# FIELD CALCULATIONS FOR THE MSU 500 MeV SUPERCONDUCTING CYCLOTRON MAGNET

H.G. Blosser
D.A. Johnson

Edited by J. Eastman

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Cyclotron Laboratory
Michigan State University
East Lansing, Michigan 48824

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The MSU Cyclotron Laboratory is completing the construction of a 500 MeV 3-sectored superconducting cyclotron magnet. First turn-on of this magnet is expected to take place in May 77. The purpose of this report is twofold, namely 1) to provide a record of the magnetic field used in orbit calculations up until now; and 2) to provide a point of comparison between the calculated field and the measured field so as to assess the adequacy of the calculation procedure for predicting fields of future magnets of this type.

The calculated magnetic field is based on two different kinds of computations, namely:

1) Cylindrically symmetric parts of the magnet are calculated using a relaxation program called TRIM[1] which solves the field equations first for potentials and then for fields. TRIM in principle finds an exact solution for a given array of soft magnetic materials, conductors and boundary conditions although in practice there are limitations based on finite grid size effects, round-off errors, etc.
2) For nonazimuthally symmetric parts of the magnet, particularly the pole tips, the field is calculated assuming all magnetic moments are fully aligned along the magnet axis.[2]

Finally, fields from the two types of components are combined to give a total field.

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[1] J.S. Colonias, UCRL-18439, Lawrence Berkeley Laboratory, CA, 1968.

[2] H.G. Blosser and D.A. Johnson, Focusing Properties of Superconducting Cyclotron Magnets. Nucl. Instr. 121 301(1974).

The calculation procedure has been tested against known fields in simplified geometries and found to be accurate to about one percent.[3] The much more complicated geometry involved in the superconducting cyclotron magnet will constitute a very interesting and much more stringent test of the procedure.

For orientation, Fig. 1 is a photograph of the yoke and coil tank of the magnet as these parts were being assembled for factory inspection, and Fig. 2 is a photograph of a wooden model of the lower pole of the magnet showing the pole tip structure. A TRIM grid indicating the shape of cylindrically symmetric magnet components (coil and yoke) is shown in Fig. 3. This cross section shows a quadrant of the magnet from center and median plane and does not include azimuthally varying components due to the 3-sectored construction of hills and valleys. Table I gives the B/H data used by the TRIM program to represent the magnetic properties of the 1020 cast steel used for construction of the magnet. Figure 4 defines the azimuthally varying parts of the magnet plotted as a function of radius. The curves give the hill width, the gap between hills ($G_h$) and the gap between valley floors ($G_v$). As examples, the greatest gap between valley floors is 36" and the smallest gap between hills is 2.5". The angular location of the hill center (the spiral angle) is given by

$$\phi_{hc}(\text{rad}) = \phi_i(\text{rad}) + r/13", \; \phi_1 = 0, \; \phi_2 = \frac{2\pi}{3}, \; \phi_3 = -\frac{2\pi}{3}.$$

Table II gives the radial dependence of 1) the air core coil field, 2) the field as calculated by TRIM (coil and yoke), 3) the azimuthal average of the total field of the magnet (i.e. the sum

[3] D. Johnson, H. Blosser, and M. Gordon, Field Calculations for 500 MeV Magnet, MSU Cyclotron Laboratory Annual Reports (1974-5, 1975-6), pp97-98.

of the azimuthally symmetric and azimuthally varying components), and 4) a computed isochronous field. Table III is a Fourier series description of the azimuthal variations in the field, giving respectively the amplitude and phase of the various Fourier components, where

$$B(r,\theta) = BAV(r) + B3(r) \cos 3(\theta - PH3) + B6(r) \cos 6(\theta - PH6) + \ldots$$

Table IV describes the field using the alternate formulation of the Fourier series in terms of sine and cosine coefficients where

$$B(r,\theta) = BAV(r) + H3(r) \cos 3\theta + G3(r) \sin 3\theta + H6(r) \cos 6\theta + \ldots$$

Table V illustrates the effect of the truncation of Fourier series by giving the directly calculated field versus angle at 20-inch radius, the truncated series approximation and the difference. Table VI gives calculated orbit properties in the isochronous field. Finally, Fig. 5 is an isogauss contour map of the calculated field.

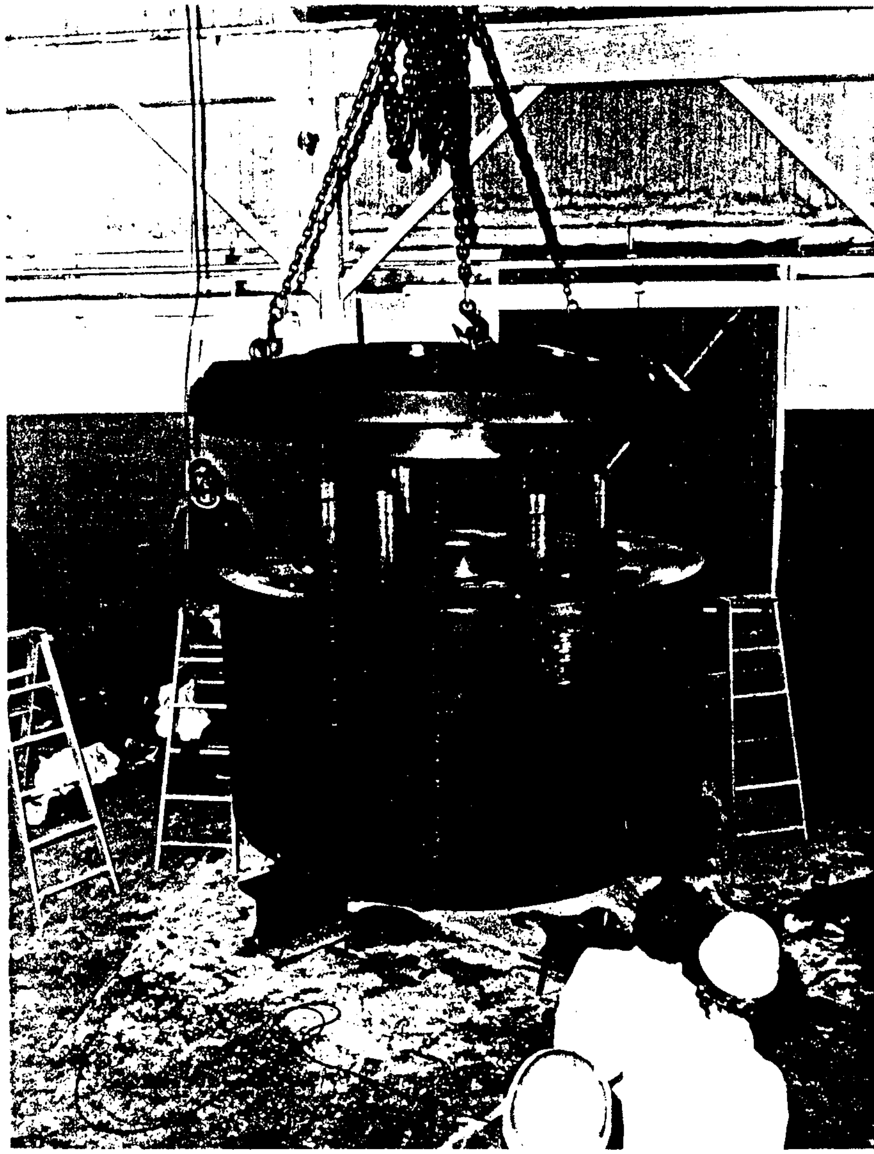

Figure 1. Yoke and coil tank for superconducting magnet; cast of 1020 steel, it weighs approximately 90 tons.

Figure 2. Wooden model of the lower pole of the magnet, showing the pole tip structure.

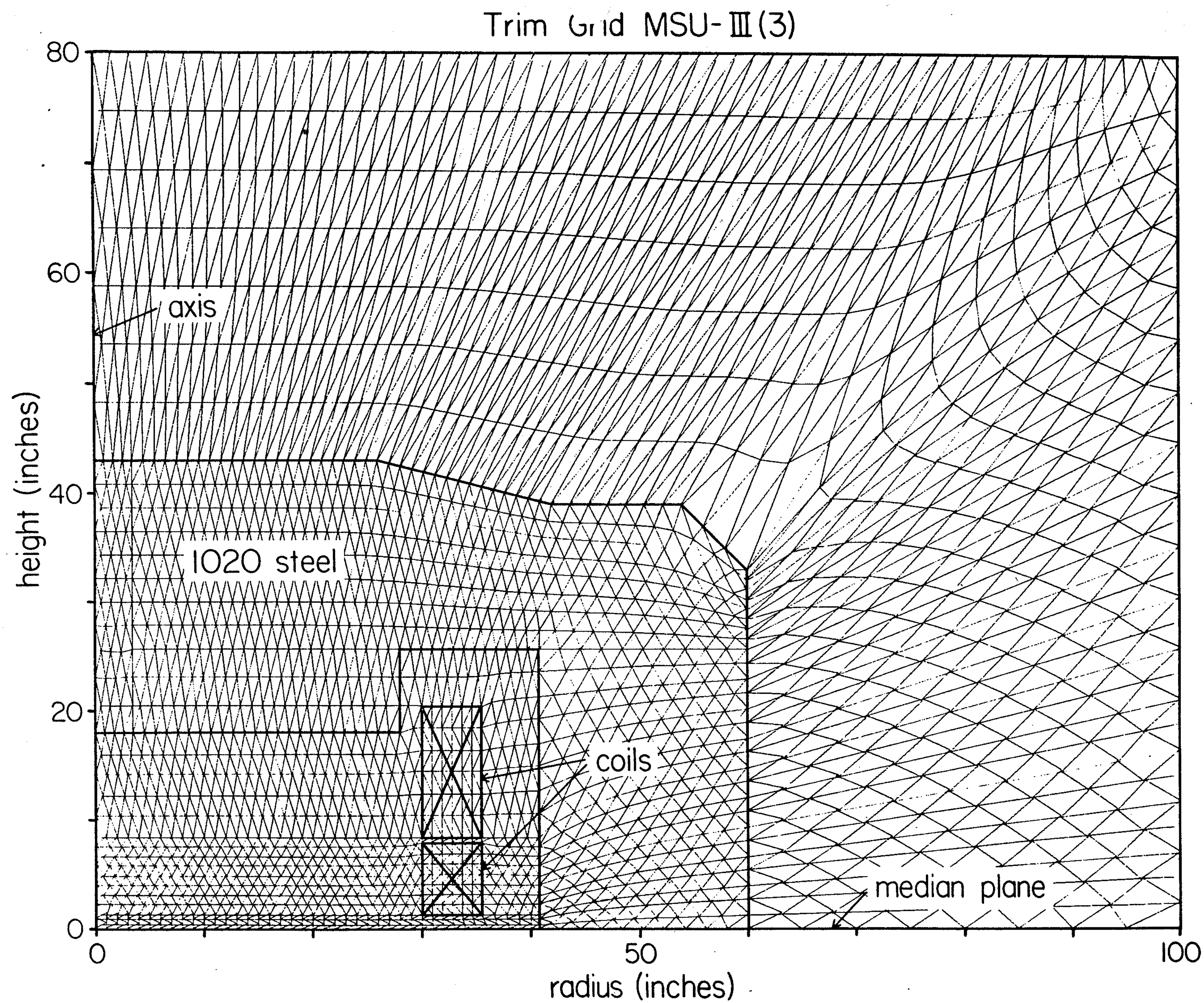


Figure 3. TRIM grid, indicating the shape of cylindrically symmetric magnet components (coil & yoke).

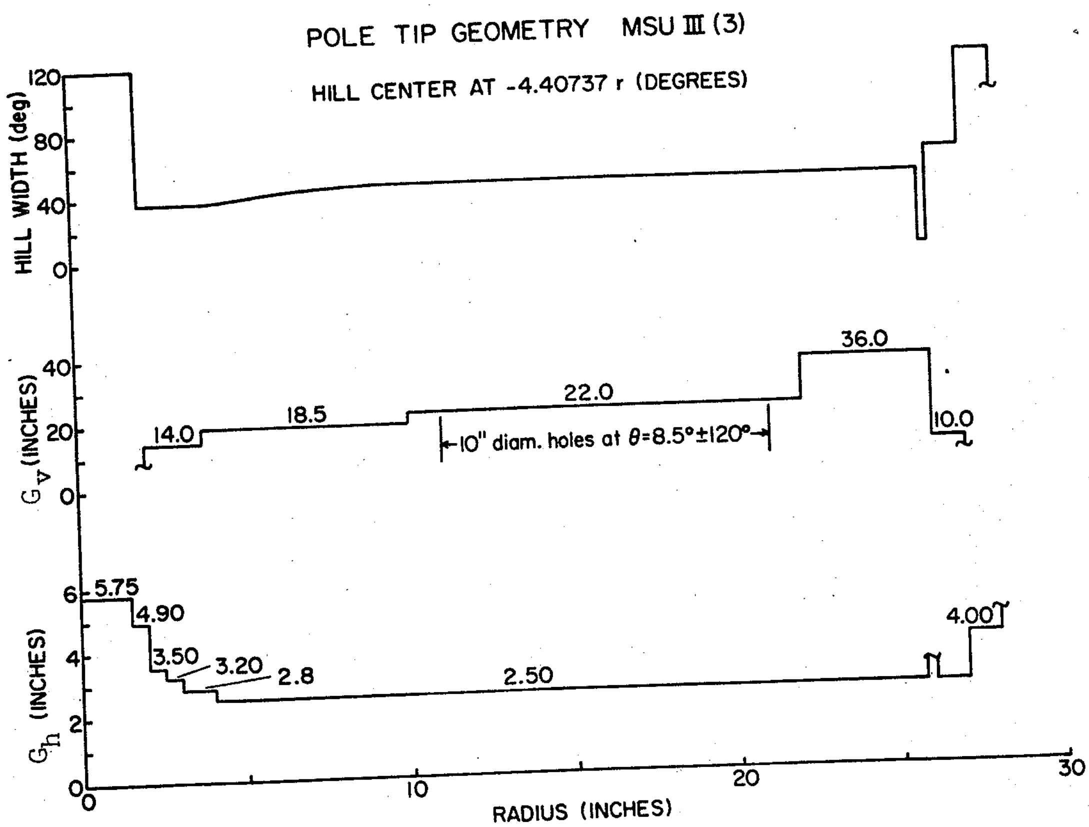


Figure 4. Description of the azimuthally varying parts of the magnet plotted as a function of radius. Top curve gives hill width in degrees, middle curve gives gap between valley floors, and bottom curve gives gap between hills.

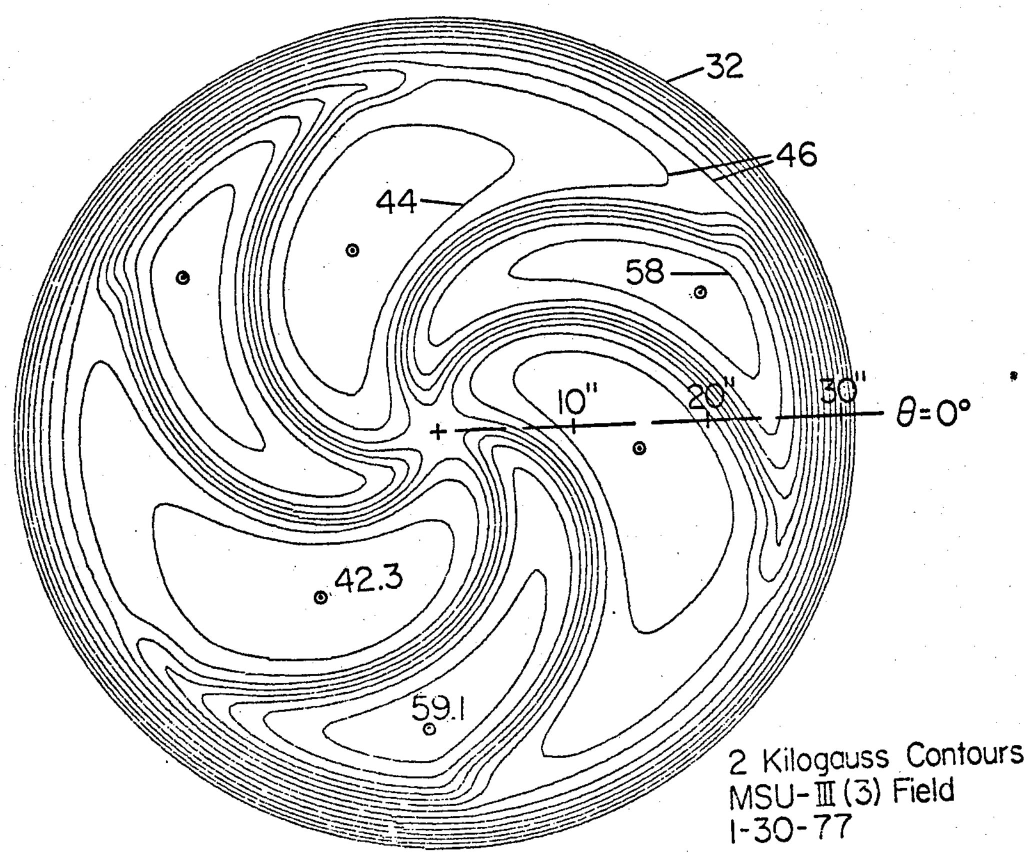


Figure 5. Isogauss contour map of the calculated field.

Table I. B/H data used by the TRIM program to represent the magnetic properties of the 1020 cast steel.

| B (tesla) | $\mu_o H$ (tesla) |
|---|---|
| .8000 | .00025 |
| 1.1000 | .00038 |
| 1.2500 | .00050 |
| 1.3300 | .00063 |
| 1.3950 | .00075 |
| 1.4500 | .00088 |
| 1.4750 | .00101 |
| 1.5000 | .00113 |
| 1.5250 | .00126 |
| 1.5550 | .00151 |
| 1.5700 | .00176 |
| 1.5820 | .00201 |
| 1.6080 | .00251 |
| 1.6280 | .00302 |
| 1.6500 | .00352 |
| 1.6750 | .00402 |
| 1.7000 | .00503 |
| 1.7350 | .00603 |
| 1.7600 | .00704 |
| 1.7800 | .00804 |
| 1.8150 | .01005 |
| 1.8550 | .01332 |
| 1.8920 | .01571 |
| 1.9200 | .01885 |
| 1.9430 | .02199 |
| 1.9720 | .02513 |
| 2.0700 | .03770 |
| 2.1420 | .05027 |
| 2.1870 | .07540 |
| 2.2530 | .12566 |
| 2.3910 | .25130 |
| 2.5170 | .37700 |
| 2.6750 | .53500 |
| 2.8533 | .71330 |
| 3.0571 | .91710 |
| 3.2923 | 1.15230 |
| 3.5667 | 1.42670 |
| 3.8909 | 1.75090 |
| 4.2800 | 2.14000 |
| 4.7556 | 2.61560 |
| 5.3500 | 3.21000 |
| 6.1143 | 3.97430 |
| 7.1333 | 4.99330 |
| 8.5600 | 6.42000 |
| 10.7000 | 85.60000 |
| 14.2667 | 12.12670 |
| 21.4000 | 19.26000 |

(Rows .37700 through 19.26000:) "saturated" $(B-\mu_o H) = 2.14$ tesla

Table II. Radial dependence of 1) the air core coil field; 2) the field as calculated by TRIM; 3) the azimuthal average of the total field of the magnet; and 4) a computed isochronous field. (Ampere-turns = 2.313 x $10^6$, radius in inches, fields in kilogauss.)

| R | COIL | COIL+YOKE | BAV | ISOC BAV | FLUTTER |
|---|---|---|---|---|---|
| 0 | 29.19044 | 43.22800 | 49.21651 | 47.77951 | .00000 |
| 1 | 29.20003 | 43.23300 | 49.14925 | 47.71532 | .00000 |
| 2 | 29.22879 | 43.24400 | 49.01660 | 47.58356 | .00018 |
| 3 | 29.27676 | 43.26300 | 48.91545 | 47.48520 | .00125 |
| 4 | 29.34396 | 43.28999 | 48.86365 | 47.43018 | .00341 |
| 5 | 29.43047 | 43.32201 | 48.83600 | 47.41652 | .00578 |
| 6 | 29.53635 | 43.36200 | 48.80191 | 47.44206 | .00774 |
| 7 | 29.66162 | 43.40900 | 48.79436 | 47.47606 | .00942 |
| 8 | 29.80635 | 43.46300 | 48.79861 | 47.51865 | .01058 |
| 9 | 29.97054 | 43.52499 | 48.80421 | 47.56938 | .01217 |
| 10 | 30.15414 | 43.59300 | 48.80135 | 47.62856 | .01332 |
| 11 | 30.35707 | 43.66701 | 48.78505 | 47.69655 | .01435 |
| 12 | 30.57909 | 43.74800 | 48.77040 | 47.77274 | .01526 |
| 13 | 30.81984 | 43.83501 | 48.77206 | 47.85834 | .01604 |
| 14 | 31.07875 | 43.92599 | 48.79474 | 47.95438 | .01665 |
| 15 | 31.35504 | 44.02200 | 48.84151 | 48.06087 | .01707 |
| 16 | 31.64754 | 44.12199 | 48.91121 | 48.17732 | .01730 |
| 17 | 31.95462 | 44.22400 | 49.00087 | 48.30273 | .01736 |
| 18 | 32.27408 | 44.32600 | 49.10747 | 48.43620 | .01729 |
| 19 | 32.60284 | 44.42700 | 49.22975 | 48.57738 | .01711 |
| 20 | 32.93674 | 44.52299 | 49.36479 | 48.72749 | .01665 |
| 21 | 33.26990 | 44.60800 | 49.50943 | 48.88911 | .01651 |
| 22 | 33.59415 | 44.67700 | 49.66281 | 49.06746 | .01601 |
| 23 | 33.89757 | 44.71700 | 49.81563 | 49.27217 | .01521 |
| 24 | 34.16231 | 44.71400 | 49.94989 | 49.52472 | .01380 |
| 25 | 34.36015 | 44.63800 | 50.00545 | 49.85086 | .01106 |
| 26 | 34.44366 | 44.44099 | 49.96092 | 49.96092 | .00706 |
| 27 | 34.32657 | 44.03500 | 48.99658 | 48.99658 | .00292 |
| 28 | 33.83519 | 43.24500 | 46.31744 | 46.31744 | .00047 |
| 29 | 32.58084 | 41.69000 | 42.84982 | 42.84982 | .00005 |
| 30 | 29.74167 | 38.55600 | 38.45082 | 38.45082 | .00000 |
| 31 | 24.68889 | 33.34900 | 32.55014 | 32.55016 | .00001 |
| 32 | 18.19980 | 26.72000 | 25.55344 | 25.55345 | .00003 |
| 33 | 11.24449 | 19.61600 | 18.25877 | 18.25877 | .00005 |
| 34 | 4.47922 | 12.72400 | 11.27655 | 11.27654 | .00013 |
| 35 | -1.32159 | 6.80600 | 5.32809 | 5.32809 | .00052 |
| 36 | -5.12032 | 2.74000 | 1.26841 | 1.26841 | .00775 |
| 37 | -6.89744 | .65500 | -.78731 | -.78731 | .01563 |
| 38 | -7.60066 | -.27800 | -1.67678 | -1.67678 | .00293 |
| 39 | -7.81150 | -.73700 | -2.08369 | -2.08369 | .00157 |
| 40 | -7.77843 | -.96700 | -2.25682 | -2.25682 | .00108 |

Table III. Fourier series description of the azimuthal variations in the field, giving respectively the amplitude and phase of various Fourier components.

| R | B3 | PH3 | B6 | PH6 | B9 | PH9 | B12 | PH12 | B15 | PH15 | B18 | PH18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .00000 | .0000 | .00000 | .0000 | .00000 | .0000 | .00000 | .0000 | .00000 | .0000 | .00000 | .0000 |
| 1 | .12652 | -14.2425 | .00147 | -12.7707 | .00000 | .0000 | .00000 | .0000 | .00000 | .0000 | .00001 | -5.9962 |
| 2 | .92878 | -15.1714 | .06338 | -13.9698 | .00152 | -13.4712 | .00023 | 1.4923 | .00003 | -1.7097 | .00001 | -4.9421 |
| 3 | 2.42360 | -17.3260 | .35618 | -16.2952 | .01867 | -15.5741 | .00717 | -1.0028 | .00247 | -3.6308 | .00034 | -5.0677 |
| 4 | 3.94859 | -20.5615 | .82217 | -19.5453 | .05153 | -17.9171 | .04075 | -4.7559 | .01817 | -7.2535 | .00281 | -8.5949 |
| 5 | 5.10998 | -24.3722 | 1.20592 | -23.2920 | .04392 | -17.7981 | .11356 | -8.5273 | .04662 | -10.8042 | .00454 | -10.2349 |
| 6 | 5.89889 | -28.5030 | 1.42801 | -27.4368 | .07158 | -11.7213 | .20198 | -12.5927 | .06840 | -14.9058 | .00935 | -9.3633 |
| 7 | 6.50096 | -32.7447 | 1.57486 | -31.7035 | .17098 | -13.5548 | .29272 | -16.7800 | .08196 | -19.0874 | .02716 | -12.4007 |
| 8 | 6.98381 | -37.0407 | 1.67654 | -36.0116 | .28194 | -17.1230 | .37743 | -21.0290 | .08605 | -23.2956 | .05167 | -16.3609 |
| 9 | 7.38425 | -41.3643 | 1.74897 | -40.3509 | .38995 | -21.0486 | .45181 | -25.3029 | .08190 | -27.5214 | .08009 | -20.4932 |
| 10 | 7.72383 | -45.6820 | 1.81084 | -44.7222 | .48156 | -25.0647 | .51346 | -29.6177 | .07268 | -32.0015 | .10567 | -24.6457 |
| 11 | 8.00891 | -49.9584 | 1.87949 | -49.0722 | .53993 | -29.1728 | .56719 | -33.9683 | .07444 | -36.7922 | .12628 | -28.9086 |
| 12 | 8.25068 | -54.2035 | 1.94566 | -53.4002 | .57786 | -33.4412 | .61835 | -38.3404 | .08373 | -41.2859 | .14393 | -33.2540 |
| 13 | 8.45395 | -58.4033 | 1.99608 | -57.7332 | .60802 | -37.7720 | .66367 | -42.7059 | .09188 | -45.6590 | .15943 | -37.6191 |
| 14 | 8.61473 | -62.5601 | 2.03045 | -62.1118 | .63290 | -42.1280 | .70281 | -47.0762 | .09875 | -50.0312 | .17353 | -41.9983 |
| 15 | 8.72828 | -66.6901 | 2.05668 | -66.5600 | .65367 | -46.5061 | .73702 | -51.4535 | .10482 | -54.4103 | .18638 | -46.3816 |
| 16 | 8.79316 | -70.8161 | 2.08594 | -71.0736 | .67201 | -50.9058 | .76732 | -55.8354 | .11020 | -58.7926 | .19796 | -50.7672 |
| 17 | 8.81272 | -74.9605 | 2.12783 | -75.6179 | .68958 | -55.3184 | .79418 | -60.2186 | .11490 | -63.1796 | .20830 | -55.1550 |
| 18 | 8.79446 | -79.1392 | 2.18635 | -80.1403 | .70715 | -59.7315 | .81751 | -64.6018 | .11913 | -67.5740 | .21746 | -59.5472 |
| 19 | 8.74735 | -83.3556 | 2.25874 | -84.5915 | .72467 | -64.1370 | .83685 | -68.9880 | .12319 | -71.9678 | .22594 | -63.9457 |
| 20 | 8.67750 | -87.5978 | 2.33725 | -88.9389 | .74233 | -68.5319 | .85220 | -73.3894 | .12669 | -76.3443 | .23432 | -68.3377 |
| 21 | 8.58275 | -91.8401 | 2.41168 | -93.1635 | .76092 | -72.9064 | .86742 | -77.8274 | .12807 | -80.7338 | .24056 | -72.7099 |
| 22 | 8.44664 | -96.0446 | 2.46900 | -97.2386 | .77990 | -77.2360 | .89631 | -82.2903 | .12956 | -85.2824 | .24291 | -77.1268 |
| 23 | 8.22728 | -100.1615 | 2.48320 | -101.0992 | .79394 | -81.5093 | .95222 | -86.6304 | .14700 | -89.8970 | .25123 | -81.6597 |
| 24 | 7.82979 | -104.1286 | 2.38870 | -104.6021 | .80161 | -85.8079 | 1.01325 | -90.5451 | .19468 | -93.5351 | .29228 | -85.9905 |
| 25 | 7.03074 | -107.8311 | 2.00679 | -107.3418 | .83380 | -90.2117 | .93959 | -93.5323 | .24758 | -94.7593 | .31906 | -89.1782 |
| 26 | 5.68446 | -111.1975 | 1.17418 | -108.4412 | .96763 | -94.0537 | .54249 | -94.8998 | .38938 | -93.4579 | .19209 | -90.6119 |
| 27 | 3.58816 | -113.1887 | .52370 | -108.0239 | .77645 | -95.7702 | .20364 | -94.8982 | .37425 | -93.3702 | .06378 | -90.7284 |
| 28 | 1.36053 | -112.8605 | .25263 | -107.3628 | .29385 | -95.7891 | .07795 | -94.5833 | .14133 | -93.5005 | .02040 | -90.5119 |
| 29 | .37181 | -111.3939 | .13701 | -106.8319 | .08586 | -94.9053 | .03344 | -94.2646 | .04148 | -93.5166 | .00728 | -90.2503 |
| 30 | .01366 | -88.4951 | .08056 | -106.4332 | .02198 | -92.2779 | .01576 | -93.9698 | .01162 | -93.5231 | .00288 | -89.9821 |
| 31 | .13186 | -51.7754 | .04991 | -106.1258 | .00873 | -84.8784 | .00797 | -93.6895 | .00280 | -93.5265 | .00124 | -89.7082 |
| 32 | .17867 | -50.3574 | .03198 | -105.8724 | .00862 | -80.6694 | .00426 | -93.4200 | .00029 | -93.4265 | .00057 | -89.4452 |
| 33 | .18878 | -49.4477 | .02091 | -105.6411 | .00759 | -79.3357 | .00238 | -93.1458 | .00032 | -81.6995 | .00028 | -89.1831 |
| 34 | .18343 | -48.7471 | .01382 | -105.4075 | .00600 | -78.7038 | .00138 | -92.8761 | .00037 | -81.7180 | .00014 | -88.9568 |
| 35 | .17174 | -48.1883 | .00917 | -105.1431 | .00452 | -78.3102 | .00082 | -92.5909 | .00029 | -81.7527 | .00008 | -88.6545 |
| 36 | .15780 | -47.7381 | .00606 | -104.8193 | .00333 | -78.0147 | .00050 | -92.3019 | .00020 | -81.7932 | .00004 | -88.4436 |
| 37 | .14349 | -47.3739 | .00396 | -104.3871 | .00243 | -77.7674 | .00031 | -91.9958 | .00014 | -81.8662 | .00002 | -88.2059 |
| 38 | .12971 | -47.0780 | .00254 | -103.7820 | .00177 | -77.5401 | .00020 | -91.6602 | .00009 | -81.9349 | .00001 | -87.9688 |
| 39 | .11682 | -46.8365 | .00159 | -102.8473 | .00129 | -77.3265 | .00013 | -91.3138 | .00006 | -81.9939 | .00001 | -87.7470 |
| 40 | .10438 | -46.6382 | .00095 | -101.3118 | .00095 | -77.1174 | .00008 | -90.9402 | .00004 | -82.0336 | .00000 | .0000 |

Table IV. Field described by using the alternate formulation of the Fourier series in terms of sine and cosine coefficients.

| R | H3 | G3 | H6 | G6 | H9 | G9 | H12 | G12 | H15 | G15 | H18 | G18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .00000 | -.00000 | .00000 | -.00000 | .00000 | -.00000 | .00000 | -.00000 | .00000 | -.00000 | .00000 | -.00000 |
| 1 | .09294 | -.08585 | .00034 | -.00143 | -.00000 | -.00000 | .00000 | .00000 | -.00000 | -.00000 | -.00000 | -.00001 |
| 2 | .65083 | -.66261 | .00682 | -.06301 | -.00079 | -.00130 | .00022 | .00007 | .00003 | -.00001 | .00000 | -.00001 |
| 3 | 1.49284 | -1.90925 | -.04798 | -.35293 | -.01434 | -.01196 | .00701 | -.00149 | .00144 | -.00201 | -.00001 | -.00034 |
| 4 | 1.87306 | -3.47641 | -.37673 | -.73078 | -.04380 | -.01656 | .02215 | -.03420 | -.00585 | -.01720 | -.00254 | -.00120 |
| 5 | 1.48406 | -4.88973 | -.92042 | -.77914 | -.04132 | -.01489 | -.02424 | -.11094 | -.04435 | -.01436 | -.00453 | .00033 |
| 6 | .46191 | -5.88078 | -1.37687 | -.37871 | -.01912 | -.06898 | -.17655 | -.09757 | -.04955 | .04716 | -.00916 | -.00186 |
| 7 | -.93104 | -6.43394 | -1.54987 | .27946 | -.09069 | -.14501 | -.27262 | .10662 | .02302 | .07866 | -.01979 | .01859 |
| 8 | -2.51667 | -6.51459 | -1.35515 | .98708 | -.25364 | -.12312 | -.11445 | .35966 | .08459 | .01578 | .02142 | .04702 |
| 9 | -4.13915 | -6.11511 | -.81824 | 1.54576 | -.38463 | .06394 | .25026 | .37617 | .04949 | -.06525 | .07913 | -.01236 |
| 10 | -5.65308 | -5.26310 | -.05268 | 1.81007 | -.33704 | .34396 | .51182 | .04107 | -.03637 | -.06293 | .01185 | -.10601 |
| 11 | -6.92719 | -4.01955 | .77742 | 1.71117 | -.06996 | .53538 | .38232 | -.41898 | -.07285 | .01533 | -.11893 | -.04245 |
| 12 | -7.87358 | -2.46587 | 1.49918 | 1.04018 | .29737 | .49545 | -.10827 | -.60880 | -.01556 | .08227 | -.07505 | .12282 |
| 13 | -8.42442 | -.70597 | 1.94011 | .46938 | .57116 | .20847 | -.58852 | -.30675 | .07516 | .05285 | .11537 | .10844 |
| 14 | -8.53745 | 1.15130 | 1.98100 | -.44538 | .59787 | -.20764 | -.63740 | .29606 | .08512 | -.05007 | .14044 | -.10192 |
| 15 | -8.19824 | 2.99530 | 1.59018 | -1.30433 | .34100 | -.55767 | -.16025 | .71939 | -.01124 | -.10422 | -.07838 | -.16910 |
| 16 | -7.42034 | 4.71785 | .83371 | -1.91209 | -.09529 | -.66522 | .49348 | .58759 | -.10474 | -.03426 | -.19224 | .04725 |
| 17 | -6.24439 | 6.21864 | -.13758 | -2.12337 | -.51138 | -.45262 | .79335 | -.03635 | -.07734 | .08498 | .01014 | .20805 |
| 18 | -4.73591 | 7.41037 | -1.12088 | -1.87717 | -.70652 | -.02982 | .46631 | -.67148 | .04771 | .10916 | .21527 | .03083 |
| 19 | -2.98221 | 8.22330 | -1.90604 | -1.21199 | -.57697 | .43847 | -.25660 | -.79653 | .12318 | .00104 | .07347 | -.21365 |
| 20 | -1.08806 | 8.60896 | -2.32283 | -.25918 | -.16967 | .72265 | -.80417 | -.28205 | .05321 | -.11497 | -.20309 | -.11685 |
| 21 | .82566 | 8.54294 | -2.28054 | .78442 | .33545 | .68299 | -.71973 | .48416 | -.08403 | -.09665 | -.15852 | .18094 |
| 22 | 2.62889 | 8.02712 | -1.79298 | 1.69741 | .70754 | .32807 | -.03935 | .89545 | -.12233 | .04269 | .15049 | .19065 |
| 23 | 4.17373 | 7.09000 | -.93639 | 2.27889 | .77174 | -.18647 | .72477 | .61760 | -.00396 | .14695 | .21785 | -.12514 |
| 24 | 5.27823 | 5.73325 | -.09951 | 2.38662 | .49053 | -.63400 | 1.00665 | -.11543 | .15553 | .11709 | -.08949 | -.27824 |
| 25 | 5.65122 | 4.18271 | .48722 | 1.94675 | -.02772 | -.63334 | .69398 | -.63343 | .23463 | .07901 | -.30849 | -.28146 |
| 26 | 5.09131 | 2.52817 | .41403 | 1.09876 | -.57657 | -.77710 | .28105 | -.46401 | .30628 | .24043 | -.18865 | .03670 |
| 27 | 3.36238 | 1.25272 | .16308 | .49765 | -.61128 | -.47876 | .10556 | -.17415 | .28900 | .23779 | -.06212 | .01447 |
| 28 | 1.26657 | .49684 | .06187 | .24494 | -.23188 | -.18050 | .04471 | -.06385 | .11213 | .08602 | -.02014 | .00327 |
| 29 | .33470 | .16193 | .02612 | .13450 | -.05980 | -.06161 | .02096 | -.02605 | .03302 | .02511 | -.00726 | .00057 |
| 30 | -.00107 | .01361 | .01205 | .07966 | -.00770 | -.02059 | .01062 | -.01164 | .00926 | .00702 | -.00288 | -.00008 |
| 31 | -.11982 | -.05504 | .00587 | .04957 | .00629 | -.00605 | .00571 | -.00557 | .00223 | .00169 | -.00123 | -.00011 |
| 32 | -.15638 | -.08642 | .00292 | .03184 | .00858 | -.00091 | .00322 | -.00280 | .00022 | .00018 | -.00056 | -.00010 |
| 33 | -.16069 | -.09908 | .00140 | .02086 | .00755 | .00079 | .00188 | -.00146 | -.00026 | -.00018 | -.00027 | -.00007 |
| 34 | -.15250 | -.10193 | .00059 | .01381 | .00588 | .00121 | .00113 | -.00078 | -.00030 | -.00021 | -.00013 | -.00005 |
| 35 | -.13993 | -.09957 | .00014 | .00917 | .00436 | .00118 | .00070 | -.00042 | -.00024 | -.00016 | -.00007 | -.00003 |
| 36 | -.12638 | -.09449 | -.00011 | .00606 | .00316 | .00102 | .00044 | -.00023 | -.00017 | -.00011 | -.00004 | -.00002 |
| 37 | -.11326 | -.08810 | -.00025 | .00396 | .00223 | .00083 | .00028 | -.00013 | -.00012 | -.00007 | -.00002 | -.00001 |
| 38 | -.10113 | -.08121 | -.00032 | .00252 | .00154 | .00067 | .00018 | -.00007 | -.00008 | -.00005 | -.00001 | -.00001 |
| 39 | -.09015 | -.07423 | -.00035 | .00155 | .00118 | .00053 | .00012 | -.00003 | -.00005 | -.00003 | -.00001 | -.00001 |
| 40 | -.08032 | -.06760 | -.00036 | .00088 | .00085 | .00042 | .00008 | -.00002 | -.00003 | -.00002 | -.00000 | -.00000 |

Table V. Scan in angle at 20" radius showing the effect of truncating the Fourier series. The first column is the angle (in degrees); the second column is the calculated field; the third column is the sum of the Fourier series from Table III; the fourth column gives the truncation error.

| | | | |
|---|---|---|---|
| 0 | 44.97729 | 44.82968 | -.14761 |
| 1 | 45.36447 | 45.28369 | -.08078 |
| 2 | 45.81863 | 45.82430 | .00566 |
| 3 | 46.35080 | 46.45007 | .09927 |
| 4 | 46.97177 | 47.15542 | .18365 |
| 5 | 47.69058 | 47.93079 | .24021 |
| 6 | 48.50964 | 48.76308 | .25343 |
| 7 | 49.42058 | 49.63639 | .21581 |
| 8 | 50.40074 | 50.53289 | .13215 |
| 9 | 51.41135 | 51.43386 | .02251 |
| 10 | 52.40659 | 52.32070 | -.08589 |
| 11 | 53.34293 | 53.17600 | -.16693 |
| 12 | 54.19084 | 53.98440 | -.20645 |
| 13 | 54.93694 | 54.73332 | -.20362 |
| 14 | 55.58015 | 55.41346 | -.16669 |
| 15 | 56.12852 | 56.01906 | -.10947 |
| 16 | 56.59360 | 56.54781 | -.04579 |
| 17 | 56.98732 | 57.00070 | .01338 |
| 18 | 57.32092 | 57.38149 | .06056 |
| 19 | 57.60413 | 57.69615 | .09202 |
| 20 | 57.84515 | 57.95216 | .10701 |
| 21 | 58.05067 | 58.15782 | .10715 |
| 22 | 58.22606 | 58.32156 | .09550 |
| 23 | 58.37563 | 58.45138 | .07576 |
| 24 | 58.50287 | 58.55441 | .05154 |
| 25 | 58.61057 | 58.63661 | .02605 |
| 26 | 58.70082 | 58.70267 | .00185 |
| 27 | 58.77538 | 58.75598 | -.01940 |
| 28 | 58.83549 | 58.79882 | -.03667 |
| 29 | 58.88216 | 58.83254 | -.04961 |
| 30 | 58.91600 | 58.85784 | -.05816 |
| 31 | 58.93756 | 58.87499 | -.06257 |
| 32 | 58.94701 | 58.88405 | -.06296 |
| 33 | 58.94440 | 58.88497 | -.05943 |
| 34 | 58.92947 | 58.87763 | -.05184 |
| 35 | 58.90195 | 58.86175 | -.04020 |
| 36 | 58.86107 | 58.83672 | -.02435 |
| 37 | 58.80598 | 58.80138 | -.00460 |
| 38 | 58.73538 | 58.75377 | .01839 |
| 39 | 58.64772 | 58.69089 | .04317 |
| 40 | 58.54103 | 58.60854 | .06751 |
| 41 | 58.41270 | 58.50127 | .08856 |
| 42 | 58.25961 | 58.36243 | .10282 |
| 43 | 58.07782 | 58.18445 | .10663 |
| 44 | 57.86235 | 57.95916 | .09681 |
| 45 | 57.60710 | 57.67839 | .07129 |
| 46 | 57.30444 | 57.33455 | .03011 |
| 47 | 56.94553 | 56.92134 | -.02419 |
| 48 | 56.52017 | 56.43443 | -.08574 |
| 49 | 56.01726 | 55.87211 | -.14515 |
| 50 | 55.42592 | 55.23574 | -.19018 |
| 51 | 54.73709 | 54.53011 | -.20698 |
| 52 | 53.94800 | 53.76349 | -.18451 |
| 53 | 53.06720 | 52.94749 | -.11971 |
| 54 | 52.11703 | 52.09660 | -.02043 |
| 55 | 51.13615 | 51.22760 | .09145 |
| 56 | 50.17021 | 50.35866 | .18845 |
| 57 | 49.26262 | 49.50836 | .24575 |
| 58 | 48.44269 | 48.69468 | .25199 |
| 59 | 47.72366 | 47.93390 | .21024 |
| 60 | 47.10640 | 47.23972 | .13332 |
| 61 | 46.58267 | 46.62249 | .03982 |
| 62 | 46.14076 | 46.08863 | -.05213 |
| 63 | 45.76852 | 45.64043 | -.12809 |
| 64 | 45.45450 | 45.27606 | -.17844 |
| 65 | 45.18866 | 44.98989 | -.19877 |
| 66 | 44.96260 | 44.77310 | -.18950 |
| 67 | 44.76936 | 44.61444 | -.15492 |
| 68 | 44.60321 | 44.50122 | -.10199 |
| 69 | 44.45946 | 44.42028 | -.03918 |
| 70 | 44.33432 | 44.35896 | .02464 |
| 71 | 44.22458 | 44.30599 | .08141 |
| 72 | 44.12766 | 44.25222 | .12457 |
| 73 | 44.04140 | 44.19106 | .14967 |
| 74 | 43.96400 | 44.11875 | .15475 |
| 75 | 43.89398 | 44.03428 | .14030 |
| 76 | 43.83005 | 43.93915 | .10910 |
| 77 | 43.77115 | 43.83686 | .06571 |
| 78 | 43.71640 | 43.73225 | .01585 |
| 79 | 43.66507 | 43.63080 | -.03427 |
| 80 | 43.61650 | 43.53787 | -.07863 |
| 81 | 43.57019 | 43.45803 | -.11216 |
| 82 | 43.52567 | 43.39449 | -.13117 |
| 83 | 43.48264 | 43.34879 | -.13384 |
| 84 | 43.44081 | 43.32057 | -.12024 |
| 85 | 43.39998 | 43.30765 | -.09233 |
| 86 | 43.36000 | 43.30632 | -.05368 |
| 87 | 43.32079 | 43.31174 | -.00905 |
| 88 | 43.28232 | 43.31849 | .03617 |
| 89 | 43.24458 | 43.32122 | .07664 |
| 90 | 43.20770 | 43.31524 | .10754 |
| 91 | 43.17175 | 43.29704 | .12529 |
| 92 | 43.13690 | 43.26472 | .12782 |
| 93 | 43.10332 | 43.21820 | .11488 |
| 94 | 43.07133 | 43.15925 | .08791 |
| 95 | 43.04123 | 43.09134 | .05011 |
| 96 | 43.01335 | 43.01927 | .00592 |
| 97 | 42.98808 | 42.94869 | -.03939 |
| 98 | 42.96594 | 42.88548 | -.08046 |
| 99 | 42.94739 | 42.83516 | -.11223 |
| 100 | 42.93301 | 42.80225 | -.13076 |
| 101 | 42.92339 | 42.78986 | -.13352 |
| 102 | 42.91908 | 42.79932 | -.11976 |
| 103 | 42.92084 | 42.83003 | -.09080 |
| 104 | 42.92928 | 42.87961 | -.04967 |
| 105 | 42.94518 | 42.94411 | -.00106 |
| 106 | 42.96927 | 43.01861 | .04934 |
| 107 | 43.00241 | 43.09774 | .09533 |
| 108 | 43.04552 | 43.17649 | .13097 |
| 109 | 43.09964 | 43.25091 | .15127 |
| 110 | 43.16591 | 43.31879 | .15288 |
| 111 | 43.24573 | 43.38021 | .13448 |
| 112 | 43.34067 | 43.43788 | .09722 |
| 113 | 43.45261 | 43.49730 | .04470 |
| 114 | 43.58377 | 43.56657 | -.01720 |
| 115 | 43.73686 | 43.65602 | -.08084 |
| 116 | 43.91522 | 43.77756 | -.13766 |
| 117 | 44.12280 | 43.94389 | -.17892 |
| 118 | 44.36467 | 44.16749 | -.19718 |
| 119 | 44.64697 | 44.45969 | -.18728 |

Table VI. Calculated orbit properties in the isochronous field.

CLOSED ORBIT AT THETA = .00 INTEGRATED OVER 1 SECTORS

| E | RAV | PHASE | R | PR | NU R | NU Z | NT | F(E) |
|---|---|---|---|---|---|---|---|---|
| .1000 | 1.216858 | -.005994 | 1.217416 | -.001473 | .998449 | .056415 | 4 | .000 |
| .2000 | 1.723247 | -.004554 | 1.725287 | -.006061 | .997884 | .069107 | 2 | -3.313 |
| .3000 | 2.112557 | -.003507 | 2.116733 | -.013541 | .998028 | .073860 | 3 | -5.846 |
| .4000 | 2.440989 | -.002748 | 2.447672 | -.023055 | .998367 | .078245 | 3 | -7.811 |
| .5000 | 2.730434 | -.002162 | 2.739658 | -.035555 | .998882 | .081910 | 3 | -9.353 |
| .6000 | 2.992167 | -.001678 | 3.003829 | -.049062 | .999366 | .085515 | 3 | -10.560 |
| .7000 | 3.232824 | -.001284 | 3.246628 | -.063597 | .999985 | .088059 | 3 | -11.490 |
| .8000 | 3.456763 | -.000964 | 3.472385 | -.078883 | 1.000824 | .087405 | 3 | -12.196 |
| .9000 | 3.666725 | -.000776 | 3.683801 | -.094601 | 1.001960 | .083514 | 3 | -12.743 |
| 1.0000 | 3.865091 | -.000656 | 3.883264 | -.110635 | 1.002624 | .084053 | 2 | -13.133 |
| 1.1000 | 4.053677 | -.000559 | 4.072559 | -.126473 | 1.003341 | .083054 | 2 | -13.575 |
| 1.2000 | 4.233626 | -.000517 | 4.252857 | -.142191 | 1.003770 | .085101 | 2 | -13.913 |
| 1.3000 | 4.406265 | -.000461 | 4.425479 | -.157525 | 1.003654 | .091816 | 2 | -14.220 |
| 1.4000 | 4.572598 | -.000353 | 4.591451 | -.172195 | 1.003375 | .099205 | 2 | -14.476 |
| 1.5000 | 4.733270 | -.000210 | 4.751414 | -.186322 | 1.003404 | .103320 | 2 | -14.653 |
| 1.6000 | 4.888632 | -.000077 | 4.905791 | -.199909 | 1.003971 | .101912 | 2 | -14.743 |
| 1.7000 | 5.038965 | .000001 | 5.054900 | -.212908 | 1.004948 | .096205 | 2 | -14.767 |
| 1.8000 | 5.184634 | .000022 | 5.199084 | -.225284 | 1.005804 | .091229 | 2 | -14.760 |
| 1.9000 | 5.326111 | .000010 | 5.338845 | -.237047 | 1.006342 | .089399 | 2 | -14.750 |
| 2.0000 | 5.463856 | -.000004 | 5.474696 | -.248247 | 1.006555 | .091033 | 2 | -14.748 |
| 4.0000 | 7.712622 | -.000018 | 7.662402 | -.373864 | 1.010648 | .118231 | 2 | -14.632 |
| 6.0000 | 9.429385 | -.000024 | 9.305900 | -.366705 | 1.014771 | .135494 | 3 | -14.896 |
| 8.0000 | 10.869787 | -.000012 | 10.680575 | -.284755 | 1.017788 | .150913 | 3 | -15.121 |
| 10.0000 | 12.132698 | -.000009 | 11.891691 | -.158847 | 1.021922 | .159514 | 3 | -15.254 |
| 12.0000 | 13.268972 | -.000014 | 12.991495 | -.006016 | 1.025223 | .168639 | 3 | -15.398 |
| 14.0000 | 14.309063 | -.000012 | 14.010714 | .163324 | 1.028344 | .175536 | 3 | -15.558 |
| 16.0000 | 15.272830 | -.000004 | 14.968639 | .342711 | 1.031383 | .181308 | 2 | -15.659 |
| 18.0000 | 16.173950 | .000007 | 15.878227 | .528012 | 1.034337 | .186267 | 2 | -15.641 |
| 20.0000 | 17.022507 | .000020 | 16.748230 | .714683 | 1.036546 | .191489 | 2 | -15.470 |
| 22.0000 | 17.825150 | -.000011 | 17.584854 | .900466 | 1.038356 | .197497 | 3 | -15.412 |
| 24.0000 | 18.588867 | -.000024 | 18.393707 | 1.080459 | 1.043012 | .199471 | 2 | -15.629 |
| 26.0000 | 19.318649 | .000000 | 19.178055 | 1.249153 | 1.046423 | .201080 | 2 | -15.775 |
| 28.0000 | 20.017792 | .000025 | 19.939560 | 1.401590 | 1.047517 | .202704 | 2 | -15.614 |
| 30.0000 | 20.687576 | -.000036 | 20.679153 | 1.529602 | 1.050696 | .197816 | 3 | -15.678 |
| 32.0000 | 21.334518 | .000003 | 21.398697 | 1.608673 | 1.054214 | .191807 | 3 | -15.881 |
| 34.0000 | 21.958405 | .000025 | 22.095596 | 1.630308 | 1.052363 | .186692 | 3 | -15.702 |
| 36.0000 | 22.560287 | -.000015 | 22.767761 | 1.574431 | 1.056243 | .162821 | 3 | -15.638 |
| 38.0000 | 23.145981 | .000068 | 23.414688 | 1.469643 | 1.056232 | .137854 | 3 | -15.303 |
| 40.0000 | 23.712570 | .000070 | 24.031921 | 1.318925 | 1.055374 | .098862 | 2 | -14.612 |
| 40.5000 | 23.851822 | .000076 | 24.181458 | 1.277822 | 1.053015 | .098302 | 3 | -14.381 |
| 41.0000 | 23.990784 | .000113 | 24.329681 | 1.235254 | 1.051949 | .097418 | 2 | -14.083 |
| 41.5000 | 24.128677 | .000141 | 24.475723 | 1.190516 | 1.052737 | .074561 | 2 | -13.684 |
| 42.0000 | 24.265106 | .000148 | 24.619263 | 1.144521 | 1.052012 | .056487 | 2 | -13.230 |
| 42.5000 | 24.401199 | .000180 | 24.761002 | 1.098590 | 1.049544 | .073527 | 2 | -12.715 |
| 43.0000 | 24.536835 | .000227 | 24.900986 | 1.050991 | 1.049878 | .072180 | 2 | -12.076 |
| 43.5000 | 24.671051 | .000250 | 25.038177 | 1.002135 | 1.051271 | .053962 | 2 | -11.327 |
| 44.0000 | 24.803970 | .000257 | 25.172745 | .952953 | 1.050812 | .065692 | 2 | -10.531 |
| 44.5000 | 24.936462 | .000282 | 25.305298 | .903848 | 1.049295 | .102709 | 2 | -9.684 |
| 45.0000 | 25.068283 | .000311 | 25.435715 | .853870 | 1.049394 | .131887 | 2 | -8.752 |
| 45.5000 | 25.199219 | .000336 | 25.563782 | .803056 | 1.047817 | .170479 | 2 | -7.735 |
| 46.0000 | 25.330139 | .000395 | 25.690552 | .751485 | 1.041597 | .223250 | 2 | -6.588 |
| 46.5000 | 25.462646 | .000544 | 25.817307 | .700070 | 1.031440 | .280695 | 2 | -5.114 |
| 47.0000 | 25.597397 | .000811 | 25.944855 | .648650 | 1.015476 | .341423 | 2 | -2.987 |
| 47.5000 | 25.735703 | .001246 | 26.075043 | .596862 | .994607 | .403042 | 2 | .243 |
| 48.0000 | 25.881226 | .001986 | 26.211609 | .544627 | .957283 | .485898 | 2 | 5.320 |
| 48.5000 | 26.040543 | .003282 | 26.359833 | .490628 | .906773 | .585603 | 3 | 13.594 |
| 49.0000 | 26.220932 | .005390 | 26.524445 | .430757 | .840575 | .707959 | 2 | 27.215 |
| 49.5000 | 26.443634 | .009108 | 26.721939 | .360491 | .726436 | .866373 | 3 | 49.989 |
| 50.0000 | 26.864471 | .020292 | 27.085236 | .256371 | .397150 | 1.118337 | 4 | 96.172 |

OFF FIELD
*STOP* 0