordinate of the center point at times $\theta_o + \tau = \nu\pi$, introducing (4) in (21), becomes

(23) $$\frac{y_c}{d} = \frac{\sin\theta_o}{\chi} - \frac{\dot{x}_o}{\chi u} .$$

In the case of d.c. injection this relation may also be written in the form

(23a) $$\frac{y_c}{d} = \frac{\sin\theta_o}{\chi} - \sqrt{\frac{2}{\chi}\frac{U_i}{U_o}} ,$$

or

(23b) $$\frac{y_c}{d} = 0.5\left(\frac{r_o}{d}\right)^2 \sin\theta_o - \frac{r_o}{d}\sqrt{\frac{U_i}{U_o}} ,$$

if one introduces the parameter

$$(24) \qquad \frac{r_o}{d} = \frac{1}{dB}\sqrt{\frac{2mU_o}{e}} = \sqrt{\frac{2}{\chi}} \;.$$

From formula (24) one can derive a lower limit for the values of χ. If the first turn is to be within the electric-field region we must have $r_o/d < 1$ or $\chi > 2$. In the case where the ions are injected with initial energy $U_i \neq 0$ one would have $\frac{2}{\chi}(1 + U_i/U_o) < 1$, i.e., for $U_i = U_o$ one would get $\chi > 4$.

Since formula (23) is valid for any $\nu = 1, 2, 3$, etc., we can draw the following conclusions: (a) The instantaneous centers of each particle trajectory with given θ_o move in such a way that they are always at the same point after each half turn (i.e., at times $\theta_o + \tau = \nu\pi$); (b) the distribution of orbit centers for a group of ions starting in a certain phase interval $\Delta\theta_o$ remains constant throughout the acceleration process; (c) the actual spread width Δy_c depends on the operating parameter χ or the ratio r_o/d; (d) injection of the ions with initial energy shifts the orbit centers in negative y direction by an amount proportional to the square root of the ratio of injection voltage to dee voltage.

Fig. 2 shows the center-point coordinate y_c/d as a function of starting phase θ_o for two cases of zero injection energy ($U_i/U_o = 0$) and three cases of d.c. injection with $\chi = 4.0$ and $U_i/U_o = 0.5$, 1.0, and 2.0. It is seen that, if the operating parameter χ is kept constant, d.c. injection does not change the spread of orbit centers. This means that the interval of acceptable starting phases is the same as for zero injection energy; only the duty cycle is increased as shown in the upper part of Fig. 2.

After this general analysis of the phase-bunching effect and associated problems in the large-gap situation we shall now check the accuracy of the uniform-field theory by comparing the results with the nonuniform-field calculations in the particular case discussed previously (Fig. 1). For this comparison we need proper values for the width 2d of the uniform-field gap and the parameter χ which is a function of d. A straightforward way of determining these parameters is to normalize both calculations at the point where the particle starting at $\theta_o = 0$ crosses the center line at the end of the first turn ($\theta = 2\pi$). In the nonuniform-field case this is at $y = 3.766$ cm, and if we introduce this value in formula (10b) we obtain

$$(25) \qquad \frac{d}{\chi} = \frac{3.766}{\pi} = 1.199 \ .$$

The parameter χ is defined by formula (1); with the specific charge of the protons and the chosen values for U_o and B_o we get

$$(26) \qquad \chi = 0.2596 \times d^2_{[cm]} \ .$$

Solving both equations leads to the values

(27) $$d = 3.2 \text{ cm} \quad \text{and} \quad \chi = 2.68 \, .$$

In Fig. 1 the uniform-field potential with $d = 3.2$ cm is marked by a dashed line, and the picture shows that this approximation of the potential distribution would also have been a natural choice from a graphical point of view. Fig. 3 compares the nonuniform-field results for the phase θ and the center-point coordinate y_c at the end of the first turn ($\theta_o + \tau = 2\pi$) with the uniform-field approximation (dashed curve) using $d = 3.2$ cm and $\chi = 2.68$. It is seen that the nonuniform-field curves are slightly asymmetric with respect to $\theta_o = 0^o$, differing mainly at negative starting phases from the uniform-field results by a small amount. Both the total pulse width as well as the total spread of orbit centers is slightly larger than predicted by the uniform-field theory: the phase is -13.3^o at $\theta_o = -90^o$, 1.3^o at $\theta_o = 0^o$, and 9.1^o at $\theta_o = 90^o$ as compared to the uniform-field values of -9.0^o, 0^o, and 9.0^o, respectively; the center-point coordinates for these starting phases are -1.35 cm, 0.09 cm, 1.21 cm respectively, while the equivalent uniform-field values are -1.20 cm, 0.00 cm, and 1.20 cm. The agreement of the results is quite satisfactory; this comparison demonstrates the accuracy and usefulness of the uniform-field theory for this type of investigation.

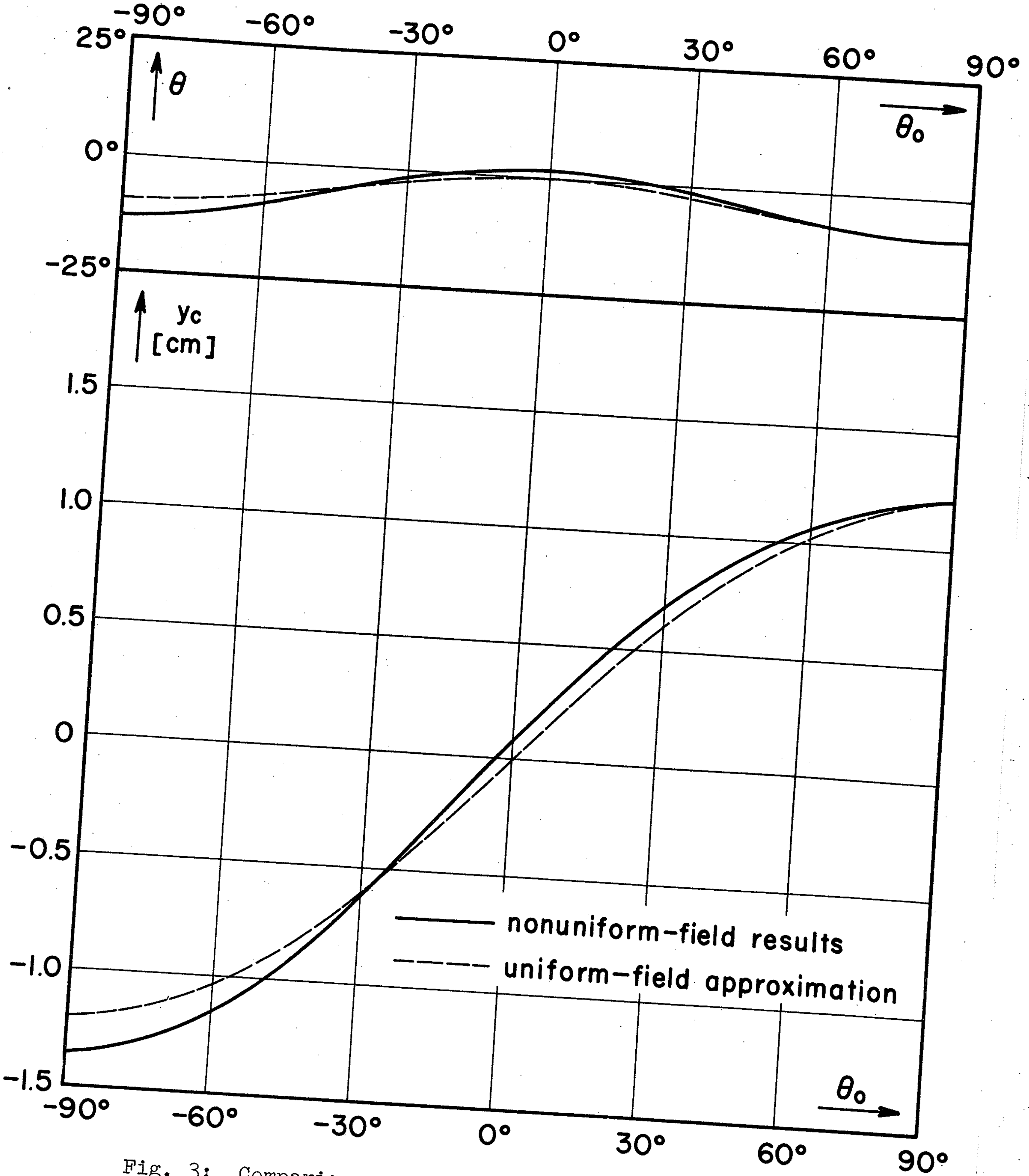

Fig. 3: Comparison of uniform-field approximation with the nonuniform-field results in the case of Fig. 1.

4. Theoretical Results for the Narrow-Gap Situation ($r >> d$).

The procedure of analyzing the central motion in the case of the narrow gap is straightforward: we simply divide the central area into the gap region with uniform electric field and the region outside the gap where the electric field strength is zero. For the calculations in this case the "COP" code (Central-Orbit Program)[19] can be employed.

The general tendency with respect to phase grouping is clear from what has been said previously: the shorter the time of gap crossing compared to the r.f. period the smaller is this bunching effect; the upper limit is reached in the case where the transit angle is negligible (infinitesimal gap) and no bunching occurs at all. Parallel with the reduction of phase bunching one expects the effect of radial grouping and degrouping to decline and hence the pattern of radial motion to become more uniform.

In regard to the spread of orbit centers a brief consideration will give an idea of what will happen. Consider the idealized case of infinitesimal gap width. In a coordinate system with the origin at the starting point of the ions the initial center-point coordinate y_c is simply given by the radius of curvature r_1 after the first gap crossing:

(28) $$y_c = - r_1 = - r_o \sqrt{U_i/U_o + \cos\theta_o} \quad .$$

For zero injection energy ($U_i = 0$) the total spread is $\Delta y_c = - r_o$ and in the case of d.c. injection

$$(29) \qquad \Delta y_c = - r_o \sqrt{U_i/U_o + 1} - \sqrt{U_i/U_o} \; .$$

This formula shows that with increased injection energy the maximum spread of initial orbit centers becomes smaller, and one can expect a higher acceptance rate if d.c. injection is employed; with $U_i/U_o = 1$, for example, one obtains $\Delta y_c = - 0.41\, r_o$, i.e., the total spread is only 41% of the amount in the normal case ($U_i = 0$).

The actual cyclotron situation is somewhere between the open-dee geometry and the infinitesimal-gap case. There is always a large fraction of the first turn within the electric-field region, although this fraction decreases quickly in subsequent turns proportional to the radius of curvature. The objective of our calculations is then to get an idea as to what extent phase bunching can be reduced in a practical situation. A general analysis as in the large-gap case would require a great number of computer runs with varied conditions. We shall instead consider the particular cyclotron specified in Section 2, calculate the initial trajectories for several conditions (normal and d.c. injection), and try to draw some general conclusions. We assume that the total width 2d of the electric-field gap is 3.0 cm, except for the source-to-puller spacing which is to be only 1.0 cm.

The first computer run was made with initial conditions $x_o = 0$, $y_o = 1.6$ cm, $\dot{x}_o = 0$, $\dot{y}_o = 0$. (The origin of the coordinate system is supposed to be in the cyclotron center and the starting point y_o for the ions was chosen such that the useful group of particles is centered at subsequent turns). Fig. 4 shows a family of calculated trajectories for this case with starting phases between -90^o and 30^o. As expected the trajectories are more uniform than in Fig. 1, and the group of particles with $-90^o \leq \theta_o \leq 0^o$ is very well bunched radially. Notice also that the radius of curvature increases faster than in the large-gap case; this is of course due to the short transit time and hence higher energy gain in the narrow gap. In Fig. 5 the phase of the particles at the end of the third gap crossing, i.e., after one full turn, is plotted versus the starting phase θ_o (curve 3). For comparison the phases for normal injection (curve 1) and a particular case of d.c. injection (curve 2) in the large-gap situation have also been plotted. These curves were calculated from the uniform-field theory assuming the values $d = 4.0$ cm and $\chi = 4.15$; the injection voltage in case 2 was chosen to be 70 kV, i.e., $U_i/U_o = 1.0$. Curve 4 is another case of narrow-gap geometry (d.c. injection) to be discussed below. Curves 2 and 3 are very similar in shape, and it is seen that in both cases the phase-bunching effect can be substantially

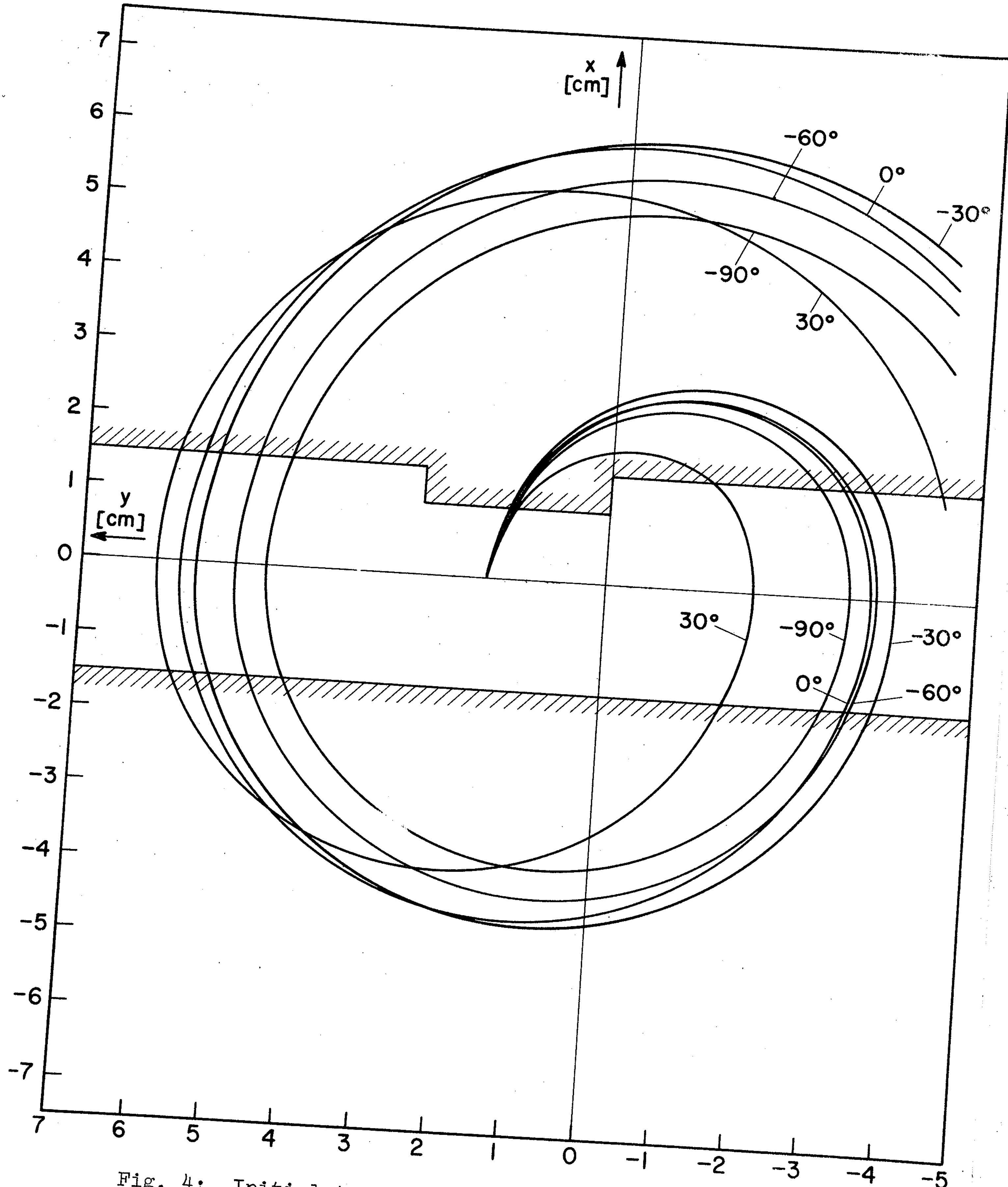

Fig. 4: Initial trajectories in the narrow-gap situation (normal injection).

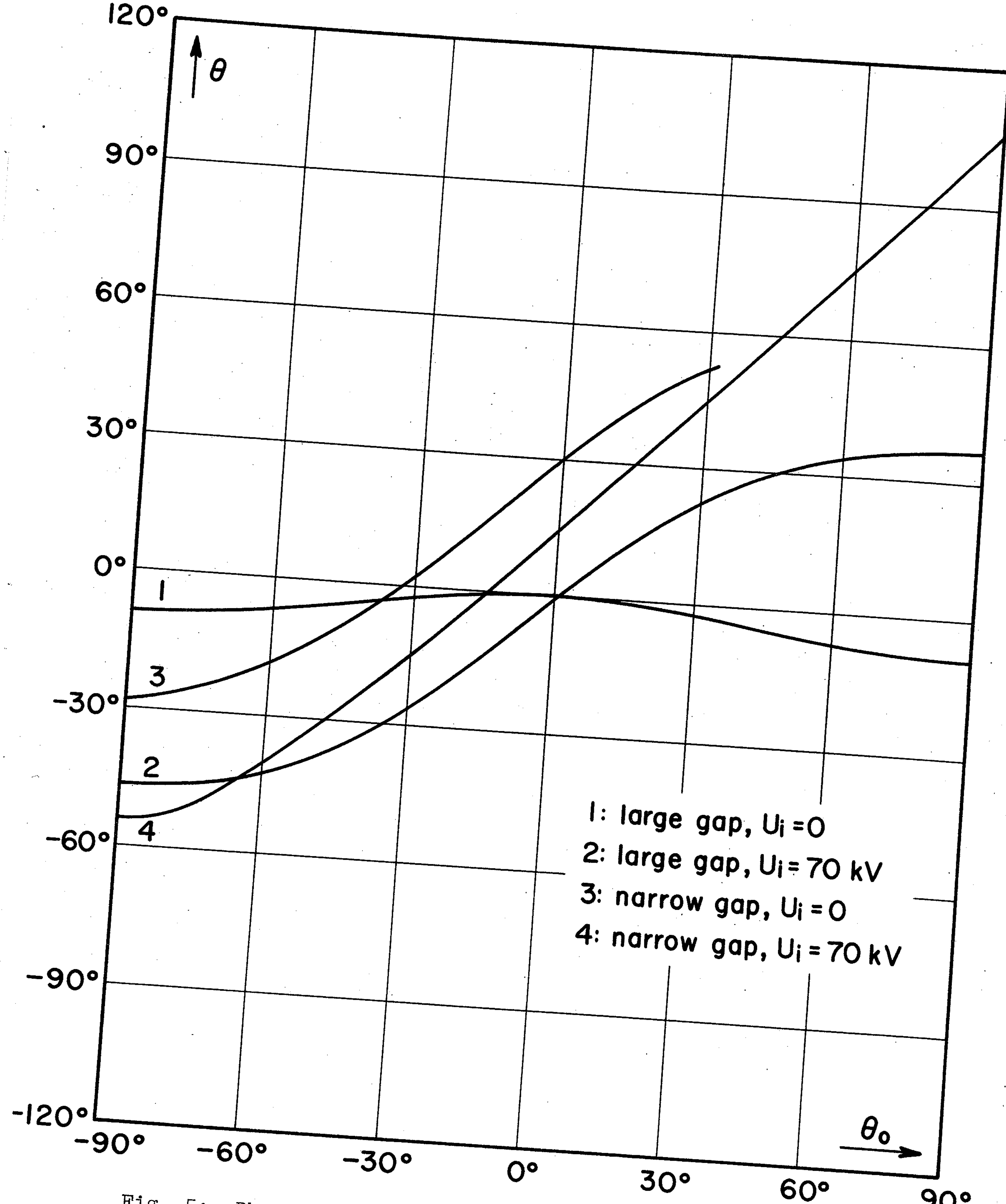

Fig. 5: Phase θ at the end of the first turn as a function of starting phase θ_o in four different cases of central geometry.

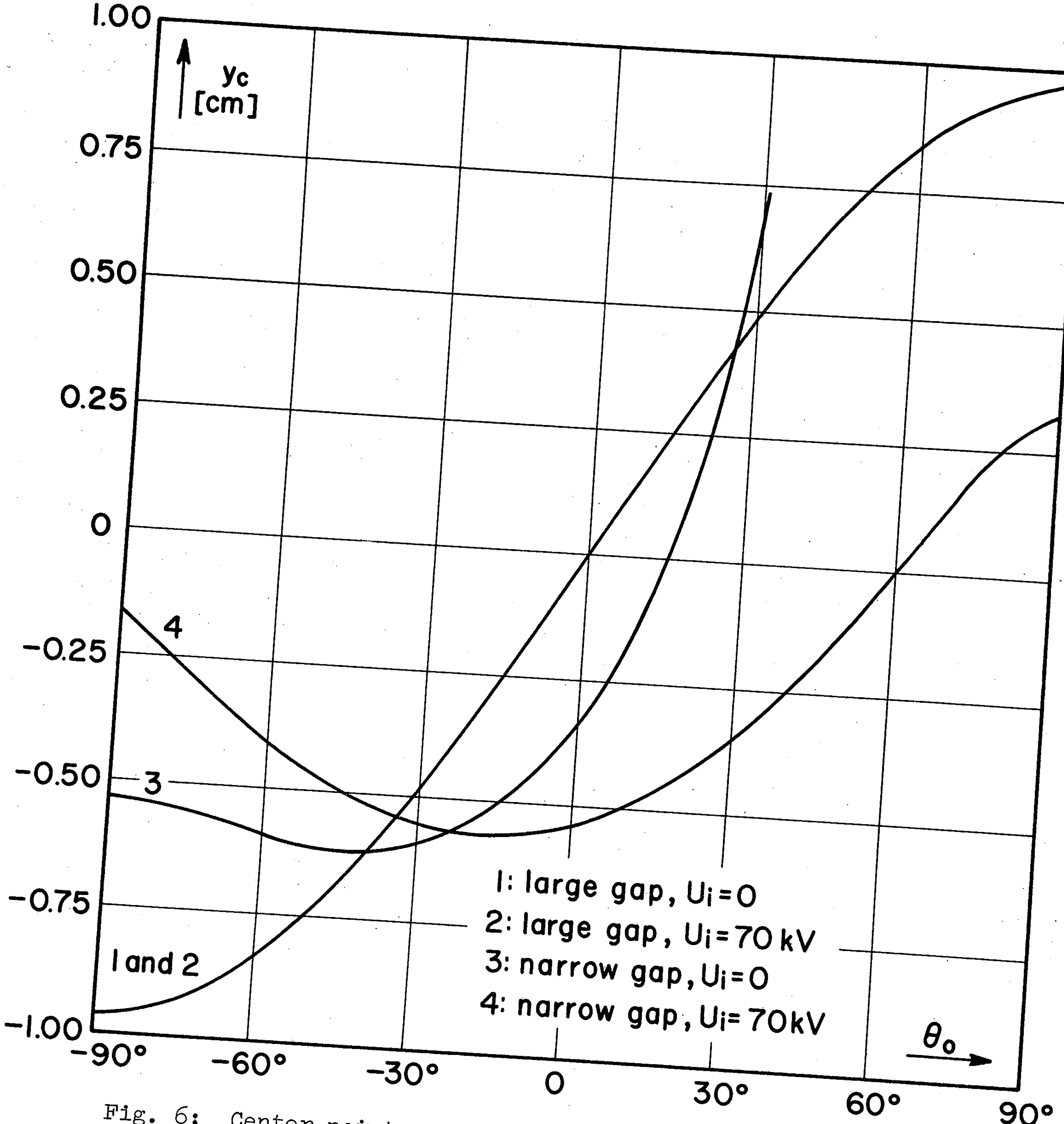

Fig. 6: Center-point coordinates y_c at the end of the first turn as a function of starting phase θ_o.

reduced for a large interval of starting phases. Fig. 6 shows the distribution of orbit centers at the end of the first turn (curve 3). Numbers 1 and 2 refer again to the two examples of large-gap geometry (both curves are drawn identically since, as we have seen, d.c. injection does not change the distribution of center points). In contrast to Fig. 5, curve 3 is here very different from the large-gap case, and indicates that reducing the gap width causes a complete redistribution of center points; a remarkable decrease in the spread of centers occurs in the range of negative starting phases, while for positive phases the spread becomes stronger than in the case of large gap width. Particles starting at phases beyond about 20° or 30° are not useful for further acceleration because of the low energy gain (they would not clear the ion source). The comparison of curves 2 and 3 shows that limiting the electric field to a narrow gap is more effective than d.c. injection in a large-gap geometry. The numerical comparison in the next section will establish this more specifically.

In the next computer run with the COP code it was assumed that the ions are injected with an energy of 70 keV, corresponding with $\dot{x}_0 = 3.66 \times 10^8$ cm/s and $\dot{y}_0 = 0$, at $x_0 = 0$, $y_0 = 3.0$ cm. In this case phase bunching is almost completely eliminated for the whole range of starting phases between -90° and 90°, as curve 4 in Fig. 5 demonstrates. Simultaneously, as one expects, the spread of orbit centers (curve 4 in Fig. 6) becomes smaller, and the center points for starting phases between -40° and 10° are strongly bunched within a narrow area of only 0.7 mm.

In the actual cyclotron one would try to program the useful part of the beam throughout the first turns and shift the phase towards positive, electrically focusing values by offsetting the extractor electrode from the dee edge[20]. Therefore two more runs were made, one with zero injection energy and a puller-to-dee angle of 25° and another run for 70-keV injection energy and a puller-to-dee angle of 15°. The results are not shown on the graphs, since they were very similar to the previous cases, but they will be considered in the final comparison in the next section.

5. Comparison of the Theoretical Results and Conclusions.

The previous analysis of initial ion motion for various conditions of central geometry and injection schemes has shown that it is possible to substantially reduce or even completely eliminate the phase-bunching effect which is one of the main causes for the low duty cycle of conventional cyclotron beams. This means that, as far as ion injection is concerned, a central beam with a pulse length of practically 180°, covering

---

20) M. Reiser, Nucl. Instr. and Meth. 18-19 (1962), 370.

the whole range of the accelerating half period of the r.f. voltage and corresponding to a duty cycle of 50% can be produced and offered for the subsequent acceleration process. The limiting factors are then only the transmission capability of the magnetic field (i.e., the magnetic stability limit Δr_{st}) and the tolerance limit for the final energy spread, both of which determine the acceptable spread of initial orbit centers. These two factors may vary to a large extent from one cyclotron to another depending on the size of the machine, design features of the magnetic field, requirements with regard to quality of the extracted beam, etc. To get an idea how these conditions and requirements effect the duty cycle in an actual case we shall consider again the negative-hydrogen cyclotron described in Section 2, with a given tolerance limit $\Delta r_{tol} = 0.5$ cm, and determine the acceptable fraction of the injected beam in the cases that have been analyzed theoretically. For this cyclotron we assume a dee voltage of 70 kV and consider normal injection ($U_i = 0$) and d.c. injection with $U_i = 70$ kV. The cases we shall compare are listed in Table 1. All the information needed is contained in Figs. 5 and 6 (except for the two cases with offset puller), and the procedure is straightforward: we determine in Fig. 5 the maximum interval of starting phases (closest to $\theta_o = 0$) where the spread of orbit centers $\Delta y_c = y_{cmax} - y_{cmin}$ is smaller than the tolerance limit of 0.5 cm and get from Fig. 6 the pulse width of this acceptable starting-phase interval at the end of the first turn.

The results of this analysis are presented in Table 1. In the first large-gap case with ideal phase-bunching conditions the acceptable starting-phase interval $\Delta\theta_o$ is only 30^o and the pulse length after one turn only 0.6^o. D.c. injection in the large-gap situation does not change the spread of center points and hence the acceptable interval $\Delta\theta_o$ in case 2 is the same as before; however, the pulse length is now considerably larger (26^o) and the corresponding duty cycle is 7.2%. The phase measurements reported so far seem to indicate that case 2 represents the situation in most cyclotrons where "feelers" or an extractor electrode are employed (which technique, as we discussed, is equivalent with d.c. injection), but otherwise the first turn and large fractions of subsequent turns are still within the electric-field region.

Case	θ_o	$\Delta\theta_o$	θ	$\Delta\theta$	Duty cycle [%]
1	-15° to 15°	30°	-0.6° to 0°	0.6°	0.17
2	-15° to 15°	30°	-13° to 13°	26°	7.2
3	-90° to 17°	107°	-28° to 44°	72°	20.0
4	-90° to 55°	145°	-55° to 70°	125°	34.7
5	-90° to 24°	114°	-8° to 77°	85°	23.6
6	-104° to 58°	162°	-46° to 91°	137°	38.1

Case 1: large gap (width 8 cm), normal injection

Case 2: large gap (width 8 cm), d.c. injection

Case 3: small gap (width 3 cm), puller angle 0, normal injection

Case 4: small gap (width 3 cm), puller angle 0, d.c. injection

Case 5: small gap (width 3 cm), puller angle 25°, normal injection

Case 6: small gap (width 3 cm), puller angle 15°, d. c. injection

Table 1: Acceptable starting-phase interval and duty cycle for different situations of central geometry with $\Delta r_{tol} = 0.5$ cm.

The next case in Table 1 shows what improvements can be expected if the electric field is limited to a narrow gap. Since in this case the spread of center points is being reduced for the useful interval of starting phases as the pulse width increases we get a gain in duty cycle by almost a factor of 3 compared to case 2. Additional d.c. injection (case 4) brings the duty cycle up to about 35%. The last two cases with offset puller are equivalent to case 3 and 4, respectively, but the duty cycle is further improved by about 3%.

The length of the acceptable starting-phase interval and its location with respect to phase $\theta_o = 0$ are a direct measure of the ion acceptance efficiency, i.e., the fraction of total ion beam which is useful for further acceleration. The table illustrates how the amount of acceptable beam increases substantially as one changes from large to narrow gaps. This amount would be especially high in the case of d.c. injection where not only $\Delta\theta_o$ is largest but also the injected current independent of the starting phase (contrary to normal injection where the current density varies roughly with the 3/2 power of $\cos\theta_o$ according to Child's Law).

In all six cases the ion-source structure was not a limiting factor. The ions that would be intercepted by the source (which was represented by a disk of diameter $D = 2.0$ cm) start all beyond the useful interval $\Delta\theta_o$ at large positive starting phases.

The results of this analysis can be summarized in the following conclusions:

(1) The phase-bunching effect, which is the main cause for the low duty factors of conventional cyclotron beams, depends very strongly on the time the particles spend in the electric-field region during their first few revolutions and can be substantially reduced if the time of gap crossing is only a small fraction of the r.f. period.

(2) Short gap-transit times can be achieved by increasing the radius of curvature on the first turns, i.e., operating at high dee voltages, not too strong magnetic fields, and, if necessary, employing a d.c. injection scheme.

(3) Another possibility is to reduce the extension of the electric field, which normally penetrates deep into the dees, by attaching diaphragms to the edges of the dees along the first turns. Since the trajectories are well grouped radially if phase bunching is reduced, it is possible to program the useful part of the beam through narrow, vertical slits similar to focusing grids that are sometimes used in cyclotrons. If electric overfocusing causes difficulties one can shape the diaphragms on the side where the ions enter the gap in a way that vertical field components are small at the beginning but increase gradually from one gap to another.

(4) Since the transit angle in the electric gap is proportional to the harmonic number $N = \omega_e/\omega$, it is essential to operate the cyclotron in the first-harmonic mode.

(5) By employing these techniques and conditions it appears possible to substantially increase the pulse length of the central beam and achieve duty factors as high as 40% to 50%, the actual value being limited only by the transmission capability of the magnetic field (stability limits) and/or the tolerable energy spread (or required beam quality) in the stripping extraction process.

6. Acknowledgements.

The author is indebted to H. G. Blosser and M. M. Gordon for many helpful discussions and suggestions. The assistance of T. I. Arnette and D. A. Johnson, in guiding the calculations with the computer, is gratefully acknowledged.