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CYCLOTRON PROJECT

Central Geometry and Initial Orbits In The MSU Cyclotron*

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1. Introduction

The basic considerations for the design of the injection system and central dee geometry of the MSU cyclotron and the preliminary orbit calculations with idealized field distributions have been discussed in previous reports and papers[1-3]. The results of these studies can be summarized as follows: (a) due to the geometric limitations in the center it is necessary to operate the cyclotron in a constant-orbit scheme where the number of turns is approximately the same in all modes of operation; (b) this requirement implies that the dee angle must be equal or close to 144°; (c) ion-source and puller positions have to be changed with the mode of operation in order to shift the group of ions with optimum starting conditions to the phase interval with maximum energy gain during the subsequent acceleration process; (d) the calculations of vertical motion indicated that the influence of the electric field is very strong and phase sensitive and that a net focusing effect can be expected for ions accelerated in the useful phase interval of maximum energy gain.

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1) M. Reiser, MSU Cyclotron Report 15, February 1963.

2) M. Reiser, MSU Cyclotron Report 16, March 1963.

3) H. G. Blosser, M. M. Gordon, and M. Reiser, Proc. of the 1963 Conference on Sector-Focused Cyclotrons, CERN 63-19, May 1963, p. 193.

These preliminary studies furnished the guidelines for the construction of an electrolytic-tank analog of the central region and an extensive program of detailed numerical trajectory calculations using measured electric and magnetic field data. The electric field was carried as a rectangular grid of data points which were measured with a self-balancing bridge and printed on IBM cards in an automated system[4]. Orbit calculations were first carried out with the Cartwheel Code[5] on MSU's former computer, the MISTIC, and later with Pinwheel[4], a modified version of Cartwheel, on the new Control Data 3600 computer.

During this program, which started in spring 1963, the first electrolytic tank was replaced by an improved model[4] and the geometry was changed several times. A detailed discussion of these modifications and of the computer results for all the various cases shall be omitted for the sake of brevity and clarity. Instead only the results for the field geometry that was finally chosen will be presented in this report.

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4) M. Reiser and J. Kopf, MSU Cyclotron Report 19, April 1964.

5) T. I. Arnette et al., Nucl. Instr. & Meth. 18, 19 (1962), 343.

2. Central Geometry

2.1 Dee System and Electric Field Distribution

The first electrolytic tank had been built with a dee angle of 144° and sector-shaped dummy dees of 36°, which arrangement minimizes the required radial displacements of the source-puller system if the mode of operation is changed[1]. As the design of the cyclotron progressed a number of considerations and requirements necessitated a modification of the dee geometry: (1) A change of the pole tips will make it possible to accelerate the ions to larger radii and energies than was originally planned (for protons it will be 50 MeV instead of 40 MeV), and to this end the dees had to be enlarged; the undesirable increase of dee capacitance and r.f. power resulting from this enlargement can be avoided by cutting back the angle of the two dees. (2) Studies of beam extraction showed that it was necessary to increase the length of the electrostatic deflector, which is located in one of the dummy dees, and this again pointed to a reduction of the dee angle. (3) The dummy dee on the opposite side of the dee stems will be isolated from ground and used as a coupling capacitor between the two resonance cavities[6]. In the push-pull mode this dummy dee will be at nearly ground potential whereas in the push-push mode it has a potential

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6) W. P. Johnson, Proc. of the 1963 Conference on Sector-Focused Cyclotrons, CERN 63-19, May 1963, p. 279.

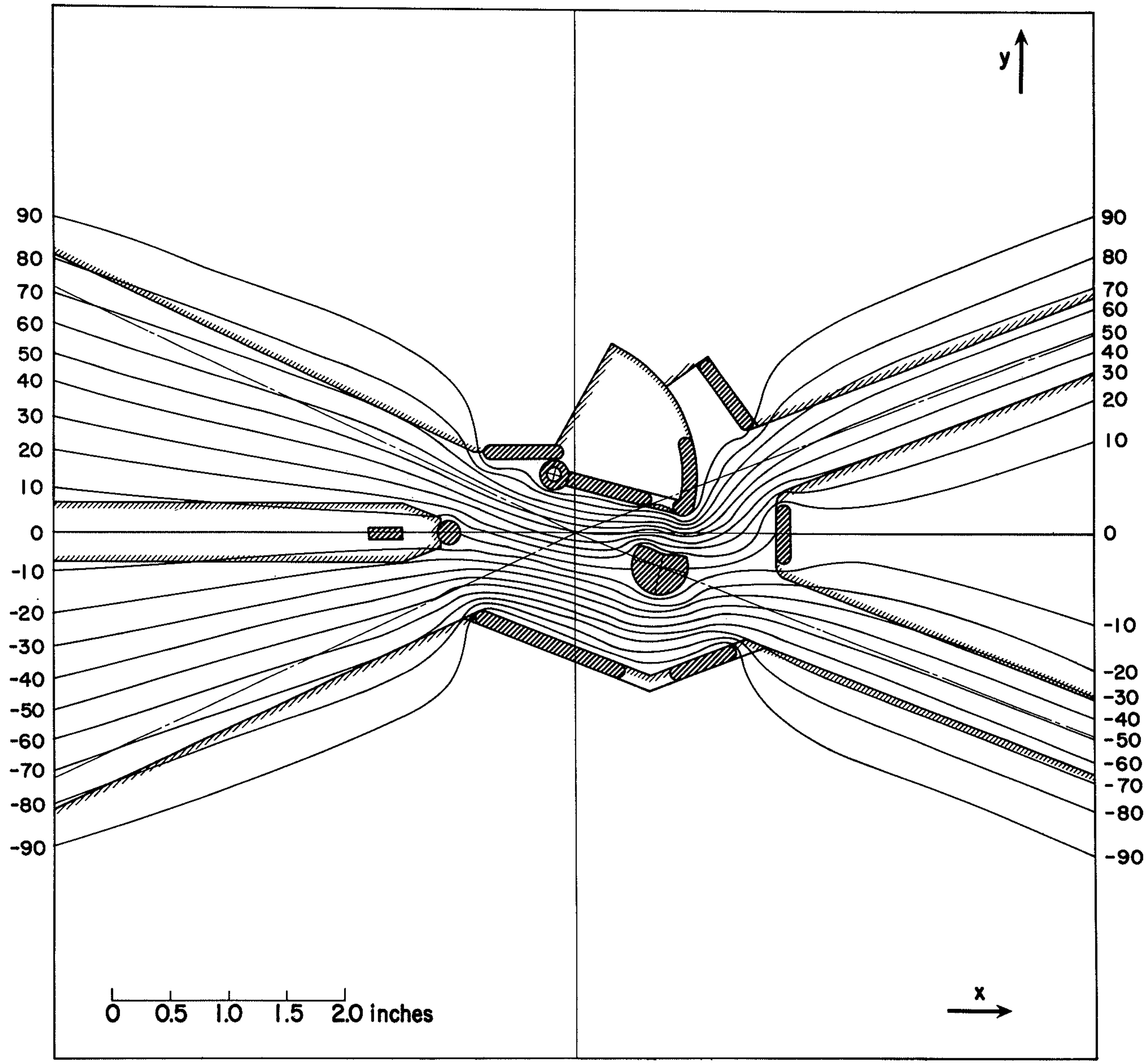

Fig. 1: Central geometry and equipotential lines in the push-pull mode of r.f. operation (case N = 1, field 1.05.1).

of about 10% of the dee-to-ground voltage. (5) The dummy dee on the stem side of the dees was reduced to a small straight plate, 0.5 inch wide, to allow easy assembly of the two dee structures.

The last version of the central dee geometry studied in the electrolytic tank is shown in Fig. 1. Ion source, puller, and beam defining slit are in the position for first-harmonic proton acceleration in the push-pull mode; the equipotentials are drawn to give a picture of the electric field configuration. The dees are asymmetric with half angles of 69° on the right and 65° on the left side; the right dummy dee, which is used as the coupling capacitor, is pie shaped and has an angle of 42°. The asymmetry in the dee angle has been introduced to balance the energy gain at larger radii as the small dummy dee on the left produces a different field distribution than the sector-shaped dummy dee on the right.

As a consequence of the change of dee geometry a modification of the puller design became necessary. In the original plan the puller had two slits to allow repositioning as the mode of operation was changed by a mere rotation about an axis which was either fixed or only slightly adjustable[1]. When the dee angle was reduced from 144° to 138° the ion source and outer puller slit had to be displaced radially by about 1/4 inch in the N = 3 mode of operation and the spacing between edge and the tip of the right dummy dee became critically small (see Fig. 22 which illustrates this situation). To avoid sparking troubles in the most important $N = 1$ mode, where the dee voltage is 70 kV, a new 1-slit puller was built; now the small spacing exists only in the $N = 3$ mode where the voltage is lower than 70 kV. A very flexible adjustment mechanism allows a large x-y displacement of the puller as a whole as well as a rotation about the puller axis.

The electrolytic tank setup for the field measurements and the Pinwheel program for the orbit calculations were discussed in a previous report[4]. The field was measured as a rectangular grid of potentials, usually covering an area of 9" x 9" of the cyclotron center, and printed on IBM cards. With a special field processing code one could obtain a printout of the data points as 3-digit numbers in proper order so that it was possible to assemble a large-scale 4 ft. x 4 ft. chart of the measured grid area. This code could also produce a contour map of the field by marking asterisks between neighboring points in each column where a particular equipotential line went through. From such a contour map the equipotentials in Fig. 1 have been drawn. This figure illustrates the situation in the push-pull mode where the dees are on opposite potentials. It is seen how the asymmetry of the electrode system leads to a slightly different field distribution on each side:

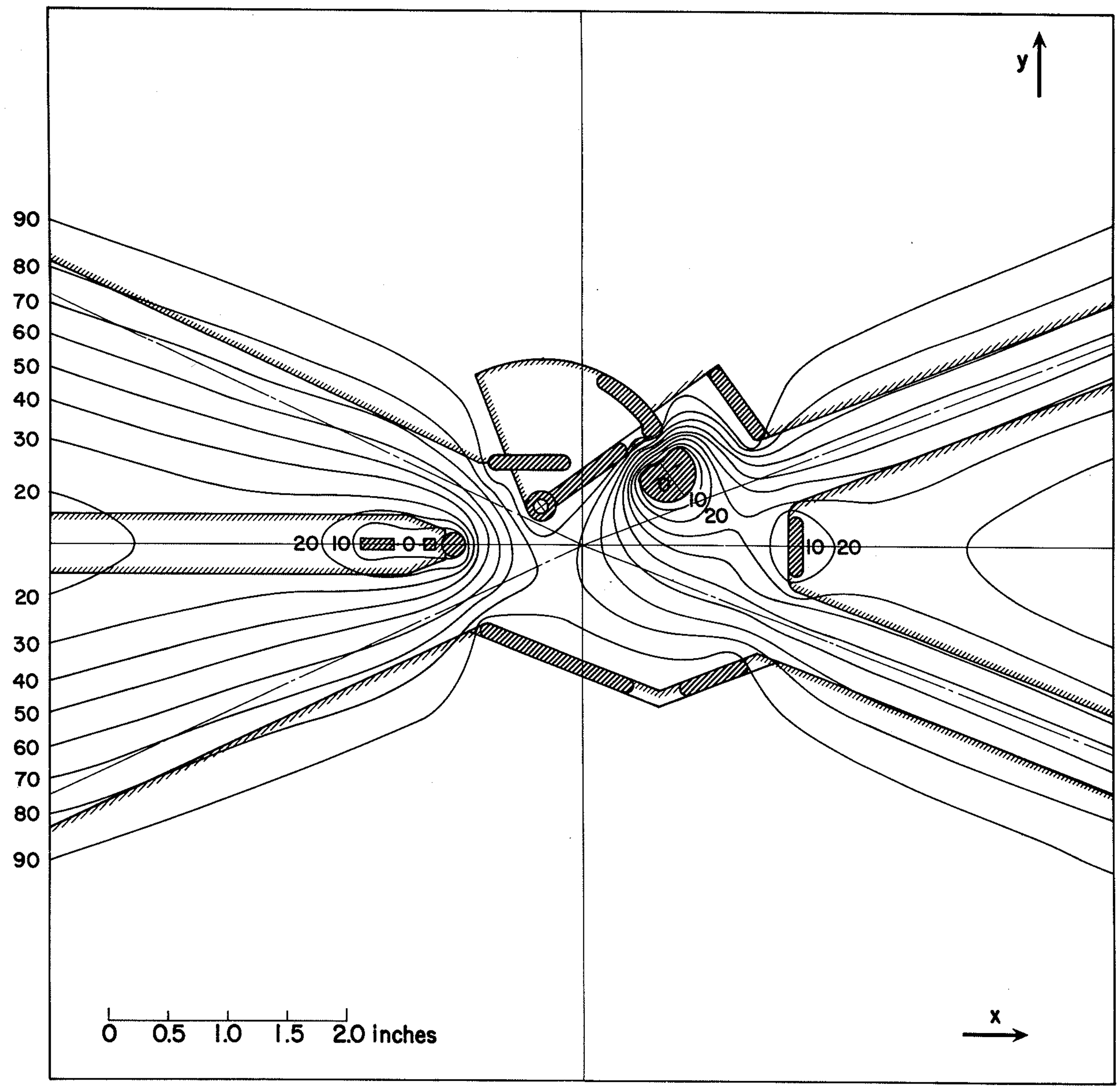

Fig 2: Central geometry and equipotential lines in the push-push mode for second-harmonic acceleration (case N = 2, field 2.04.1).

on the right the pie-shaped dummy dee localizes the field in the gap region, with the 50% potential lines following closely the gap center lines; on the left side, where the dummy dee is formed by a small straight plate, the field spreads out over the whole region and the 50% equipotential is closer to the center line of the dummy dee than it is on the other side.

The situation in the push-push mode, where both dees have the same potential, is markedly different as is shown in Fig. 2. In this case the right dummy dee has a potential of 10%, and the contour map has a saddle point in the center, with ion source and beam defining slit forming the deepest points in the potential "valleys." The figure clearly depicts the importance of the beam slit and the electrode structures at the tip of each dummy dee: they prevent the overlapping of the fields from the two dees and thus increase the potential difference seen by the particles on the first turns.

In the first electrolytic-tank setup slits had been placed along the first turn on the downstream side of the gaps and in the right dummy dee[1]. After measurements of the field without slits and orbit calculations showed that, except for the third-harmonic mode of operation, there was little difference between the two cases, the slits were abandoned. In the present arrangement only the adjustable beam-defining slit in the left dummy dee remains. The shielding electrodes in the dee behind the ion source, in the dummy dee tips and to the right of the puller are still in place. With the modification of the puller another electrode structure was added and placed near the puller axis (see Figs. 1 and 2). This electrode has the function of preventing the field from penetrating deep into the dee when the puller is in the far-right $N = 3$ position.

2.2 The Magnetic Field in the Center of the Cyclotron

The magnetic field in a 3-sector cyclotron is usually represented in the form of a Fourier series, i.e.,

$$(1) \quad B(r,\phi) = B_o[\bar{B}(r) + \sum_{n=3,6,9} H_n \cos(n\phi) + \sum_{n=3,6,9} G_n \sin(n\phi)],$$

where the average field $\bar{B}(r)$ and the coefficients $H_n(r)$ and $G_n(r)$ are in units of the central isochronous field B_o. In the case of the MSU cyclotron the field is either measured in the model-magnet setup as a rectangular grid of data points or in the actual cyclotron magnet in a polar grid of radius r and azimuth ϕ. Then it is analyzed by a field processing code which prints out tables of the Fourier coefficients $\bar{B}$, H_n, and G_n for consecutive radii, usually in steps of one inch.

For the central-orbit calculations in the Pinwheel program only the third-harmonic coefficients H_3, G_3 are used. Formula (1) may then be written in the form:

$$(2) \quad B = B_o[\bar{B} + B_3 \cos 3(\phi - \delta)]$$

The hill peaks of the field are located at angles δ, $\delta + 120°$, and $\delta - 120°$, where δ is given by the formula

$$(3) \quad \tan 3\delta = \frac{G_3}{H_3}$$

The line $\phi = 0$ of the polar coordinate system employed in the magnetic-field representation coincides with the positive x axis of the cartesian coordinate system used in the electrolytic-tank measurements and in the Pinwheel calculations (see Fig. 1). In the MSU cyclotron the orientation of the pole tips with respect to the dee system is such that the first hill peak is located at an angle of about 60°, which means that the pie-shaped dummy dee on the right is in a valley, whereas the left, straight dummy dee is in the middle of a hill.

In the past, two alternatives for the central magnetic field in the MSU cyclotron have been under investigation: (a) the case of isochronous field, and (b) a bump field which provides a radial fall-off to improve vertical focusing. Either possibility can easily be realized by properly shaping the tip of a 6-inch diameter plug in the center of the magnet. In the Pinwheel calculations at first a flat magnetic field was assumed until a satisfactory solution for the ion-source and puller position was found. Then the computer runs were repeated with a nonuniform field, and, if necessary, the starting conditions were corrected to optimize the orbit geometry and beam quality. In the latest computer runs, which are discussed in this report, two nonuniform fields were employed: the isochronous field 28.4 and the bump field 187F6. Most of the calculations were done with field 28.4 which was derived from measurements in the actual cyclotron magnet. The average value $\bar{B}$ and the two third-harmonic Fourier coefficients H_3, G_3 of field 28.4 are plotted in Fig. 3 for the first radii. $\bar{B}(r)$ is the azimuthal average taken along a concentric circle of radius r. In a sector field $\bar{B}(r)$ is smaller than the average value taken along the orbit since the ions spend more time in a hill than in a valley. This explains why $\bar{B}(r)$ in Fig. 3 drops below 1 in the transition region from the flat central plateau to the flutter field. The isochronous field averaged along the orbit is of course practically constant and equal to unity in this region since the relativistic mass increase is not yet large enough to have a noticeable effect. The main Fourier coefficient H_3 in field 28.4 rises from zero at the center to the value of -0.2794 at

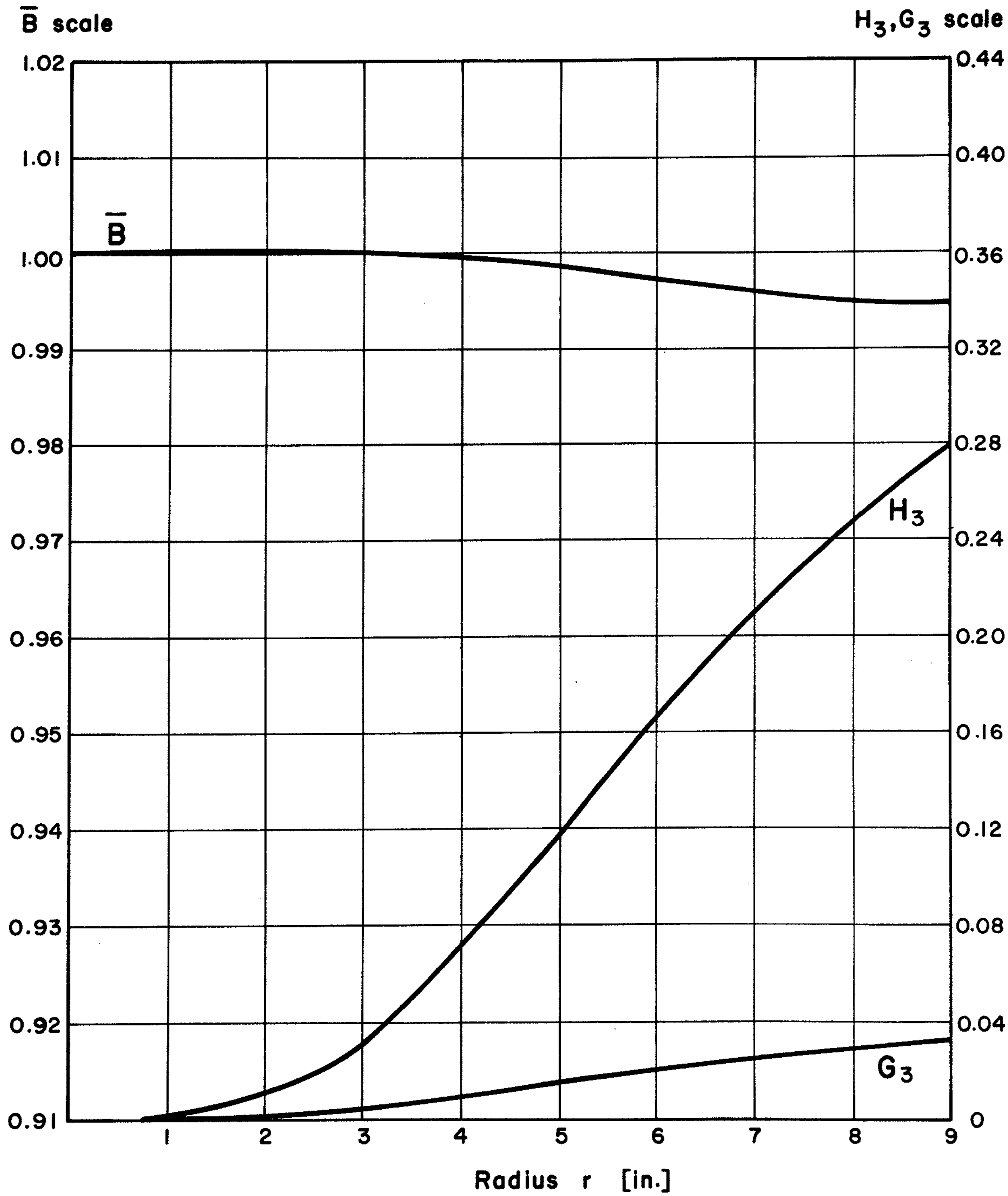

Fig. 3: Average value $\bar{B}$ and third-harmonic Fourier coefficients H_3, G_3 of isochronous field 28.4 in the central region.

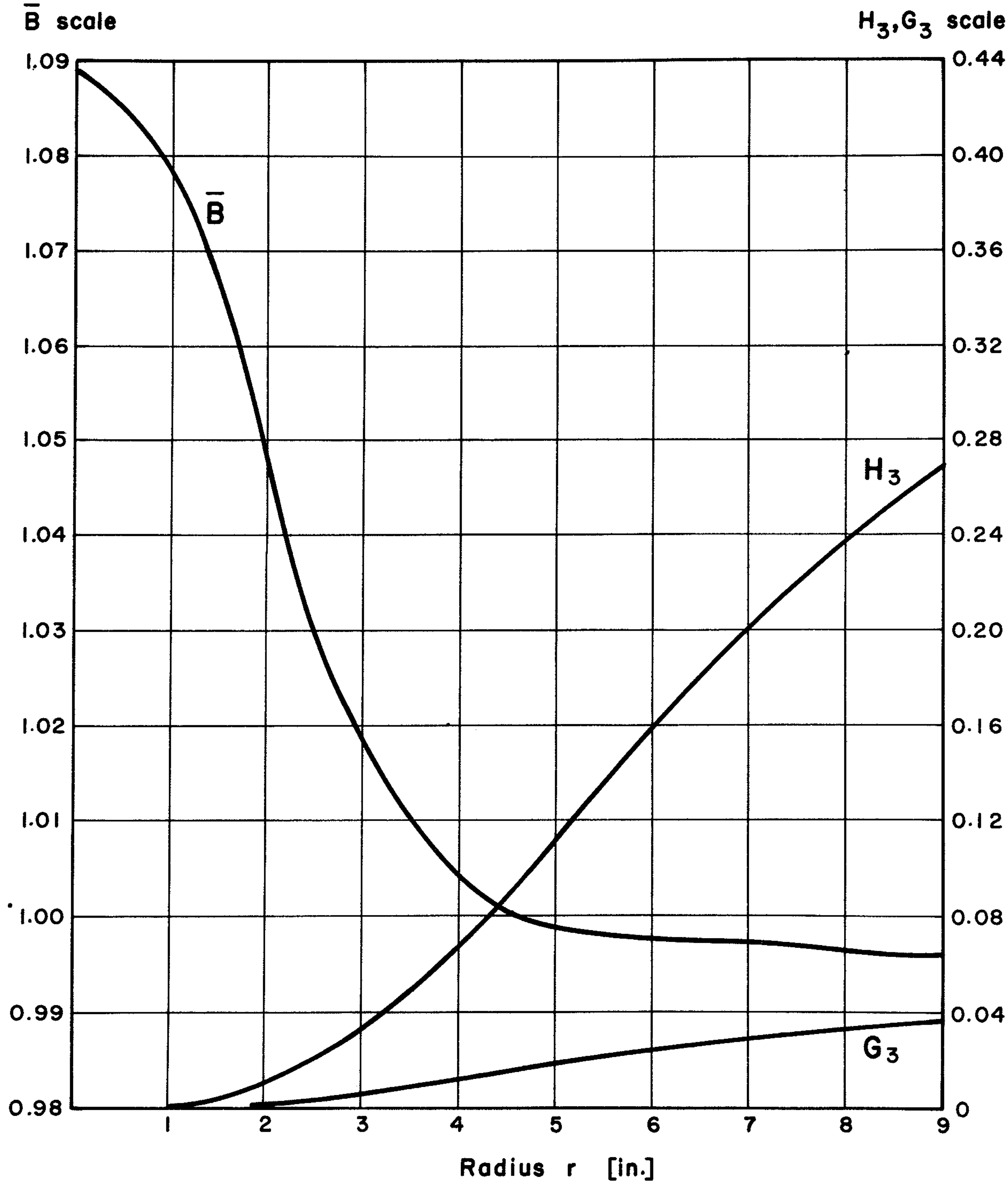

Fig. 4: Bump field 187F6.

a radius of 9 inches, while G_3 is only -0.0324 at this point. With these numbers the hill-peak angle is about 63° at $r = 9$ in.

The bump field 187F6, which is plotted in Fig. 4, has a peak of 8.9% in the center and drops to 1, i.e., $\bar{B} = B_o$, at a radius of 5 inches. The two Fourier coefficients H_3 and G_3 are not much different from the corresponding values in field 28.4.

3. Median-Plane Motion in the First-Harmonic Mode of Cyclotron Operation

3.1 Computer Results in the Case of Isochronous Field 28.4

The final Pinwheel runs for the protons in first-harmonic acceleration were carried out with the electric field 1.05.1 shown in Fig. 1 and (for the isochronous case) with magnetic field 28.4 (Fig. 3). The peak dee voltage was set at 70 kV and the electric frequency at 21 Mc/s which corresponds to a cyclotron unit of $a = 89.4514$ in. Usually two types of computer runs were made: one where the particles started at the center of the ion-source output slit with zero initial momentum at different phases θ_o with respect to the r.f. (θ_o is negative if the instantaneous dee voltage is increasing and zero at the peak voltage), and another one where a group of particles started with different r, p_r values at a given phase θ_o. The first class of runs provides information on the general orbit pattern, energy gain, and phase grouping as a function of r.f. phase while the second type of calculations shows the effect of radial focusing and beam transmission in radial phase space. In the latter case four particles were considered: two starting at the center of the source slit with initial directions of 45° and -45°, and the other two starting 40 mils and -40 mils off center with 0° angle. The total momentum of the protons was chosen to correspond to an initial energy of 2.3 eV, which is believed to be a realistic estimate for ions emerging from a low-pressure arc plasma such as in a cyclotron source.

In the first calculations with field 1.05.1 it was found that the orbits were too much off center. Better results could be achieved by placing the x coordinate of the source 0.1 inches closer to the origin. Such displacements could easily be simulated without resorting to new electrolytic-tank measurements by picking up the particles in the field-free region inside the puller

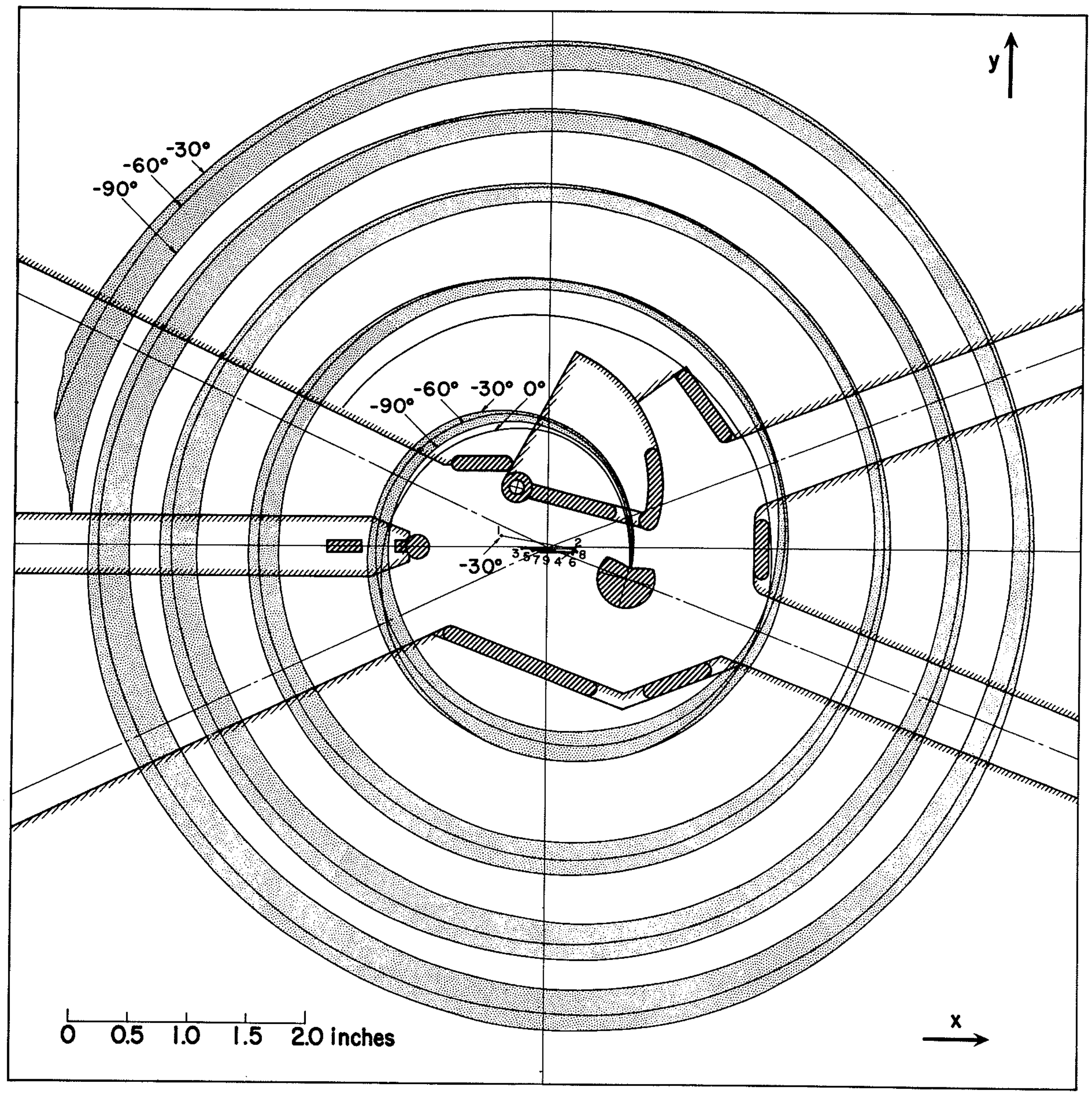

Fig. 5: Central trajectories of protons with different starting phase (Run 1.37, electric field 1.05.1, isochronous field 28.4).

and transforming the coordinates (or, if necessary, the momentum components) as desired for optimizing the orbit pattern. The computer results obtained for the protons after the 0.1-inch displacement in Pinwheel Run 1.37 are plotted in Fig. 5 which shows the trajectories of particles starting at different θ_o. Source and puller are placed at the positions corresponding to the corrected starting conditions. The starting point is at $x = 0.675$ in., $y = -0.175$ in. It is seen that the group of particles leaving the source in the phase interval between -90° and -30° (shaded beam) is well behaved on the first four turns that were calculated. The 0° particle is slightly off with its trajectory, but it is still acceptable if a phase-insensitive extraction scheme, such as negative-ion stripping, is used. Beyond 0° starting phase, particles are unacceptable because of rapidly decreasing energy gain in the first gap. The maximum phase transmission is therefore about 90°, corresponding to a duty factor of 25%. If a phase-selective extraction method is employed, however, only a narrow phase interval of a few degrees is acceptable. In this case the beam defining slit in the left dummy dee can be used to cut off most of the unwanted beam and vertical slits[3)] can be employed for fine selection, as will be discussed below.

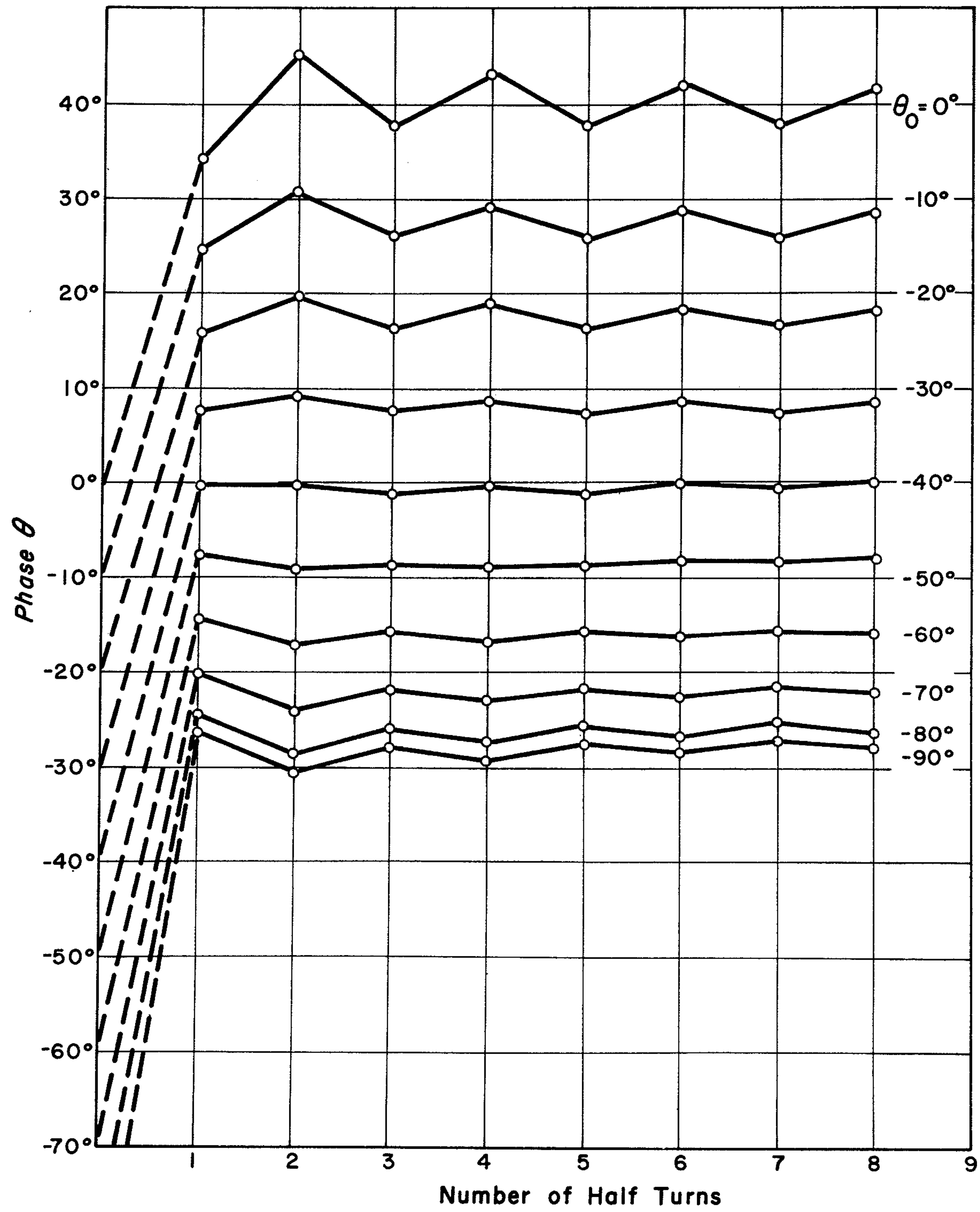

Fig. 6: Phase history of protons with different starting phase (Run 1.37).

The phase θ of the ions with respect to the accelerating peak voltage is plotted in Fig. 6 at successive half turns, i.e., each time they cross the x axis. Each curve belongs to a particular starting phase θ_o as indicated. It is seen that the useful starting-phase interval $-90° \leq \theta_o \leq 0°$ is shifted to $-30° \leq \theta \leq 40°$ with a weak bunching effect taking place. The particle with optimum starting phase $\theta_o = -30°$ (maximum energy gain in the first gap) assumes a phase of about 9° where adequate vertical focusing can be expected. If the cyclotron operates with a phase-selective extraction mechanism any small phase interval in the range of positive, vertically focusing θ values, i.e., at starting phases $\theta_o > -40°$ can be chosen for acceleration depending on how effective electrical focusing shall be. In high-duty-factor operation, however, it is unavoidable that a small portion of the beam, namely particles starting earlier than -40°, experience a certain amount of defocusing. Fortunately the majority of ions leave the source at times close to the peak-voltage phase so that this effect should not lead to appreciable intensity losses.

One of the main objectives in the optimization of starting conditions is to center the orbits of the useful group of ions. Since one is dealing with accelerated orbits in a magnetic sector field one has to define what one understands by centering. The best way of approaching this problem is to check the particle's radial position and momentum at a given kinetic energy and azimuth and compare it to the equilibrium orbit at this azimuth and energy. The particle's orbits are centered according to this description if the displacements from the equilibrium orbits approach zero after a large number of turns. In the Pinwheel program, however, which does not calculate or contain equilibrium-orbit properties, an approximate method was invoked to obtain some information on orbit centering. The program simply calculates and prints out the instantaneous center points at 90° azimuthal intervals, i.e., each time the ions cross the x or y axis[4]. For computational simplicity the actual magnetic field at the 90° azimuths is replaced in the calculation by the central field B_o so that this approximation is only good in the very center of the cyclotron where the flutter is negligible or very small. Due to the geometry of the accelerating gaps the center points oscillate mainly along the x axis on the first turns while the y coordinates change only little. This can be seen in Fig. 5 where the center points at successive crossings of the y axis (middle of dees) are marked for the particle starting at -30°. A better display of the center-point motion can be obtained by plotting the x coordinate at successive crossings of the y axis. This is done in Fig. 7 for the -90°, -60°, -30°, and 0° particles of

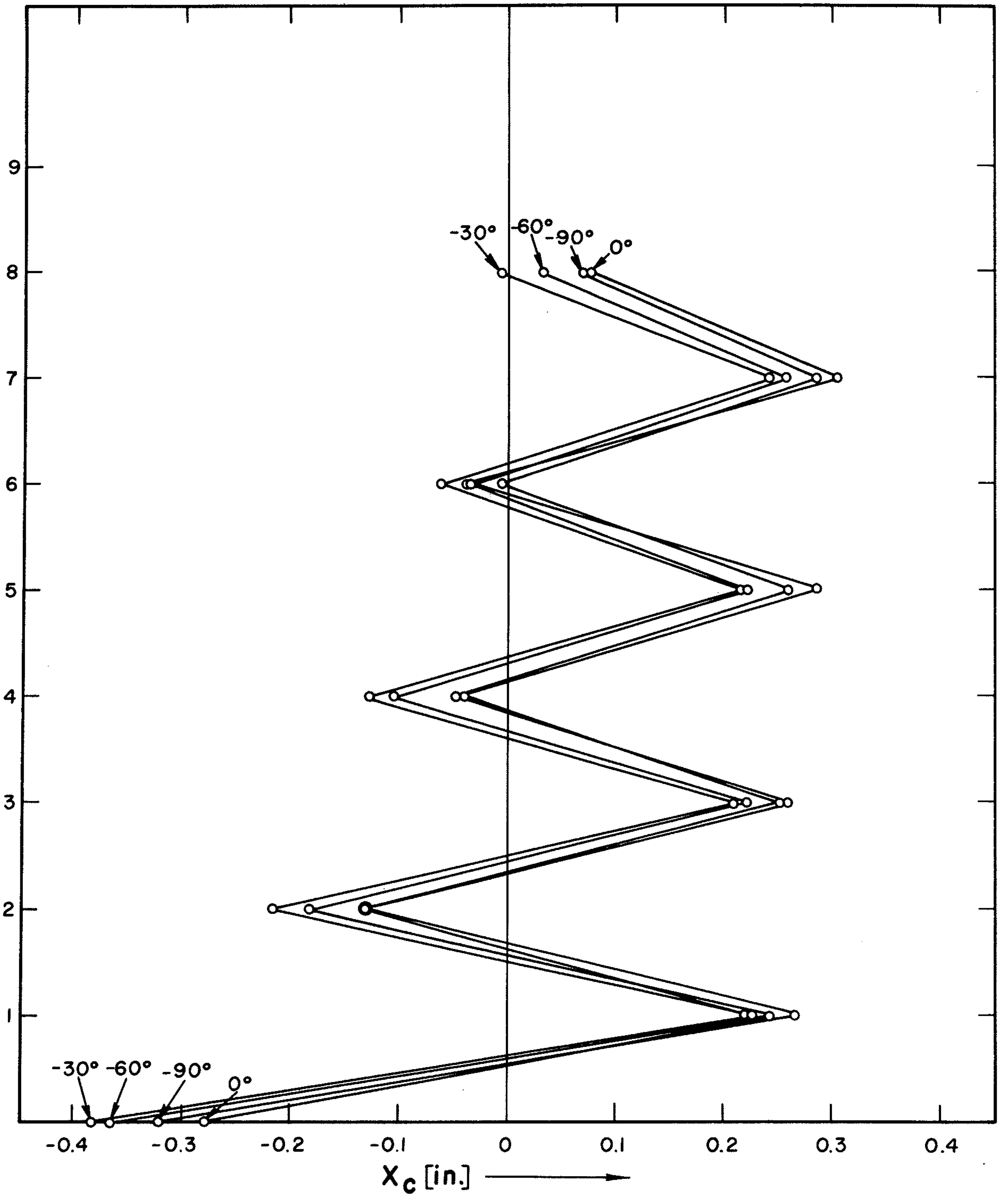

Fig. 7: x coordinates of instantaneous center points at 90° and 270° azimuths at consecutive half turns in Run 1.37.

Run 1.37. This graph exhibits two significant features: first, the center points of all four particles are well grouped, and second, there is an obvious off-center drift in positive x direction. While grouping of the orbit centers of particles with different phases is an expected result of properly positioning the source-puller system, the drift away from the origin is an undesirable effect. There are essentially three possibilities that would explain such a drift, namely: (a) an asymmetric energy gain, (b) the gap-crossing resonance which was investigated by Gordon[7], and (c) the increasing flutter of the magnetic sector field. Only the first two effects would produce a genuine drift of the orbits as a whole whereas the last assumption would simply mean that the increasing flutter causes a displacement of the instantaneous center points at the 90° and 270° azimuths and in addition implies a breakdown of the approximate center-point formulas employed in the Pinwheel program. The computer results in Run 1.37 do not show a systematic asymmetry in the energy gain of the ions; the energy gain in the two gaps on each side of the dee system is fairly balanced with slight fluctuations depending on the phase of the ions. Further evidence against the assumption of asymmetric acceleration rates comes from

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7) M. M. Gordon, Nucl. Instr. & Meth. 18, 19 (1962),268.

the fact that the drift does not occur in a flat magnetic field as computer runs with otherwise identical conditions show. The gap-crossing resonance, on the other hand is an inherent effect in this type of cyclotron and arises from the interaction of the three-sector magnetic-field configuration with the two-fold symmetry of the dee system. Blosser continued the Pinwheel calculations to higher energies with the General-Orbit Program (which calculates the deviations from equilibrium orbits) and found that the drift due to the gap crossing resonance reaches about 0.4 inches at larger radii and then levels off. The off-center motion on the first turns, however, does not go in the same direction as the instantaneous center points in Fig. 7. One concludes, therefore, that the drift in Fig. 7 is not the genuine gap-crossing resonance effect but mainly a manifestation of the increasing flutter.

The next picture (Fig. 8) shows the effect of radial focusing due to the magnetic and electric fields. The trajectories belong to a group of particles starting at the initial phase $\theta_o = -30°$ with different r, p_r values as described earlier. Only one of the two ions starting with $\pm 45°$ inclination in the center of the output slit is shown since there is practically no difference between the two trajectories. It is seen that the beam pattern exhibits some sort of an oscillation with a periodicity corresponding roughly to the magnetic betatron frequency

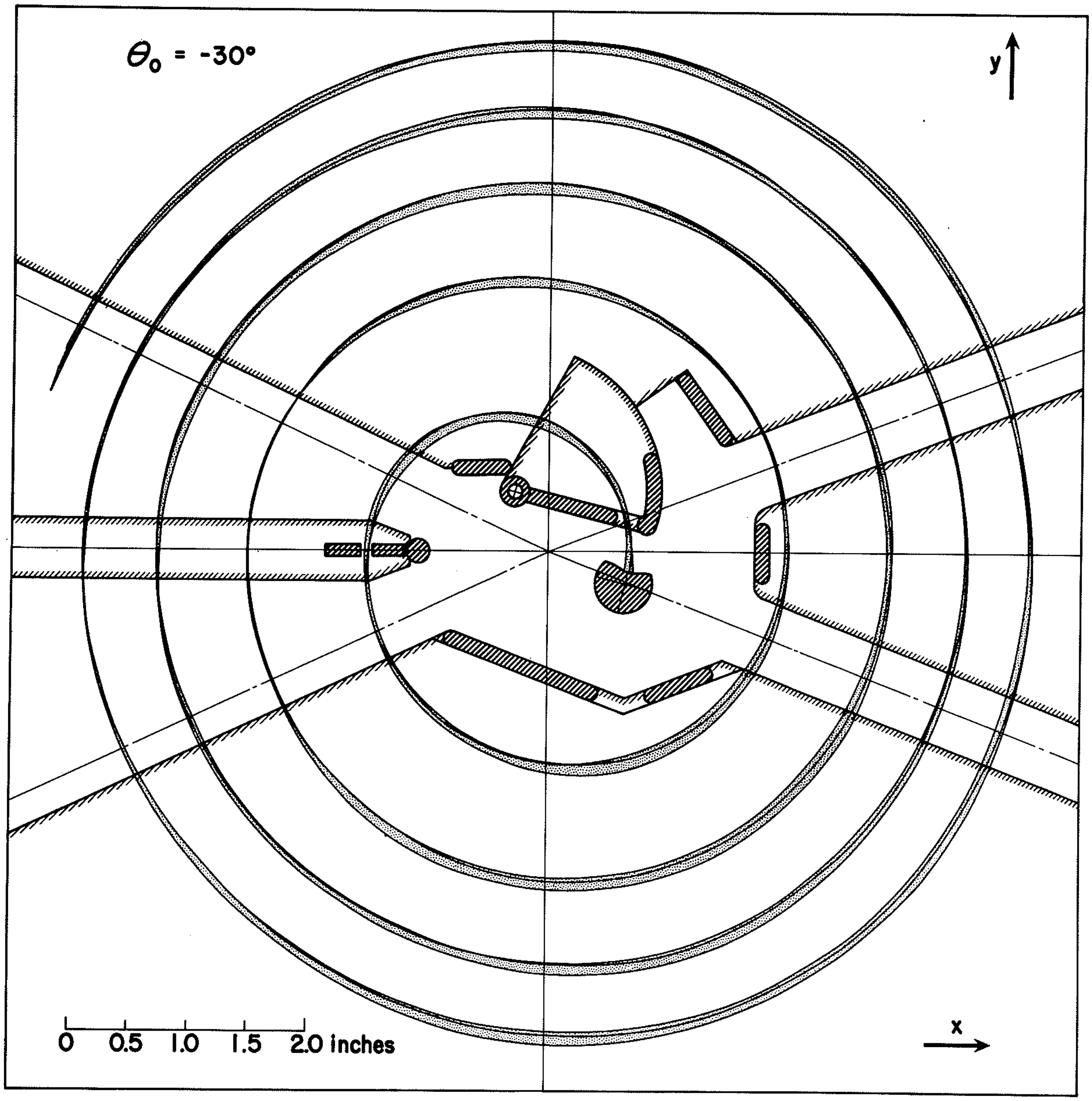

Fig. 8: Radial-motion pattern for group of protons starting at same phase of -30° (Run 1.38).

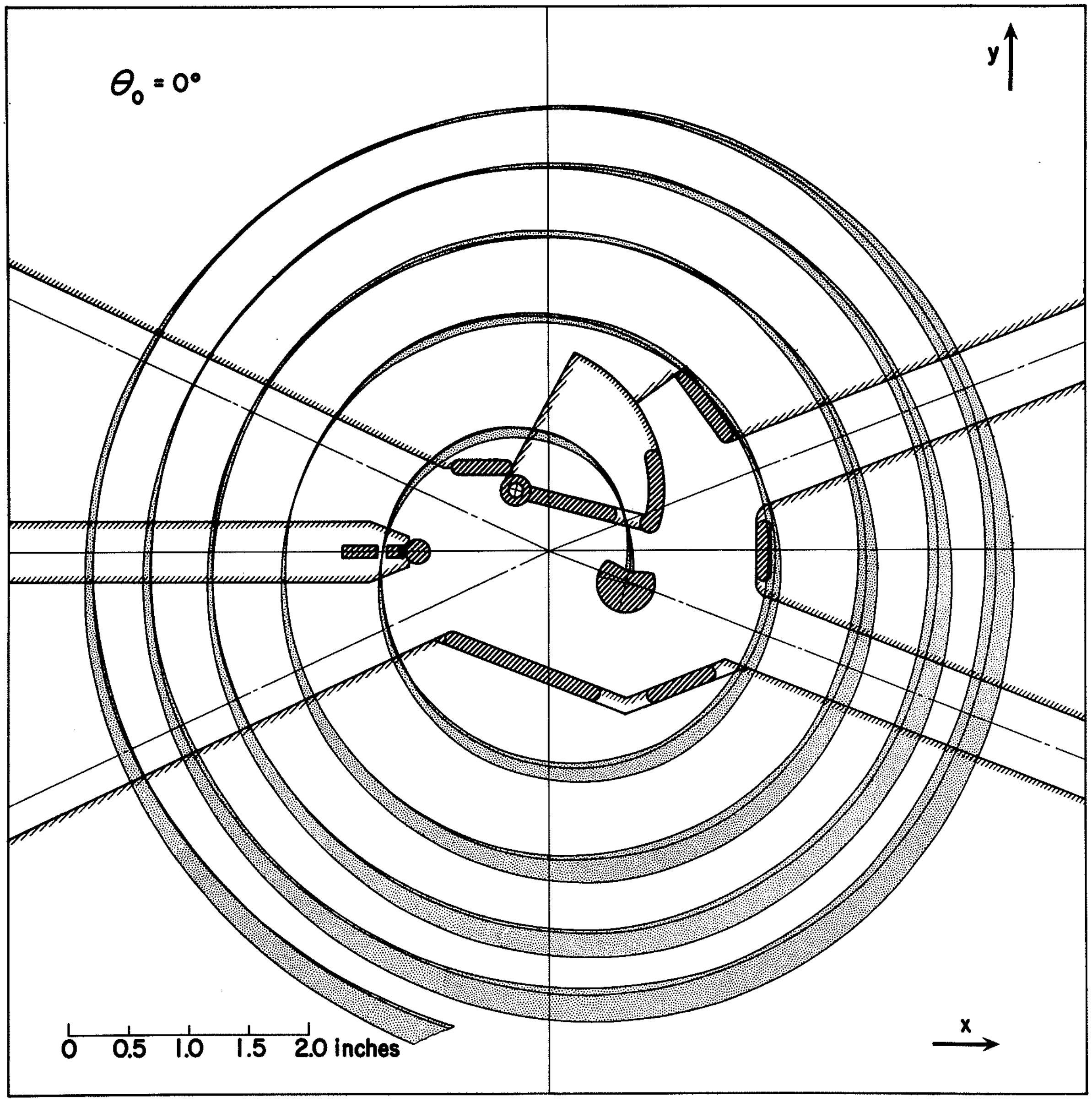

Fig. 9: Radial motion in the case of 0° starting phase (Run 1.38).

which is practically unity in this region. However, the influence of the electric field modifies this simple picture of linear oscillations quite distinctly. There is a first crossover as the beam enters the puller slit; this is a result of the ion source geometry (see reference 2 for details). The next focal point occurs right behind the beam defining slit at an azimuth of about 195°. After that the "nodes" of the beam pattern are less clearly defined as they take the form of extended focal lines rather than distinct points. This shows, of course, that the picture of a radial oscillation with basic frequency $\nu_r = 1$ slightly modulated by the electric field is an oversimplification. There are non-linear effects as well as coupling from radial motion into energy-time effects so that Lionville's Theorem cannot be applied to the two-dimensional radial phase space r,p_r. Particles starting at the same time and with same initial momentum experience a different energy gain and phase history. This becomes more obvious if one calculates the median-plane motion for particles starting at other phases than -30°. The beam pattern for the ions with 0° starting phase seen in Fig. 9, for example, is markedly different from the previous picture. The first focal point occurs almost exactly at 180°, i.e., at the beam defining slit; this indicates that radial focusing in this region is stronger than it was in the case of the -30° ions. The next node of oscillation occurs at an azimuth of about 20° at the end of the first turn and is quickly followed by another one at 155°; from then on only one focal point per turn (located near 150° azimuth) occurs.

Similar results have been obtained for other starting phases: the trajectories of the -90° particles, for example, also exhibit this type of oscillatory pattern with focal lines occuring at about 270° but extending over large angular regions. The -60° beam pattern is somewhere in the middle between -90° and -30°. The conclusion from these results is that the radial motion is—like the vertical motion—highly influenced by the electric field and depends strongly on the phase of the particles with respect to the r.f.

In normal cyclotron operation the major fraction of the ion beam emerging from the source—such as ions starting at wrong phase, molecular ions, etc.—is not acceptable or not desired for acceleration to the final radius and extraction from the magnetic field. It is of great importance that these ions are stopped as early as possible to avoid unnecessary loading of the r.f. system, cooling and radiation problems. In the MSU cyclotron the adjustable beam defining slit in the left dummy dee is designed to accomplish this task. An ideal defining-slit system should perform two functions: (a) accomplish a phase selection, i.e., cut off all ions with undesirable phases to the accelerating voltage, and (b) define radial width and momentum of the useful portion of the beam to match the optics of the magnetic field and extraction system of the cyclotron. Theoretically, depending on the resolution required, one may need a whole number of slits properly placed on the central turns and all adjustable. The deliniation of radial momentum and width of a group of particles with same phase, for example, requires two slits; for phase selection additional slits may be necessary. At MSU the philosophy is to start out with one adjustable slit for coarse beam selection and add more slits for higher resolution if desired. Actually the puller performs the function of a preselector or first beam stopper as only particles in a limited interval of starting phases (protons starting earlier than about 10°) and r, p_r values are transmitted through the slit. The main task of the beam defining slit in the dummy dee is to further narrow down the desired phase interval and radial width of the beam in the case of low-duty-factor operation of the cyclotron with high energy resolution. To get an idea of how good a phase selection may be expected from this slit, the radial position and width of the beam at the 180° azimuth has been plotted as a function of starting phase in Fig. 10 using the computer results from Run 1.38. In the calculations it had been

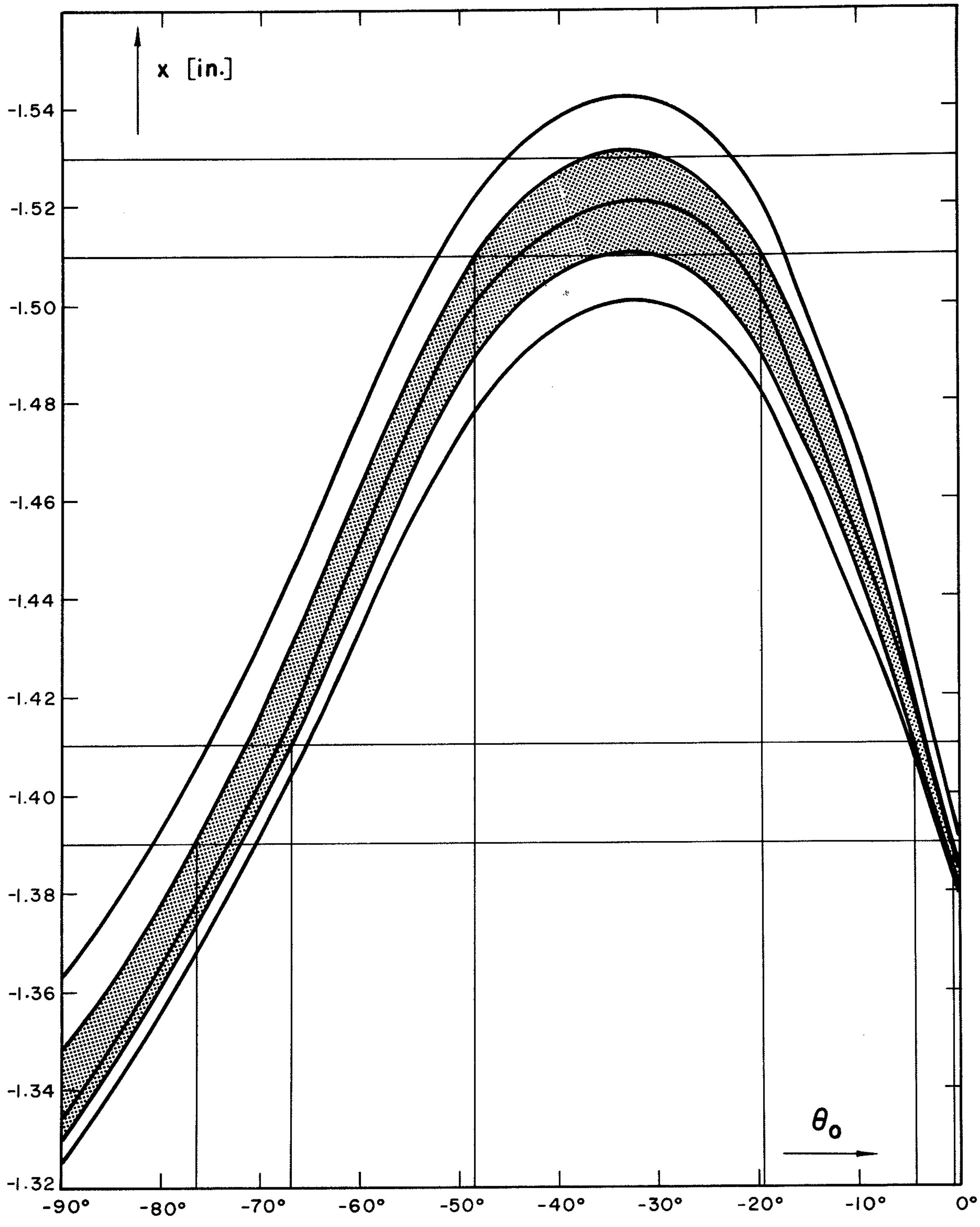

Fig. 10: Radial position and width of beam at defining slit (180° azimuth) as a function of starting phase (Run 1.38).

assumed that the beam starts at the source with an initial width of 80 mils so that one could get a better display of the focusing properties of the electric and magnetic fields. The inner and outer curves in Fig. 10 belong to the ions starting at +40 mils and -40 mils displacements, respectively, the central curve belongs to an ion starting in the middle. The actual beam width, however, is only about half that big since the output slit of the ion source is only 40 mils wide, and so the shaded area in Fig. 10 is a better representation of the true situation. From this graph one may now get an estimate of the resolution (in terms of the phase) to be expected if the beam defining slit is used as a phase selector. For this estimate the thickness of the slit jaws and other effects shall be neglected. First one sees that the beam width is less than half of the value at the source for all starting phases. This is due to the fact that in all cases a node of oscillation is located near the 180° azimuth as was discussed previously; towards0° phases the beam becomes smaller since the focal points are almost exactly at 180° for these particles.

The most useful group of particles, which start at phases near -30°, have the maximum radius, as one expects, but they also have the largest width at the 180° azimuth. Consequently, a phase selection is least effective in this region: as is illustrated in Fig. 10 a phase interval of 30° would pass through a slit with 20-mils opening. The resolution improves substantially, however, if one shifts away from the maximum radius. A 20-mils slit centered at $x = -1.40$ in., for example, would select a 10° interval at about -70° starting phase and a 4° interval near 0° starting phase. In this case one would have two small pulses of ions, and it would be desirable to have another slit which stops one of them at a point where their radial position no longer coincides.

The above estimates are, of course, only qualitatively correct since one does not know the true source-emittance characteristics in radial (and vertical) phase space and since the graph in Fig. 10 is based on only four trajectories; besides, space-charge effects are not included in the calculations. Nevertheless, the figure is a good approximation of the actual situation and may serve as a guide in placing and adjusting of the beam defining slit.

3.2 Computer Results with the Bump Field 187F6

Before the Pinwheel results with field 187F6, which has a central bump with 8.9% peak strength, will be discussed, a brief estimate shall be made as to what is to be expected in comparison to the isochronous-field situation. The main difference to the isochronous case is that the bump causes a large phase shift which has a strong effect on the radial motion if one looks at particles with different starting phases. For a rough estimate of the phase shift one may use the formula:

$$(4)\quad \sin\theta - \sin\theta_o = - \frac{2\pi N E_k}{eVr^2} \int_o^r \frac{\Delta B}{B_o}\, dr^2$$

where N is the harmonic number, E_k the kinetic energy at radius r, and eV the maximum energy gain per turn. If one plots $\frac{\Delta B}{B_o}$ versus r^2 one can graphically integrate, and with $N = 1$, $r = 5$ in., $\theta_o = 0°$, one obtains for the total phase shift in the bump region a value of 47°. With such a large phase shift the particles starting in the phase interval close to -90° are driven into the decelerating half period of the r.f. voltage and hence getting lost for the acceleration process which means that one does not get the large duty factor of the isochronous field operation. Another effect of the bump is that (compared to the isochronous case) it causes a shrinkage of the orbit radii since (a) ions of same starting phase experience a smaller total energy gain due to the phase shift, and (b) ions of same kinetic energy have a smaller radius due to the higher field in the bump case. (At a radius of 2 in., for example, where B is 5% above B_o according to Fig. 4, the orbit radius is 0.10 in.

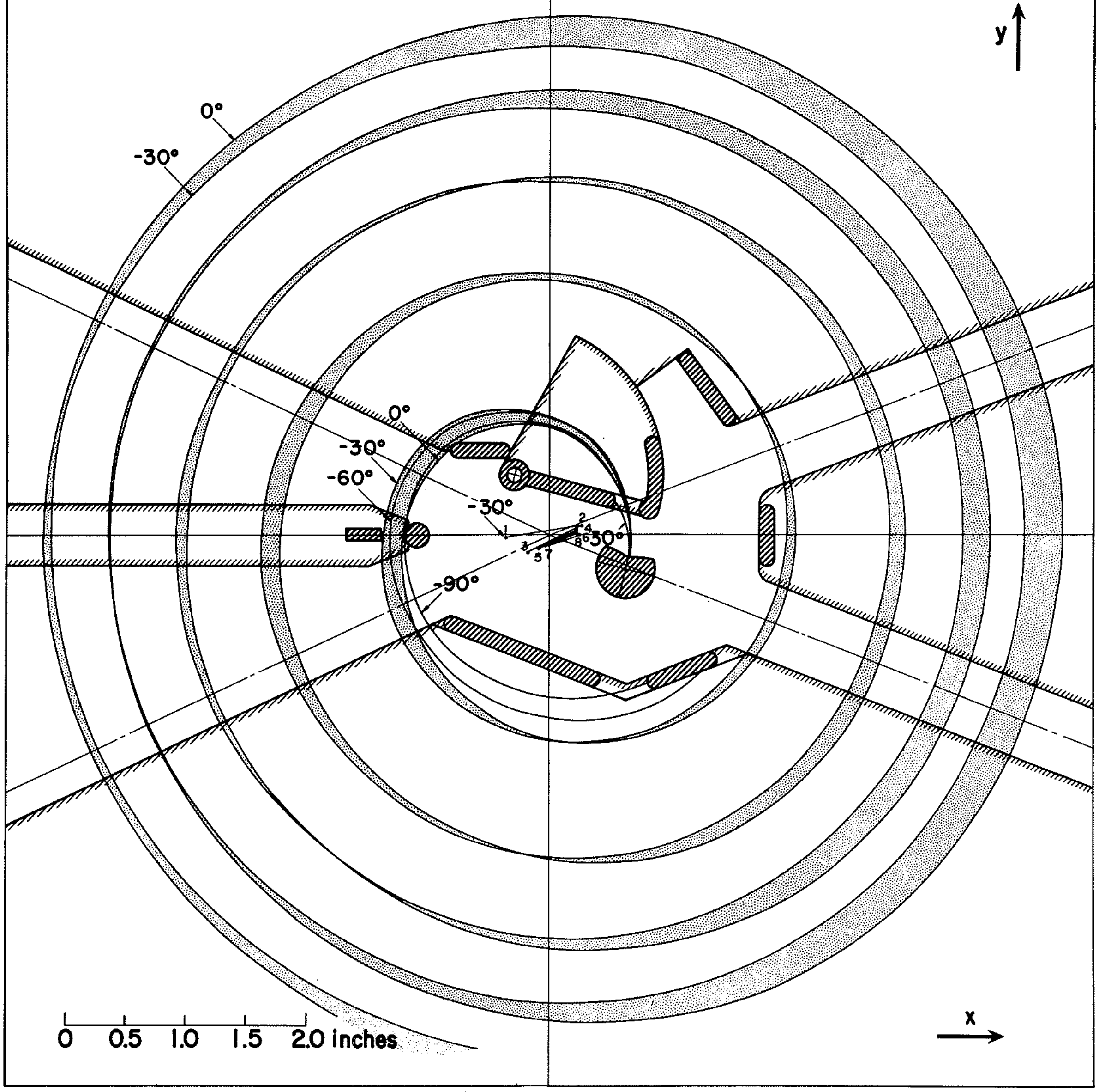

Fig. 11: Proton trajectories in the case of bump field 187F6
(Run 1.42, electric field 1.05.1).

smaller than it would be in the isochronous field.)

In the Pinwheel runs that were made with field 187F6 the same electric field (1.05.1), same operating and starting conditions were used as in the previously discussed runs 1.37 and 1.38 to allow a direct comparison with the isochronous-field situation. Fig. 11 shows the results of Pinwheel Run 1.42 where protons with different starting phases have been calculated in the bump field. It is seen that only the -30° and 0° trajectories are well behaved. As expected the -90° particle gets quickly out of phase and hits the screening electrode in the lower dee. Likewise the -60° particle despite its better starting phase does not clear the electrode structures on the first turn; the calculations show that even if it could continue its path it would not be useful for further acceleration. Beyond 0° a phase interval of about 30° would be acceptable as far as acceleration is concerned but ions starting in this region do not gain sufficient energy in the first gap to get through the puller slit and clear the electrode structures on the first half turn; this is demonstrated for the 30° particle whose trajectory ends at the puller.

What remains then is that only the protons starting in the phase interval between about -40° and 0° are useful for further acceleration. But even in this acceptable phase interval the trajectories are not as nicely bunched

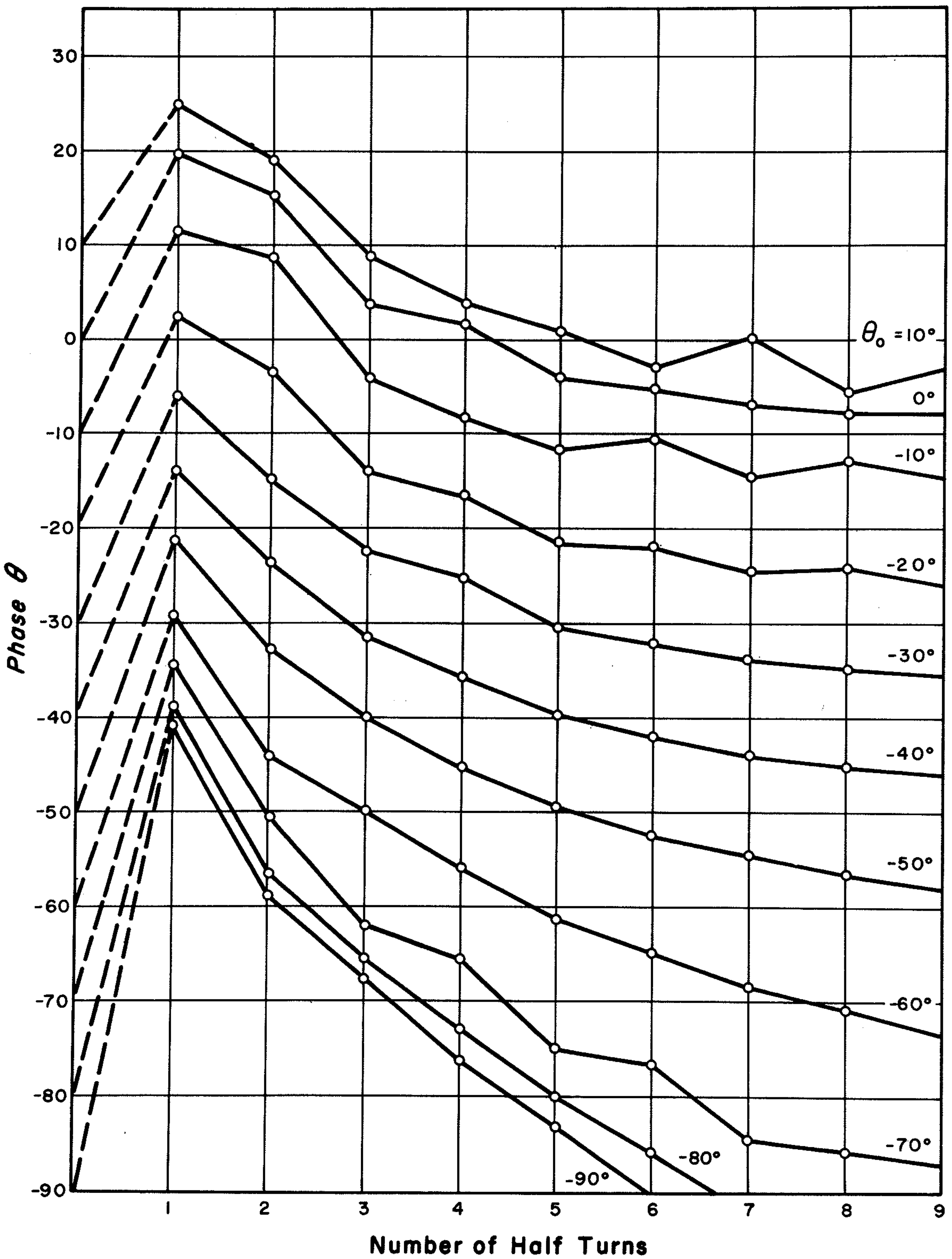

Fig. 12: Phase history in field 187F6 (Run 1.42).

in radius as was the case in Run 1.37 for the isochronous field. This is readily seen if one compares the -30° and 0° trajectories in Fig. 5 with those in Fig. 11 where an increasing spread in radius is taking place on the right side due to the difference in phase history for the two ions.

The center points of the -30° protons at successive crossings of the y axis are marked in Fig. 11. Center points of protons starting at other phases behave similar but the bunching effect, i.e., grouping of orbit centers for ions with different phase, is much weaker than it was in Fig. 7 for Run 1.37.

The phase history of protons with different starting phases is plotted in Fig. 12 for the first nine consecutive half turns in Run 1.42. The total phase shift in the bump region can be determined by a comparison with the isochronous-field results. In Run 1.37 the particle starting at -40° was shifted to phase 0° by the azimuthal displacement of source and puller (see Fig. 6). Fig. 13 shows that the same particle in the bump field has a phase of -14° at the end of the first half turn and is down at a phase of about -46° after nine half turns. The total phase shift due to the bump is therefore in good agreement with the estimate based on formula (4). Fig. 12 also confirms what had been said previously: that the ions starting earlier than about -40° are shifted either

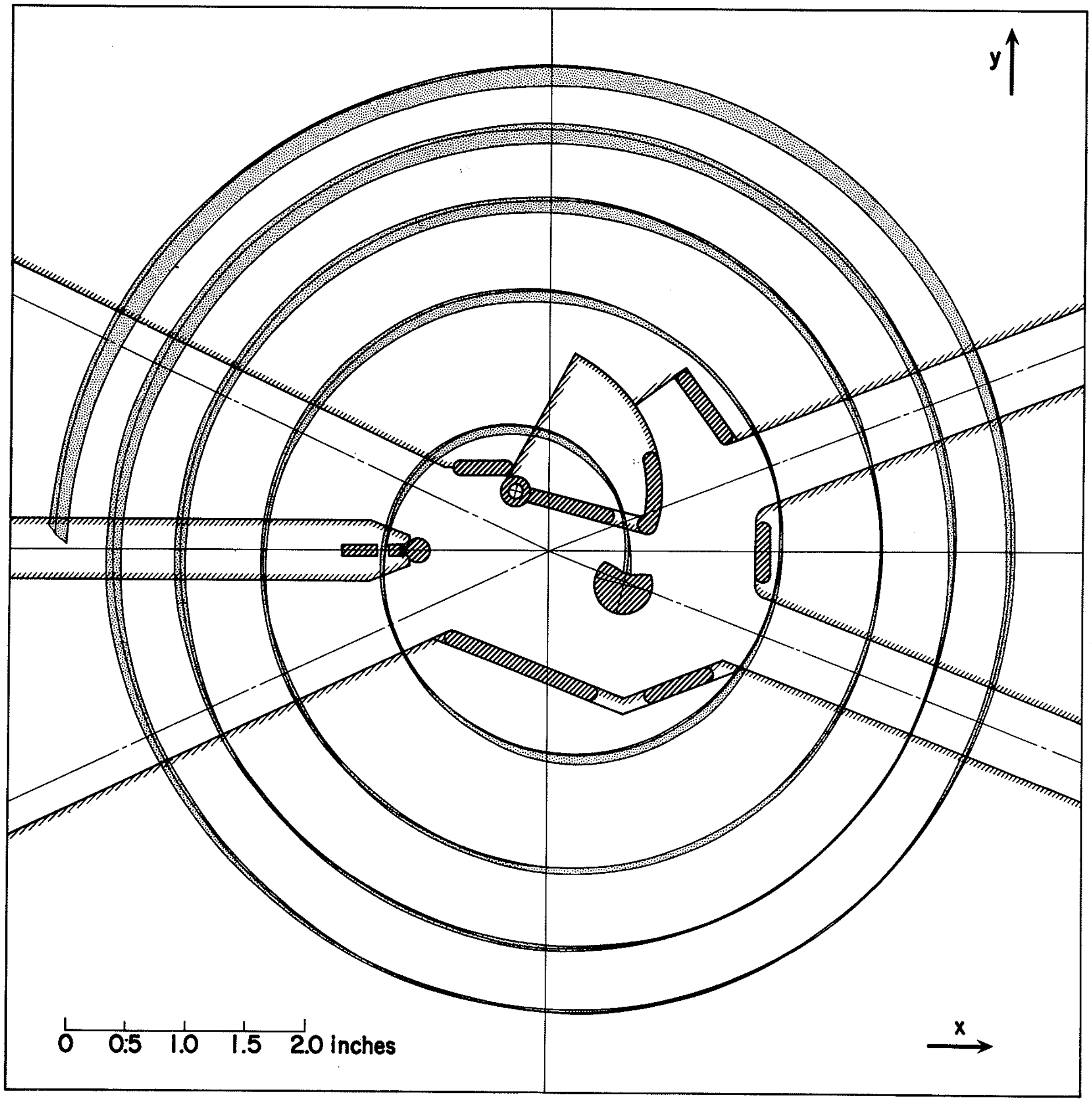

Fig. 13: Radial motion for group of protons starting at -30°
(Run 1.42).

into the decelerating half cycle of the r.f. or towards phases with insufficient energy gain.

The next picture (Fig. 13) shows the results of Run 1.43 where the effects of radial focusing in field 187F6 have been calculated for a group of ions starting at -30°. In this case the overall situation does not differ very much from the isochronous-field results. Focusing is quite effective although the pattern of oscillation is somewhat different than it was for the -30° particles in Run 1.38. But this difference is no surprise as the -30° protons of Run 1.43 due to the phase shift in the bump region behave more like the -60° protons of the isochronous case. On the other hand, the 0° particles of Run 1.43 display a similar pattern than the -30° group in Run 1.38 as the calculations show; however, the amplitudes of the radial oscillations are somewhat larger than in the equivalent isochronous case.

In the final analysis and comparison one may summarize the computer results for median-plane motion in the isochronous field 28.4 and in the bump field 187F6 as follows: (1) If one looks at the "quality" of the beam in terms of radial bunching, energy gain, etc., for particles of different starting phases the isochronous field yields substantially better results than the bump field. (2) As a result of this the phase acceptance is better and hence the maximum obtainable duty factor higher, in the case of isochronous field. It should be mentioned, however, that some improvement of the situation in the bump field should be possible by further off-setting the source-puller system to balance the phase shift.

(3) With respect to radial focusing for ions starting with different r, p_r at same phase both fields are comparable with minor differences due to the different phase history in each case. This implies that in low duty-factor operation of the cyclotron where only a narrow phase interval is transmitted, both schemes are equally effective as far as radial focusing and radial beam quality is concerned.

If vertical motion is included in the comparison the isochronous field still offers the advantage of larger duty factor, better over-all quality of the beam, and hence higher intensity despite weaker vertical focusing. In low-duty-factor operation, on the other hand, it was shown that it is possible to select a reasonably small phase interval above or below the peak-energy phase with the beam defining slit. As the resolution increases towards 0° starting phase one would work at some phase between -30° and 0° where electric focusing is sufficiently strong. But while it is easy to select phases with efficient vertical focusing in either case, the bump field offers the additional advantage of the phase shift which may be utilized to bring the phase to the maximum-energy-gain position after the electric-focusing region is traversed by the ions. This phase shift needs not to be as large as it is in field 187F6, and a small bump of 2 to 3% strength would be sufficient for this purpose.

There is another argument which also points towards a smaller bump or isochronous field. Field 187F6 had been designed to work for protons in first-harmonic operation. It is not suitable for higher-harmonic modes of operation as the phase shift exceeds the tolerable limit of 90° if N in equation (4) becomes larger than 1. The solution which was now chosen for the central magnetic field of the MSU cyclotron is a compromise which offers great flexibility: the iron plug in the center is shaped to produce a bump of about 3% peak strength; in addition the inner trimming coil will be employed to neutralize or enhance the iron field so that any desired field shape—from isochronism to strong bump—can be realized.

4. Higher-Harmonic Acceleration

4.1 Pinwheel Results for the Second-Harmonic Mode of Operation (N = 2)

As was pointed out previously the strong bump of field 187F6 is not feasible for higher-harmonic acceleration since it would cause an excessive phase shift. Only very weak bumps, if any, or isochronous field configurations can be employed in these cases. For these reasons the Pinwheel studies for the N = 2 and N = 3 modes of operation were restricted to field 28.4.

In the N = 2 case, where the two dees are oscillating in phase (push-push), the latest calculations were carried out with electric field 2.04.1 (see Fig. 2). The ions accelerated in this mode are those with charge-to-mass ratios of 1/2 compared to protons, i.e., deuterons, alpha particles, etc. The Pinwheel computations were performed for deuterons, but the results are, of course, applicable to any other particle of same specific charge. The dee voltage was set at 48.8 kV which corresponds to the value for constant-orbit geometry[1)] in a 138° dee system. Electric frequency and magnetic field level are the same as for the protons while the cyclotron unit is twice the proton value.

In the first calculations with field 2.04.1 it was found that source and puller had to be displaced to

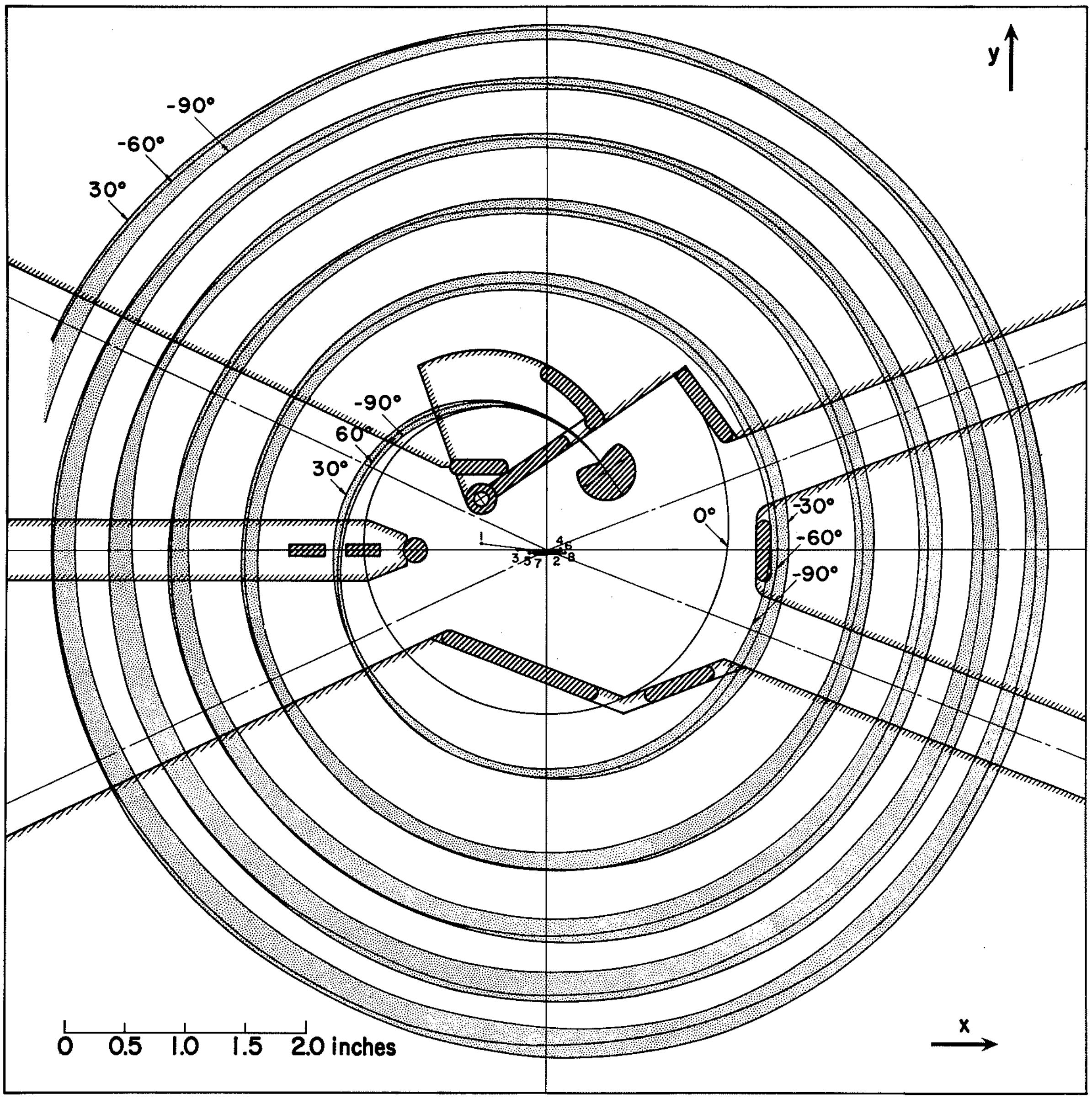

Fig. 14: Central orbits in the case of second-harmonic acceleration (Run 2.24, fields 2.04.1 and 28.4).

obtain the desired beam behaviour. The best conditions were achieved in Run 2.24 (with isochronous magnetic field 28.4) and Fig. 14 shows the turn pattern for a group of deuterons with different starting phases as calculated in this run. The source output slit and starting point for the particles is at $x = 0.44$ in. and $y = 0.74$ in. Due to the displacement of the puller towards the left side the shielding electrode behind the puller had to be cut to 0.5 in. (see Fig. 2 for comparison). As was mentioned in the beginning this electrode is necessary to prevent field penetration into the dee in the third-harmonic mode where the puller is far to the right. Since this electrode is an obstacle for the positioning of the puller it should be taken out when the cyclotron is operated in the $N = 1$ or $N = 2$ mode.

The acceptable phase interval in the second-harmonic mode of operation is a little smaller than it was in the $N = 1$ case and ranges from starting phases of -90° to about -20°. As Fig. 14 shows the particles leaving the source in this useful interval are accelerated with sufficient energy gain on well separated orbits and are strongly bunched in radial width. Particles starting later are not acceptable due to insufficient energy gain in the first gap and increasing divergence of radial motion as is demonstrated for the case of 0° starting phase.

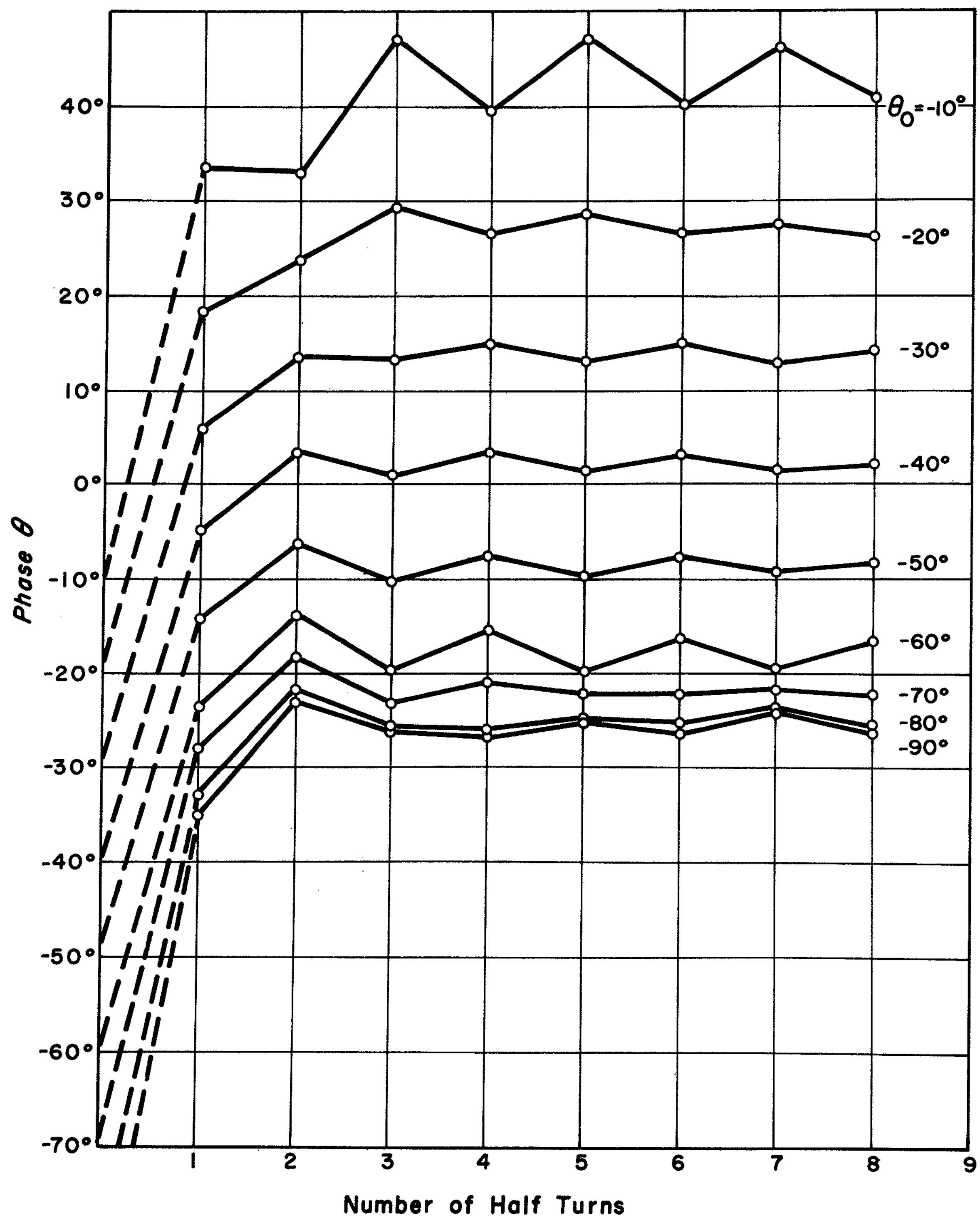

Fig. 15: Phase history in Run 2.24.

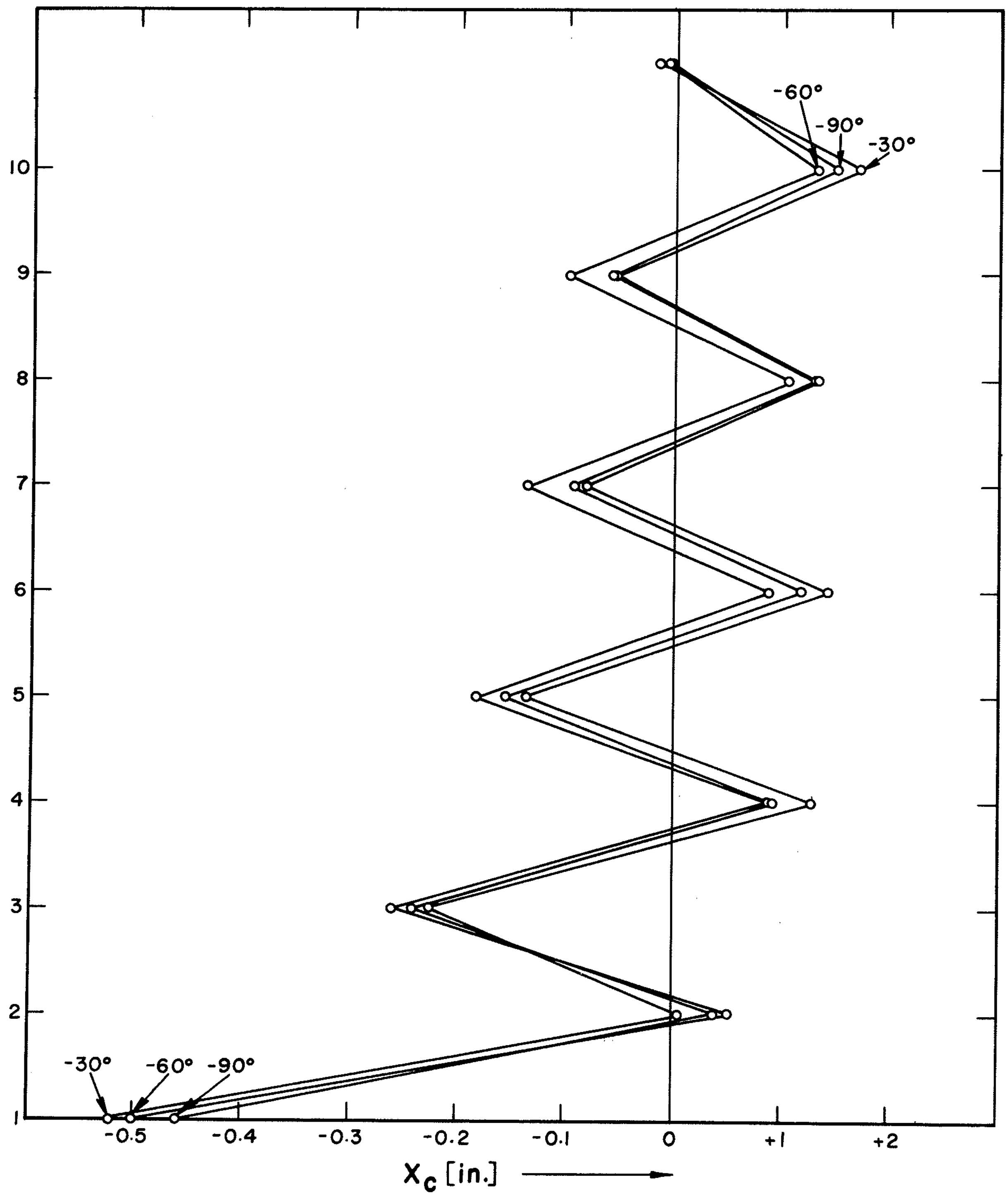

Fig. 16: Center-point coordinates at 90° and 270° azimuths (Run 2.24).

The phase-history plot in Fig. 15 shows that the useful group of ions is being centered at phase 0°(maximum energy gain) due to the reduction of dee angle between first and second gap. In addition the figure depicts a phase grouping effect for particles starting close to -90°: the 30° interval between -60° and -90° is contracted to only 10°. This bunching effect essentially takes place in the first gap between ion source and puller.

The orbit centers of different particles in Run 2.24 are very well grouped as is to be seen in Fig. 16 where the x coordinates of the center points are plotted at consecutive crossings of the y axis. The figure exhibits the same drift which was discussed in the proton case; the only difference is that the starting point in Run 2.24 is further off center than it was the case in Run 1.37.

The next picture (Fig. 17) displays the effects of radial focusing for a group of deuterons starting at a phase of -30°. The situation is quite similar to the proton case although the oscillation pattern differs somewhat in a few details. The main difference to the corresponding $N = 1$ pictures shown previously is that there is no crossover in the puller slit, and on most of the first turn the pattern resembles closely that of a parallel beam. The first distinct focal point occurs

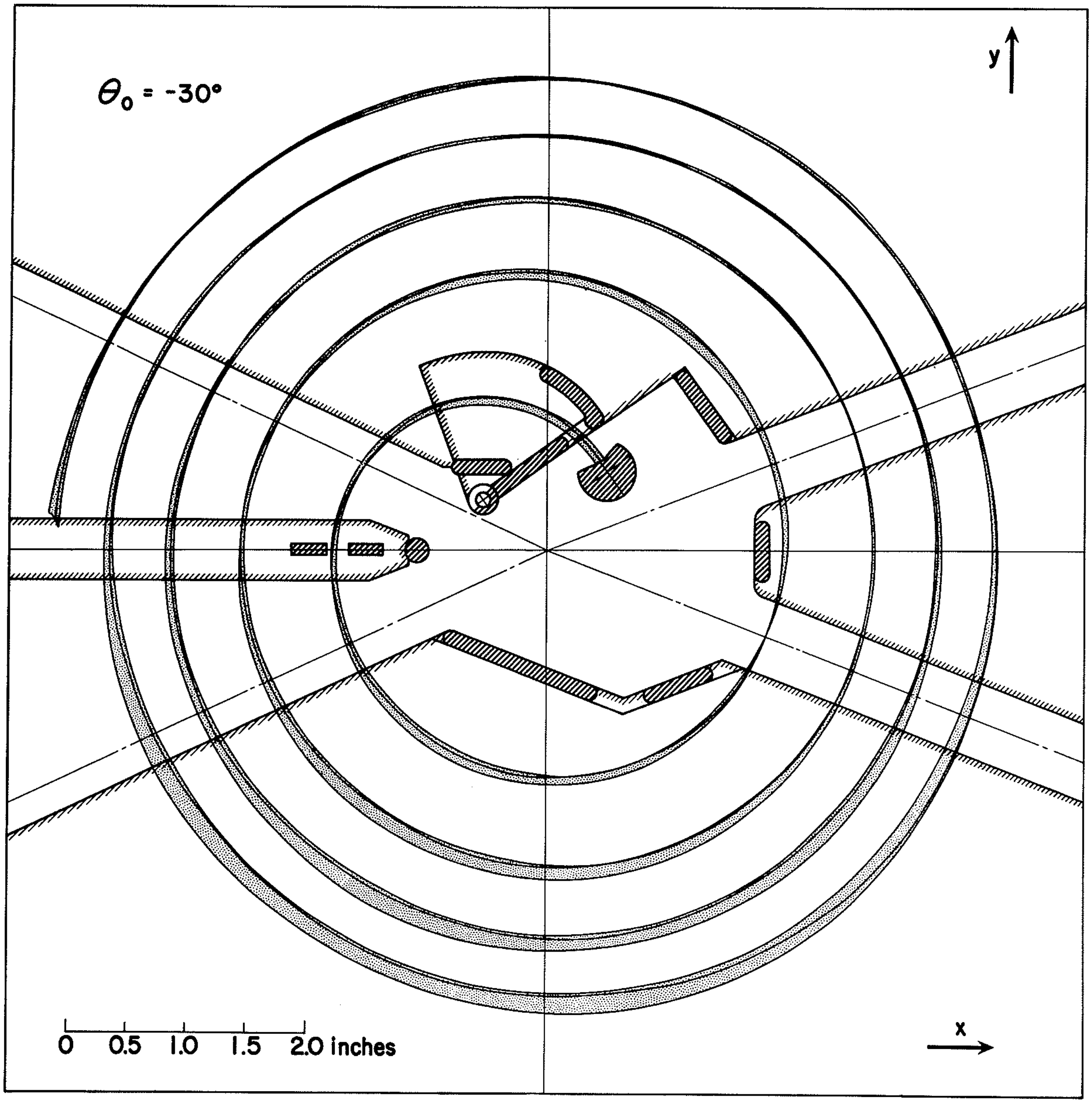

Fig. 17: Radial-motion pattern for the ions starting at -30° in the case N = 2 (Run 2.25).

before the end of the first turn at an azimuth of -25° and is followed by another, less accentuated one at about 160°. Further out the pattern shifts from two nodes per turn to a focal line in the upper dee and very distinguished antinodes in the lower dee.

4.2 Computer Results for Third-Harmonic Acceleration (N = 3)

The third-harmonic mode of operation posed the greatest difficulties to obtaining a satisfactory solution for the source position and the central orbits. One reason for this difficulty has already been mentioned previously: Due to the reduction of the dee angle from the optimum value of 144° to 138° the dee voltage had to be raised from 38 kV to 48 kV to maintain the constant-orbit requirement; this results in a higher energy gain for the ions in the first gap and therefore the ion source must be placed further off center than before. Another reason is that in third-harmonic acceleration the time (or fraction of the r.f. period) the ions need to traverse an electric-field gap is three times as large as in the first-harmonic mode. Consequently the ions may experience deceleration as they travel through the field even though the phase is correct. To achieve the same relative energy gain as in first-harmonic acceleration one would have to reduce the width of the field regions or accelerating gaps seen by the particles so that the transit angles are the same as in the $N = 1$ mode. As the transit angle is determined by the ratio of gap width to orbit radius the most critical conditions exist on the first revolution.

The changes in the dee system of the MSU cyclotron (reduction of dee angle, modification of left dummy dee, elimination of slits along first turn) had no serious ill-effect in the first- and second-harmonic modes but aggravated the situation in the $N = 3$ case considerably. The greatest problem was to find an acceptable position for the source-puller system and to get enough energy gain on the first turn so that the ions would clear the electrode structure in the tip of the right dummy dee. In the $N = 3$ case the ions suitable for acceleration are those with charge-to-mass ratios of 1/3 relative to the protons, such as ${}_{12}C^{4+}$ which was chosen as the test particle. In the computer runs for the case $N = 3$ the electric frequency and the magnetic field were again the same as in the $N = 1$ and $N = 2$ modes while the dee voltage was 48 kV. The first computer runs with electrolytic-tank field 3.04.1, which is similar to the field geometry shown in Fig. 1 except that source and puller are at different positions, were negative: starting from the source slit at $x_o = 1.16$ in., $y_o = -0.15$ in.

the ions did not gain sufficient energy to clear the electrode structures on the first turn. Then source and puller were displaced towards the right dummy dee as much as was tolerable from the standpoint of sparking safety; the new position corresponded to a starting point x_o = 1.25 in., y_o = 0, but the results were still negative. The next step was to change the dee voltage in several steps from 48 to 70 kV. But this too did not change the situation since higher energy gains in accelerating regions were neutralized by higher energy losses in decelerating regions of the electric field. Finally a sequence of backward runs with various dee voltages and phases of the ions were made to find out where the source should be located if the ions are to clear every electrode on the first turn. This search led to the source-puller position shown in Fig. 18, and the trajectories in this picture were calculated in Run 3.24 with a dee voltage of 48 kV using the 3.04.1 electric field data and magnetic field 28.4. The figure clearly illustrates the geometrical problems in this situation. The ion source almost touches the tip of the dummy dee, but this, of course, poses no problem since source and dummy dee are both at ground potential. However the critical distances between puller, which is at dee potential, and dummy dee is certainly smaller than can be tolerated and would probably result in serious breakdown and sparking troubles.

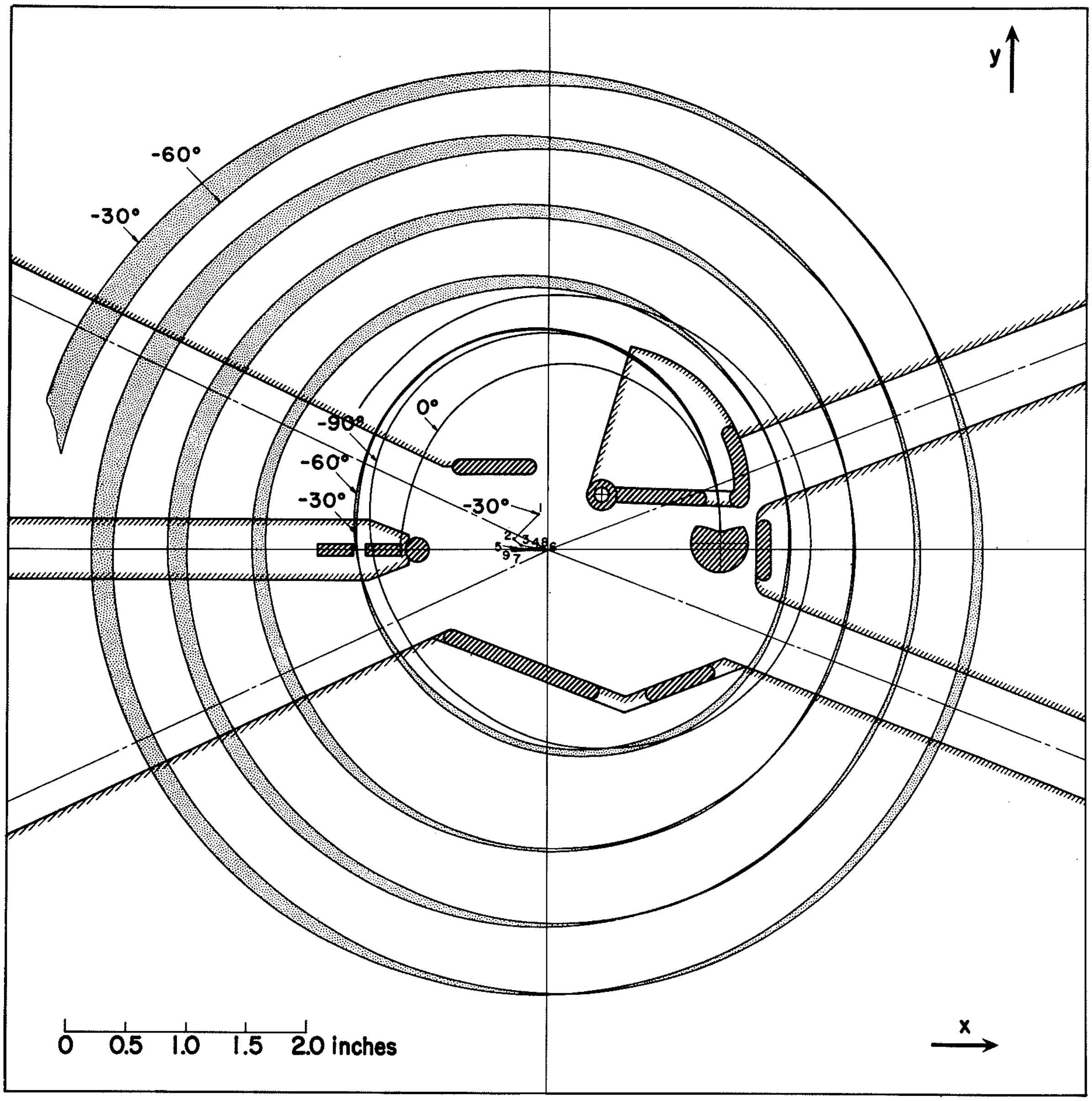

Fig. 18: Source-puller position and initial trajectories in the third-harmonic mode (Run 3.24, electric field 3.04.1, magnetic field 28.4).

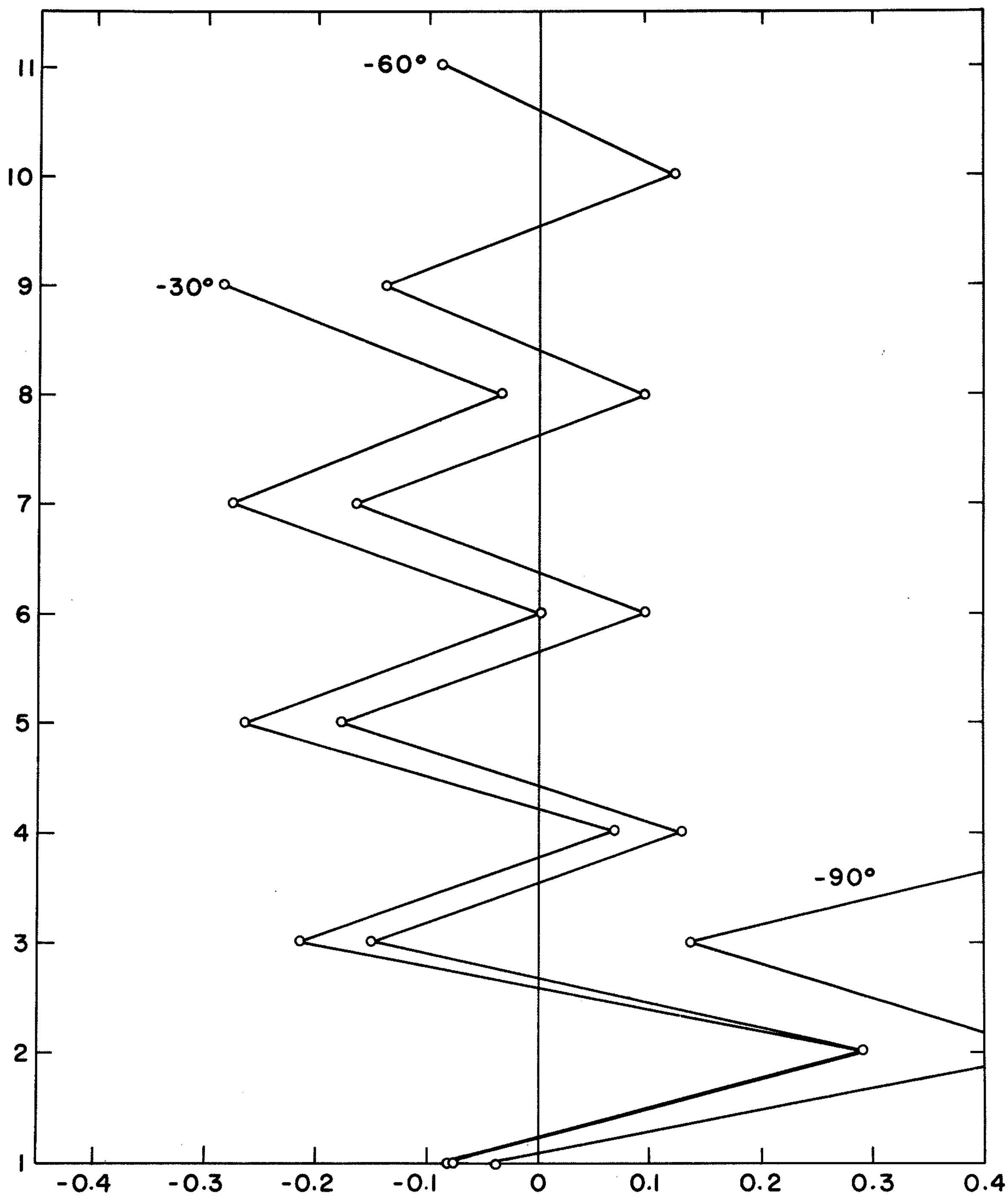

Fig. 19: Center-point coordinates at successive half turns (azimuths 90° and 270°) in Run 3.24.

Besides the puller is located right on top of the shielding electrode in the dee, and in the cyclotron one would either have to disassemble this electrode or change the puller. A further disadvantage is that the outer puller edge produces a strong radial field distortion which adversely effects the trajectories of the ions. To some extent this effect can be seen in Fig. 18: the particles are strongly pulled towards the left as they travel through this region. However, since the Run 3.24 calculations are based on field 3.04.1 where source and puller are further away from the dummy dee, this effect is not shown as strongly as it would be in the actual situation.

The acceptable range of starting phases is very small. As Fig. 18 shows only the ions starting between -60° and -30° are well behaved. The -90° particle is quickly driven off center and out of the magnetic stability limit, and in the case of the 0° particle the situation is similar. The center-point plot in Fig. 19 is somewhat different from the pictures in the $N = 1$ and $N = 2$ cases: The -60° ion is well centered and does not experience any off-center drift on the first five turns; the -30° particle, on the other hand, seems to move in negative x direction on the first turns while the -90° ion quickly bounces off center towards the right. This unusual behaviour in the $N = 3$ case is, of course, explained by the asymmetric energy gain in this mode of operation.

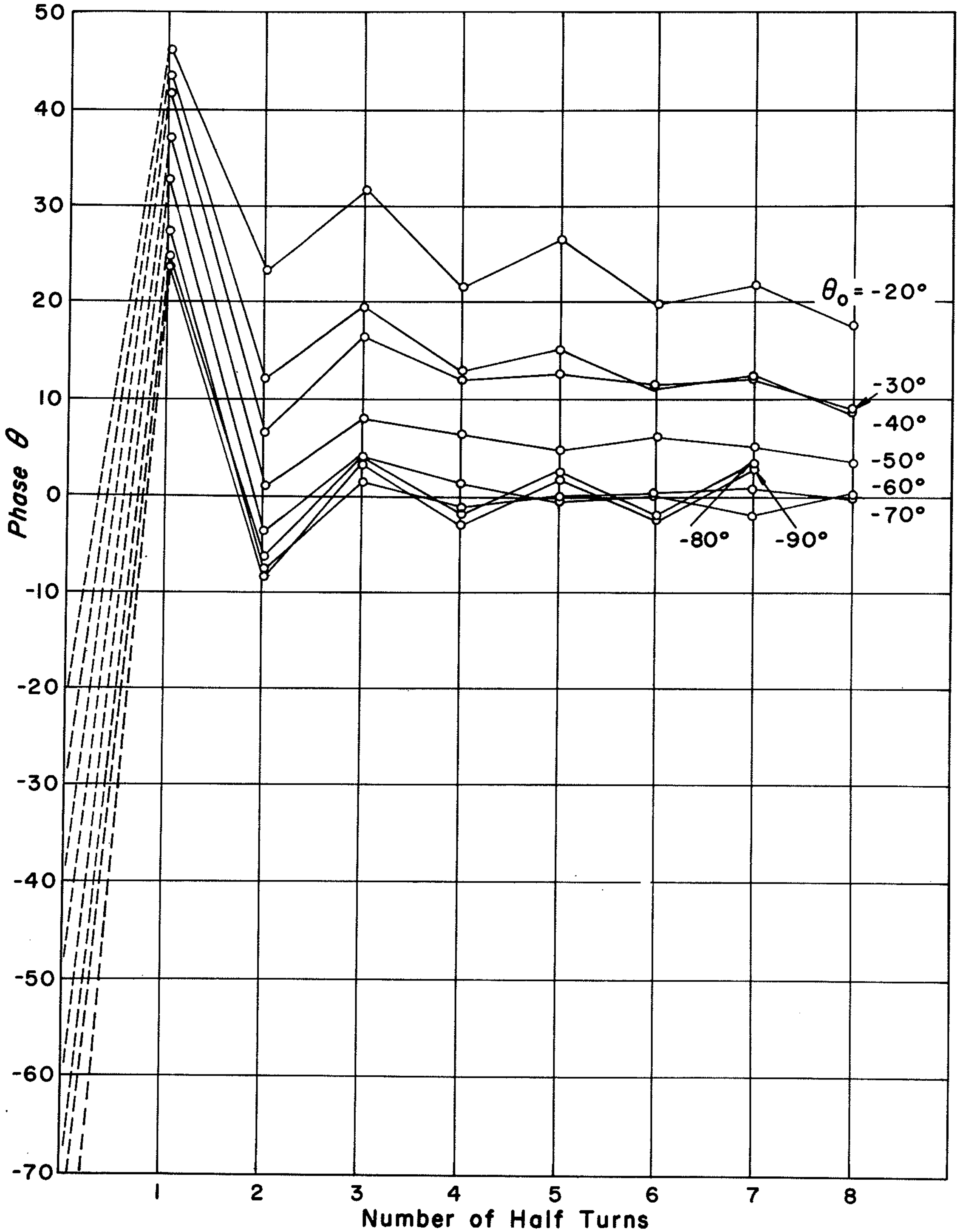

Fig. 20: Phase history in the N = 3 case (Run 3.24).

The phase-history plot in Fig. 20 exhibits a very strong phase-bunching effect which essentially takes place during the first gap crossing and which is very desirable as only low-duty factor operation is anticipated in this mode (no stripping extraction for heavy ions!). The interval of starting phases $-90° \leq \theta_0 \leq 20°$ is contracted to only 23° at the end of the first half turn. Another notable feature of the phase plot is the large phase excursion into the positive region between 20° and 45° during the first half turn and the subsequent shift of about 25° back towards the 0° phase. This effect was not observed in the first- and second-harmonic modes; it comes from the fact that the orbits are off center in positive y direction (see Fig. 18) during the first turn, i.e., the angle subtended by the orbits on the first half turn is larger than the angle on the second half turn. The useful group of particles starting between -60° and -30° end up with slightly positive phases where one can expect sufficient electric focusing. No computer runs on the effect of radial focusing in field 3.04.1 have been made at this time.

In summary one can conclude that the results of Run 3.24 for the third-harmonic mode do not compare very well with the situation in the cases $N = 1$ and $N = 2$ as only a very small group of particles are acceptable. However, this does not mean that the $N = 3$ results are unsatisfactory: since for heavy ions a phase sensitive conventional extraction scheme must be employed, one can only utilize a narrow interval of starting phases and is not interested in a broad range of acceptable phases. The only point of concern is the geometrical problem of source and puller positions. The situation shown in Fig. 18 is certainly not practicable. There are, however, many possibilities to overcome the difficulties. First, leaving source and puller where they are, one could modify the tip of the right dummy dee to increase the distance to the puller edge. Another possibility is to search for acceptable starting conditions where puller and source are further away from the dummy dee; for example, it may be possible to reduce the puller-to-dee angle somewhat and at the same time shift the starting point a small distance closer to the center.*) A third alternative, which will be discussed briefly, is to reduce the angular spread of the electric field on the first half turn by inserting extra electrodes in this region or by increasing the dee angle. At present the dee angle is asymmetric, as was discussed in the beginning: 69° on the right half and 65° on the left. Previously a symmetric dee geometry with a 69° angle on either side had been investigated. In this case additional slits had been placed at the lower dee where the ions enter on the first turn and behind the tip of the right dummy dee. The trajectories in the $N = 1$ and $N = 2$ modes were little different from the results that were obtained later with the asymmetric dees. However, the situation in the third-harmonic mode was markedly better in the

*) See footnote at end of this chapter.

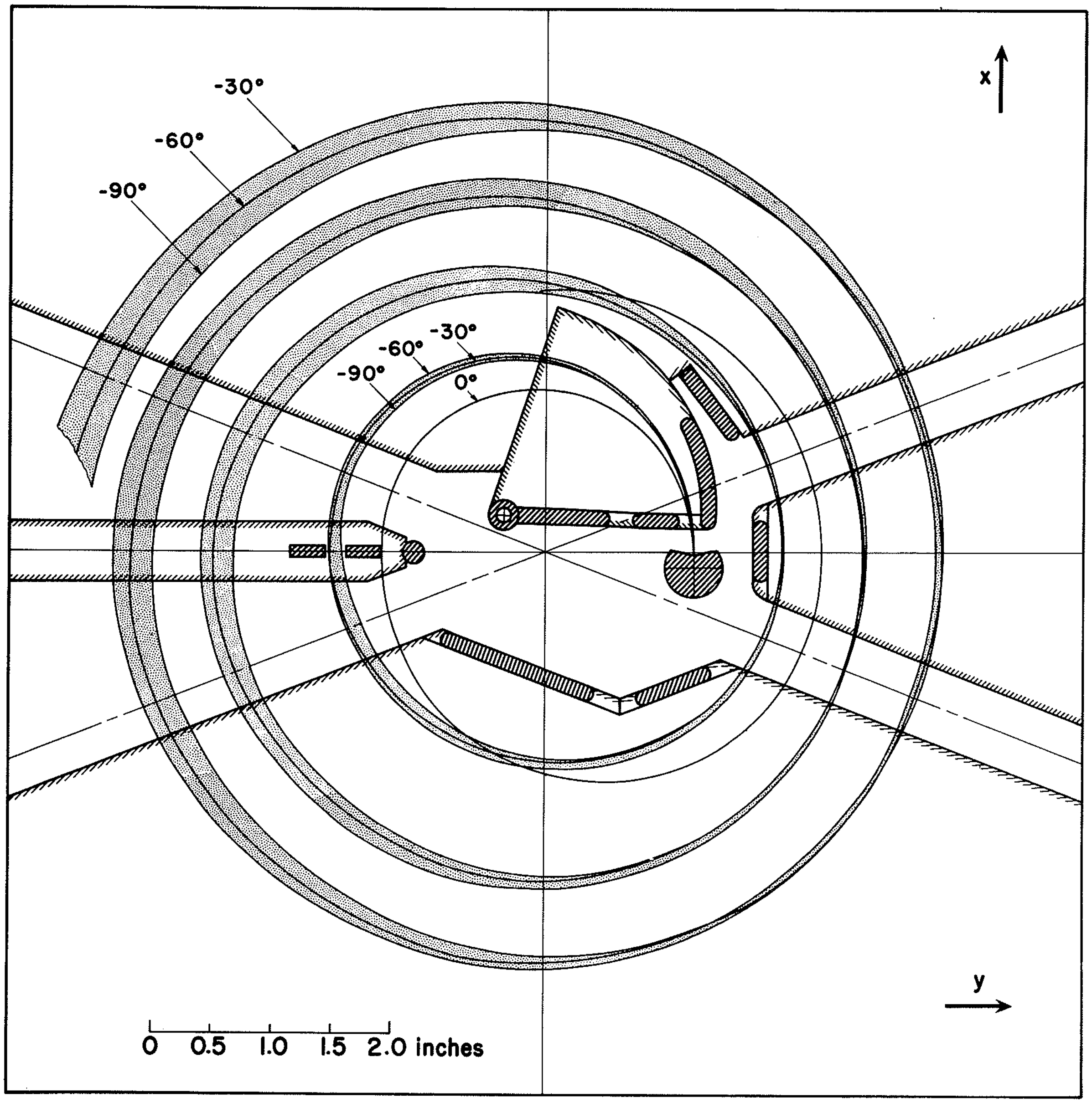

Fig. 21: Source-puller positions and central orbits for third-harmonic operation in field 3.03.2 (Run 3.19).

symmetric geometry (field 3.03.2). This is demonstrated in Fig. 21 which shows the trajectories for particles with different starting phases as calculated in Run 3.19; it is seen that the acceptable interval of starting phases is larger than in the case of Run 3.24 and includes the -90° phase while the orbits as a whole are more symmetric and better behaved. This is also illustrated by the phase-history and center-point plots which are not shown here. The conclusion from the comparison of the two geometries is that the reduction of the dee angle and the elimination of the two additional slits, especially that at the entrance of the lower dee, change the energy-gain history on the first turn sufficiently to cause the difficulties that were encountered in the computer runs with field 3.04.1. (In Fig. 21 the additional slits have been omitted by mistake.)

More important than the larger interval of acceptable starting phases (which situation is not particularly desirable for third-harmonic acceleration) is that in the symmetric-dee case source and puller are at positions which can be realized without any changes in the central geometry. The 2-slit puller, of course, can be replaced by the modified 1-slit design with the additional shielding electrode employed in field 3.04.1. What the comparison between Runs 3.24 and 3.19 suggests then is to build the dees with symmetric half angles of 69° in the very center and reduce the angle on the left to the desired 65° at a convenient radius (about 3 or 4 in.) so that the first three turns are unaffected. In addition, a slit should be placed at the entrance of the lower dee. If this solution is chosen source and puller could remain in the present positions for the N = 1 and N = 2 case, while in the N = 3 mode they would be located at the position shown in Fig. 21.

5. Conclusion

For a retrospective valuation of the results of the computer studies for the central region of the MSU cyclotron it may be helpful to restate briefly the over-all objectives and boundary conditions for this program and point out some of the problems that are not yet completely settled. The first and major goal of this study was to design the injection system and central geometry in a way that an ion-optically clean, well defined and centered beam could be produced with a maximum acceptance rate; in addition means should be provided for efficient phase selection as well as separation of unwanted particles. Such a scheme would make it possible to operate the cyclotron with either large duty factor, high intensity

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Footnote: After completion of this report the search for a better source-puller location in the field 3.04.1 geometry was successful. It was possible to match backward-run data to a previous forward run (3.20) with a displacement of source and puller such that the starting point is at x_o = 1.32 in., y_o = 0.40 in. and the puller is a safe distance of about 0.4 in. away from the tip of the dummy dee.

and acceptable quality of extracted beam (in case of stripping extraction, for example) or with small duty factor and high energy resolution.

The second requirement was to find a solution where a change from one of the three basic modes of cyclotron operation to another could be accomplished by mechanical readjustments of movable elements (source, puller, slits) from the outside and without major changes of the central dee geometry, i.e., without the necessity of disassembling and altering the dee system. These geometric boundary conditions restricted the search for possible solutions to the frame of a constant-orbit scheme.

In the preliminary survey of injection possibilities in a double-mode dee system it was found that (a) to achieve the desired beam quality large azimuthal and radial readjustments of the source-puller system are necessary when the mode of operation is changed, and (b) to minimize the radial displacements the dee angle has to be in the neighborhood of 144°. The first electrolytic-tank and computer studies with a symmetric 144° geometry furnished positive results in all three modes of operation (these studies were not discussed in this report). However, the calculations were limited to the first two turns and flat magnetic field as the old computer at MSU, the MISTIC, had not enough memory capacity. Besides the electrolytic tank itself represented a too small region of the center and had a number of other shortcomings. As the cyclotron design work got into more details a modification of the dee system became necessary and a new, improved electrolytic tank was built. The results of the final computer studies with this new geometry were presented in this report.

The change of the dee geometry produced no ill-effects on the beam quality in the first- and second-harmonic mode but complicated the situation in the $N = 3$ case considerably, the difficulties being mainly of a geometric nature. A way out of the dilemma is to have a symmetric dee angle in the center and/or to insert additional shielding electrodes. One crucial point that remains open regards the electrode which was placed into the puller dee to prevent field penetration when the 1-slit puller is at the $N = 3$ position. This electrode represents an unnecessary obstacle (even after it is cut shorter) in the $N = 1$ and $N = 2$ case, while in the $N = 3$ situation it is not placed and shaped as well as one would like it. The best solution is to take it out if the cyclotron is operated in the first- or second-harmonic mode and put a more suitable structure in its place when the cyclotron operates in the $N = 3$ mode. This, of course, requires the disassembly of the dee and hence a compromise of the second requirement stated above, but the third-harmonic operation is least important and does not occur very often as heavy-ion work can better be done with the new tandem accelerators.

One important point where more information and computations would be helpful is the problem of vertical focusing. The present attempts to build first-order vertical motion into the Pinwheel program using the median-plane data of the electrolytic-tank measurement would, if successful, close one of the major gaps that still exist in the theoretical analysis of ion motion in the cyclotron. The Silax calculations[3)] are not rigorously valid on the first and second turn and, as in the case of radial motion, this is the most crucial region. More accurate results for the vertical motion would not only answer the question to which extent a bump is necessary but also furnish the information needed if vertical slits shall be employed for fine phase selection. With respect to the central magnet field the results of the Pinwheel studies have indicated that a strong bump with 9% peak strength has an unfavorable effect on the radial motion causing a reduction of the acceptance rate in the first-harmonic mode and is unfeasible for higher-harmonic acceleration because of excessive phase slip. The best solution seems to be a small bump which brings the phase of the useful ion beam back to 0° after the electric-focusing region is traversed, as is now planned.

One other interesting result of these studies was that the electric field has a very strong effect on radial focusing. The central magnetic field alone would produce an oscillation about the equilibrium orbit with one cycle per revolution. This simple betatron oscillation with frequency 1 is distinctly changed if the electric field and energy gain is included in the calculations. The Pinwheel results show that the nodes and antinodes of the oscillatory pattern vary widely from one mode of operation to another and depend strongly on the phase of the ions with respect to the r.f. In addition there is coupling from radial phase space into the energy-time phase space. The phase sensitivity of the radial focusing effect allows an effective phase selection with the beam defining slit in the left dummy dee. The resolution depends on the phase interval at which selection shall be accomplished and differs from one mode to another. However, it seems that the defining slit can be employed for this task to a much greater extent than was expected. Where higher resolution is required the envisaged vertical-slit techniques[3)] may supplement the coarse action of the beam defining slit.

An important question not yet mentioned so far is how accurate the positioning of the source-puller system and the alignment of the dees must be. The answer is that the most crucial parameter is the source-to-puller spacing, followed in importance by the source position and puller angle, with the dee alignment being the least critical factor. The highest accuracy is required in the third-harmonic mode, the lowest for first-harmonic acceleration. The source-to-puller spacing and the field geometry in this region determine the energy gain, transit time, optimum starting phase, and the initial focusing effect. The mechanical design should allow minimum adjustments of 5 to 10 mils. As for the position of the source-puller system relative to the center and basic reference system the computer runs indicate an almost linear dependence of the orbit centers from the starting point within a certain limit of about 0.25 inches, i.e., a displacement of the starting point by 0.1 inches leads to a shift of the center points by about the same amount. The freedom of moving the starting point and the orbits, however, is restricted by the shielding electrodes, especially the one in the tip of the right dummy dee: this structure determines the minimum radius for the source position in the $N = 1$ and $N = 2$ mode and the maximum radial displacement of the puller in the $N = 3$ case. The severest restrictions occur in the third-harmonic mode where only a narrow radial "channel" for beam transmission exists due to the geometry.

Before concluding this report it should be pointed out that space-charge effects have not been considered in the calculations. The action of the repulsive Coulomb forces under the conditions existing in the cyclotron—plasma boundary at the ion source, magnetic field, and time varying electric field—poses the greatest difficulties to a mathematical treatment, aside from the fact that the true conditions at the ion source, where molecular ions, negative ions and electrons emerge from the plasma, are not known. One can only make rather crude estimates on the basis of simplified assumptions or rely on experimental results, and so far little has been done in either respect. At any rate, the median-plane calculations discussed in this report are accurate in the limit of low current densities.

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